Available online at www.sciencedirect.com

ScienceDirect

Procedia CIRP 75 (2018) 27–32

www.elsevier.com/locate/procedia

15th CIRP Conference on Computer Aided Tolerancing – CIRP CAT 2018

# Robust-Design-Optimization of mechanisms based on kinematic requirements considering uncertainties

Björn Heling[a,*], Benjamin Schleich[a], Sandro Wartzack[a]

[a]*Engineering Design, Friedrich-Alexander-University Erlangen-Nürnberg (FAU), Martensstr. 9, 91058 Erlangen, Germany*

* Corresponding author. Tel.: +49-9131-85-23220; fax:+49-9131-85-23223.*E-mail address:* heling@mfk.fau.de

**Abstract**

Today the market environments require products to be developed by multidisciplinary teams, which have to consider contradictory requirements in drastically reduced times-to-market. In this context, the importance of robust design ambitions, which strive for developing products that are insensitive to variations from manufacturing, assembly and use processes, increases. In this paper, a novel methodology for the robust-design optimization of a mechanism is introduced, which allows optimizing the mechanism so that contradictory requirements are met despite the presence of geometrical deviations and uncertainties during use. The result of this optimization process are mechanisms, which do not only fulfill kinematic requirements, but are also insensitive to geometrical part deviations and uncertain ambient parameters.


Peer-review under responsibility of the Scientific Committee of the 15th CIRP Conference on Computer Aided Tolerancing - CIRP CAT 2018.

*Keywords:*Tolerance-CostOptimization; Skin Model Shapes; Process-oriented Tolerancing

## 1. Introduction

Product developers are nowadays confronted with various challenges. Complex products that are manufactured and assembled worldwide, should fulfill contradictory demands under different operating conditions, while the product development times are constantly shortened. To remain competitive at international markets, the approach “right-first-time” gains in importance for modern companies.

As a consequence, the need for high quality products and processes, which enable higher customer satisfaction, increases. Grouped under the heading “Robust Design Methodology” (RDM), different principles, tools, methods, and metrics have been established in the research as well as in the industrial sector [1,2].

All these RDM-approaches have in common that they support engineers in designing processes or products, which are insensitive to varying influencing factors. Although there are several different approaches, only a few approaches deal with the behavior of mechanisms [3].

Designing robust mechanisms at an early design stage is, however, essential if part-tolerances should not be strictly tightened at a later stage – which would inevitably increase the manufacturing costs [4].

In this contribution, a new methodology for parameter optimization for mechanisms is introduced. This methodology is based on the idea that the deviations of mechanisms are influenced during its whole life cycle. As shown in Fig. 1, all these influencing factors go along with uncertainties, which have to be considered during the development of products. Therefore, the presented methodology considers these uncertainties and their interactions so that robust products can be designed at an early stage.

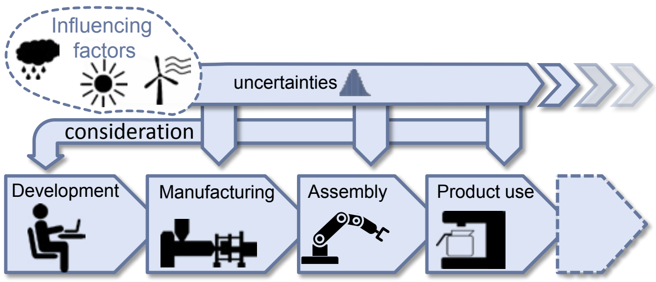


Fig. 1. Influences on part deviations

2212-8271 © 2018 The Authors. Published by Elsevier B.V.
Peer-review under responsibility of the Scientific Committee of the 15th CIRP Conference on Computer Aided Tolerancing - CIRP CAT 2018.
10.1016/j.procir.2018.04.048

## 2. State of the art

In this section, the fundamentals of Robust Design and Kinematic Design are briefly described. Furthermore, the tolerancing of a mechanism and the connection of Robust Design and Kinematic Design is outlined. Besides that, the principles of the particle swarm optimization, used for the parameter optimization, are provided.

### *2.1. Taguchi's Robust Design*

TAGUCHI calls a system "robust" if it is insensitive towards scattering influencing factors. Geometric dimensions, different operating temperatures or varying manufacturing process parameters could be such influencing factors. In Fig. 2, the connection of scattering input- and output-variables is shown. According to TAGUCHI, system A is more robust since even a broader spread width of the influencing variable leads to a smaller scatter of the output variable [5].

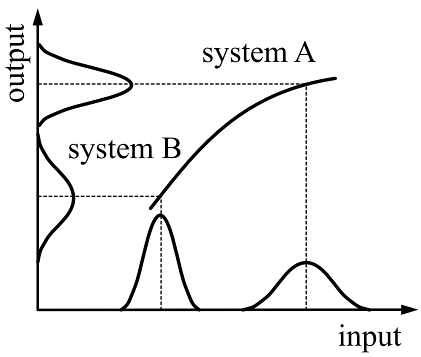


Fig. 2. Assessment of the Robustness according to [5]

To achieve a design, which is insensitive to scattering input variables, Taguchi introduced in [5] a three-step approach:

1.) Systems design
The kind of system and its components are outlined at the conceptual level.
2.) Parameter design
Nominal values of dimensions are defined.
3.) Tolerance design
Permitted deviations from the nominal values are restricted by tolerances.

For all these steps, there are different methods, which can be used to design a more robust mechanism. In [6] significant principles and their advantages respectively disadvantages are discussed.

### *2.2. Kinematic Design*

According to MYSZKA [7], the system design of a mechanism is one of the most important tasks of the design process, because the fundamental properties of a system are defined during this process. One of the most important properties of a mechanism, which has to be considered from the start, is its mobility, respectively its degree of freedom. The number of independent inputs that are required to position all links with respect to a reference is called the degree of freedom. For a planar mechanism, which we will present as a case study in this paper, the degree of freedom can be calculated with the GRUEBLER equation:

$$M = 3(n - 1) - 2j_p - j_h \quad (1)$$

Therefore, the total number of links (n), the total number of primary joints ($j_p$) and the total number of higher-order joints ($j_h$) has to be considered. While primary joints, such as pin joints, restrict two degrees of freedom, higher order joints, like cam joints, only restrict one degree of freedom. Depending on the movability (M), a mechanism can be classified into three groups:

$M>0$ under-constrained
$M=0$ fully constrained
$M<0$ over-constrained

To avoid jamming and ensure that a mechanism is able to move without additional forces, a mechanism with one degree of freedom should be pursued [7]. Nevertheless, calculating the movability has one major disadvantage. The dimensions of each link are not considered. While a mechanism may have one degree of freedom, according to the GRUEBLER equation, it may jam nevertheless due to deviations from ideal part dimensions.

### *2.3. Tolerance analysis and synthesis for mechanisms*

The motion behavior of a mechanism is significantly influenced by the geometric deviations of its components. Therefore, different authors have dealt with the effects of geometry deviations on mechanisms.

In the context of tolerance analyses, random [8] and systematic part deviations [9] as well as their overlapping [10] were examined. Moreover, the consideration of form defects in the motion tolerancing of a mechanism is enjoying increasing attention in tolerancing research [8,11–13].

While different forms of tolerance representation are used in the area of tolerance analysis [11,14–16], vector models are predominantly used in tolerance synthesis and tolerance cost optimization due to the shorter calculation time [10].

### *2.4. Linkage of Robust and Kinematic Design*

LI and WU picked up the idea that parameters as well as their tolerances should be considered simultaneously. In [17] they introduced a single stage approach for parameters and tolerances optimization.

SÖDERBERG also deals in [18] with the connection of Robust Design and tolerancing. He introduces a two-step procedure, which increases the geometrical assembly robustness. The goal of the first step is the minimization of parameters, which influence a certain characteristic. In the second step, tolerances are allocated, keeping the general robustness in mind.

GÖHLER also worked in a similar field and focuses in [19] on the connection of functional requirements and design parameters. Based on this connection, optimal tolerances can be allocated.

Nevertheless, the above-mentioned approaches cannot easily be adapted to kinematic systems since they do not allow the consideration of time-depending properties.

In [3] a classification of different Robust Design methods and a classification between "new" and "existing" RDMs is shown. The authors point out that the main focus of current methods is the analysis in contrast to synthesis. Furthermore, they show the possibilities and the application of Robust Design principles whereas their focal point lies exclusively on the mobility of mechanisms. Consequently, time-depended effects are not considered at all.

In [20] a basic concept of parameter optimization of a mechanism was presented by considering uncertainties. Unlike the methodology introduced in the following, no environmental influences can be considered.

## 3. Need for action

In summary, it can be stated that there are promising approaches that combine Robust with Kinematic Design. Nevertheless, one important question remains: How should nominal dimensions of a mechanism be determined so that kinematic requirements can be guaranteed, while every dimension is subject to deviations?

In the following, a novel methodology for the Robust-Design-Optimization of a mechanism is introduced. Here, a mechanism is designed in such a manner that it is insensitive to varying noise factors, while partly contradictory kinematic requirements can be fulfilled.

## 4. Methodology for Robust-Design-Optimization of a mechanism

The methodology, which is shown in Fig. 3, is based on a concept of a mechanism. This means that, for example, the number of links and the kind of joints are already defined, but no detailed engineering has taken place so far. To clarify the system requirements, the goal of the optimization process as well as kinematic boundary conditions have to be described. In addition, it is necessary to set a kinematic model of the mechanism to describe the kinematic behavior of the mechanism.

In addition, the influencing factors that can occur during production as well as during product use must be collected. Tool wear or variations during the assembly process could be such influencing factors. In addition, varying environmental influences such as temperature differences are gathered during this step.

Based on a selection of these influencing factors, which cannot be neglected, aleatory uncertainties [21] are derived. In order to make these aleatory uncertainties usable for optimization, they must be subsequently described with the help of statistical moments. Which uncertainties can be taken into account strongly depends on the information available. If, for example, the range of temperatures during the use of a product can be quantified, this information can be used. The main focus nevertheless lies on geometric part deviations, since many fluctuations in production overlap here.

Considering these statistical moments, the optimization problem can be formulated and solved using a suitable algorithm. During this fourth step, it is crucial how kinematic boundary conditions and uncertainties are taken into account. The authors suggest kinematic boundary conditions as constraints in the optimization process, whereby individual scrap rates are optionally permitted.

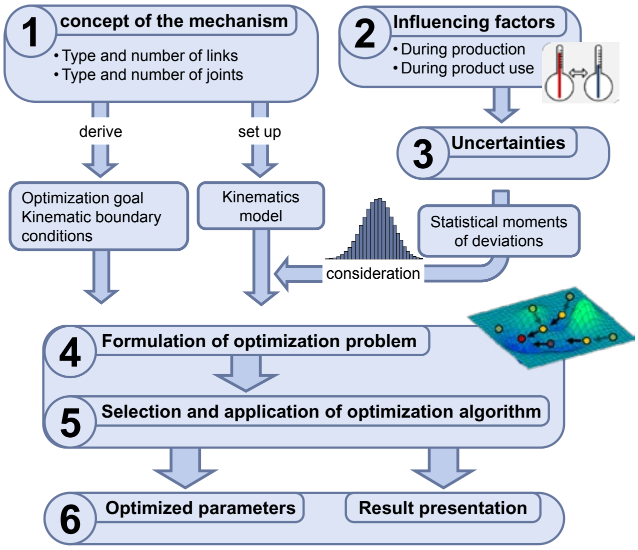


Fig. 3. Methodology for Robust-Design-Optimization of a mechanism

Uncertainties should be taken into account in the form of a sampling. It has been shown that it makes sense to sample random numbers and to use the same sample of random numbers for each iteration of the optimization process. If dimensions are sampled again during each iteration, different deviating dimensions may result for each iteration during the optimization. This can lead to an optimization algorithm without ever finding a suitable solution.

In the fifth step, an optimization algorithm is selected and applied to the problem. It has become apparent that metaheuristic methods are well suited for comparable problems [4,20].

Finally, in the sixth step the optimized parameters are received and the result can be displayed.

## 5. Case study

In the following, the kinematic of a windscreen wiper is used as a case study. As shown in Fig. 4, the kinematic of the windscreen wiper is driven by an electric motor. With several levers, the rotational movement is converted to a movement of the wipers which describe a circular arc on the windscreen.

Since the windscreen wiper is used to remove rain or snow from the windshield, its reliability under varying operating conditions is crucial. Consequently, varying temperatures between -30°C and +90°C are considered during the optimization process. Although the whole system is subject to several more uncertainties, geometric deviations are considered as second uncertainty.

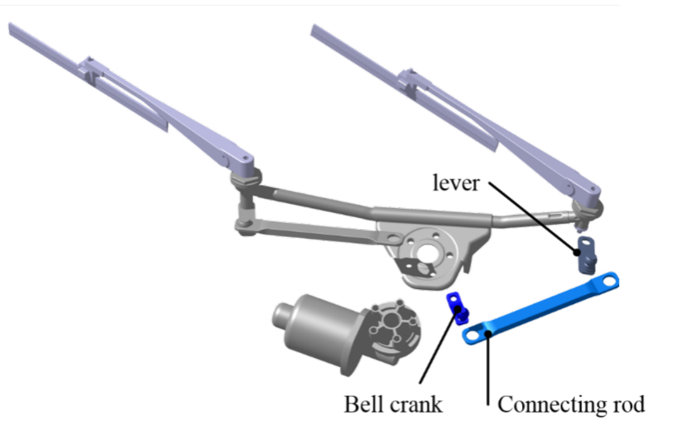


Fig. 4. Kinematic of a windscreen wiper

### *5.1. Concept of the mechanism and main requirements*

For reasons of comprehensibility and for a better traceability, only the right half of the mechanism is considered in the following optimization process. It is represented with a vectorial kinematic model shown in Fig. 5.

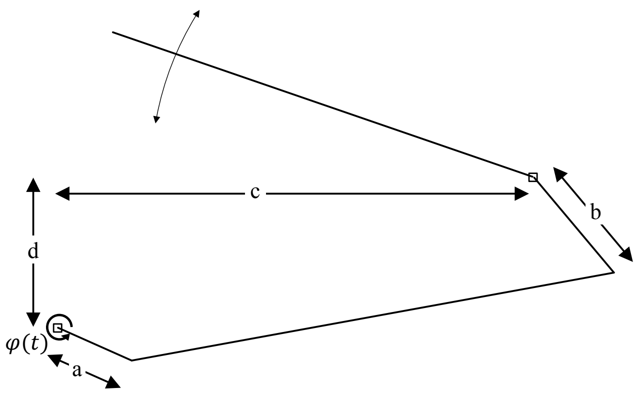


Fig. 5. Kinematic model of the mechanism

The simplified mechanism consists of a lever, a connector rod and a bell crank driven by a motor. The bell crank and the lever, which drives the wiper arm, are both connected to the wiper transmission. In the case study, geometric uncertainties caused by geometric tolerances and the influence of varying temperatures are considered. Consequently, the position of the top of the wiper in *x*- and *y*-direction can be described as a function, which is influenced by the time-depending rotation angle φ(t) of the motor and part-dimensions which are influenced by tolerances ($T_i$) and varying temperatures ($\Delta T$):

$$\begin{bmatrix} x_p \\ y_p \end{bmatrix} = f(\varphi(\mathrm{t}), \mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \Delta\mathrm{T}, \mathrm{T}_a, \mathrm{T}_b, \mathrm{T}_c, \mathrm{T}_d) \tag{2}$$

The corresponding properties of the single parts are shown in Table 1. Since the mounting plate is made of aluminum while the bell crank and the lever are made of steel, the particular linear thermal expansion coefficients are considered. Furthermore, different probability distributions are taken into account. The temperature is assumed as uniformly distributed during the sampling because the windscreen wiper should fulfill its function under different environmental conditions. This assumption represents a very conservative estimate, in which each temperature is assumed to be equally likely. In contrast to that, the geometric tolerances are considered as normally distributed. This is based on the assumption that manufacturing processes are subject to influences that lead to random deviations from the target value.

Table 1. Properties of parts

| Dimension | Initial dimension (mm) | Bounds (mm) | Material | Tolerance (mm) |
|---|---|---|---|---|
| a | 61.5 | [30, 40] | steel | ±1.5 |
| b | 30.5 | [30, 70] | steel | ±1.8 |
| c | 235 | [70, 80] | aluminum | ±1,5 |
| d | 79 | [70, 80] | aluminum | ±2 |

During the optimization, one primary goal is to be fulfilled, while different constraints are considered at the same time. The goal of the optimization is the reduction of the time while the top of the wiper is in the primary field of view (see Fig. 6). This goal ensures that the driver has the best visual conditions while using the windscreen wiper.

Additionally, one important condition is that the wiper covers the whole field of vision, while the jamming of the mechanism can be ruled out at any time. In the next section, these requirements are transferred into mathematical requirements, which are necessary for the optimization.

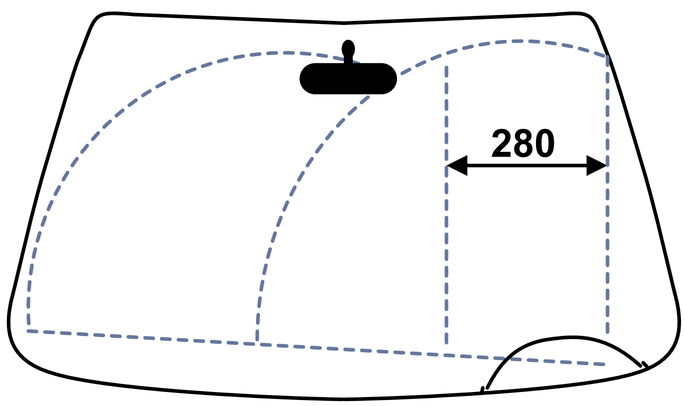


Fig. 6. Windscreen with primary field of view

### *5.2. Formulation of functional interaction*

The above-mentioned constraints and the goal of the optimization are transferred into mathematical relationships, which are necessary for the optimization process.

The first step in this process is to set up the kinematics model, based on the concept of the mechanism. As shown in equation (2), the *x*- and *y*-position of the top of the wiper can be described by a relationship between dimensions, which are influenced by tolerances and varying temperatures.

The aspired goal is the minimization of the time while the top of the wiper is in the primary field of view. Consequently, this leads to a maximization of the velocity in the primary field of view:

$$\max\left\{ \left\| \begin{bmatrix} x_p \\ y_p \end{bmatrix} \frac{d}{dt} \right\| \right\}, \, for\ x\ \varepsilon\ [\mathrm{x}_{\max} - 280, \mathrm{x}_{\max}] \tag{3}$$

Although the degree of freedom of the mechanism is M=1 (Eq. 1), the mechanism can jam if part dimensions deviate from their ideal. This requirement can be guaranteed as long as all equations can be solved, without using imaginary numbers. If imaginary numbers occur during the optimization process, the optimization solution will be considered as invalid. This condition is taken into account in the form of a constraint during optimization.

To obtain useful results from the optimization, the range of motion has to be restricted. Therefore, the left and the right turning points of the movement have to be restricted. Finally, the quality level has to be defined, whereby a level of $\pm 3\sigma$ is chosen.

### *5.3. Application and results of the optimization*

Based on the formulation of the optimization goal and the constraints, which have to be considered during the optimization, the optimization can be set up. For this purpose, different mathematical optimization algorithms can be used. However, in this contribution, a particle swarm optimization method is employed. The particle swarm optimization (PSO), which was invented by KENNEDY and EBERHART, is a population-based stochastic algorithm [22], which imitates the swarming behavior of a fish swarm. One of the advantages of the PSO is the capability of finding global minima of an objective function, while highly non-linear constraints and scattering input variables can be considered. Furthermore, the algorithm can be used for constrained optimization (e.g. in [4]).

This means that some variables of an objective function are optimized, while other constraints are considered simultaneously. The parameter-optimization is achieved by using a particle swarm optimization in MATLAB [14] with the following settings.

Table 2. P*arameter settings for the PSO Toolbox*

| parameter | value |
|---|---|
| Population size | 10 |
| Termination tolerance on the constraint violation | 1e-3 |
| Maximum number of nearly identical generations | 100 |
| Maximum number of iterations | 10 |

Since there are no generally valid parameter settings when selecting the optimization parameters, the parameters were selected based on different preliminary investigations. Generally speaking however, the conflicting goals between diversification and intensification must be resolved during the optimization. Diversification represents the ability to distribute itself as well as possible over the search space, whereby an exclusion of local minima is to be achieved. The opposite is the concentration on individual areas in which good solutions for the optimization problem have been found. This ability is called intensification.[23]

During the optimization, then nominal dimensions are changed until the goal is reached. To consider the uncertainties at the same time, for each possible solution a Monte-Carlo based sampling with n=20,000 samples is performed. For each sample generated this way, the compliance of the constraints is checked. The results of this optimization process are listed in Table 3.

Table 3. Resulting parameters of the optimization

| Dimension | Initial dimension (mm) | Optimized dimensions (mm) | Robust dimensions (mm) |
|---|---|---|---|
| a | 69.8 | 56.5 | 67.9 |
| b | 30.5 | 37.7 | 33.1 |
| c | 235 | 246.8 | 250.0 |
| d | 77.5 | 78.8 | 76.3 |

To illustrate the effects of the optimization-process, the velocity curves before and after the optimization are shown in Figure 7. For the initial as well as for the optimized parameters the curves for all deviation-afflicted constellations are shown. The curves for the ideal dimensions (without the uncertainties) are shown in red, whereas the scatter is shown in blue.

By optimizing the parameters, the maximum velocity was increased by almost 70%. As can be seen in Fig. 7, the increase in speed is accompanied by an increase of the reversal point. The standard deviation increases by 67 %. However, this increase is permissible since no restriction of the permissible deviations from the auxiliary conditions was provided for during optimization.

In order to show that an increase in velocity is inevitably accompanied by an increase of the standard deviation, an additional parameter constellation ('robust parameters') is added.

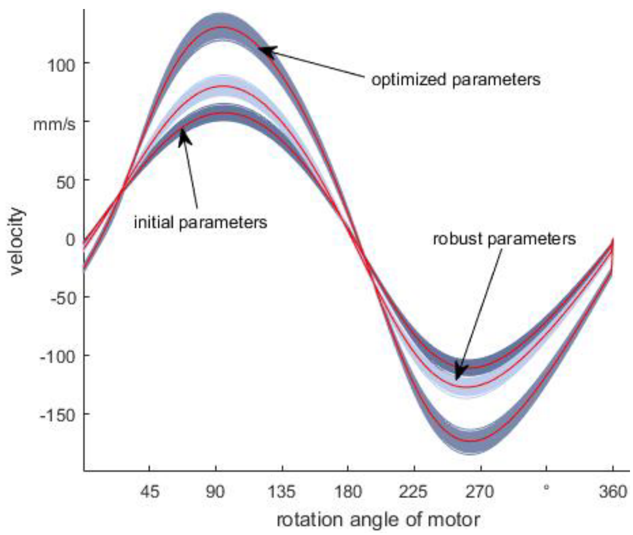

Fig. 7. Velocity curves of windscreen wiper

This parameter constellation represents one possible middle way between an increase in speed and a corresponding increase in the standard deviation. The robust parameters and the influence on the motion behavior are shown in Table 3 and 4.

Table 4. Resulting properties of the mechanism

| | Initial dimension | Optimized dimensions | Robust dimensions |
|---|---|---|---|
| Maximum velocity (mm/s) | 107.6 | 180.9 | 130.4 |
| Maximum standard deviation(mm/s) | 2.1 | 3.5 | 2.4 |

## 6. Summary and Outlook

The aim of this paper is to present a novel methodology for the parameter optimization of mechanisms considering different uncertainties. It is demonstrated that it is possible to fulfill defined requirements while different uncertainties affect the behavior of a mechanism. Using the kinematic of a windscreen wiper as a case study, the advantages of the parameter optimization are illustrated. It has been shown that an optimization of the velocity goes hand in hand with an increase of the scatter of the Key Characteristic, looking at the case study. For this reason, a further optimization solution, which allows an increase in speed with a small increase of the standard deviation was added.

The presented methodology opens up the possibility for the designer to design a robust mechanism far more efficiently, since time-consuming iterations and expensive tolerance restrictions at a later stage can be avoided. Moreover, influencing factors and their variations can be considered from the beginning.

Based on the successful parameter optimization of this relatively simple mechanism, it is appealing to use the presented methodology for a more complex mechanism. Since the tolerances were not changed during the optimization process, it is also possible to integrate tolerance optimization within the parameter optimization. In addition to these two aspects, the simultaneous optimization of competing targets is an interesting question for further research.

## Acknowledgment

The authors thank the German Research Foundation (DFG) for supporting the research project WA2913/19-1.

### References

[1] Eifler T., Ebro M., Howard T.J. A classification of the industrial relevance of robust design methods. The 19th International Conference on Engineering Design 2013.

[2] Gremyr I., Arvidsson M., Johansson P. Robust Design Methodology: Status in the Swedish Manufacturing Industry. Qual. Reliab. Engng. Int. 2003;19:285–293.

[3] Ebro M., Howard T., Rasmussen J.J. The foundation for robust design: Enabling robustness through kinematic design and design clarity 2012;70:pp. 817–826.

[4] Heling B., Aschenbrenner A., Walter M., Wartzack S. On Connected Tolerances in Statistical Tolerance-Cost-Optimization of Assemblies with Interrelated Dimension Chains. Procedia CIRP 2016;43:262–267.

[5] Taguchi G. Taguchi on robust technology development: Bringing quality engineering upstream. ASME Press series on international advances in design productivity. New York: ASME Press; 1993.

[6] Ebro M., Howard T.J. Robust design principles for reducing variation in functional performance. Journal of Engineering Design 2016;27:75–92.

[7] Myszka D.H. Machines and mechanisms: Applied kinematic analysis, ed. 4. Upper Saddle River, N.J.: Pearson Prentice Hall; 2012.

[8] Bruyère J., Dantan J.-Y., Bigot R., Martin P. Statistical tolerance analysis of bevel gear by tooth contact analysis and Monte Carlo simulation. Mechanism and Machine Theory 2007;42:1326–1351.

[9] Dupac M., Beale D.G. Dynamic analysis of a flexible linkage mechanism with cracks and clearance. Mechanism and Machine Theory 2010;45:1909–1923.

[10] Walter M., Spruegel T.C, Wartzack S. Least Cost Tolerance Allocation for Systems with Time-variant Deviations. Procedia CIRP 2015;27:1–9.

[11] Schleich B., Wartzack S. Tolerance Analysis of Rotating Mechanism Based on Skin Model Shapes in Discrete Geometry. Procedia CIRP 2015;27:10–15.

[12] Schleich B., Wartzack S. Motion Tolerancing Based on Skin Model Shapes by Form Deviation Parametrization and Meta-Modelling. In: ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference. p. V001T02A007.

[13] Schleich B., Wartzack S. A discrete geometry approach for tolerance analysis of mechanism. Mechanism and Machine Theory 2014;77:148–163.

[14] Heling B., Hallmann M., Wartzack S. Hybrid Tolerance Representation of Systems in Motion. Procedia CIRP 2017;60:50–55.

[15] Ballu A., Plantec J.-Y., Mathieu L. Geometrical reliability of overconstrained mechanisms with gaps. CIRP Annals 2008;57:159–162.

[16] Lê H.-N., Ledoux Y., Ballu A. Experimental and Theoretical Investigations of Mechanical Joints With Form Defects. J. Comput. Inf. Sci. Eng. 2014;14:41004.

[17] Li W., Wu C.FJ. An Integrated Method of Parameter Design and Tolerance Design. Quality Engineering 1999;11:417–425.

[18] Söderberg R., Lindkvist L. Two-Step Procedure for Robust Design UsingCAT Technology. In: F. van Houten et al. (eds.), Global Consistency of Tolerances. p. 231–40.

[19] Göhler S.M, Husung S., Howard T.J. The Translation between Functional Requirements and Design Parameters for Robust Design. Procedia CIRP 2016;43:106–111.

[20] Heling B., Wartzack S. Methode zur Parameteroptimierung komplexer Mechanismen hinsichtlich kinematischer Anforderungen. Design for X - Beiträge zum 27. DfX-Symposium 2016:111–122.

[21] Oberkampf W.L, DeLand S.M, Rutherford B.M, Diegert K.V, Alvin K.F. Error and uncertainty in modeling and simulation. Reliability Engineering & System Safety 2002;75:333–357.

[22] Kennedy J., Eberhart R. Particle Swarm Optimization. IEEE International Conference on Neural Networks 1995:1942–1948.

[23] Nesmachnow S. An overview of metaheuristics: Accurate and efficient methods for optimisation. IJMHEUR 2014;3:320.