

15th CIRP Conference on Computer Aided Tolerancing – CIRP CAT 2018

Toward a Classification of Partitioning Operations for Standardization of Geometrical Product Specifications and Verification

Nabil Anwer^a, Paul J. Scott^b, Vijay Srinivasan^{c*}^a*LURPA, ENS Cachan, University Paris-Sud, Université Paris-Saclay, 94235 Cachan, France*^b*University of Huddersfield, Huddersfield, U.K.*^c*National Institute of Standards and Technology, Gaithersburg, MD, U.S.A.** Corresponding author. Tel.: +01-301-975-3508; fax: +01-301-258-9749. E-mail address: vijay.srinivasan@nist.gov

Abstract

Partitioning is one of the four major operations used in the international standards issued by ISO/TC 213 on geometrical product specifications and verifications (GPS, for short). The ISO GPS standards define both tolerancing (for specification) and metrology (for verification) practices that are of critical importance to manufacturing industry. Collectively, these emerging ISO GPS standards have the potential to be some of the key enablers of digitization of manufacturing (also known as smart manufacturing, cyber-physical manufacturing, cyber-physical production systems, and Industrie 4.0). In simple terms, partitioning operations divide a surface (ideal or measured) into subsets that are then subjected to further GPS operations. This paper addresses the classification problem of partitioning operations to provide a science-based solution for the development of ISO GPS partitioning standards. Such a classification is considered to be one of the first tasks that must be completed before detailed standards on partitioning can be developed.

© 2018 The Authors. Published by Elsevier B.V.

Peer-review under responsibility of the Scientific Committee of the 15th CIRP Conference on Computer Aided Tolerancing - CIRP CAT 2018.

Keywords: ISO; GPS operations; partitioning; tolerancing; metrology; standards; manufacturing; digitization

1. Introduction

Partitioning is one of the four major operations used in the international standards issued by ISO/TC 213 on Dimensional and Geometrical Product Specifications and Verification (GPS, for short). The ISO GPS standards define both tolerancing (for specification) and metrology (for verification) practices that are of critical importance to manufacturing industry. Collectively, these emerging ISO GPS standards have the potential to be some of the key enablers of digitization of manufacturing (also known as smart manufacturing, cyber-physical manufacturing, cyber-physical production systems, and Industrie 4.0).

ISO GPS standards [1] define four major operations that can be described informally as follows:

1. **Extraction.** Creation of a computable representation of the boundary of an imperfect-form model (also known as the skin model). The computable representation can be a (potentially dense) set of discrete points, or some interpolated (for example, linearly tessellated) surface.

2. **Partitioning.** Division of an extracted skin model of a work-piece into pieces, each of which corresponds to a surface feature on the boundary of a nominal model of ideal form. The boundary of a nominal model of ideal form is also subjected to partitioning operations to define features of interest for specifications.
3. **Filtration.** Creation of a scale-dependent computable representation of (potentially a portion of) a skin model. This may precede or succeed the extraction operation.
4. **Association.** Fitting a mathematically perfect-form surface (or curve) to an extracted and filtered set (which may be discrete or continuous) of points, using an optimization process.

These four operations are applicable for specifications as well as verification. The major differences are that (1) the specification operations work with continuous sets of points, and (2) the verification operations will have measurement uncertainties – including those introduced by discrete sampling to create discrete sets of points – that should be carefully

accounted for. ISO/TC 213 is actively engaged in producing a series of documents on these ISO GPS operations.

Classifications of filtration and association operations exist, and they are used for ISO standards development. For example, filtration operations can be applied to profiles or surfaces, and they can be Gaussian, spline, wavelet or morphological filters. In the same spirit, association operations can be, for example, Gaussian (least-squares), Chebyshev, maximum inscribed, or minimum circumscribed fittings; most of these association operations can also be defined using the L_p -norm, for $p = 1, 2$, and ∞ . Such science-based classifications have been used by ISO/TC 213 to develop a series of standards for filtration and association operations.

Similar classifications do not exist for extraction and partitioning operations. It is generally acknowledged that the partitioning operation is performed after the extraction operation. This paper provides a scientific basis toward a classification of ISO GPS partitioning operation. This can then be used for the development of ISO GPS partitioning standards.

The rest of the paper is organized as follows. Section 2 describes a science-based classification of perfect-form surface features that is used in the ISO GPS standards. A science-based classification of features of size is explained in Section 3. A general set of operations that were originally developed rigorously for selective geometric complexes is outlined in Section 4. These lead to Section 5 that deals with a classification of ISO GPS partitioning operations. The results are summarized and some concluding remarks are made in Section 6.

2. Classification of Perfect-form Features Based on Symmetry Groups

ISO GPS standards define seven invariance classes shown in Table 1 [1]. For geometrical product specifications, the boundary of a nominal solid should be partitioned into surface patches, each of which belongs to one of the seven invariance classes. These seven classes are derived from a science-based classification of continuous symmetry groups of surfaces in three-dimensions (3D) [2]. The use of group theory for advancing the study of geometry dates back to the pioneering works of Sophus Lie, Felix Kline, and Henri Poincaré.

The seven classes shown in Table 1 also provide the classification of lower-order kinematic pairs in 3D. For example, a spherical surface feature belongs to the spherical invariance class in Table 1 because the spherical surface remains invariant (from a set theoretic perspective) with respect to all three rotations about the center of the sphere. Equivalently, a ball-and-socket kinematic joint belongs to the spherical kinematic pair because the ball in the kinematic joint can execute a spherical motion relative to the socket in that kinematic joint. Hence a ball-and-socket joint has three kinematic degrees of freedom corresponding to three rotations about their common center.

The relationship between the invariance classes and kinematic pairs depicted in Table 1 had led to a notion of ‘local slippable motion’ that has been used in some point-cloud (or skin model) partitioning algorithms. In a local slippable motion, an infinitesimal displacement of a point on a surface under a

kinematically constrained motion (derived for that surface from Table 1) will be tangential to that surface at that point. Therefore, any partitioning algorithm that is based on invariance classes (equivalently, slippable motions) will be of importance to ISO GPS partitioning standards. This notion will be discussed further in Section 5.2.2.

Table 1. ISO GPS invariance classes and their relationship to kinematic pairs.

	ISO GPS Invariance Classes (Lower-order Kinematic Pairs)	Kinematic Degrees of Freedom
1	Planar	3 Translations
2	Cylindrical	1 Translation and 1 Rotation
3	Helical	1 Translation and 1 Rotation, linked by pitch
4	Spherical	3 Rotations
5	Revolute	1 Rotation
6	Prismatic	1 Translation
7	Complex	None

3. Classification of Perfect-form Features of Size

Features of size play an important role in ISO GPS standards. They are subjected to size tolerances, and they are also used for datum establishment. Any feature of size must satisfy two basic criteria: (1) it must belong to a one-parameter family of features, and (2) it must obey the monotonic containment property. The monotonic containment property refers to the fact that a feature with larger parameter value (size) should contain (or be contained in) the feature with a smaller parameter value (size).

ISO GPS standards define two types of features of size – one for linear sizes and the other for angular sizes. For example, a cylinder is a feature of size with a linear size (e.g., its diameter). On the other hand, a cone is a feature of size with an angular size (e.g., its apex angle). Both cylinder and cone satisfy the two criteria mentioned above: each belongs to a one-parameter family of surfaces and each satisfies the monotonic containment property. Any feature of size also belongs to an invariance class enumerated in Table 1.

There is an important relationship between features of size and a science-based classification of quadric surfaces (i.e., surfaces defined by second degree implicit algebraic equations). Table 2 shows some of the popular one-parameter family of real quadric surfaces that are used in industry and are also part of the features of size [2].

The last column of Table 2 lists the curvature properties of the quadric surface, which is also a feature of size. Such curvature information has been used in some point-cloud (or skin model) partitioning algorithms. Given the importance of features of size, any curvature-based algorithm that can perform the partitioning into these size features will be of importance to ISO GPS partitioning standards. This idea will be discussed further in Section 5.2.2.

Table 2. Popular one-parameter families of real quadrics and their curvature properties.

	Surface Type*	Principal Curvatures (at any point on the surface)
Non-degenerate, Natural Quadrics	Sphere (Spherical)	$k_1 = k_2 = 1/\text{radius} = \text{const. everywhere}$
	Right-circular Cylinder (Cylindrical)	$k_1 = 0, k_2 = 1/\text{radius} = \text{const. everywhere}$
	Right-circular Cone (Revolute)	$k_1 = 0$ everywhere, and k_2 varies inversely as the distance from the apex.
Degenerate Quadrics	Parallel Planes (Planar)	$k_1 = k_2 = 0$ everywhere.
	Intersecting Planes (Prismatic)	$k_1 = k_2 = 0$ everywhere, except along the intersection edge.

* The invariance class is indicated in parenthesis

4. Classification of Operations Based on Selective Geometric Complexes

Selective geometric complexes (SGC) provide a general methodology to represent 3D objects with internal structures [3]. It generalizes the notion of solid modeling. Three major operations – called selection, subdivision, and simplification – have been defined rigorously for processing SGCs. These operations are also useful for partitioning 3D nominal models or two-dimensional (2D) drawings for the purpose of ISO GPS. In fact, these operations are already used in the ISO GPS standards (as described in more detail in Section 5.1), even though they have not been referred to by the same names.

In SGC, geometrical objects of different dimensionality are referred to as 3-cells (volumes embedded in 3-manifolds), 2-cells (surface patches embedded in 2-manifolds), 1-cells (curved edges embedded in 1-manifolds), and 0-cells (point vertices that are 0-manifolds). Informally, the SGC operations can be described as follows:

1. **Selection:** Identification of which geometrical objects (that is, which cells) should be considered for further operations.
2. **Subdivision:** Splitting selected geometrical objects into more cells, retaining compatibility (that is, no overlaps) between adjacent cells.
3. **Simplification:** Combining several cells (that are not necessary adjacent) to create a simpler (that is, reducing combinatorial count) object.

Thus far, the operations of selection, subdivision and simplification have been used in the ISO GPS specifications. It is important to mimic these operations in the ISO GPS verification as well.

5. Classification of GPS Partitioning Operations

ISO GPS standards deal with partitioning of nominal models as well as skin models (or point-clouds). Figure 1 presents a high-level classification of ISO GPS partitioning operations. This classification diagram contains the keys to various sections

in this paper, and will form the basis for further discussion and exploration.

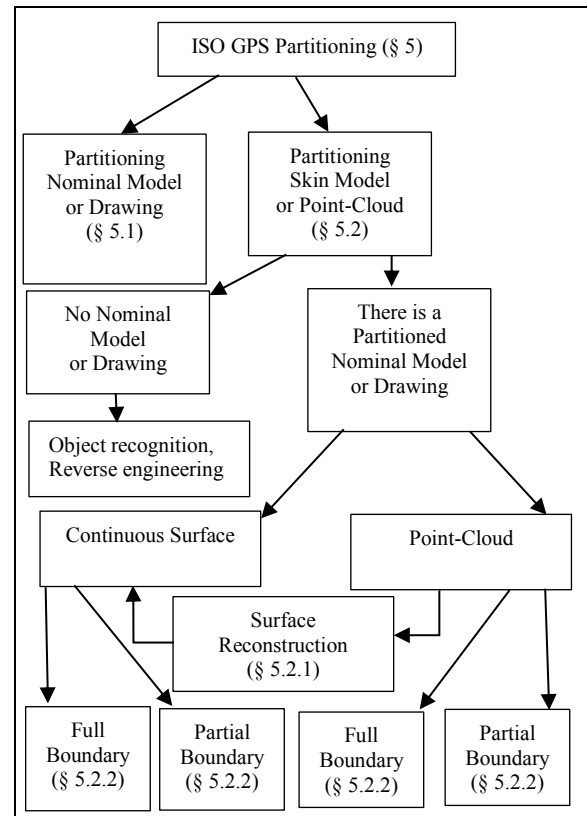


Fig. 1. Classification diagram for ISO GPS partitioning operations

5.1 Partitioning nominal model or drawing

Partitioning operation can be applied on 2D drawings or on 3D CAD models. The SGC operations described in Section 4 can be used for this purpose. Figure 2 illustrates the three SGC operations on nominal 3D models using examples from a recent ISO GPS standard [4].

In Fig. 2(a) an annular planar surface feature is identified using leader and extension lines, and the feature so selected is then subjected to (as yet undisclosed) tolerance specifications. This is a common type of selection operation that is used throughout ISO GPS specifications on nominal 3D models or 2D drawings. It is implicitly assumed that all nominal 3D models and 2D drawings are already constructed using computable representations of entities such as faces, edges, and vertices that partition the boundary of a 3D solid. So, a set of standardized graphical elements pointing to a partitioned entity, as shown in Fig. 2(a), invokes a selection operation.

A planar surface feature on the boundary of a nominal 3D model in Fig. 2(b) is further subdivided as indicated by hatching, and this hatched region is selected as the feature for specification of parallelism tolerance. This is a typical example of the subdivision operation. It also shows how subdivision and selection operations can be combined for tolerance specifications.

Figure 2(c) illustrates the simplification operation. Here three separate surface features on the boundary of a nominal 3D model are combined using long-and-short dashes, and this

combined feature can then be subjected to a tolerance specification, such as a profile tolerance. So, the simplification operation can also be viewed as a subset of ‘collection operation’ defined in ISO GPS standards [1].

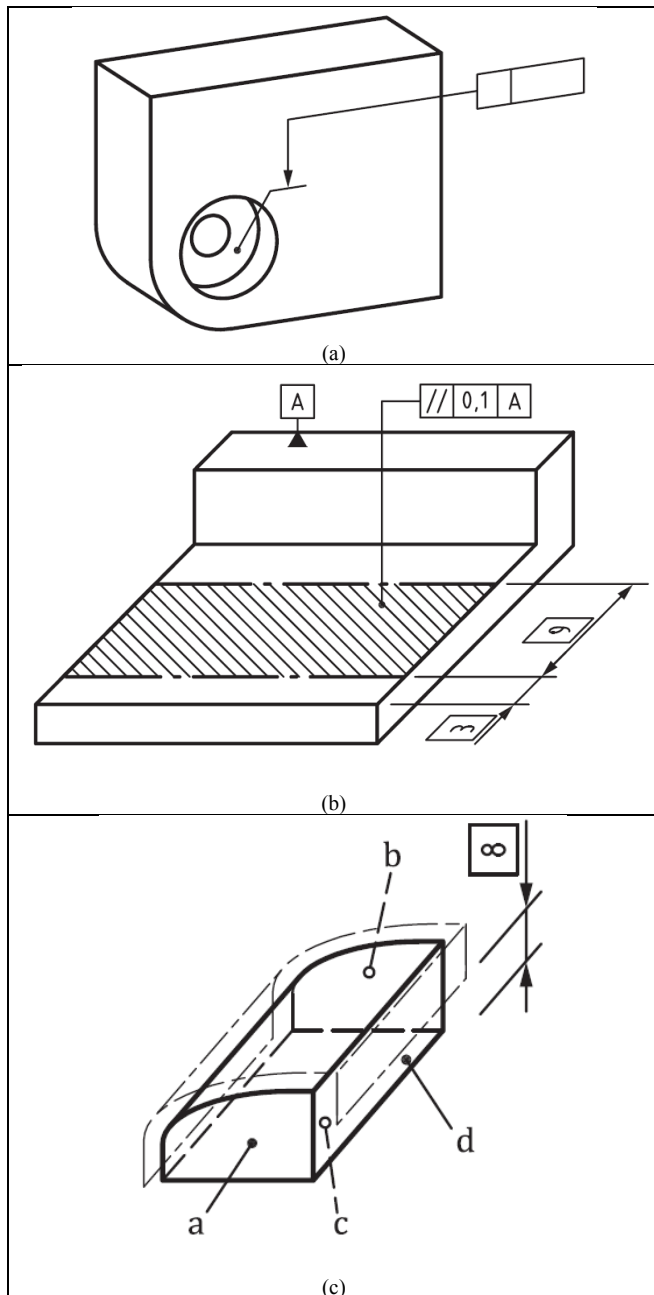


Fig. 2. Illustration of SGC operations used in ISO GPS standards [4].

5.2 Partitioning skin model or point-cloud

The next major classification of ISO GPS partitioning operations shown in Fig. 1 is the partitioning of skin model or point-cloud. For ISO GPS *specification* purposes, it can always be assumed that a skin model (with continuous surfaces or surface patches) is available. However, for ISO GPS *verification* purposes, the availability of only discrete points in the form of point-clouds should also be considered.

It is important to point out that the problem of partitioning skin models and point-clouds has been addressed in research literature without any consideration of an underlying nominal

model and its partitioning. Such problems come under the category of object recognition and reverse engineering. These are important engineering problems. However, within the context of ISO GPS, it can always be assumed that a partitioned nominal model or drawing is available. This assumption will be maintained in the rest of this paper.

Mathematically speaking, a skin model of a manufactured part is a continuous surface that is closed (compact and without boundary) and orientable. It is also the boundary of the solid model of the manufactured part under consideration. From a topological point of view, the skin model is a two-dimensional manifold (2-manifold) supplied with defined differentiable and metric structures. As a 2-manifold, the skin model possesses a finite triangulation. This fundamental property admits a common framework, as shown in Fig. 1, to handle the boundary of a solid model as (1) a triangulated (more generally, a mesh-based) continuous surface, or (2) a point-cloud.

From a point-set topology perspective, the skin model can be separated into disjoint components having boundaries while preserving dimensionality invariance (that is, still a set of surface patches). These surface patches can then have lower dimensional boundary entities, such as edges and vertices. The existence of different dimensional topological and combinatorial entities (such as surface patches, edges, and vertices) makes partitioning a challenging problem.

To address this challenge, Section 5.2.1 deals with surface reconstruction from point-clouds. Then Section 5.2.2 addresses current approaches to partition meshes and points.

5.2.1 Surface reconstruction from points

Since a point-cloud does not provide much point-set topological information, a mesh structure may be first constructed to organize the points and reconstruct the underlying topology. A mesh construction builds a first order approximation of the boundary surface, and it often relies on combinatorial structures such as Voronoi diagram and Delaunay triangulation. These structures are particularly useful for visualization and geometric modelling purposes.

For example, it has been shown that a sub-space of the Delaunay triangulation, called the Restricted Delaunay, is a very good approximation of the targeted surface with geometric and topological guarantees [5]. There are also other surface reconstruction methods reported in literature, such as nearest neighbours, α -shapes, cocones, and Wrap [6]. Their topological and geometric guarantees are affected by sampling density, noise, outliers, and sharp features [7].

The topic of surface reconstruction from points in the context of ISO GPS raises a couple of the following questions:

- (1) Does the surface reconstruction operation belong to the *partitioning* operation, or does it belong to the *extraction* operation?
- (2) In many measurement operations (e.g., tactile sensing with spherical probes), an estimation of the coordinates of contact points also involves an estimation of the surface normals at these points. Shouldn't such information in the form of (point, normal) pair be preserved and exploited in a subsequent surface reconstruction operation?

These questions motivate further discussion and research.

5.2.2 Current approaches for mesh and point-cloud partitioning

Various methods and algorithms have been developed to partition 3D point clouds and meshes into homogeneous regions based on geometrical and topological criteria. At present, these partitioning approaches can be classified into one of four categories shown in Table 3, and are explained below.

Table 3. Classification of current mesh and point-cloud partitioning approaches.

	Approach	Comments
1	Edge detection	Popular in image processing literature, more recently in machine learning applications.
2	Region growing	Further classified as bottom-up and top-down approaches.
3	Attribute clustering	Uses special attributes relevant to the domain of interest.
4	Hybrid approach	Combines the best of other approaches.

(1) **Edge detection:** Edges are usually defined by points where the local surface attributes change rapidly to exceed a given threshold. The partition is then realized by detecting the edges to mark the boundaries of different regions and then grouping points inside the boundaries [8]. Partition methods by edge detection are known to get inaccurate results in case of noise and irregular density of point clouds [9].

(2) **Region growing:** Partition by region growing is achieved by combining points in a given neighbourhood that have similar attributes, such as normal and curvature (for example, as classified in Table 2). The growing process is repeated until a set of termination or growing criteria is satisfied, and then different regions characterized by dissimilarity are obtained. There are two major categories of region growing methods: (a) seeded-region (or bottom-up) methods, and (b) unseeded-region (or top-down) methods [10].

The performance of seeded-region methods is highly dependent on the selection of seed points. Inaccurate selection of seed points will cause under- or over-partitioning. Moreover, the partition results may be sensitive to the chosen threshold. The main difficulty of unseeded-region methods lies in deciding where and when to subdivide the regions. These require a lot of previous knowledge, such as object models and number of regions. Fortunately, such information is available for ISO GPS in the form of partitioned nominal models, as described in Section 5.1.

(3) **Attribute clustering:** In attributes clustering methods, attributes are described and calculated first for each point based on geometrical characteristics. Thereafter, the points are clustered into segmented regions by comparing their attributes. The points belonging to each cluster have similar attributes and are labelled as a unique region. The most important factor that determines the segmentation results is the criterion function used for clustering. Since attributes of individual points are usually described using points in the local neighbourhood, this partition method is sensitive to the noise in the data and the definition of neighbourhood.

A common example of point attribute is the

(estimated) curvature at that point. But there can be other attributes that are relevant to the particular domain of interest. The ISO GPS is one such engineering domain of interest, whose invariance attributes are depicted in Tables 1 and 2. An attribute clustering based on slippage motions for partitioning has been explored in literature [11, 12]. In this approach, points (with estimated normals) are classified according to Table 1. Thereafter, hierarchical clustering can be used to merge points to form larger regions based on slippage similarity scoring, as shown in Fig. 3. This method can be improved by guided region growing method based on better estimates of point normals.

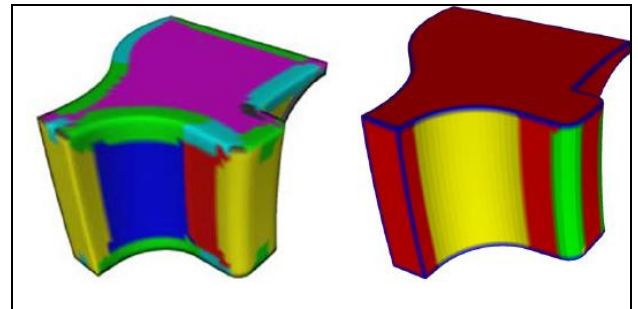


Fig. 3. Example of partitioning using slippage analysis [11,12].

(4) **Hybrid approach:** A hybrid approach combines the best of the previous three approaches to achieve better results. For example, edge detection method and attribute clustering method can be combined for mesh segmentation [13,14]. This hybrid approach consists of two steps: (1) detection and extraction of sharp edges and high curvature regions by curvedness and, (2) clustering and refining of vertices by shape index.

Figure 4 shows two examples of the edge detection results. In these examples, other vertices are grouped into ten different clusters according to their local surface types that are defined by the value of shape index, as illustrated in Fig. 5. Finally, a refining algorithm is applied iteratively to improve the clustering quality (Fig. 6).

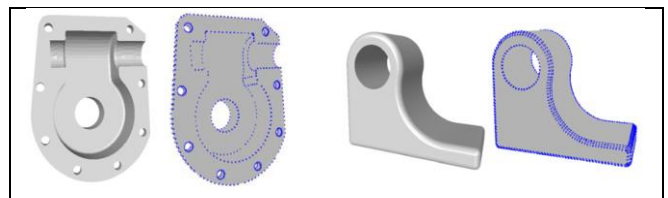


Fig. 4. Examples illustrating sharp edges and high curvature regions [13,14].

The four partitioning approaches outlined above can benefit from an *a priori* knowledge of the partitioned nominal model. This will enable the establishment of one-to-one mapping between the features of the nominal model and the features of the skin model (or point-cloud). Such mapping is critical for further verification purposes. In some cases, the nominal model may not carry explicit boundary representation for features such as chamfers and fillets; such features may be indicated only symbolically. It is then necessary to account for them while partitioning the skin model.

Finally, there are still two possibilities, as illustrated in Fig. 1: (1) the entire work-piece has been scanned (and full boundary skin/points extracted) for verification, or (2) only a part of the work-piece has been scanned (and partial boundary skin/points extracted) for verification. The latter case is more common than the former. In both cases, the partitioning approaches outlined above can be applied.

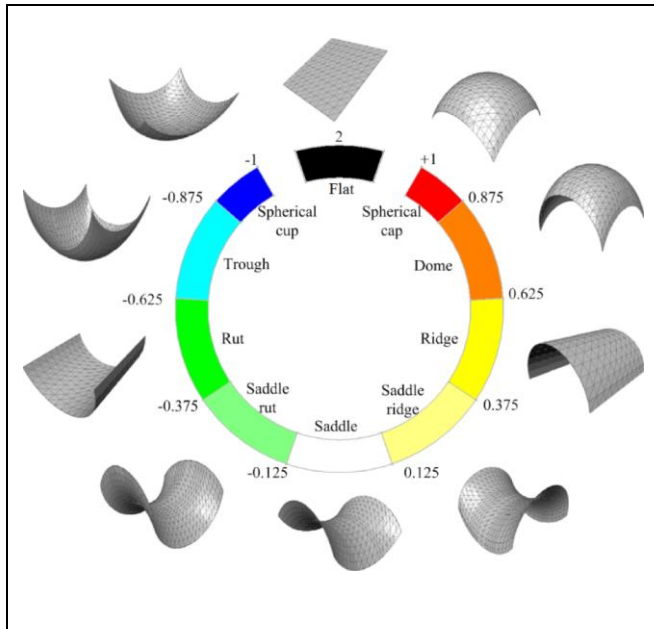


Fig. 5. Shape indices for clustering [13,14].

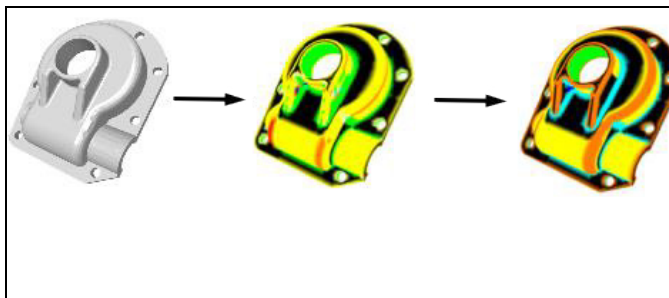


Fig. 6. Cluster refinement to improve partition quality [13,14].

6. Summary and Concluding Remarks

This paper has taken some initial steps toward a science-based classification of partitioning operation, which is one of the four ISO GPS operations. Such a classification is one of the first tasks that must be completed before detailed standards on partitioning can be developed to support digitization of manufacturing.

The work presented in this paper is consistent with current efforts in standardization of partitioning operations spearheaded by the ISO/TC 213/AG 12 (Mathematical Support Group). The proposed series of standards on partitioning (ISO 18183 series) have been decomposed into three parts: Part 1 deals with basic concepts, Part 2 is related to nominal model partitioning, and Part 3 focuses on the methods for specification and verification.

As mentioned above, these are only the initial steps. More research is needed for detailed exploration of the partitioning of skin models and point-clouds addressed in Section 5.2. Newly reported research on machine learning based segmentation, and geometric deep learning will provide new insights in partitioning [15–17]. Industrial test cases should be developed and subjected to various partitioning algorithms to evaluate their performance in practice. The international research community in computer aided tolerancing can play an important role in this exciting and important effort.

References

- [1] ISO 17450-1:2011. Geometrical product specifications (GPS) – General concepts – Part 1: Model for geometrical specifications and verification. International Organization for Standardization, Geneva, 2011.
- [2] Srinivasan, V., Theory of Dimensioning: An introduction to parameterizing geometric models, Marcel-Dekker, New York, 2004.
- [3] Rossignac J, O'Connor M, SGC. A domain independent model for pointsets with internal structures and incomplete boundaries. In: Geometrical modeling for product engineering, eds. Wozny M, Turner J, Preiss K, North Holland, Amsterdam, 1990.
- [4] ISO 1101:2017. Geometrical product specifications (GPS) – Geometrical tolerancing – Tolerances of form, orientation, location and run-out. International Organization for Standardization, Geneva, 2017.
- [5] Edelsbrunner H, Shaw NR. Triangulating topological spaces. *Int J Comput Geom Appl*, 1997;7:365-78.
- [6] Dey TK. Curve and surface reconstruction: algorithms with mathematical analysis, Cambridge University Press, 2006.
- [7] El-Hayek N, Nouria H, Anwer N, Damak M, Gibaru O. Reconstruction of freeform surfaces for metrology, *Journal of Physics: Conference Series* 2013;483, doi:10.1088/1742-6596/483/1/012003, 2013
- [8] Rabbani T, Van Den Heuvel F, Vosselmann G. Segmentation of point clouds using smoothness constraint. *Int arch photogramm remote sens spat inf sci* 20016;36:248-53.
- [9] Huang J, Menq CH. Automatic data segmentation for geometric feature extraction from unorganized 3-D coordinate points. *IEEE Trans Robot Autom* 2001;17:268-79.
- [10] Nguyen A, Le B. 3D point cloud segmentation: A survey, 6th IEEE Conference on Robotics, Automation and Mechatronics (RAM), 2013
- [11] Gelfand N, Guibas LJ. Shape segmentation using local slippage analysis. *Proceedings of the 2004 Eurographics/ACM SIGGRAPH symposium on Geometry processing*, 214-223, 2004
- [12] Yi B, Liu Z, Tan J, Cheng F, Duan G, Liu L. Shape recognition of CAD models via iterative slippage analysis. *Comput Aided Des* 2014;55:13-25.
- [13] Zhao H, Anwer N, Bourdet P. Curvature-based Registration and Segmentation for Multisensor Coordinate Metrology. *Procedia CIRP* 2012;10:112-8.
- [14] Anwer N, Mathieu L. From reverse engineering to shape engineering in mechanical design. *CIRP Ann - Manuf Technol* 2016;65:165–8.
- [15] Rodrigues RSV, Morgado JFM, Gomes AJP. Part-Based Mesh Segmentation: A Survey. *Computer Graphics Forum* 2018. doi:10.1111/cgf.13323
- [16] Yi L, Su H, Guo X, Guibas L. SyncSpecCNN: Synchronized spectral CNN for 3D shape segmentation. In : *Computer Vision and Pattern Recognition (CVPR)* 2017.
- [17] Bronstein MM, Bruna J, LeCun Y, Szlam A, Vandergheynst P. Geometric deep learning: going beyond euclidean data. *IEEE Signal Process Mag* 2017;34:18-42.