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Tolerance Analysis - Key Characteristics Identification by Sensitivity Methods

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Abstract

During the design of manufactured products, functional requirements that need to be fulfilled are defined; they are expressed in terms of the dimensions. Functional requirements are considered as critical if they have a major impact on the performance of the system. In the current industrial practice, the dimensions involved in a critical functional requirement are identified as Key Characteristics. This study aims to rank the influence of the dimensions involved in a critical functional requirement with respect to their impacts on the non-conformity of a mechanical system. This paper presents a methodology for the identification of Key Characteristics by combining the Sobol' global sensitivity method (based on the variance decomposition) with the Advanced Probability-based Tolerance Analysis approach (APTA). The method is applied on an electrical plug having a functional requirement with a linear formulation. The results showed a possibility to prioritize some dimensions and neglect the effect of the others.

Keywords: Key Characteristics; Sensitivity Analysis; Tolerance Analysis

1. Introduction

The products manufactured in the industries are all subjected to geometrical imperfections. The designers specify the nominal dimensions, the tolerance intervals, the capability requirements that need to be fulfilled. When these requirements are not respected, unfavourable effects may occur on the performance of the mechanical system. Therefore, designers indicate the dimensions that have to be considered as Key-Characteristics (sometimes indicated by a diamond on the drawing, especially in the French automotive industry), i.e. the dimensions with a first order influence on the functionality.

Numerous definitions are associated with the Key Characteristics (KCs). They differ from a company to another. A general definition is given by Thornton in [1], and it states the following: *"Key characteristics are the product, sub assembly, part, and process features that significantly impact the final cost, performance, or safety of a product when the key characteristics vary from nominal"*. During manufacturing, the dimensions specified as KCs require more attention because of the following reasons:

- They are more frequently controlled.
- The derogations are not allowed.

It is therefore necessary to guarantee that all the KCs have a significant impact on the performance of the mechanical system, in order to reduce the waste, the costs and efforts associated with manufacturing.

The industrial standards consist of using the Failure Mode Effects and Criticality Analysis (FMECA) [2] in order to identify the critical safety, functional and assembly aesthetic requirements, which could be potentially associated with a high Non-Conformity-Rate (NCR). An upper and a lower bound are associated with each functional requirement. FMECA is a risk-based approach: the probability of non-functionality is considered as well as the consequence of the non-functionality. The requirements are presented as mathematical functions involving the dimensions of the parts. FMECA does not provide a method to identify the dimension with a first order contribution to the NCR. A possible industrial practice is setting all the dimensions involved in a critical requirement as KCs, even though some of them may have a reduced contribution with respect to the others. In addition, a method named Analysis of Causality, Failures and Effects (ACDE) for the identification of the key characteristics was developed in LCFC [3][4]. This method is based on the FMECA method [2] and the KC flowdown [1,5]. Other methods exist in the literature, where the influence of the part tolerance on the assembly clearance is estimated using sen-

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sitivity analysis methods [6]. In addition, the global sensitivity analysis is implemented in convex hull based tolerancing techniques such deviation domains [7].

The focus of this paper is the implementation of a method which allows designers to identify the dimensions with a high or a reduced impact on the NCR in order to reduce the total number of KCs indicated on their drawings without loss of quality. Three main points should be identified to define an appropriate strategy for the identification of the KCs:

1. The model of uncertainty of the dimensions.
2. Geometrical modelling.
3. The identification of a suitable numerical model associated with the quality of a system.

The model mentioned in the first point is with two levels of uncertainties, that is the uncertainty in the dimensions and in the corresponding statistical parameters. The geometrical model is not investigated in depth in this paper, but the method is sufficiently flexible to be applied on any geometrical model such as linear stack-up, non-linear functions or over-constrained mechanisms with gaps. The NCR is used as a quality indicator.

The proposed procedure is based on sensitivity analysis. The formulation of the NCR is done using the advanced probability based tolerance analysis method and it is detailed in section 2. In another hand, the sensitivity method considered is the Sobol' method described in section 3. The formulation of the problem combining the two methods is given in section 4. An application example of an electrical plug is presented in section 5. Finally, conclusions are given in section 6.

2. Formulation of the Non Conformity Rate

In industry, the quality of the product is one of the main measures checked to satisfy the needs of the customer. According to the technical requirements and specifications imposed by the customer, a list of functional requirements is defined for dimensional chains. A functional requirement can be expressed as a function of input parameters $\mathbf{X}$, these parameters are the dimensions of the parts included in the dimensional chains. In the case of a function *f*, the dimensional chain is expressed as:

$$Y = f(\mathbf{X}) \tag{1}$$

The mathematical formulation of this function can be linear such as the case of a linear stack-up, non-linear for problems of greater complexity. When the system studied is an over-constrained system, the function *f* is implicit [8]. It is tolerated that a small number of systems are non-functional, as long as this occurs seldom. The Non conformity rate therefore provides suitable metrics to measure the quality. It can be interpreted as the probability that an assembly is non-functional, or as the frequency of manufacturing non-functional components. The NCR, defined by Eq.(2), is the probability that a functional requirement is outside the permissible bounds. It is often expressed in parts per million (ppm).

$$NCR = P(Y \notin [LSL_Y; USL_Y]) \tag{2}$$

LSL_Y is the lower specification limit and USL_Y is the upper specification limit. They are called the functional variable bounds.

Several approaches exist in the literature for the estimation of the NCR. They are based on the reliability methods used originally for the calculation of the probability failure of the systems. The Monte Carlo method is used in the context of tolerancing [9], however this method is time consuming. Some authors tried to overcome this problem by using the First Order Reliability Method (FORM) instead [9]. Another method based on the Taylor developments was also used for the same purpose and it is the Second Order Tolerance Analysis method (SOTA) [10].

2.1. Formulation for statistical tolerance analysis

In the manufacturing process, the capability and performance measures can be considered to check if the products are acceptable according to the given criteria. The definitions of the performance and the capability measures are stated in the standard ISO 22514-1 [12]. The capability measure is defined as the quality measure of the outcome of the process in a state of statistical control, however, the performance measure is considered when the process is not in the statistical control state. In the scope of this paper, it is assumed that the process is mature, so it is under the statistical control. The produced batches should meet the capability requirements. The capability indices controlling the manufacturing process are C_p and C_{pk} [11]. C_p is the measure of the capability of a process to meet given requirements. C_{pk} takes into account the non centred data. Generally, the dimensions of the manufactured parts X_i are defined with a target value T_i, a tolerance interval t_i having respectively a lower specification limit LSL_i and an upper specification limit USL_i and two required capability indices $C_{pi}^{(r)}$ and $C_{pki}^{(r)}$. For a given batch, the dimensions of the parts are assumed to be random variables with a normal distribution having a standard deviation σ_i and mean shift $\delta_i = \mu_i - T_i$, where μ_i denotes the mean value of the dimension. Two capability requirements are associated to each dimension included in the batch, they are defined as:

$$C_{pi} = \frac{t_i}{6\sigma_i} \tag{3}$$

$$C_{pki} = \frac{t_i/2 - |\delta_i|}{3\sigma_i} \tag{4}$$

The given formulas of the capability indices are only used when the variables follow a normal distribution. More general definitions of C_p and C_{pk} are provided in [12] applied for different types of distributions. A part is acceptable if the value of its dimension is within the tolerance interval ($X_i \in [LSL_i, USL_i]$). The corresponding production batch includes multiple components. Off-tolerance dimensions are tolerated as long as they are not too frequent. Threshold limits are set for C_p and C_{pk}. They monitor the statistical parameters of the dimensions such that the probability of the occurrence of off-tolerance dimensions is sufficiently low. The capability requirements to be satisfied are that $C_p \geq C_p^{(r)}$ and $C_{pk} \geq C_{pk}^{(r)}$. Based on these conditions, a conformity domain V_D corresponding to each dimension X_i can be defined. It is represented on a σ_i-δ_i diagram. The conformity domain has a triangular shape when $C_{pi}^{(r)} = C_{pki}^{(r)}$ and a trapezoidal shape when $C_{pi}^{(r)} \geq C_{pki}^{(r)}$ as shown in Fig.1. The hatched part of the triangle is part of the conformity domain that it is not unattainable since the standard deviation cannot have such low values. It is assumed that a manufacturing process can also be characterized by the lower limit of the standard deviation. For example, it is not reasonable to assume that that the standard deviation of a forged or a cast part is as low as 1 μm (in case the surfaces are not machined).

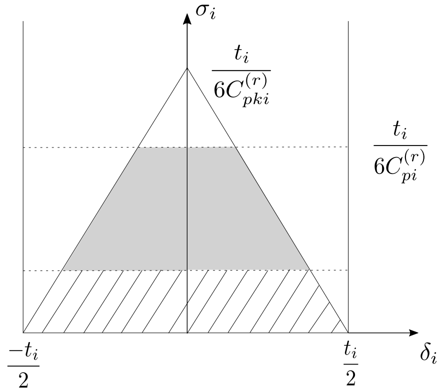

Fig. 1. Representation of the conformity domain V_D (grey area), the solid lines represent the boundary associated with $C_{pki}^{(r)}$, the dash lines represent the boundaries associated with C_p and the lower bound of the standard deviation. The hatched area of the triangle is associated with compliant batches, but the standard deviation is too low to be achieved.

2.2. Non Conformity Rate based on APTA method

The manufacturing process differs from day to day depending on the conditions affecting the machines' performance. According to [13] the statistical parameters δ_i and σ_i are variable. The variability of these parameters have a remarkable influence on the NCR and should be taken into account in the analysis [14]. The methods used for the estimation of the NCR already mentioned in section 2 do not take into consideration the variability of the statistical parameters such as the mean shifts and the standard deviations. The Advanced Probability-based Tolerance Analysis (APTA) is an approach used for an accurate estimation of the NCR. It is originally formulated in [15]. The APTA method is applied independently to all the dimensions. A mean shift and a standard deviation is associated to each dimension. The mean shifts are noted as $\boldsymbol{\delta} = (\delta_1, ...\delta_n)$ and the standard deviations as $\boldsymbol{\sigma} = (\sigma_1, ...\sigma_n)$, where n is the number of dimensions. The statistical parameters are random variables, therefore the NCR is a random variable as well, it is expressed in terms of $\boldsymbol{\delta}$ and $\boldsymbol{\sigma}$.
The dimensions of the parts are assumed to have a normal distribution such as $X_i \sim N_i(\mu_i, \sigma_i)$. The statistical parameters δ_i and σ_i associated for each dimension are considered as random variables, linked to the appropriate capability requirements, and defined by a joint density function $h_{\delta,\sigma}(\delta_i, \sigma_i)$. This density function is defined over the conformity domain V_D shown in Fig.1. The NCR based on APTA is computed as the following:

$$NCR_{APTA} = \int_{V_D} NCR_{|\boldsymbol{\delta},\boldsymbol{\sigma}}(\boldsymbol{\delta}, \boldsymbol{\sigma}) \prod_{i=1}^{n} h_{\delta,\sigma}(\delta_i, \sigma_i) d\delta_i d\sigma_i \quad (5)$$

where $NCR_{|\boldsymbol{\delta},\boldsymbol{\sigma}}(\boldsymbol{\delta}, \boldsymbol{\sigma})$ is the conditioned or the short term Non Conformity Rate. It is dependent only on the standard deviation values $\boldsymbol{\sigma}$ and the mean shifts values $\boldsymbol{\delta}$. When APTA method is applied to systems, the variability of the statistical parameters of the dimensions ($\boldsymbol{\sigma}$,$\boldsymbol{\delta}$) is considered. Therefore it is possible to compute the long term NCR (NCR_{APTA}).

3. Sensitivity Analysis: Sobol' method

The Sensitivity analysis is defined as *"the study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input"*[16] . Consider a numerical model y with input variables represented as a vector $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_n)$ and that the output variable is represented by $y(\boldsymbol{\theta})$. This function represents the relation between the input and the output of the model. The methods used in the sensitivity analysis can be classified as local and global methods. In the local sensitivity methods, a reference point is chosen and then the variation is considered around this point. The local methods are not considered in this work because the reference point may not be representative and the distribution of the input variables cannot be taken into consideration. In the global methods the variation over the whole domain can be studied. The Sobol' method which is considered as a global sensitivity method is used in the work of this paper. It is based on the decomposition of the variables.

Sobol' showed that an integrable function y can be expressed as the sum of different dimensions [17]. In other words y is decomposed in elementary functions as shown in Eq.(6).

$$\begin{aligned} y(\boldsymbol{\theta}) =& y_0 + \sum_{i=1}^{n} y_i(\theta_i) + \sum \sum_{1 \leqslant i \leqslant j \leqslant n} y_{ij}(\theta_i, \theta_j) \\ &+ ...y_{12...n}(\theta_1, \theta_2, \ldots, \theta_n) \end{aligned} \quad (6)$$

where y_0 is a constant, $y_i(\theta_i) = E_{\sim\theta_i}(y \mid \theta_i)$, similar relations are obtained for higher order terms of the decomposition.
Sobol' indices are based on variance decomposition and determine the part of the variance of the output resulting from each variable [17]. They measure the influence of each input parameter and their interactions to the whole model output variance. The ANOVA decomposition is obtained by taking into account that the input parameters θ_i are random and independent [18]. The variance of y can be expressed as in Eq.(7)

$$V = \sum_{i=1}^{n} V_i + \sum \sum_{1 \leqslant i \leqslant j \leqslant n} V_{ij} + ...V_{12...n} \quad (7)$$

where V is the total variance of model output; V_i is the first order contribution of the i^th model parameter: $V_i = V[y_i(\theta_i)]$; V_{ij} is the effect of the interaction of the i^{th} and j^{th} parameters.
Sobol' sensitivity indices are defined as: the first order sensitivity index, also known as the main effect, is the direct impact of the variable on the variable of the output of the model. The equation corresponding to the first order index is given by Eq.(8).

$$S_i = \frac{V_i}{V} \quad (8)$$

If the value of S_i is zero, it can be deduced that the function y does not depend on θ_i. In another hand if S_i is equal to one, it means that y depends only on θ_i.
The second order sensitivity indices gives the sensitivity of the variance of the output to the interaction between the input factors θ_i , θ_j. It is expressed as:

$$S_{ij} = \frac{V_{ij}}{V} \tag{9}$$

If n is the number of the model parameters, $2^n - 1$ indices are to be estimated.
The total Sensitivity index, measures the interaction of each variable with the other variables [19]. All the terms of the decomposition in Eq.(7) where i is involved are used. It is expressed as:

$$S_{Ti} = S_i + \sum_j S_{ij} + \sum_{j,k} S_{ijk} + \dots \tag{10}$$

It can be also expressed as:

$$S_{Ti} = \frac{V_{-i}}{V} \tag{11}$$

Where V_{-i} is the contribution of all except the i^{th} parameter. There is absence of interaction between X_i and other variables, if the value of the total effects is equal to the value of the main effects. The Monte Carlo methods are used for the estimation of the Sobol' indices [19]. One drawback of this method is the need of a high number of samples for a precise approximation of the results.

4. Sensitivity Analysis of the statistical parameters on the Non Conformity Rate

In the scope of this paper, a method to study the sensitivity of the NCR estimation to the uncertainty of the statistical input parameters is presented. The NCR is estimated using the APTA method already discussed in section 2.2. Sobol' indices also presented in section 3 are calculated to measure the influence of the uncertainty in the input parameters on the NCR. To do a sensitivity analysis, the input parameters, the output function and the geometrical model should be defined. The input parameters are the statistical parameters σ_i and δ_i corresponding to the dimensions. The output function is the NCR of the system. The NCR gives an approximation of the number of defect products during the process. In addition, the reduction of the defect products is a main concern in industry to assure a high quality production, which explains the choice of the output of this model. The choice of the input parameters is due to the fact that the production of the products is not perfect, therefore there is a variation in the mean shifts and standard deviation of the dimensions. This variability has non negligible impact on the NCR. The choice of the Sobol' method which is a variance based method is based on the fact that it is a quantitative global method.
As mentioned in section 3, the application of the Sobol' method is only applicable when the input parameters are independent. Values for the mean shifts and the standard deviations are to be associated for each dimension following a normal distribution. It can be deduced that σ_i and δ_i are dependent variables. The dependency in the input parameters imposes to group these parameters together in order to be able to apply the Sobol' method. The formulation of the problem is done by considering the NCR as the function to be decomposed such that:

$$\begin{aligned} NCR(\boldsymbol{\delta}, \boldsymbol{\sigma}) = &\sum_i^n NCR_i(\delta_i, \sigma_i) \\ &+ \sum \sum_{1 \leqslant i \leqslant j \leqslant n} NCR_{ij}(\delta_i\sigma_i, \delta_j\sigma_j) \\ &+ \dots NCR_{12\dots n}(\delta_1\sigma_1, \delta_2\sigma_2, \dots, \delta_n\sigma_n) \end{aligned} \tag{12}$$

Then the variance decomposition in applied as in Eq.(7), and the partial variances are expressed as:

$$V_i = V_{\delta_i\sigma_i}\left[E_{\sim\delta_i\sigma_i}[NCR \mid \delta_i\sigma_i]\right] \tag{13}$$

$$V_{ij} = V_{\delta_i\sigma_i\delta_j\sigma_j}\left[E_{\sim\delta_i\sigma_i,\delta_j\sigma_j}[NCR \mid \delta_i\sigma_i, \delta_j\sigma_j]\right] - V_i - V_j \tag{14}$$

E is the mathematical expectation. Similar relations can be obtained for the higher order terms of the decomposition. Once the total and the partial variances are well defined, the Sobol' indices can be computed. In this work the first and the total order indices are estimated. These indices aggregate the contributions of the dimension to the variability of the NCR. Dimensions with high Sobol' indices need additional attention, as they can be associated with large variations of the NCR.

5. Application Example

5.1. Description of the problem

An example of an electrical plug is shown in Fig.2. A functional requirement is associated with this example, the inclination Y of the electrical plug should be below a threshold value. Y is a function of fourteen parameters which are the dimensions of the parts involved in the dimensional chain. This function is linearised in a way to have the following relationship:

$$Y = f(D_i) \approx a_0 + \sum_0^{14} a_i D_i \tag{15}$$

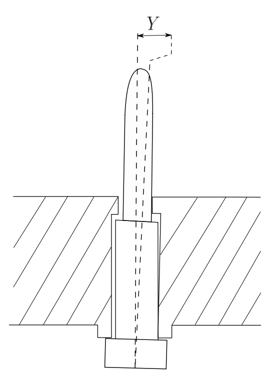

Fig. 2. Electrical Plug

The chain of dimensions associated with this example is set as a critical chain. Following the procedure of the FMECA, the parts involved in this chain are all set as KCs. The objective of this application is to classify the dimensions depending on their influence on the output and to be able to reduce the number of KCs as much as possible. An average value and a standard deviation and a tolerance interval are associated for each dimension. The capability requirements $C_{pi}^{(r)}$ and $C_{pki}^{(r)}$ are also given for each dimension. Having all the data needed as presented in Table 1, the conformity domain where the average values and the standard deviations should lie into is determined, it is a triangular domain since $C_{pi}^{(r)} = C_{pki}^{(r)}$. The input parameters are both the variable standard deviations and the mean shifts values associated for each dimension. The output is the Non-conformity rate estimated by the APTA method already described. The sensitivity method applied is the Sobol' method. The first order and the total order Sobol' indices are computed, they correspond respectively to the main effect of each parameter and to the interaction of each parameter with the others. The strategy adopted to estimate the Sobol' indices is based on Monte Carlo method, and it is detailed in [20]. The number of samples taken is N=30000. The mean shifts and standard deviations cannot be separated. They are grouped during the calculation as discussed in section 4. They are assumed to have a uniform repartition on the triangular domain,because no other information was available. The distribution may be adjusted if data is available. The different steps done during the calculation are summarized in Fig.3.

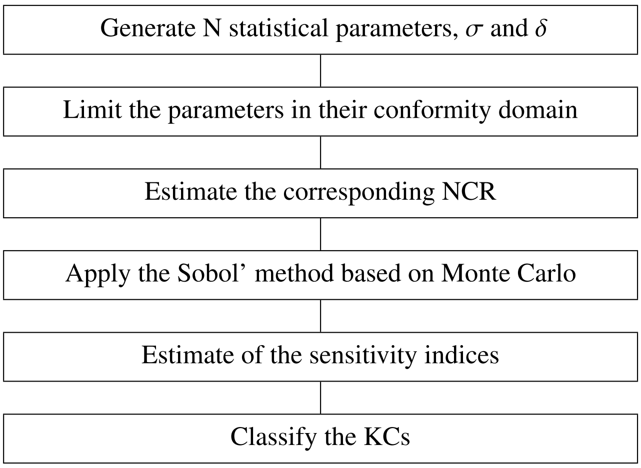

Fig. 3. The different steps of the proposed method

5.2. *Results*

The graph presented in Fig.4 shows that the main effects of the mean shifts and standard deviation of the dimensions D2 and D5 are the most important. In addition, it is noticed that the total indices associated with the same dimensions have a high value comparing to others. It can be concluded that these two dimensions have a major importance in the dimensional chain and should be considered as Key-Characteristics. The effects of other dimensions such as D1, D7, D8 D9 and D10 are almost zero so they can be neglected. The remaining dimensions with intermediate sensitivity values cannot be classified as KCs or non-influencing parameters. The decision on how to treat these dimensions remains outside of the scope of this paper.

Table 1. Input Data.

Dimensions	a_i	T_i	t_i	$C_p^{(r)} = C_{pk}^{(r)}$
D1	-0.04	10.53	0.2	1.1
D2	-0.5	0.1	0.2	1.1
D3	-0.5	0	0.06	1.1
D4	1.14	0.643	0.015	1.1
D5	0.91	0	0.06	1.1
D6	0.91	0.72	0.04	1.1
D7	1.10^{-4}	1.325	0.05	1.1
D8	0.05	0.75	0.04	1.1
D9	1.10^{-4}	0	0.04	1.1
D10	0.13	3.02	0.06	0.86
D11	-1.4	0.72	0.04	0.86
D12	-1.15	0	0.04	0.86
D13	-0.9	0.97	0.04	0.86
D14	0.13	0.4	0.06	0.86

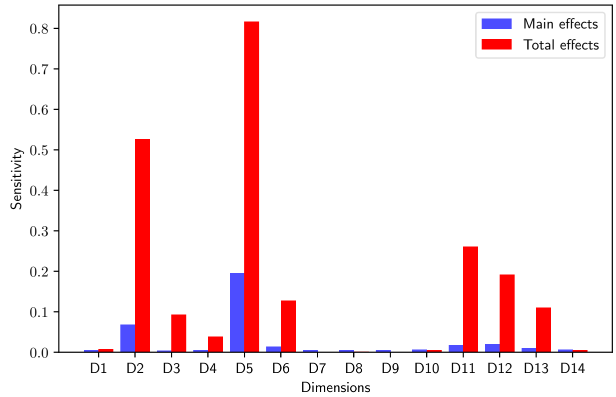

Fig. 4. First and Total order Sensitivity indices

6. Conclusions

This paper presents a new methodology that can be used for the identification of the Key Characteristics by combining the Sobol' sensitivity analysis method with the APTA method. Both methods are described in this paper. An application example with the case of a linear functional requirement is also presented. In this example, the effects of the variation of the mean shifts and standard deviations of each dimension on the NCR are measured. The results show that the variability of the statistical parameters of some dimensions have an important impact on the non conformity rates. It can be concluded that the corresponding dimensions should be manufactured with special attention.

Based on the FMECA method, the parts involved in a critical dimensional chain are all set as KCs. The application of the proposed methodology allows the reduction of the number of parts considered as KCs and consequently the efforts dedicated to checking them.

The sensitivity analysis is done on two levels of uncertainty, the uncertainty in the production batches and on their statistical parameters.

The proposed method could be developed in later steps for the identification of the Key Characteristics for more complex systems such as non linear and over-constrained systems. In addition, the considered variables are assumed to have normal distributions. It could be interesting to see the behaviour of the method when dealing with different types of distributions.

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References

[1] Thornton, A.C.. Mathematical framework for the key characteristic process. Research in Engineering Design - Theory, Applications, and Concurrent Engineering 1999;11(3):145–157.
[2] Space product assurance-Failure modes, effects (and criticality) analysis (FMEA/FMECA). NF EN 16602-30-02 2015;.
[3] Serin, C., Hassan, A., Dantan, J.Y., Siadat, A., Serin, C., Hassan, A., et al. Développement d ' un outil pour l 'identification des caractéristiques clés à inspecter. 2015,.
[4] Mirmadi, S., Dantan, J.Y., Martin, L.. Système d'aide à la décision pour la génération des processus d'inspection par la fédération des expertises métier. Ph.D. thesis; Laboratoire de Conception Fabrication Commande; Metz, France; 2014.
[5] Thornton, A.C.. Variation Risk Management, Focusing Quality Improvements in Product Development and Production. Hoboken, New Jersey: John Wiley & Sons; 2004.
[6] Ziegler, P., Wartzack, S.. A statistical method to identify main contributing tolerances in assemblability studies based on convex hull techniques. Journal of Zhejiang University: Science A 2015;16(5):361–370.
[7] Ziegler, P., Wartzack, S.. Sensitivity Analysis of features in tolerancing based on constraint function level sets. Reliability Engineering and System Safety 2015;134:324–333.
[8] Beaucaire, P., Gayton, N., Duc, E., Dantan, J.Y.. Statistical tolerance analysis of over-constrained mechanisms with gaps using system reliability methods 2012;15:29–41.
[9] Nigam, S.D., Turner, J.U.. Review of statistical approaches to tolerance analysis. Computer-Aided Design 1995;27(1):6–15.
[10] Glancy, C.G., Chase, K.W.. A Second-Order Method for Assembly Tolerance Analysis. Proceedings of the 1999 ASME Design Engineering Technical Conferences 1999;(1):1–8.
[11] Kane V, . Process Capability Indices_1986. Journal of Quality Technology 1986;18:41–52.
[12] Statistical methods in process management-Capability and performance-Part 1: general principles and concepts. NF ISO 22514-1 2014;.
[13] Scholz, F.. Tolerance Stack Analysis Methods A Critical Review 1995;.
[14] Greenwood, W.H., Chase, K.W.. A New Tolerance Analysis Method for Designers and Manufacturers. Journal of Engineering for Industry 1987;109(2):112–116.
[15] Gayton, N., Beaucaire, P., Bourinet, J.M., Duc, E., Lemaire, M., Gauvrit, L.. APTA: Advanced probability-based Tolerance Analysis of products. Mecanique et Industries 2011;12(2):71–85.
[16] A. Saltelli, S. Tarantola, F. Camplongo, M.R.. Sensitivity Analysis in Practice, A Guide to Assessing Scientific Models. John Wiley & Sons; 2004. ISBN 0470870931.
[17] Sobol', M.. Sensitivity Estimates for Nonlinear Mathematical Models. Mathematical Modeling and Computational experiment 1993;1(4):407–414.
[18] Efron, B., Stein, C.. The Jackknife Estimate of Variance. Ann Statist 1981;9(3):586–596.
[19] Homma, T., Saltelli, A.. Importance measures in global sensitivity analysis of nonlinear models. Reliability Engineering & System Safety 1996;52:1–17.
[20] Saltelli, A., Ratto, M., Tarantola, S., Campolongo, F.. Sensitivity analysis practices: Strategies for model-based inference. Reliability Engineering & System Safety 2006;91(10):1109–1125.