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Tolerance analysis of annular surfaces considering form errors and local surface deformations

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Abstract

This paper proposes a tolerance analysis method of annular surfaces considering form errors and local surface deformations. Specifically, form errors are properly considered based on skin model shapes. Local surface deformations are calculated based on a Conjugate Gradient-Fast Fourier Transform (CG-FFT) method, which is a classic Boundary Element Method (BEM). Based thereon, tolerance analysis of annular surfaces is conducted based on Monte Carlo simulation, and efficiency of the process is also demonstrated.

Keywords: Tolerance analysis; Form errors; Local surface deformations

1. Introduction

Tolerance design plays an important role in guaranteeing quality of mechanical products through improving functional performance and manufacturing efficiency during the early design stage of mechanical products [1, 2]. In order to provide theoretical foundation and practical technology for prediction and evaluation of the performance of a product, tolerance modeling and analysis have been widely studied based on the given Geometric Dimensioning and Tolerancing (GD&T) scheme [3, 4].

Over the past thirty years, lots of researchers have devoted themselves to solving tolerancing related problems. Various different tolerance modeling methods have been proposed [5-10]. Using these tolerance models, many researchers have presented different tolerance analysis methods. The most typical and recognized methods range from the variational method [11], the matrix method [12], the vector loop method [13] to the Jacobian-torsor method [14-16], and the T-Map based method [17]. However, those traditional tolerance analysis methods mentioned above just ignored the effect of form errors, which are ubiquitous during manufacturing process. To be more specific, these methods simply regarded the geometric deviations as translational and rotational variations of nominal features. Because more and more researchers have realized that this kind of simplification may cause inaccuracy of tolerance analysis results, consideration of form errors have increased interests among peers [18-20]. To properly consider the effects of form errors during tolerance analysis, the idea of skin model shapes was put forward [21, 22]. Because skin model shapes can integrate various modeling methods of form errors [23, 24] and have the ability to access assembly simulation and tolerance analysis process [25], they have shown great potential in Computer-Aided Tolerancing (CAT) applications [26, 27]. However, the assembled parts are inevitably to have deformations during real assembly process due to the external forces [28]. Existed research on skin model shape-based methods has not yet considered the combined effects of form errors and local surface deformations, which may have considerable impacts on tolerance analysis results in some specific conditions. Concerning the effects of deformations, Grandjean et al. [29] modeled the local surface deformations by a purely plastic behavior of the material during tolerance, but it did not conform to real deformations behavior. Guo et al. [30] presented a linear equivalent model, but their model requires FEM to calculate the stiffness matrix, which restricts its efficiency. Hence, the integration of form errors and local

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surface deformations still remains a challenge, due to the complexity of interactions between them.

In short, previous work on tolerance analysis has not appropriately considered the combined effects of form errors and local surface deformations. Recently, the authors proposed a novel tolerance analysis framework, which aims to consider the interactions between form errors and local surface deformations during assembly simulation and tolerance analysis. The implementation of the framework is based on skin model shapes and a classic Boundary Element Method (BEM), named the Conjugate Gradient-Fast Fourier Transform (CG-FFT). The proposed framework only conducted the tolerance analysis process of rectangular surfaces because of the restriction of the adopted BEM calculation method. To further extend the application of the framework, the current paper focuses on the deformations calculation and tolerance analysis of annular surfaces. It is structured as follows: Section 2 briefly illustrates the tolerance analysis framework. Section 3 introduces a method for generating skin model shapes of annular surfaces considering form errors. Section 4 describes the BEM calculation of local surface deformations of annular surfaces under external forces. Section 5 presents the tolerance analysis process. Finally, Section 6 presents our conclusions and future prospects.

2. The tolerance analysis framework

For sake of completeness, the tolerance analysis framework considering the combined effects of form errors and local surface deformations is briefly introduced in this section. As shown in Fig.1, the first step in this framework is to determine the key mating surfaces for a given assembly. This can be realized by the experience of designers and/or through a quick CAT/FEM calculation. Specifically, the surfaces whose imperfections can have significant effects on the key functional requirements can be determined by a quick CAT calculation, and the surfaces that may bear relatively high compressive pressure can be determined by a quick FEM calculation. It has to be mentioned that the quick FEM calculation here is only conducted once, by using the solid part models with ideal sizes and shapes. Then, tolerance analysis is conducted by a complete Monte Carlo-based assembly simulation. During Monte Carlo simulation, skin model shapes are first generated. After that, the difference surface method or point cloud registration methods [25] are used to obtain the relative positioning of mating surfaces with the consideration of form errors. For key mating surfaces obtained in the first step, the relative positioning has to be further adjusted based on the calculation of local surface deformations. After all of the parts and mating surfaces have been evaluated, the assembly simulation is complete. After repeating the above assembly simulation hundreds of times, i.e., a Monte Carlo simulation, the tolerance analysis procedure can be complete.

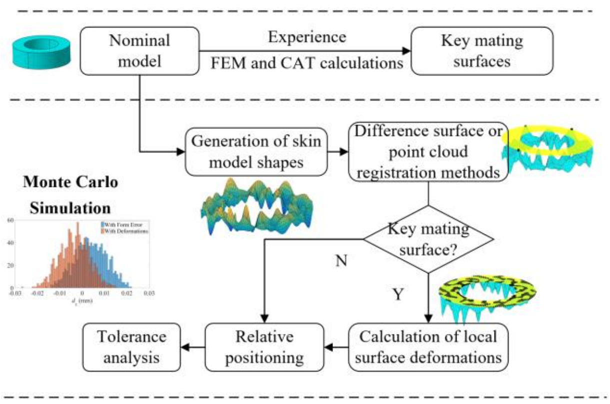

Fig. 1. Illustration of the tolerance analysis framework (Basic idea of the framework has been elaborated in a submitted journal article, but limited to rectangular surfaces. The illustration here is an extension to annular surfaces).

3. Generating skin model shapes for annular surface

Generation of skin model shapes with position, orientation, and form errors is an essential step in the proposed tolerance analysis framework. This section introduces a method of modeling skin model shapes for annular surfaces with position, orientation, and form errors.

3.1. Modeling position and orientation errors

As shown in Fig. 2, tolerance t_p constrains the position and orientation of the top surface; thus the top surface can deviate only within the corresponding tolerance zone. Based on SDT theory [7], deviation of the top surface can be expressed by three parameters, $[d_z \ \theta_x \ \theta_y]$, which represent the translation and rotations of the tolerance surface. The range of the three parameters can be obtained according to the extreme positions of the surface.

With consideration of position and orientation errors, the nominal surface deviates from its ideal position and becomes inclined, called a 'biased surface'. The equation of the biased surface can be obtained based on the specific values of $[d_z \ \theta_x \ \theta_y]$. Then, the z-coordinates of sampling points on the biased surface can be expressed as:

$$S_p = f(X,Y) \tag{1}$$

where S_p represents the z-coordinates of sampling points, f indicates the equation of the biased surface, and X and Y represent the x- and y-coordinates, respectively, of the sampling points.

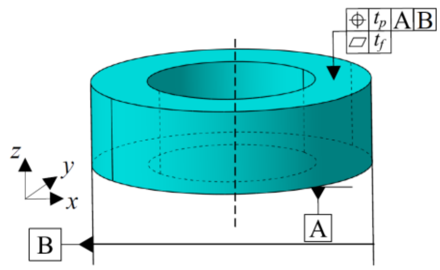

Fig. 2. Geometric design of a simple part.

3.2. Modeling form errors based on Zernike polynomials

Similarly, the form tolerance zone is constrained by two parallel annular surfaces with distance t_f, as shown in Fig. 3. Various methods have been proposed to model form errors [24]. Here, the Zernike polynomials [31] are adopted to model the form errors based on the design value of form tolerance. It should be noted that the Zernike polynomials modeling method is not the only choice; other methods [24] are also viable.

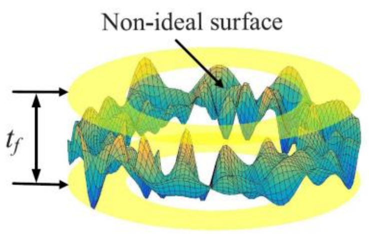

Fig. 3. Tolerance zone of the form error.

The representation of form errors can be regarded as a linear combination of a series of basis functions. The Zernike polynomials are used to generate the basis functions; this can be expressed as:

$$U_n^t(\rho,\theta) = R_n^t(\rho)\Theta_n^t(\theta) \tag{2}$$

where

$$\begin{cases} R_n^t(\rho) = R_n^{n-2m}(\rho) = \sum\limits_{s=0}^{m}(-1)^s \dfrac{(n-s)!\,\rho^{n-2s}}{s!(m-s)!(n-m-s)!} \\ \Theta_n^t(\theta) = e^{\mathrm{i}\,t\theta} = \begin{cases} \cos(n-2m)\theta, & n-2m<0 \\ \sin(n-2m)\theta, & n-2m \ge 0 \end{cases} \end{cases} \tag{3}$$

where $R_n^t(\rho)$ is a radial function and $\Theta_n^t(\theta)$ is an angle related function; t and n are nonnegative integers with $n \ge t$; θ is the azimuthal angle.

After obtaining the basis functions, a surface with a specific form error can be generated by the linear combination of the basis functions as follows:

$$S_f = \sum \lambda_i g_i \tag{4}$$

where S_f represents the coordinates of points on the surface with form errors, g_i represents the basis functions, and λ_i represents the coefficients of the basis functions.

3.3. Modeling skin model shapes

After obtaining S_p and S_f, modeling of the skin model shape for an annular surface with position, orientation, and form errors can be performed by summing S_p and S_f, as shown in Fig.4, which can be expressed as:

$$S_t = S_p + S_f \tag{5}$$

where S_t represents the coordinates of the points on the skin model shape.

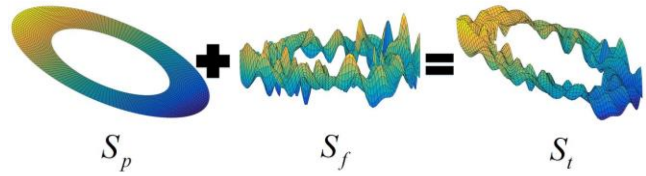

Fig. 4. Modeling skin model shapes.

However, the direct summation of S_p and S_f may violate the constraint of tolerance specification t_p. To avoid this problem, S_t, obtained directly from Eq. (5) should be checked to ensure that it conforms to the constraint of t_p; i.e., that it is a valid skin model shape. The invalid skin model shapes are discarded and re-generated until S_t conforms to the tolerance specification.

4. Calculating local surface deformations

For many mechanical products, the combined effects of form errors and local surface deformations on assembly deviations are non-negligible, which deserve a comprehensive research. However, considering the combined effects of them is relatively difficult in the field of tolerance analysis, especially for Monte Carlo simulations based on randomly generated skin model shapes. However, much research in tribology has focused on contact problems between non-ideal surfaces, and BEM is known to be an accurate and efficient means of solving contact problems [32-35]. In this section, the CG-FFT method [36, 37], a classical BEM method, is introduced to calculate local surface deformations.

CG-FFT is a classical BEM calculation method, and its accuracy and efficiency have been widely demonstrated [36, 37]. Moreover, it requires only discrete point data on surfaces (i.e., boundaries), which makes the combination of skin model shapes and CG-FFT a perfect match. Under the assumption of frictionless non-adhesive contact, the contact between two non-ideal surfaces can be replaced by the contact between a rigid ideal surface and an equivalent non-ideal surface [32].

The BEM methods are typically used for contact problems of rectangular surfaces. The elastic displacement of discrete points on the rectangular surface, according to contact mechanics, can be calculated as:

$$u_{i,j} = \sum_{k=1}^{M}\sum_{l=1}^{N} K_{i-k,j-l}\, p_{k,l} \tag{6}$$

where M and N represent the number of discrete points in the x- and y-directions, respectively; p represents the contact force on each discrete point; and K denotes the matrix that collects the influence coefficients; it is related to the effective composite elastic modulus , E', which can be obtained by:

$$\frac{2}{E'} = \frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2} \tag{7}$$

with E_1 and E_2 being the Young's moduli of elasticity and υ_1 and υ_2 being the Poisson's ratios for the two contacting parts.

Based on the principle of minimizing the total complementary potential energy, the calculation of a contact problem can be transformed into a quadratic programming problem [37]. The key issue of calculating local surface deformations is solving the quadratic programming problem. Here, the CG-FFT method is used, which applies the Conjugate Gradient algorithm to search the gradient direction, and the Fast Fourier Transform method to compute Eq. (6) efficiently. A more detailed explanation of the CG-FFT method can be found elsewhere [36, 37].

However, the CG-FFT method is only suitable for local surface deformations calculations of rectangular surfaces. To make the calculation appropriate for annular surfaces as well, a method that transforms the annular surface into a rectangular one is proposed here. Based on the radius difference between the inside and outside circle of the annular surface, two different kinds of solutions are proposed to get with the calculation process.

For conditions where the radius difference is relatively large comparing with the radius of the inner circle, i.e., $R-r \gg r$, the annular surface is expanded to a rectangular surface as shown in Fig. 5(a), where cyan points are valid sampling points on the annular surface while the red points are dumb ones. Then, *z* coordinates of these red points are set to very small values while the cyan ones are set to the coordinates of the toleranced annular surface. This method does not cause too much computational efficiency loss because the portion of the dumb points is relatively small.

For conditions where the radius difference is relatively small comparing with the radius of the inner circle, i.e., $R-r \ll r$, which is very common in mechanical products, the annular surface is approximately transformed into a rectangular surface as shown in Fig. 5(b).

After transforming the annular surface into a rectangular one, local surface deformations can be calculated using the introduced CG-FFT method.

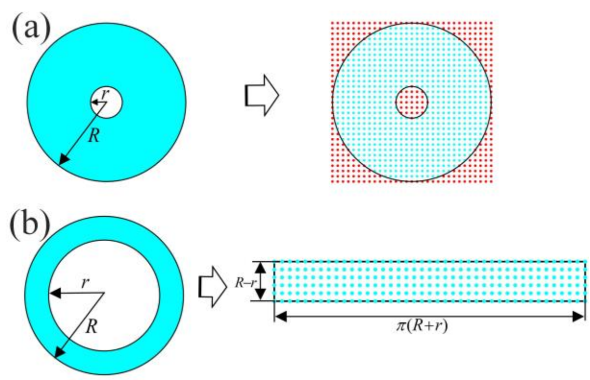

Fig. 5. (a) $R-r \gg r$; (b) $R-r \ll r$.

5. Tolerance analysis

This section conducts the tolerance analysis process based on Monte Carlo simulation. As mentioned in Section 2, the key issue in this process relies on the calculation of the relative positioning of key mating surfaces. Here, the traditional method is termed as Case 1, and two other Monte Carlo simulations are conducted to show the effects of form errors and local surface deformations. One considers only the effects of form errors, i.e., Case 2, and the other considers the effects of both form errors and local surface deformations, i.e., Case 3. The relative positioning of mating surfaces can be depicted by SDT parameters. The simple part shown in Fig. 2 is used to conduct the simulation, which can be regarded as the condition shown in Fig. 5(b). A total of 1000 simulations is conducted, and the sampling size $M \times N$ is 161×21 after transforming the annular surface into a rectangular one based on the method proposed in Section 4. In the simulation, specific parameters of the simple example include: $r = 15$ mm, $R = 25$ mm, $t_p = 0.05$ mm, and $t_f = 0.02$ mm. In addition, simulations were conducted using the commercial mathematics software, Matlab® 2016.

5.1. Monte Carlo simulation

<u>*Case 1: traditional method*</u>

In traditional tolerance analysis, the mating surfaces are assumed to be ideal and rigid; i.e., the effects of form errors and local surface deformations are not included. In this case, the SDT parameters simply need to satisfy the restriction of the tolerance zone. Based on that, a theoretical tolerance domain can be obtained, as shown in Fig. 6(a). 1000 valid combinations of the SDT parameters are also generated, i.e., the green points in Fig. 6(a).

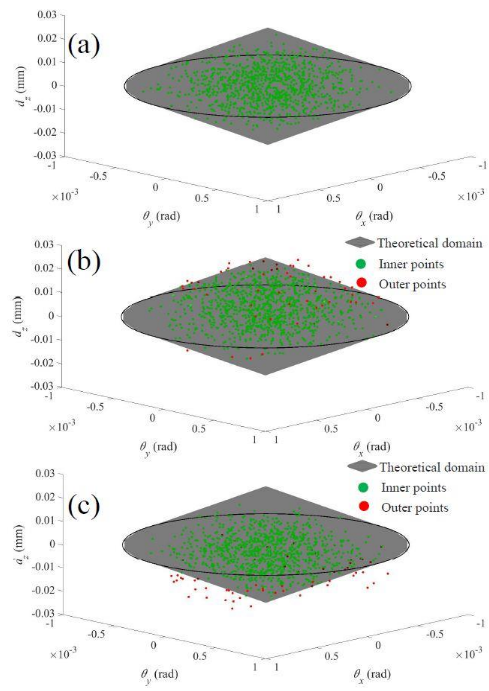

Fig. 6. (a) theoretical domain; (b)with form errors; (c) with deformations.

<u>*Case 2: considering form errors*</u>

To consider the effects of form errors, skin model shapes of the top surface of the part shown in Fig. 2 are randomly generated based on the method presented in Section 3. A convex hull-based assembly simulation method [25] is used to identify the relative positioning of the assembled parts. The contact points are indicated by three black dots (Fig. 7(a)).

After conducting 1000 simulation runs, the distribution of SDT parameters can be obtained, as shown in Fig. 6(b). Some of the points are not in the theoretical deviation domain any more (the red ones), which shows the effects of form errors.

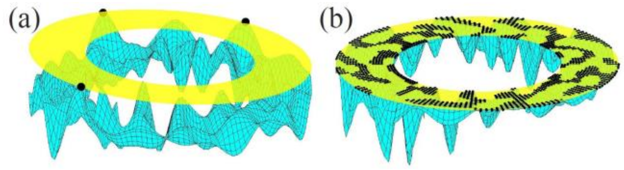

Fig. 7. (a) with form errors; (b)with deformations.

Case 3: considering deformations

To demonstrate the effects of local surface deformations on the SDT parameters of the relative positioning of mating surfaces, local surface deformations are calculated based on the CG-FFT method presented in Section 4. In this calculation, the same skin model shapes obtained in Case 2 are used. The parameters of the materials include $E_1 = E_2 = 210$ Gpa and $\upsilon_1 = \upsilon_2 = 0.34$, which are typical parameters of steels. The external force is assumed to be 1000 N here. The direction of the external force is assumed to be perpendicular to the rigid surface. With consideration of the local surface deformations, Fig. 7(b) shows the contact area based on the same skin model shape shown in Fig. 7(a). The non-ideal surface is flattened by the force applied and the contact area is no longer three points, but several small, flat facets.

The distribution of SDT parameters can be also obtained after conducting 1000 simulation runs, as shown in Fig. 6(c). The distribution of outer points in Fig. 6(c) is different from that in Fig. 6(b), which shows the effects of local surface deformations.

5.2. Comparison of the results

To quantitatively evaluate the effects of form errors and local surface deformations, the percentages of outer points in Figs. 6(b) and 6(c) are calculated, which is called the noncompliance rate (NCR) [29]. For the two cases, the NCRs are 5.1% and 6.1%, respectively. This indicates that both form errors and local surface deformations have considerable effects on the results of tolerance analysis.

To further show the effects of form errors, the SDT parameters calculated solely by translational and rotational errors (i.e., a traditional tolerance analysis method) from the same 1000 simulation runs are also obtained, and their distributions are shown in Fig. 8 for comparison. It can be seen that form errors influence all of d_z, θ_x, and θ_y. In addition, comparison of the SDT parameter distributions of 1000 simulations of Cases 2 and 3 is also shown in Fig. 9. It can be seen that the local surface deformations have a considerable effect on d_z.

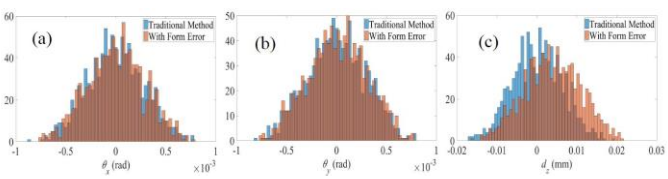

Fig. 8. SDT parameters (a) θ_x; (b) θ_y; (c) d_z.

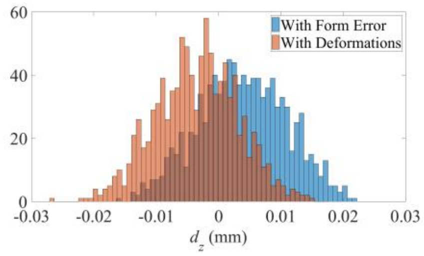

Fig. 9. SDT parameters d_z.

Moreover, table 1 lists the CPU times of the different cases of simulations. Here, the CPU time is the total time for 1000 Monte Carlo simulation runs. The simulations are quite efficient: even for Case 3, in which local surface deformations are calculated, the average time required for one simulation run is less than 0.5 s. The simulations were conducted on the author's ordinary desktop computer (Windows 7 operating system, 3.20 GHz Intel Core i7-6900K CPU and 64 GB RAM). Therefore, the efficiency of the proposed tolerance analysis framework is acceptable.

Table 1. Comparison of the results.

	Case 1	Case 2	Case 3
Considering form errors?	No	Yes	Yes
Considering deformations?	No	No	Yes
NCR	-	5.1%	6.1%
Time	0.05s	41.64s	448.66s

6. Conclusions and future prospects

Form defects and local surface deformations may affect product performance considerably. However, traditional tolerance analysis methods almost ignore the combined effects of form errors and local surface deformations. Hence, with a comprehensive consideration of them, tolerance analysis of annular surfaces is conducted in this paper. Specifically, form errors are properly considered based on skin model shapes. Local surface deformations are calculated based on a CG-FFT method, which is a classic BEM calculation method. Based thereon, tolerance analysis of annular surfaces is conducted based on Monte Carlo simulation, and efficiency of the process is also demonstrated. It is expected that the consideration of the combined effects of form errors and local surface deformations can help ensure the product performance more effectively.

Because the current research only treats the annular surfaces and simple force conditions. Further research should focus on improving the proposed tolerance analysis framework to ensure its broad applicability and ability to solve more complex tolerancing problems. First, other surface types especially cylindrical surface should be considered. There are some research outcomes on this issue [38], but the calculation of deformations of cylinders is still challenging. Second, the current research only treat conditions where the external force is a uniform loading force. Implementing an accurate and efficient calculation for complex force conditions also needs further studies. At last, to make the calculation of deformations more accurate, plasticity of the materials and density of the sampling points should also be considered.

Acknowledgements

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References

[1] Cao YL, Mathieu L, and Jiang J. Key research on computer aided tolerancing. J Zhejiang Univ-Sc A 2015;16(5):335-40.
[2] Zhang ZQ, Liu JH, Ding XY, Jiang K, Bao QW. Searching multibranch propagation paths of assembly variation based on geometric tolerances and assembly constraints. J Mech Des 2017;139(5):051701.
[3] Chen H, Jin S, Li ZM, Lai XM. A comprehensive study of three dimensional tolerance analysis methods. Comput Aided Des 2014;53:1-13.
[4] Cuo CY, Liu JH, Jiang K. Efficient statistical analysis of geometric tolerances using unified error distribution and an analytical variation model. Int J Adv Manuf Tech 2016;84(1):347-60.
[5] Requicha AAG. Toward a theory of geometric tolerancing. Int J Robot Res 1983;2(4):45-60.
[6] Desrochers A, Clement A. A dimensioning and tolerancing assistance model for cad-cam systems. Int J Adv Manuf Tech 1994;9(6):352-61.
[7] Clement A, Bourdet P. A study of optimal-criteria identification based on the small-displacement screw model. CIRP Ann-Manuf Techn 1988;37(1):503-6.
[8] Teissandier D, Couetard Y, Gerard A. A computer aided tolerancing model: Proportioned assembly clearance volume. Comput Aided Des 1999;31(13):805-17.
[9] Davidson JK, Mujezinovic A, Shah JJ. A new mathematical model for geometric tolerances as applied to round faces. J Mech Design 2002;124(4):609-22.
[10] Giordano M, Samper S, Petit JP. Tolerance analysis and synthesis by means of deviation domains, axi-symmetric cases. In: Models for computer aided tolerancing in design and manufacturing, Netherlands: Springer; 2007, p. 85-94.
[11] Gupta S, Turner JU. Variational solid modeling for tolerance analysis. IEEE Comput Graph Appl 1993;13(3):64-74.
[12] Desrochers A, Riviere A. A matrix approach to the representation of tolerance zones and clearances. Int J Adv Manuf Tech 1997;13(9):630-6.
[13] Wittwer JW, Chase KW, Howell LL. The direct linearization method applied to position error in kinematic linkages. Mech Mach Theory 2004;39(7):681-93.
[14] Salomons OW, Haalboom FJ, Poerink HJJ, van Slooten F, van Houten F, Kals HJJ. A computer aided tolerancing tool II: Tolerance analysis. Comput Ind 1996;31(2):175-86.
[15] Chase KW, Gao JS, Magleby SP, Sorensen CD. Including geometric feature variations in tolerance analysis of mechanical assemblies. Iie Trans 1996;28(10):795-807.
[16] Laperriere L, Ghie W, Desrochers A. Statistical and deterministic tolerance analysis and synthesis using a unified Jacobian-torsor model. CIRP Ann-Manuf Techn 2002;51(1):417-20.
[17] Singh G, Ameta G, Davidson JK, Shah JJ. Tolerance analysis and allocation for design of a self-aligning coupling assembly using tolerance-maps. J Mech Design 2013;135(3):14.
[18] Polini W, Moroni G. Manufacturing signature for tolerance analysis. J Compu Inf Sci Eng 2015;15(2):5.
[19] Forslund A, Madrid J, Soderberg R, Loof J, Knuts S, Isaksson O, Frey DD. Bridging the Gap between Point Cloud and CAD: a Method to assess form error in aero structures. In : 18th AIAA Non-Deterministic Approaches Conference. Diego, California (USA). 2016.
[20] Corrado A, Polini W, Moroni G, Petro S. 3D tolerance analysis with manufacturing signature and operating conditions. In: 14th CIRP Conference on Computer Aided Tolerancing. Gothenburg (Sweden). 2016.
[21] Schleich B, Anwer N, Mathieu L, Wartzack S. Skin model shapes: A new paradigm shift for geometric variations modelling in mechanical engineering. Comput Aided Des 2014;50:1-15.
[22] Anwer N, Schleich B, Mathieu L, Wartzack S. From solid modelling to skin model shapes: Shifting paradigms in computer-aided tolerancing. CIRP Ann-Manuf Techn 2014;63(1):137-40.
[23] Zhang M, Anwer N, Stockinger A, Mathieu L, Wartzack S. Discrete shape modeling for skin model representation. P I Mech Eng B-J Eng 2013;227(B5):672-80.
[24] Yan X, Ballu A. Generation of consistent skin model shape based on FEA method. Int J Adv Manuf Tech 2017;1-14.
[25] Schleich B, Wartzack S. Approaches for the assembly simulation of skin model shapes. Comput Aided Des 2015;65:18-33.
[26] Schleich B, Wartzack S, Anwer N, Mathieu L. Skin model shapes: Offering new potentials for modelling product shape variability. In: International design engineering technical conferences and computers and information in engineering conference. New York. 2015.
[27] Schleich B, Anwer N, Mathieu L, Wartzack S. Status and prospects of skin model shapes for geometric variations management. In: 14th CIRP Conference on Computer Aided Tolerancing. Gothenburg (Sweden). 2016.
[28] Liu T, Cao YL, Wang J, Yang JX. Assembly error calculation with consideration of part deformation. In: 14th CIRP Conference on Computer Aided Tolerancing. Gothenburg (Sweden). 2016.
[29] Grandjean J, Ledoux Y, Samper S. On the role of form defects in assemblies subject to local deformations and mechanical loads. Int J Adv Manuf Tech 2013;65(9-12):1769-78.
[30] Guo JK, Li BT, Liu ZG, Hong J, Wu XP. Integration of geometric variation and part deformation into variation propagation of 3-D assemblies. Int J Prod Res 2016;54(19):5708-21
[31] Zhu LJ, Sun PC, Bartsch DU, Freeman WR, Fainman Y. Wave-front generation of Zernike polynomial modes with a micromachined menbrane deformable mirror. Appl Opt 1999;38(28):6019-26.
[32] Yastrebov VA, Anciaux G, Molinari JF. From infinitesimal to full contact between rough surfaces: Evolution of the contact area. Int J Solids Struct 2015;52:83-102.
[33] Putignano C, Carbone G. A review of boundary elements methodologies for elastic and viscoelastic rough contact mechanics. Phys Mesomech 2014;17(4):321-33.
[34] Putignano C, Afferrante L, Carbone G, Demelio G. A new efficient numerical method for contact mechanics of rough surfaces. Int J Solids Struct 2012;49(2):338-43.
[35] Wang ZJ, Wang WZ, Meng FM, Wang JX. Fretting contact analysis on three-dimensional elastic layered half space. J Tribol-Trans ASME 2011;133(3):8.
[36] Wang WZ, Hu YZ, Wang H. Numerical solution of dry contact problem based on fast fourier transform and conjugate gradient method. Chin J Mech Eng 2006;42(7):14-8.
[37] Wang ZJ, Wang WZ, Hu YZ, Wang H. A numerical elastic-plastic contact model for rough surfaces. Tribol T 2010;53(2):224-38.
[38] Homri L, Goka E, Levasseur G, Dantan J-Y. Tolerance analysis - form defects modeling and simulation by modal decomposition and optimization. Comput Aided Des 2017;91:46-59.