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Tolerance analysis of hyperstatic mechanical systems with deformations

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Abstract

The standard methodologies for tolerance analysis, consider that the mechanism is infinitely rigid. This hypothesis impose very small tolerances to ensure functional requirements and parts fittability. To increase the tolerance values, the proposed methodology takes into account the deformation of the parts in the tolerance analysis. Firstly, the geometric specifications with ISO tolerancing using maximum material conditions for fittability and least material condition for functional requirements are employed. Then, a modelling based on the mechanical behaviour of the parts is developed into the calculation of the dimensional chains. The proposed methodology is applied on a hyperstatic mechanism.

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Keywords: Tolerance analysis; hyperstatic mechanism; deformation; mechanical model

1. Introduction

The conventional approach of dimensioning hyperstatic assemblies introduces important clearances in the joints to compensate for the geometrical deviations and to guarantee the assembly of all the parts. However, these clearances are very penalizing on the functional requirements. For this reason, the solution presented is to ensure the mounting ability by allowing the deformation of the parts, and thus to limit the maximum clearance. This approach requires the preliminary study of the mechanical behaviour of the parts to deduce their abilities to deform during the assembly procedure.

This paper develops a methodology for calculating dimension chains of hyperstatic systems taking into account the deformation of parts. The originality of this approach lies in the analysis of hyperstatic system where contacts are not previously known.

First, the tolerance analysis methodology will be proposed. It fellows part of the ISO tolerancing process for hyperstatic systems. It will allow to take into account highly hyperstatic mechanisms. The goal is to be able to deploy this approach on mechanisms where potential contacts between parts are not known. In a second part, this tolerance analysis methodology will be deployed on assembly inspired by the oil industry.

2. Methodology of tolerance analysis taking into account deformations

Rouetbi et al. has proposed an approach to tolerancing of hyperstatic systems where the contacts between the parts are known [1]. In this methodology, to ensure the assembly and functional requirements, the abilities of parts, composing a hyperstatic assembly to deform, is exploited.

The objective of this methodology is to determine a mechanical modelling. This modelling gives a mathematical relation between the tolerance values and the necessary forces to fit all the assembly parts. Several studies have been considering contact interaction in tolerancing analysis. [2-3] propose a methodology to study the propagation of the effects of assemblies consisted of flexible parts. The analysis the contacts between the different parts and the influences of assembly means models by finite element methods. [4] takes into account thermomechanical strains into tolerancing analysis. An analysis of contact between compliant parts is proposed by finite element methods to estimate the thermomechanical influences.

In this article, a new approach is proposed to take into account mechanisms whose contacts are not previously known. The logic diagram of Fig. 1 presents the evolution of the

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approach of [1]. This method is based on the construction of a mechanical model of the assembly to take into account all possible configurations of contact.

This methodology is divided into two parts:

- ISO tolerancing of hyperstatic assembly,
- Tolerance analysis of hyperstatic assembly

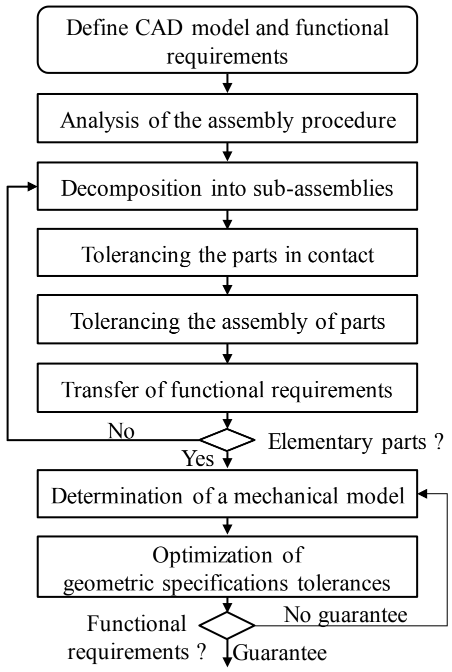

Fig. 1. Logic diagram of proposed method

2.1. ISO Tolerancing of hyperstatic mechanical

The tolerancing process usually takes place after the creation of the CAD model. The CAD model is composed of geometrically perfect parts without manufacturing and assembly defects. To ensure assembly and functional requirements, a tolerancing process is then conducted. This approach breaks down into three stages:

- tolerancing the parts in contact,
- tolerancing the assembly of parts,
- transfer of functional requirements.

The knowledge of the assembly process, giving the order of the parts positioning and the adjustment operations, is necessary to ensure the assembly requirements.

The CLIC method [5] (French acronym for "Location Tolerancing with Contact Influence") developed by B. Anselmetti allows to respect the assembly requirements. The ISO tolerancing [6] with the maximum material requirement [7] is used in this method. The assembly requirements, guaranteeing the mounting ability of parts in contact, are ensured by geometric specifications.

The analysis lines method [5] and assembly conditions with polytopes [8] are some various mathematical models allow calculating dimensioning chains.

The transfer of functional requirements is carried out using the same tools as for the transfer of assembly requirements.

The unfavourable clearance is limited by geometrical specifications with reference systems at the least material requirements [7]. The tolerancing analysis will provide mathematical relations resolved by the analysis lines in Quick GPS [9], polytopes [10], deviation domains [11], T-map [12]...

Each assembly is then decomposed into sub-assemblies or elementary parts. Then the assembly requirements become requirements. The three steps are then carried out while the system is not broken down into elementary parts.

2.2. Tolerance analysis of hyperstatic assembly

Analysis of tolerance of a hyperstatic system is divided into two steps as shown in Fig. 1:

- determination of a mechanical model,
- optimization of tolerances.

As a first step, the mechanism must be examined in order to choose the most appropriate mechanical model. A finite element model can be too complicated. Simple beam or plate type models can be defined by studying the geometry of the various parts of the assembly. The goal is to have a simple model to connect efforts to displacements. In this model, the displacements will be defined from the geometric deviations of the parts.

After choosing a simple mechanical model, the geometric deviations are then introduced as the data of the problem. The geometric deviations are defined by the different geometric specifications. Then several iterations can be done to optimize the values of the geometrical specifications which aims at maximizing them, while respecting the functional requirements.

3. Application

3.1. Description of the mechanism

In this paper, a measurement tool used in the oil industry inspires the assembly studied (see Fig. 2). These tools are composed of two sub-assemblies. The first sub-assembly is the frame, which is a stack of slender shafts of about 100mm diameter, noted Pi, as illustrated in Fig. 1. The two shafts (P1 and P2) support all electronic equipment with lengths up to a few meters. As for the two shafts (P0 and P3), they are of shorter lengths. This frame is inserted in the second sub-assembly, which is the protective tube. The two covers are mounted at the two ends of the frame and the tube to ensure the tightness of the assembly and limit the axial flutter of the frame relative to the tube.

The maximum clearance between the frame and the tube is the main parameter of good operation. For this reason, the designer creates a CAD model of the assembly by defining a reduced clearance between perfectly shaped parts and nominal dimensions. In return, a very small clearance induces significant insertion forces to insert the frame in the protective tube.

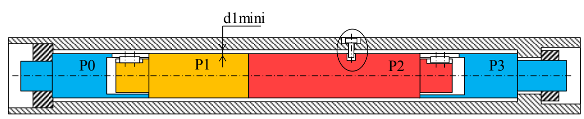

Fig. 2. Hyperstactic model studied.

The straightening of the frame by the protective tube is a function of the geometric defects permissible by the tolerances. The tolerance analysis, considering the parts as rigid, is limited. A mechanical and geometric coupled model offers a mathematical relationship between straightening forces and tolerances.

In the assembly studied, the coupling of the geometrical model (proposing multiple configurations of straightness defects of the frame) and the mechanical model (based on a model of beam type) is necessary to determine a direct mathematical relation between the forces and the admissible tolerances.

In this example, the contacts are potentially unknown (the clearance is defined constant between the frame and the protection tube). The geometrical defects of the slender shaft induce a defect of the straightness of the frame. The straightness defect involves random contact points and variable distances d1, as shown in Fig. 3.

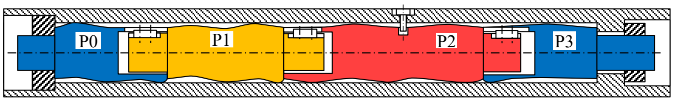

Fig. 3. Frame with deviations and variable d1 distances.

The assembled frame, as well as the tube, are long parts. Both non-rigid sub-assemblies admit deformations. To reduce straightening efforts of the frame, it is necessary to describe two envelopes, limiting the straightness defects in the free state of the two sub-assemblies. These two envelopes are defined by straightness specifications (2) on the frame (Fig. 4) and (5) on the tube (Fig. 5).

From a mechanical point of view, the frame and the tube are considered rigid below a length L. In order to guarantee the local mounting ability of each rigid section of length L, two sliding envelopes must be defined on the two sub-assemblies. These two envelopes are defined by the sliding straightness specifications (2) on the frame (Fig. 4) and (5) on the tube (Fig. 5).

To ensure the proper functioning of the mechanism, the maximum clearance must be limited. The two sliding straightness specifications on the length L at the maximum material requirement (1) on the frame (Fig. 4) and (2) on the tube (Fig. 5) ensure this function.

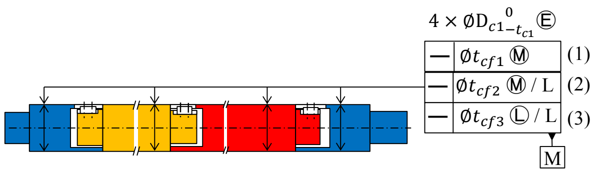

Fig. 4. Frame requirements.

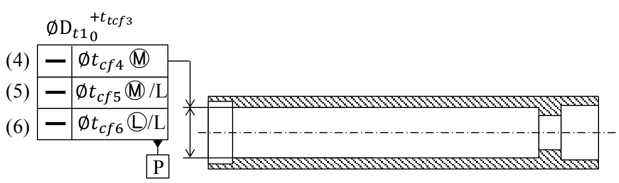

Fig. 5. Protection tube requirements

3.2. Iso tolerancing of assembly

The frame is an assembly of parts. All functional requirements and assembly requirements, illustrated in Fig. 4 and in Fig. 5, are transferred to the different parts. The Fig. 6 illustrates this transfer on part of the frame. The functional tolerancing is performed with CLIC method [5].

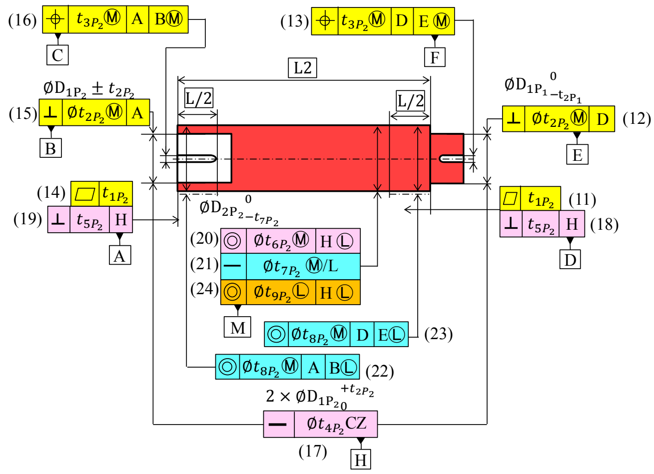

Fig. 6. Geometric specifications on a part.

The specifications (11) to (16) proposed in Fig. 6 are deduced from the CLIC method to guarantee the assembly with the neighbouring parts.

The geometric specifications (17) to (20) are the result of the transfer of the functional requirement of straightness (1). Each frame shaft is a long part, the primary position is provided by the two short centering H. A shaft is then centred between the two parallel symmetrical side planes. This positioning imposes the straightness (17) and the two perpendicularities (18) and (19) of Fig. 6. Locally, the axis of each shaft are thus defined by H. The surface M must be coaxial (20) according to the maximum material requirement with respect to H. The modifier Ⓛ is necessary on the reference H, because the clearance is unfavourable.

The sliding straightness (2) is transferred on the surface M of each shaft as a sliding straightness (21) on the same length L. When this sliding straightness is located on two parts of the frame, the transfer cannot be directly on the surface M. The transfer of straightness has to take into account the geometrical deviations of the contact interface between two adjacent shafts. The shape of slender shafts must be specified to length L/2. This form generates the coaxiality specifications (22) and (23).

For the transfer of the sliding straightness (3), it is better to locate the surface M with respect to H; which gives the coaxiality (24) while keeping the modifier Ⓛ of the requirement. There is a clearance on H. The reference H is therefore at the least material requirement.

The specifications of the other parts are deduced by analogy to the transfer process on the P2 shaft.

3.3. Analysis tolerance with a beam model

The hypotheses concerning the geometry of the different parts of the assembly make it possible to reduce the modelling to a plane problem. A model of deformable beam type is then chosen for modelling.

The protective tube is built from a stiffer material than the frame to protect the sensors and electronic components from severe operating conditions. By hypothesis, the protection tube is always considered as infinitely rigid. However, the tube may have geometric deviations. It is possible to transfer all the geometrical deviations of the tube to the frame. For the study of the mechanical behaviour of the assembly, the tube is thus "perfect". The frame carries the cumulative deviations of the tube and the frame.

No relative slip between the contact interface planes and gap between the shafts are allowed. For this, the fixing screws between two slender shafts are tightened with sufficient torque.

The measurement tools operate mainly vertically, the effects of gravity are not taken into account in the calculation of the contact forces applied to the frame. The assembly studied is designed as a cylinder inserted in a tube. The points of contact are only a function of the deviations of the shape of the frame and the tube. A frame with straightness defects must deform to be assembled in a tube.

In a first approach, the geometrical deviations of the frame are modelled by geometrical deviations in the junctions between the different parts. The parts have a perfect geometry between two junctions. The Fig. 7 illustrates this hypothesis.

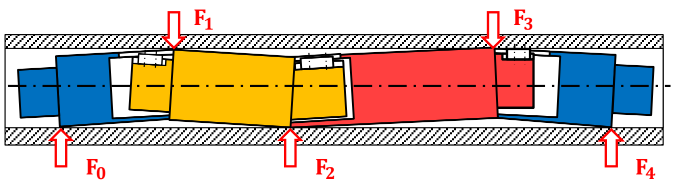

Fig. 7. Geometric deviations and straightening forces.

The main hypothesis of this first model is based on the location of the contacts between the frame and the tube. Indeed, this modelling requires that the contacts between the frame and the tube are at the different junctions of the frame. The straightening forces induced by the frame are then imposed at this level see Fig. 7.

The objective of this modelling is to propose a mechanical model making it possible to have a direct relation between the straightening forces noted F_i and the positional deviations noted δ_i induced by the deviations of the straightness of the tube and the frame. This modelling will also make it possible to optimize the tolerances to reduce the local clearance without increasing the straightening forces.

The Fig. 8 illustrates the mechanical modelling according to the known contact hypothesis.

Here infinitely rigid contacts model metal-metal contacts. The displacements, noted δ_i, are therefore directly imposed on the frame beam model. This model is developed for a frame composed of n slender trees with $n + 1$ points of contact. For the rest of this step, the displacement δ_0 is defined as zero.

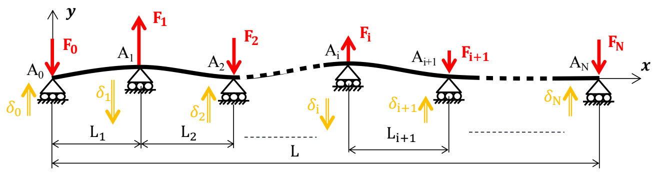

Fig. 8. Beam model.

The first theorem of Castigliano says: "If the strain energy of a linearly elastic structure can be expressed as a function of generalised force $\boldsymbol{F}$ then the partial derivative of the strain energy with respect to generalised force gives the generalised displacement δ_k in the direction of F_k" as indicated in equation (1).

$$\frac{\partial W}{\partial F_k} = \delta_k \tag{1}$$

The proposed study, in this paragraph, is based on the determination of the strain energy of the frame as a function of the contact forces. The strain energy of the frame, denoted W_{frame}, is the sum of the deformation energies of the shafts from P1 to PN, written as follows:

$$W_{frame} = \sum_{j=1}^{N} W_{\mathrm{P}_j} = \sum_{j=1}^{N} \frac{1}{2\mathrm{E}_j \cdot \mathrm{I}_j} \int_{\mathrm{X}_{j-1}}^{\mathrm{X}_j} \left(\sum_{i=j}^{N} \mathrm{F}_i \cdot (\mathrm{X}_i - x) \right)^2 \mathrm{d}x \tag{2}$$

By applying Castigliano's theorem, the derivation of the deformation energy of the frame gives the equation of geometric deviations δ_k, at the points of contact with $k = \{1, \dots, N\}$. By writing these equations in the matrix form, the matrix M of transition between the vector of the contact forces $\boldsymbol{F}$ and the vector of the positional deviations $\boldsymbol{\Delta}$ is defined by:

$$\boldsymbol{d} = \boldsymbol{M} \cdot \boldsymbol{F} \tag{3}$$

With:

- $\boldsymbol{d}$: Vector geometric differences of the points of contact at points $\mathrm{A_i}$,
- $\boldsymbol{F}$: Vector of contact efforts at points $\mathrm{A_i}$,
- $\boldsymbol{M}$: lower triangular transition matrix,

The matrix M is a lower and invertible triangular matrix. So:

$$\boldsymbol{F} = \boldsymbol{M}^{-1} \cdot \boldsymbol{d} \tag{4}$$

For the point of contact $\mathrm{A_0}$, the static equilibrium gives the equation of the contact force:

$$\mathrm{F_0} = -\sum_{i=1}^{N} \mathrm{F_i} \tag{5}$$

This modelling makes it possible to define a direct relationship between the imposed displacements characterizing the geometric differences and the contact forces. The imposed displacements correspond to the geometric variations of the frame. To determinate theses geometric variations, it is necessary to do a tolerance analysis to estimate the straightness of this frame. The worst case of straightness is the one that directs all the geometrical deviations in the same direction of analysis and defines the most important deviation of straightness of the frame. This worst case gives the positional deviations δ_i of the junctions according to the tolerances, in the form:

$$\delta\_i = \sum_{j=1}^{i}\left(\frac{t_{5P_j}\times L_j}{D_{2P_j}} + \frac{t_{4P_j}}{2} + \mathrm{Clearence}_{P_{j-1}/P_j}Ⓛ\right) \tag{6}$$

With:

$$\mathrm{CLearance}_{P_{j-1}/P_j}Ⓛ = \left(D_{1P_i} + t_{2P_i}\right) - \left(D_{1P_{i-1}} - t_{2P_{i-1}}\right) \tag{7}$$

To determine the worst of the mechanical cases, with the largest straightening forces, a 3d statistical study of a set of 3d straightness deviation configurations must be performed. For this, a model for generating orientation defects of the contact interfaces according to the $\boldsymbol{Oxy}$ and $\boldsymbol{Oxz}$ plans for generating 3d straightness deviations is used.

The Fig. 9 illustrates an exploitation of the results of this study. This histogram classifies the different solutions studied into the functions of the geometrical defects of the frame.

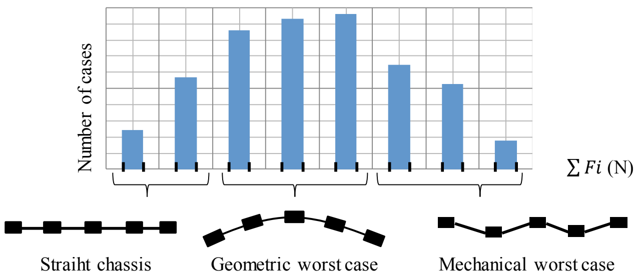

Fig. 9. Histogram of configurations in straightening efforts.

This modelling makes it possible to obtain an approximate value of straightening efforts. However, to be able to use this proposed beam type model, it is necessary to have two hypotheses:

- The parts of the frame have low straightness deviations compared to clearances in the contact interfaces between them,
- The clearances between the frame and the tube is very small compared to the global deviation of straightness of the frame.

On the other hand, the shafts have very variable lengths. The length of some shafts is a few hundred millimetres while other shafts have lengths of a few meters. The hypothesis of shafts with zero straightness is very restrictive. In addition, the deformation of these long shafts can cause random contacts between the frame and the tube.

3.4. Tolerance analysis and measurement analysis by modal model

In the previous model, a strong hypothesis concerning zero straightness defects of shafts was made. This assumption requires that the points of contact are at the junctions between the parts of the frame. The objective of this section is to lift this assumption and to assume that the contact points are indeterminate a priori. The determination of these points of contact between the frame and the protective tube proves difficult, given the possible straightness deviations of the frame and the protective tube but also the deformation of the frame.

In this part, the straightness deviations of the frame can be derived from simulation of deviations or measurement. The use of the modal decomposition of the straightness deviation of the frame is used. A new exploitation of this modal decomposition is then proposed. For each mode, straightening forces are calculated, then the different modes are superimposed to determine the straightening forces necessary to straighten the frame. The straightness deviations of the frame and the tube can be cumulated on a single subset. Subsequently, only the straightness deviations of the frame are taken into account.

The drilling is generally done vertically. So gravity does not interfere with the assemblies studied. To guarantee the assembly with residual stresses between and in the parts after assembly, it is necessary to be freed from the gravity. There are two solutions: measurement by putting the frame in a vertical position or measurement by reversal.

The reversal methodology is often used in particular for the standard rules [13, 14]. The reversal assumption is that the arrow coin has the same value in both directions. In each section, the difference of the measurements by inversion gives the distance d from the center to the axis of rotation of the part around these two levels. These measurements are made in the $\boldsymbol{Oxy}$ plane and in the $\boldsymbol{Oxz}$ plane, to obtain the location of the centers of each measured circle. The set of distances gives the straightness of the frame. Then, the outer diameter of the frame is raised in each section [15, 16]. This makes it possible to characterize the external geometry and straightness of the frame axis (Fig. 10).

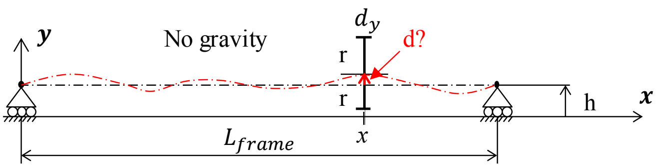

Fig. 10. Measure of straightness under the zero gravity effect.

The modal decomposition consists of breaking down the straightness deviation of the axis in a modal base deduced from the free-state bending modes Q_i and obtaining the influence coefficients λ_i of each bending mode.

The influence coefficients λ_i correspond to the amplitude of the deformation of each mode. The study of transverse vibration of the strings gives the equation of Q_i:

$$Q_i = \sin\left(\frac{i.\pi}{L_{châssis}}\cdot x\right) \tag{8}$$

The measure of straightness, noted $\delta_{straightness}$, is then written:

$$\delta_{straightness} = \sum_{i=1}^{n}\lambda_i\cdot Q_i = \mathbf{Q}\cdot\boldsymbol{\lambda} \tag{9}$$

With:

- $\mathbf{Q}$: Matrix of the modes of bending,
- $\boldsymbol{\lambda}$: Vector of modal coefficients, obtained by Fourier series decomposition.

Each bending mode is associated with a bending model of a straight beam. The N mode is associated with a model of bending $(N+2)$ contact points. Fig. 11 illustrates the location of the contact points for the first three bending modes.

The objective is to correct each mode of influence of amplitude λ_i. The points of contact are at the level of the extremums of each mode and at the ends of the frame. At the extremums of each mode, a straightening effort F_i is imposed, as illustrated on the first 3 modes in Fig. 11. Static equilibrium gives the forces at both ends.

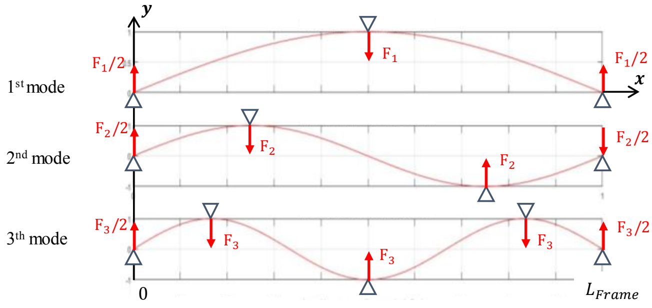

Fig. 11. Points of contact for the first 3 bending modes.

The first mode of bending is a model of 3-point bending of a beam of length L corresponding to the length of the frame. It is possible to study the straightening of each mode i as the straightening of a stack of i sections having a model of bending 3 points of beams of length $L_i = L/i$. The Fig. 12 can

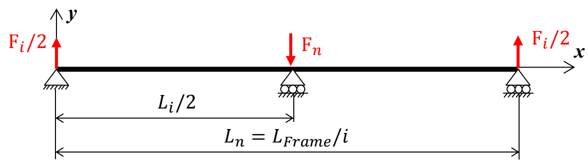

Fig. 12. 3-point bending model of a section of the mode i.

The contact force F_i, for a beam of Young's modulus E and moment of inertia I_{Gz}, is of the form:

$$F_i = \lambda_i \cdot \frac{48 \cdot i^3 \cdot E \cdot I}{L^3_{châssis}} \tag{10}$$

The contact forces F_i for each mode are now known. An algorithm is simulated on Matlab® to determine the contact forces of each mode. By superimposing the constructed beam type models, the straightening forces needed to assemble the frame into the protection tube can be estimated.

The interest of this modelling lies in the management of the contacts and the deformation of the parts. Indeed no hypothesis on the location of contacts is made. In addition, this new modelling also makes it possible to obtain an approximate value of the efforts of straightening the frame in the tube. However, this modelling is only valid when the geometrical defects of the frame are larger than the clearance between the frame and the tube.

4. Conclusions

This article proposes an extension of the ISO geometric dimensioning process for flexible mechanisms. Different approaches have been proposed to determine the forces required to straighten the frame according to the allowable geometric deviations by the tolerances.

Two geometric models are then proposed to estimate the values of straightening efforts. The first model is based on the assumption of points of contact imposed at the junctions between the stacked shafts of the frame. Thus, a mechanical model of beam type was built with known rigid supports. The resolution of this model gave first approximate values of the insertion forces necessary to assemble the stack of shafts in the tube. The second model is based on a modal decomposition in bending modes to associate simple beam type models for each mode. By superimposing these models, the necessary insertion effort is determined. As the actual geometry is very complex, a modelling by finite element simulation companion of experimental tests were realized to validate these models.

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