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Statistical Tolerance Analysis Technique for Over-constrained Mechanical Systems

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Abstract

The aim of this paper is to provide an overview of a developed statistical tolerance analysis technique for over-constrained mechanism. Statistical tolerance analysis is a more practical and economical way of looking at tolerances and works on setting the tolerances so as to assure a desired functionality; however, most of statistical tolerance analysis techniques are dedicated to isoconstrained mechanisms and simple over-constrained mechanisms, they need an explicit assembly response function. The developed technique is based on:

- the formalization of the geometrical behavior of the mechanism by an implicit assembly response function,
- the formalization of the tolerancing requirement by the quantifier notion,
- the coupling of some optimization techniques and reliability techniques.

Keywords: Tolerance analysis, Statistics, Overview

1. Introduction

The aim of tolerance analysis is to study the influence of features geometric variations on the behavior of a mechanical system. It consists to check that geometric tolerances of the components ensure the compliance of a mechanical system in terms of functional requirements. It is primordial to integrate the tolerance analysis process in all stages of product lifecycle in order to improve product quality, to reduce its cost and to ensure its reliability [1]. Generally, tolerance analysis can be done using deterministic or statistical methods. Most of mechanical systems are considered as over-constrained however most of statistical approaches are dedicated for systems which are iso-constrained or basically over-constrained [5, 6]. An over-constrained mechanism is an assembly which has more than six degrees of freedom fixed in three dimensions [2]. In such mechanism, the response function of the tolerance analysis is not explicit anymore [3] and the assembly problems occur [4]. Generally, the tolerance analysis of the over-constrained mechanisms is based on: geometrical models to represent deviations due to the manufacturing process and to characterize gaps between parts potentially in contact, behavior models in order to simulate the behavior of the over-constrained mechanisms considering the deviations, the defects and the gaps and finally mathematical techniques to assess the reliability level. The statistical method should be adapted considering these three main issues that is the goal of this paper. The main contribution of the paper is to give an efficient statistical tolerance analysis approach combining the quantifier notion, constrained optimization and reliability (FORM ...) techniques.

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In the first part, an overview of the formalization of the geometrical behavior models is given; the second part deals with the formalization of the tolerancing requirement by introducing the quantifier notion. Some optimization and reliability techniques are then detailed. An example of the tolerance analysis of an over-constrained mechanism is described at the end of the article.

2. Geometrical behavior modeling

The geometric features of a manufactured part will never be exactly the same as the nominal characteristics of which they are a specific physical realization. Geometric defects limits in a part are defined by geometric constraints. Real surfaces, derived from manufacturing process, are modeled by substitution surfaces [7]. A substitution surface is an ideal surface that illustrates a specific physical representation of the nominal surface. Furthermore, the substituted-surface representation is used to simulate the geometric deviations of a real surface. The geometric deviations can be mathematically represented with the help of the vectors [8], by a small displacement torsor SDT [9], by a matrix [10] or by the Skin model Shape concept [11]. In the case of a penitential contact between two surfaces, the deviations can be formalized based on the SDT concept.

The geometrical behavior modeling of a mechanical system consists on the determination of how its features interact. This behavior modeling is defined by displacements' accumulation and can be written by mathematical formulations. These relations depend on the topological structure of the system. This structure is identified by a joint graph that includes the features, their links, the gaps and the functional conditions [12]. For example, let an assembly made by the interaction between two parts 1 and 2 at any surface a. the interaction can be expressed mathematically by:

$$\{\mathbf{d}_{1/2}\} = \{\mathbf{d}_{1/1a}\} + \{\mathbf{g}_{1a/2a}\} + \{\mathbf{d}_{2a/2}\} \quad (1)$$

Where Here $\{\mathbf{d}_{1/1a}\}$ and $\{\mathbf{d}_{2a/2}\}$ are the deviation torsors associated with parts 1 and 2 at the surface 'a' and $\{\mathbf{g}_{1a/2a}\}$ is the gap torsor illustrating the gap between the two parts nominally in contact.

In this section, we focus on the mathematical formulation of a mechanical system's behavior and that by defining some sets of constraints on parameters (deviations, dimensions, gaps, conditions). Different constraints can be thus defined:

- Compatibility constraints: linear equalities that constraint parts deviation $\mathbf{x} = (x_1, \cdots, x_n)$ and gaps $\mathbf{g} = (g_1, \cdots, g_n)$. The equalities derive from the different combinations of admissible small displacements of surfaces of parts in the different topological loops in the graph. Therefore, the definition of the compatibility constraints is based on the SDT concept. Globally, the compatibility constraints verify $C_c(\mathbf{x}, \mathbf{g}) = 0$,
- Interface constraints: inequalities that characterize the non-interferences between substituted surfaces which are nominally in contact. They limit the gaps between two surfaces. In the case of floating contact, the relative positions of substitute surfaces are constrained technologically by the non-interference. In the case of slipping and fixed contact, the relative positions of substitute surfaces are constrained technologically in a given configuration by a mechanical action [15]. Generally, the interface constraints are expressed as: $C_i(\mathbf{x}, \mathbf{g}) \leq 0$, the interface constraints should be linearized if are not linear. A method for linearization of quadratic interface constraints is proposed in [2].
- Functional constraints: inequalities which constraint functional requirements, i.e. they limit the orientation and the location displacements between surfaces, which are in functional relation. The constraints can be written as: $C_f(\mathbf{x}, \mathbf{g}) \leq 0$. Most of joints have functional gap. These gaps induce displacements between parts. Each relative position defines a configuration of the joint. A configuration is a particular relative position of parts of an assembly depending of gaps without interference between parts. The behavior of a mechanism with gaps is controlled through at least one identified functional characteristic,

The formulations of the geometrical behavior modeling of any mechanical system are required for the tolerance analysis of over-constrained mechanisms via statistical techniques. The aim of displacement accumulation is to simulate the influences of deviations and gaps on the geometrical behavior of the mechanical system. In this case, the assembly response function constrains the geometrical relationships between the assembly resultants and the manufactured variables. The explicit assembly function can has the form:

$$Y = f(\mathbf{x}, \mathbf{g}) \quad (2)$$

Where Y denotes the output assembly response (characteristic such as gap or functional characteristics) of the assembly. The function f could be an explicit analytic expression or an implicit analytic expression. In a particular relative configuration of parts of an assembly consisting of gaps without interference between parts, the composition relations of displacements in some loops of the assembly graph permits to determine the function f. For an over-constrained mechanism, the determination of an explicit function is no longer possible. The formulation is constrained on deviations $\mathbf{x}$, gaps $\mathbf{g}$ and also tolerances in the parts.

3. Formalization of the tolerancing requirements based on quantifier notion

The major functional requirements and technical or assembly requirements allow determining the geometrical product requirements which are geometrical constraints between product's surfaces. These requirements must be respected in a tolerance analysis procedure. In this section, the quantifier notion is used to give a new formalization of the requirements expressed by constraints as shown below.

The quantifiers $\forall$ "for all" and $\exists$ "there exists" provide a univocal expression of the conditions corresponding to the geometrical product requirements. Dantan et al. [13] provided a new formalization of the geometrical requirements (assembly and functional).

- The mathematical formulation of a functional requirement is: *"for all admissible gap configurations of the mechanism and for all acceptable deviations, there exists a functional characteristic such that the geometrical behavior and the functional requirements are respected".*

$$\forall \mathbf{g} \in \left\{ \mathbf{g} = \left(g_1, \cdots, g_n\right) \in \mathbf{R}^n : C_c\left(\mathbf{x},\mathbf{g}\right) = 0 \cap C_i\left(\mathbf{x},\mathbf{g}\right) \leq 0 \right\};$$
$$C_f\left(\mathbf{x},\mathbf{g}\right) \geq 0 \tag{3}$$

- The mathematical formulation of an assembly requirement is: "*for all admissible geometrical deviations, there exists a gap configuration such that the assembly requirement modeled by interface constraints and the compatibility constraints are verified*".

$$\exists \mathbf{g} \in \left\{ \mathbf{g} = \left(g_1, \cdots, g_n\right) \in \mathbf{R}^n : C_c\left(\mathbf{x},\mathbf{g}\right) = 0 \cap C_i\left(\mathbf{x},\mathbf{g}\right) \leq 0 \right\} \tag{4}$$

In the case of the quantifier $\exists$, if there exists one configuration of the mechanism such as the value of the functional characteristic is less than or equal to the tolerance, then the geometrical product requirement is respected. In the case of the quantifier $\forall$, if for all configurations of the mechanism, the value of the functional characteristic is less than or equal to the tolerance, then the geometrical product requirement is respected.

The introduction of the quantifier notion in the formalization of the tolerancing requirements allows making more consistent the statistical tolerance analysis approach which is based on optimization techniques and reliability techniques. The statistical tolerance analysis is able to compute, based on the formulation of requirements by quantifiers, the probability of assembly and the probability of functionality or the failure of the product.

The mathematical formulation of the assembly requirement results in optimization: minimizing an interface constraint subject to all the constraints characterizing the behavior model including the interface constraint to be minimized. The objective function is not too important, only the existence or not of a solution matters. A solution means that there is a configuration of the gaps such that the constraints are respected:

$$\begin{aligned} &\min_{\mathbf{g}} \quad C_i^1\left(\mathbf{x},\mathbf{g}\right) \leq 0 \\ &\text{subject to} \quad C_c^l\left(\mathbf{x},\mathbf{g}\right) = 0,\ l \in \left\{1,\cdots,N_c\right\} \\ &\qquad\qquad\quad C_i^k\left(\mathbf{x},\mathbf{g}\right) \leq 0,\ k \in \left\{1,\cdots,N_i\right\} \end{aligned} \tag{5}$$

Where N_c is the number of compatibility constraints and N_i is the number of interface constraints.

Based on this definition, the probability of failure can be expressed as:

$$P_{failure} = P\begin{pmatrix} \nexists \max\limits_{\mathbf{g}} \quad \sum\limits_{\mathbf{g}} \mathbf{g} \\ C_c^l\left(\mathbf{x},\mathbf{g}\right) = 0,\ l \in \left\{1,\cdots,N_c\right\} \\ C_i^k\left(\mathbf{x},\mathbf{g}\right) \leq 0,\ k \in \left\{1,\cdots,N_i\right\} \end{pmatrix} \tag{6}$$

For the functional requirement, it is not possible to consider all the admissible configurations of the mechanism and, for all, to check whether the functional condition is respected or not. It is better to find the configuration of the mechanism that provides the worst value of the functional characteristic. If this worst value of the functional characteristic indicates that the mechanism is functional then it will be for all other admissible configurations. The formulation of the constrained optimization problem is given therefore by:

$$\begin{aligned} &\min_{\mathbf{g}} \quad C_f\left(\mathbf{x},\mathbf{g}\right) \leq 0 \\ &\text{subject to} \quad C_c^l\left(\mathbf{x},\mathbf{g}\right) = 0,\ l \in \left\{1,\cdots,N_c\right\} \\ &\qquad\qquad\quad C_i^k\left(\mathbf{x},\mathbf{g}\right) \leq 0,\ k \in \left\{1,\cdots,N_i\right\} \end{aligned} \tag{7}$$

The probability of non-functionality of a mechanism can be then expressed as:

$$P_{nfonctionnal} = P\begin{pmatrix} \min\limits_{\mathbf{g}} \quad C_f\left(\mathbf{x},\mathbf{g}\right) \leq 0 \\ \text{with } C_c^l\left(\mathbf{x},\mathbf{g}\right) = 0,\ l \in \left\{1,\cdots,N_c\right\} \\ C_i^k\left(\mathbf{x},\mathbf{g}\right) \leq 0,\ k \in \left\{1,\cdots,N_i\right\} \end{pmatrix} \tag{8}$$

4. Statistical tolerance analysis

The statistical tolerance analysis allows to study the influence of the geometrical deviations and the gaps on the assemblability of the mechanical system and its function. The probability of defect and the probability of non-assembly are generally assessed. We propose in this section to review the mathematical expressions of the two probabilities, based on the constraints formulation and the quantifier notions. The idea is to study the global behavior of the system affected by deviations. Two methods are proposed: (i) a Monte Carlo simulation combined with optimization that the main objective is to find the worst configuration of the functional requirement or to find a gap configuration that satisfy all constraints modeling the behavior, (ii) a reliability technique such the First Order Reliability Method (FORM).

4.1 Optimization technique

The main objective of the optimization technique is to determine the components of the clearance torsor associated to the position between surfaces (3 translations x 3 rotations) potentially in contact and in order to avoid interferences. If the components are found, the best configuration for the assembly is thus found and thereafter the functioning requirements can be checked. The objective function of this optimization is composed of the components of the clearance torsors subjected to the respect of the compatibility equations and the interface constraints. The optimization problem is described in the following equation:

$$\begin{array}{ll} \min\limits_{\mathbf{g}} & f(\mathbf{g}) \\ \text{subject to} & C_c(\mathbf{x},\mathbf{g}) = 0 \\ & C_i(\mathbf{x},\mathbf{g}) \le 0 \end{array} \tag{9}$$

The mathematical formulations of the probabilities of non-assemblability and non-functionality can be adapted for a simulation method such as Monte Carlo. One technique coupling Monte Carlo simulation and an optimization algorithm has been previously proposed by Dantan [15]. All constraints are therefore linearized in order to make resolvable the optimization problem. Therefore, a linear optimization problem such a simplex algorithm could be used. The impact of linearization on the accuracy of the results was discussed in [14].
Let N_{MC} the number of the realizations of the variables randomly generated. The optimization algorithm makes it possible to determine whether the mechanism is functional or not for all iterations. Globally, N_{MC} samples of optimization are considered. The probability of failure is estimated through the use of the indicator function:

$$\begin{gathered} \tilde{P}_{df} = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} 1_{D_f}(\mathbf{x}_i) \\ 1_{D_f}(\mathbf{x}_i) = \begin{cases} 1 \text{ if (7) is satisfaid} \\ \quad 0 \text{ if not} \end{cases} \end{gathered} \tag{10}$$

The method is discussed in the case study in the following section.

4.2 Reliability technique

Every constrained optimization problem can be transformed to a simple optimization problem without constraints. This transformation requires the definition of the Lagrangian formalism. The mathematical definition according to equation (7) is expressed as:

$$L(\mathbf{x},\mathbf{g},\boldsymbol{\lambda}) = C_f(\mathbf{x},\mathbf{g}) + \sum_{k=1}^{N_i} \lambda_k C_i^k(\mathbf{x},\mathbf{g}) \tag{11}$$

Where $\boldsymbol{\lambda} = (\lambda_1, \cdots, \lambda_{N_i})$ are the Lagrange multipliers. $(\mathbf{x}^*, \mathbf{g}^*)$ is a solution of the optimization problem if and only if $(\mathbf{x}^*, \mathbf{g}^*)$ is a saddle point or minimax point.
Let the following linear program modelling a tolerance analysis problem and expressed as matrix form:

$$\begin{gathered} \min_{\mathbf{g}\in\mathbb{R}^n} C_f(\mathbf{x},\mathbf{g}) = \mathbf{a}^T\mathbf{x} + \mathbf{b}^T\mathbf{g} + c_0 \\ \text{with } C_i^k(\mathbf{x},\mathbf{g}) = d_k^T\mathbf{x} - e_k^T\mathbf{g} + c_k \le 0 \end{gathered} \tag{12}$$

For all $k = (1, \cdots, N_i)$, the Lagrangian is defined according to the equation:

$$L(\mathbf{x},\mathbf{g},\boldsymbol{\lambda}) = \left(\mathbf{b} - \sum_{k=1}^{N_i} \lambda_k e_k\right)^T \mathbf{g} + \sum_{k=1}^{N_i} \lambda_k \left(d_k^T \mathbf{x} + c_k\right) + \mathbf{a}^T\mathbf{x} + c_0$$

The dual of the optimization problem can be written by:

$$\begin{gathered} \max_{\lambda\in\mathbb{R}^{N_i}} C_{f,dual}(\mathbf{x},\boldsymbol{\lambda}) = \left(\mathbf{d}^T\mathbf{x} + c\right)^T \boldsymbol{\lambda} + \mathbf{a}^T\mathbf{x} + c_0 \\ \text{with } \sum_{k=1}^{N_i} \lambda_k e_k = \mathbf{b} \\ \boldsymbol{\lambda} \ge 0 \end{gathered} \tag{13}$$

The dual form is interesting because the objective-function is no longer a function of gaps. The optimization constraints are now made up of n equalities to be satisfied and the Lagrange multipliers to be positive. The number of N_i interface constraints being very large compared to the n dimension of $\mathbf{g}$, the dual problem of optimization is under-constrained: there are more Lagrange multipliers λ than equality constraints. In order to find the admissible values of the Lagrange multipliers satisfying the constraints, all possible combinations must be tested. There are then N_{as} possible solutions to the problem of maximization which can be written with accordance to the equation:

$$C^{*}_{f,dual}\left(\mathbf{x}\right)=\max\left(C^{s_1}_{f,dual}\left(\mathbf{x}\right),\cdots,C^{s_{N_{as}}}_{f,dual}\left(\mathbf{x}\right)\right) \tag{14}$$

The two optimization problems admit the same solution: $C^{*}_{f,dual}\left(\mathbf{x}\right)=C^{*}_{f}\left(\mathbf{x}\right)$. The main goal is to determine the probability of the functional failure (non-functionality). This probability can be finally written as follows and the assessed by FORM method:

$$\begin{aligned} P_{df} &= \mathrm{P}\left(\max\left(C^{s_1}_{f}\left(\mathbf{x}\right),\cdots,C^{s_{N_{as}}}_{f}\left(\mathbf{x}\right)\right)\leq 0\right) \\ &= P\left(\left\{C^{s_1}_{f}\left(\mathbf{x}\right)\leq 0\right\}\cap\cdots\cap\left\{C^{s_{N_{as}}}_{f}\left(\mathbf{x}\right)\leq 0\right\}\right) \\ &= P\left(\bigcap_{i=1}^{N_{as}} C^{s_i}_{f}\left(\mathbf{x}\right)\leq 0\right) \end{aligned} \tag{15}$$

5. Case study : 3D tolerance analysis of a pump

This application is to assess the functionality of a variable flow pump from Pierburg Pump Technology France. The mechanism is composed of a frame (1), a control disc (2) and nine fins. The control disk is used to manage the opening of the fins. Indeed, the disc can rotate around its axis, the rotation causes a movement of the fins with a small lug located on the lower part of the fin.

The functional requirement of the pump relates to the relative position of the fins between them, and this regardless of the position of the control disk. Since the actual expression of this requirement is confidential, it is assumed to be a function of the relative angle of the fins between them. To simplify the study, only two fins are considered in the mechanism as well as a single adjustment of the control disk. The figure 1 and figure 2 illustrate the studied mechanism. In the figure 3, the assembly graph is given.

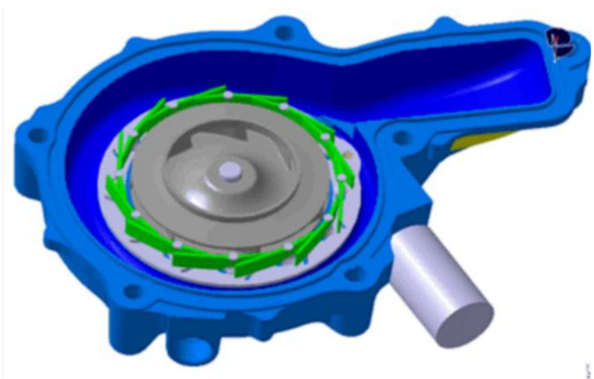

Fig. 1. Studied mechanism

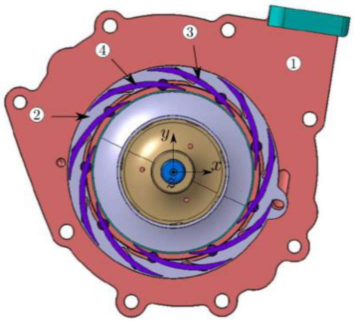

Fig. 2. CAD model of the pump

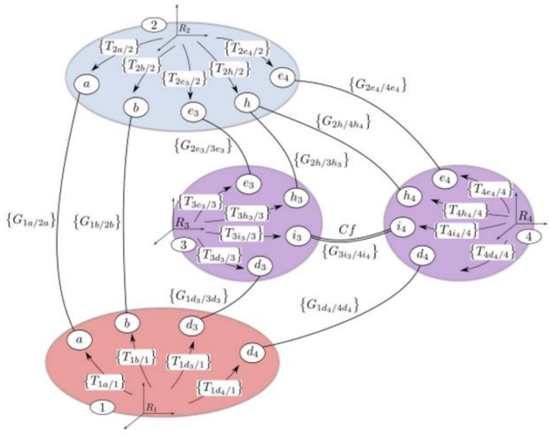

Fig. 3. Graph of the assembly

According to the previous graph, the functional condition is explained by the functional clearance torsor as:

$$\left\{\mathbf{g}_f\right\}=\left\{\begin{matrix} - & - \\ - & - \\ \gamma_{3i4i} & - \end{matrix}\right\} \tag{16}$$

To ensure the functionality of the pump, the feature γ_{3i4i} must be within an interval: $\gamma_{3i4i}\in\left[-r,r\right]$. two functional requirements are derived from equation 6:

$$\begin{aligned} C^{1}_{f}\left(\mathbf{x},\mathbf{g}\right) &= r-\gamma_{3i4i}\geq 0 \\ C^{2}_{f}\left(\mathbf{x},\mathbf{g}\right) &= \gamma_{3i4i}-r\geq 0 \end{aligned} \tag{17}$$

Based on the graph of assembly, the compatibility equations and the interface constraints are defined. Globally, there are 8 joints defined in the mechanism. 30 compatibility equations are then developed regarding the topological loops in the graph. The interface constraints are also developed based on the type of joints.

Considering the loop: 2h (part 2, surface h)/3h/3i/4i/4h/2h and equation 1, the compatibility equation including the functional characteristic is defined by:

$$\begin{aligned}\{\mathbf{d}_{1/1}\} &= \{\mathbf{g}_{2h/3h}\} + \{\mathbf{d}_{3h/3}\} + \{\mathbf{d}_{3/3i}\} + \{\mathbf{d}_{3i/4i}\} \\ &+ \{\mathbf{d}_{4i/4}\} + \{\mathbf{d}_{4/4h}\} + \{\mathbf{g}_{4h/2h}\} \\ &= \{0\}\end{aligned} \tag{18}$$

Where for example: $\{\mathbf{g}_{2h3h}\} = \begin{Bmatrix} - & u_{2h3h} \\ - & v_{2h3h} \\ \gamma_{2h3h} & - \end{Bmatrix}$ and

$$\{\mathbf{d}_{3h/3}\} = \begin{Bmatrix} \alpha_{3h3} & - \\ \beta_{3h3} & - \\ - & w_{3h3} \end{Bmatrix}.$$

The results of the application of the proposed statistical tolerance method on the pump are given in the following table.

Table 1. Comparison of the methods

Method	**FORM system**	**MC & Optimization**	**Commercial software**
Rate of non-compliance	84 ppm	83 ppm	237 ppm
Confidence interval (95%)		2,9 ppm	
Number of simulations	9341	10^7	
Computing time	**10min.**	**5 days**	

The results show that the method FORM system makes it possible to obtain a very close estimation of the probability comparing to that obtained by the reference method Monte Carlo combined with optimization (MC & Optimization). The computing time of the FORM system method is considerably reduced, about ten minutes, while it takes more than five days with the Monte Carlo method.

6. Conclusion

The paper deals with a statistical tolerance analysis of over-constrained mechanisms. A formulation of the behavior of a mechanical system is given; the behavior is modeled by a set of constraints derived from the different topological loops in the graph defining the structure of the system. A formulation of the tolerancing requirements in the mechanism is then discussed. This formulation allows the computation of the probability of non-assembly and the probability of non-functionality of a designed product. Some techniques are used such us optimization technique and reliability technique which are finally coupled for the probability of failure assessment. Comparing the accuracy of the results and the computing time of the techniques is the main goal of the application section.

To summarize, a global approach modeling the geometrical behavior of over-constrained mechanisms using constraints and deviations and gaps parameters and providing the probability of failure using efficient methods is here proposed.

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