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Reference data simulation for L∞ fitting of aspheres

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Abstract

In the field of dimensional metrology of optical components, fitting algorithms play an essential role. Performing data fitting according to L∞-norm indicates about form errors since it directly minimizes peak-to-valley (PV). In parallel, reference data (softgauges) are used to assess the performance of the developed algorithms and must incorporate multiple parameters related to points distribution, manufacturing process and measurement errors, to be as close as possible to the real measured data. In this paper, a developed algorithm for surface fitting based on L∞-norm is presented and tested on simulated reference data of aspheres.

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Keywords: computational metrology, fitting, reference data

1. Introduction

Coordinate measuring machines (CMMs) are essential elements of the quality control process in which the manufactured parts are measured and compared to design tolerance specifications. The task of measurement underlies two phases, namely data acquisition and data processing. Data acquisition is achieved by the mean of a probing system (tactile or optical) attached to the CMM, while data processing needs to include software that perform calculation tasks such as error compensation, filtering, fitting and evaluation.

ISO 10360-2:2009 specifies guidelines for acceptance tests for verifying the performance of coordinate measuring machines [1]. The calibration of CMMs makes use of gauge blocks, calibration spheres, laser interferometry, etc. Reference software and reference data are used to check the accuracy of the output of embedded software. ISO 5436-2:2012 defines two types of software measurement standards named F1 and F2 for the validation of the measuring instruments software [2]. This standard is in fact restricted to surface texture software but the principle of F1 and F2 standards could be extended to any measurement software.

Type F1 measurement standards also called reference data is a set of input data with a known measurand value. Type F2 is traceable computer software to which software in a measuring machine could be compared.

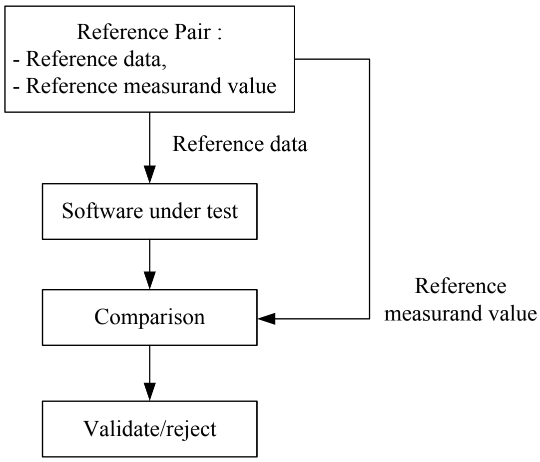

Fig. 1. Type F1software validation

According to this philosophy, two validation methods could be applied either using reference data or reference software. In the first method,

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a reference pair (reference data and its corresponding reference measurand value) is generated. Reference data is then inputted to the software under test, the resulting value is compared to the reference measurand value then the software is validated if the difference is within the maximum permissible error (fig.1). For the second method, a common data (not necessary reference data) is submitted into both reference software and software under test, the two resulting outputs are then compared (fig.2).

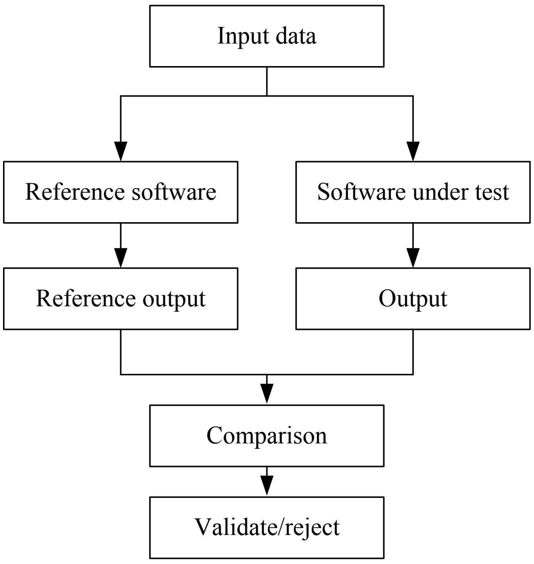

Fig. 2. Type F2 software

At the level of National Measurement Institutes (NMIs) a number of platforms were set for generating reference data for a specific application. As for surface texture, the Physikalisch-Technische Bundesanstalt (PTB) developed a web-based reference software for roughness analysis [3]. The National Institute of Standards and Technology (NIST) also developed an algorithm testing system for the same purpose where the user can either download virtual data surface files and corresponding parameters from a database or use the embedded software to make calculation on its own surface topography [4]. In 2008, the National Physical Laboratory (NPL) carried a project named SoftGauges for the evaluation of areal surface texture parameters. The aim of the project is to develop consistent definitions, specifications and reference software for the evaluation of areal surface texture parameters [5]. A detailed comparison of the three platforms and improvement recommendations is given in [6].

Reference data generation for least squares (LS) fitting problems was the subject of the ISO 10360-6:2001 [7].This standard presents a method for generating reference data that could be used for the testing of software dedicated to the determination of associated Gaussian features to measured data. Among others, this standard specifies the required number of data sets, the file format and the detailed procedure for data generation. However, the presented method is only applicable to simple geometries such as lines, planes, circles, spheres, cylinders, cones and torus. Moreover, the described method doesn't guarantee that the reference value of measurand matches the generated data. PTB provides an online service called TraCIM that offers on-demand reference data for both least squares and Chebyshev (also called min-max or L_∞-norm) fitting. Generation of reference data and evaluation of obtained results conformed to some requirements specified by TraCIM association [8]. In [9], Forbes *et.al* give practical methods in order to generate reference data for both least squares and Chebyshev fitting.

In this article, an existing method for reference data generation is applied to obtain reference data for Chebyshev fitting of aspherical shapes. In section 2, a formulation of the fitting problem is presented. The considered reference data generation method is detailed in section 3. In section 4, additional requirements of generated data are discussed. In the last section this approach is used to validate an algorithm for the Chebyshev fitting of aspherical shapes.

2. Formulation of the Chebyshev fitting problem

Chebyshev fitting is recommended by ISO standards of Geometrical Product Specifications to determine the minimum zone (MZ). Minimum zone is important in form metrology since it indicates about the form deviation of the specified part. Minimum zone could be regarded as the least value of peak-to-valley (PV). PV is defined as the difference between the maximum and the minimum deviations $\{d_i\}$ of the dataset and

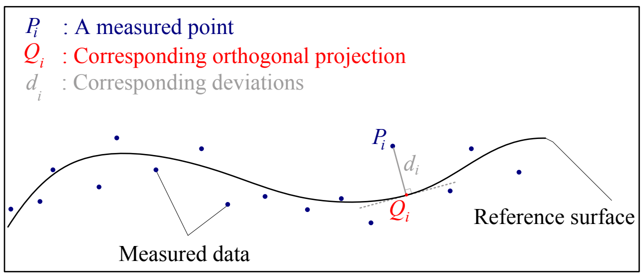

Fig. 3. Definition of deviations

the reference surface. Deviations are defined as orthogonal distances between the measured points $\{P_i\}$ and the reference surface.

Minimum zone determination could be formulated as a minimization problem. Let $f(\widehat{\boldsymbol{X}}, \boldsymbol{s}) = 0$ be the generic equation describing the shape of the measured part with $\widehat{\boldsymbol{X}}$ is the space vector ($\widehat{\boldsymbol{X}} = (\hat{x}, \hat{y}, \hat{z})$ in Cartesian coordinates) and $\boldsymbol{s}$ denotes the shape parameters, $\{\boldsymbol{P_i}\}_{1\le i\le N}$ is the set of N measured points and $\{\boldsymbol{Q_i}\}_{1\le i\le N}$ are the corresponding projections into the surface described by $\boldsymbol{s}$.

$$\min_{\boldsymbol{x}} \max_{1\le i\le N} \|\boldsymbol{P_i} - \boldsymbol{Q_i}\| \text{ where } \boldsymbol{x} = \{\boldsymbol{m}, \boldsymbol{s}\} \tag{1}$$

The minimum zone determination problem is given in eq. (1). $\|.\|$ denotes the Euclidean norm, $\boldsymbol{x} \in \mathrm{R}^n$ could be either the set of intrinsic shape parameters $\boldsymbol{s}$, or the motion parameters $\boldsymbol{m}$: rotation and translation applied to $\{\mathbf{P_i}\}$.

For the case of aspheric shapes, this problem could be addressed using a number of methods, namely smoothing optimization techniques, interior point methods, genetic algorithms, etc. [10-12]. A detailed comparison of two main algorithms is given in [13].

Most of the previous methods rely on least squares fitting as an initial solution. A recent work has been conducted that aims at developing robust least squares fitting algorithms for aspheric shape. These algorithms are based on an enhanced BFGS algorithms [14]. In the same context a number of physical artefacts were suggested for measurement validation [15, 16].

Reference data generation for complex shape is still a non-completely addressed problem. Efficient reference data generation methods exist only for simple geometries.

3. Generation of reference data

In this paper, the method presented in [9, 13] is applied to aspherical shapes in order to generate reference data for the Chebyshev fitting.

This method is not restricted to simple geometries (such as line, plane, sphere, etc.) and could be applied to more complex shapes as freeforms. The main idea of this approach is to formulate the Chebyshev minimization problem as a nonlinear programming problem and then generate data based on the Karush-Kuhn Tucker optimality conditions [18]. This method has also the advantage of generating data with global minimum. In fact, Chebyshev fitting problem has the following property: for a set of data points Σ, if a solution x^* satisfies optimality conditions for a subset $\Gamma \subset \Sigma$ and stays feasible for Σ then it is the global solution for Σ.

Expressing the Chebyshev fitting problem as a nonlinear programming results in formulation given in (2).

$$\begin{gathered} \min_{x} e \\ \text{Subject to: } c_i(\boldsymbol{x}) \geq 0 \ \forall\, i \epsilon [|1, N|] \\ \text{With: } c_i(\boldsymbol{x}) = e - d_i(\boldsymbol{x}) \end{gathered} \tag{2}$$

e refers to the parameter to minimize in order to obtain the minimum zone value. For the case where shape parameters are supposed known (only motion parameters are sought), first order optimality conditions could be expressed as:

$$\begin{gathered} \sum_{i \in I^+ \cup I^-} \lambda_i = 1 \\ \sum_{i \in I^+} \lambda_i \, \mathrm{n_i} = \sum_{i \in I^-} \lambda_i \, \mathrm{n_i} \\ \sum_{i \in I^+} \lambda_i X_i \times \mathrm{n_i} = \sum_{i \in I^-} \lambda_i X_i \times \mathrm{n_i} \end{gathered} \tag{3}$$

where λ_i refers to Lagrangian multipliers [19], $\{X_i\}$ a set of sampled points belonging to the nominal surface, $\{n_i\}$ are the corresponding normal directions and I^+ (resp. I^-) is the set of points for which $d_i(x) = e$ and lying above (resp. below) the nominal surface.

The different steps for generating Chebyshev reference data for motion parameters are given as follows.

Step1: Given a nominal surface $f(\widehat{\boldsymbol{X}}, \boldsymbol{s}^*) = 0$ (the motion parameter $\boldsymbol{s}^*$ is supposed known) and a fixed value of the minimum zone e^*, seven points X_i^*, $i \in I = \{1, \dots, 7\}$ lying exactly on the nominal surface are determined such as the corresponding Jacobian matrix associated to the chosen points given in eq. (4) must be full rank.

$$\mathrm{J} = [X_i^*, X_i^* \times n_i^*] \tag{4}$$

Step2: Solve the optimality conditions given in eq. (3) for all possible partitions $I = I^+ \cup I^-$ until finding the partition $I^* = I^{*+} \cup I^{*-}$ for which all Lagrange multipliers are positive.

Step3: Set $X_i = X_i^* + e^* n_i/2$ for $i \in I^{*+}$ and $X_i = X_i^* - e^* n_i/2$ for $i \in I^{*-}$

Step4: Generate additional points lying on the nominal surface X_i^* and a random form error $\delta_i \in [-e^*, e^*]$, $i \in \{8, \dots, N\}$ and set $X_i = X_i^* + \delta_i n_i$ with n_i is the corresponding normal direction to the nominal surface at X_i^*.

This generation method could be applied for simple geometries as well as for more complex shapes. However, the main drawback is that generated solutions are vertex solutions only. When attempting to generate non-vertex solution, the Karush-Kuhn Tucker second order optimality conditions must be satisfied as well. In the case of complex geometries such as aspherical shapes, this is not straightforward. In [18], a practical method was presented for generating non-vertex solutions for cylinders but further development is still needed for more complex geometries.

4. Requirements of reference data

In this section we attempt to answer the following question:

"*What are the required properties of the generated reference data?*"

A summarized answer to the previous question is that generated reference data must be task-specific. This means that reference data must satisfy input requirements specified in the description of the algorithm under test and this may include:

Data extraction

For some CMMs using tactile probes, recorded data has a constant sampling step along X and Y directions. The included software needs the value of the sampling step to perform calculations. The considered reference data must also follow this specification and a set of data with a varying sampling step must not be taking as a reference data for this software.

Uniqueness of the solution

Reference data must have a unique associated reference measurand value. This problem was outlined by Shakarji *et.al* in [19] for the problem of maximum inscribed circle determination.

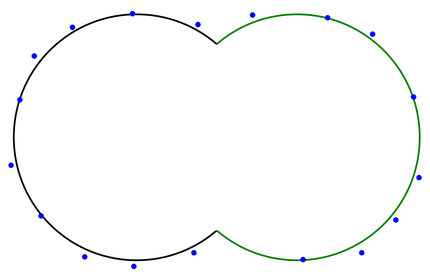

Fig. 4. Maximum inscribed circle problem with non-unique solution

In figure 4, the set of data points (blue) have two maximum inscribed circles (green and black). These situations must be avoided when generating reference data.

Number of points

The number of points contained in the reference data set must be taken according to the software specifications. Some algorithms could only be applied if the number of recorded points is not high.

Texture

Is it necessary that the generated data reproduce the topography of "physical" measured surface where adjacent form deviations are correlated or exhibit non-correlated random values?

The choice of either strategy depends whether the algorithm under test is designed to work for a specific topography.

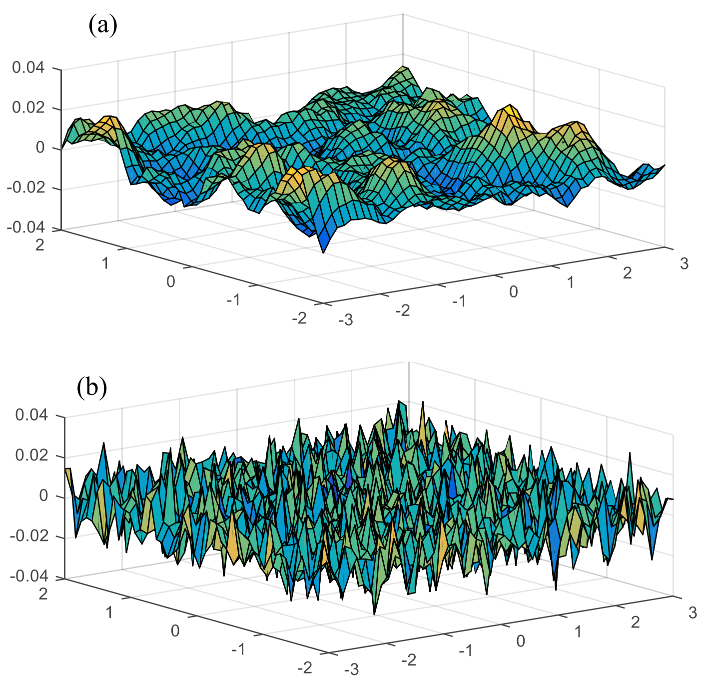

Fig. 5. Texture generated using: (a) fractional Brownian motion process, (b) Gaussian noise

Uncertainty

Uncertainty of the reference value of the measurand must also be given in order to make comparison to results obtained by software under test. The associated uncertainty to the reference data is purely numerical. It is based on uncertainties of the different steps of the generation algorithm. The final uncertainty could be calculated by the mean of uncertainty propagation rules when the generation algorithm is relatively simple. Monte Carlo simulations could also be used in other cases.

Stability

Designed reference data must be stable. In fact, a small perturbation in the designed data must not affect the reference value of the measurand too much.

Vertex, non-vertex solution

Generating reference data for Chebyshev fitting is based on formulating optimality conditions of the problem given in (2). When optimality conditions are expressed, the number of active constraints i.e. for which $c_i(x) = 0$ is either equal to the number of unknowns (vertex solution) or lower than the number of active constraints (non-vertex solution) as for the case of flatness. For the formal case, the solution lies in the vertex of the feasible domain. Some algorithms might be designed to target vertices of feasible domain to rapidly converge to the solution. Hence, they could not find non-vertex solution. Designed reference data must be aligned with this requirement and give either vertex or non-vertex solution according to algorithm specification.

Deviation error value

In some cases, fitting algorithms could not be used if the value of form error exceeds a given value. Generally this is expressed as a percentage of a characteristic dimension of the artefact. In least squares fitting for instance, calculation of the Hessian matrix is based on neglecting second order derivatives of residuals. For the case where the deviations values become high, this approximation is no longer valid and the resulting Hessian matrix is not accurate. Therefore, the returned results are not exact.

Initial alignment value

Initial position of measured data relatively to reference shape highly affects performances of fitting algorithms. Most fitting software underlie inner routines to perform a rough alignment to make measured data as close as possible to reference model. Some algorithms could only perform fitting if the input data points are well aligned with the model. Calculation of foot points which is a crucial step in approximately all fitting algorithms is highly affected by the initial position of inputted data.

Error-free

Validation of algorithms must be performed in the perfect operator approximation. In fact, when testing a given algorithm, the submitted data must be error-free. This means that measuring errors must not be embedded in designed data. Taking input data that include measuring errors make decision making difficult since we cannot tell whether inaccuracy results from measurement or processing.

5. Application on aspherical shape fitting

Aspherical surface could be defined as a rotationally symmetric surface with a radius of curvature that varies gradually from the center of the lens. The most widely used mathematical formulation to describe an aspheric surface is given in ISO 10110-12:2009 [20]. This formulation is expressed in cylindrical polar coordinates and given in eq. (5):

$$z(r) = \frac{r^2}{R\left(1+\sqrt{1-(1+\kappa)\frac{r^2}{R^2}}\right)} + \sum_{m=0}^{M} a_{2m+4}\, r^{2m+4} \tag{5}$$

with

z	→	sag of surface
r	→	radial distance
R	→	radius of curvature at the vertex
κ	→	conic constant
a_{2m+4}	→	monomial coefficients

The aperture size of the lens R_{max} defines the domain of r: $0 \le r \le R_{max}$ for which equation (5) is valid. The number of monomial terms M depends on the targeted accuracy.

Chebyshev reference data generation for aspherical shapes is performed according to the steps described in section 3. Only motion parameters are regarded. Since aspherical shapes are rotationally symmetric, six points are required in step 1 and only the first five columns of the Jacobian matrix must be linearly independent.

Here, we test an implementation of a given fitting algorithm for aspherical shapes. The used algorithm is based on Primal-Dual Interior Point solving method [12]. This algorithm was implemented using MATLAB® and is referred as "*Algorithm_Under_Test*". It is claimed that the algorithm is able to process data with the following constraints:

- Data must be given in x, y, z format.
- The number of points must not exceed 1.000 points.
- Minimum zone value must be less than 1% of the value of aperture R_{max}.
- Initial alignment relative to the solution must not exceed:
  - 10° rotation around x and y axis;
  - A translation vector with magnitude less than 5% of the aperture value R_{max}.
- The solution must be a vertex-solution.
- A relative uncertainty of 10^{-9} is associated to returned results.

For the performed tests, five data sets were generated with the characteristics represented in tables 1 and 2. For each set of data, a combination of nominal shape values was selected to represent different shapes. Values of initial position parameters (rotation and translation) were not represented in the table but were chosen to be within the specified intervals and to represent various cases.

Table 1. Reference data sets characteristics

	Number of points	Minimum zone value, MZ_{ref} (%)	Relative error of MZ_{ref} (%)
Set 1	50	0.01	10^{-6}
Set 2	250	0.05	10^{-6}
Set 3	500	0.1	10^{-6}
Set 4	750	0.5	10^{-6}
Set 5	1000	1	10^{-6}

Table 2. Nominal shape coefficients values (in mm or corresponding power)

	R	κ	a_4	a_6	a_8	R_{max}
Set 1	137.09	-1	-1.26 10^{-11}	-6.33 10^{-22}	-3.16 10^{-22}	200
Set 2	2.62	0.8	6.57 10^{-12}	3.28 10^{-12}	1.64 10^{-12}	20
Set 3	7.66	-1	-9.09 10^{-12}	-4.54 10^{-12}	-2.27 10^{-12}	40
Set 4	5.74	-0.9	-6.81 10^{-11}	-3.41 10^{-11}	-1.70 10^{-11}	30
Set 5	32	-1	8.78 10^{-09}	4.37 10^{-09}	2.21 10^{-08}	5

Figure 6 shows a representation of generated data points. Table 3 gives values of the reference measurand for each set in comparison to results returned by the algorithm under test. The two values are then compared and the decision is made whether the obtained values are within maximum permissible error.

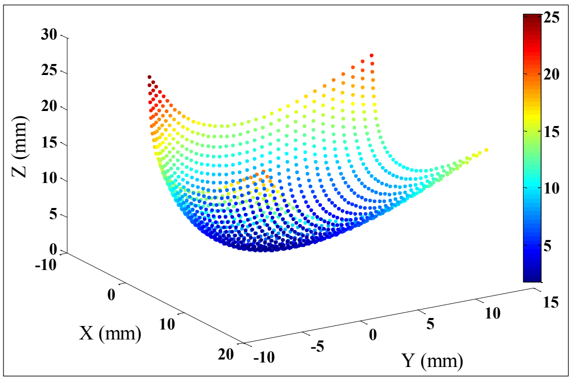

Fig. 6. Representation of generated reference data

Table 3. Comparison of the reference measurand values and results of the algorithm under test.

	Measurand reference value (mm)	Algorithm result (mm)	Validate/Reject
Set 1	$0.02\times(1\pm10^{-6})$	$0.01999995\times(1\pm10^{-9})$	Validate
Set 2	$0.01\times(1\pm10^{-6})$	$0.01000000\times(1\pm10^{-9})$	Validate
Set 3	$0.04\times(1\pm10^{-6})$	$0.04999998\times(1\pm10^{-9})$	Validate
Set 4	$0.15\times(1\pm10^{-6})$	$0.15000001\times(1\pm10^{-9})$	Validate
Set 5	$0.05\times(1\pm10^{-6})$	$0.04999999\times(1\pm10^{-9})$	Validate

All the returned values obtained using the algorithm under test are in conformance with the reference measurand values.

6. Conclusion

In this paper, reference data generation for Chebyshev fitting of aspherical shapes is discussed. An existing method for reference data generation was adopted and applied in order to obtain softgauges for the test of Chebyshev fitting algorithms. This method is based on formulating optimality conditions for the Chebyshev fitting and then inferring reference data. Five reference data sets were generated and submitted to the algorithm under test. Results show the conformance of returned values with the reference measurand ones.

Requirements of the generated reference data were also discussed in this paper. In fact, generated data must be aligned with characteristics of software under test. Data points' extraction, the number of points, stability of the solution, etc. must be taken in account when generating data.

Most of the considered requirements need further development. For instance, no method for generating non-vertex solution for Chebyshev fitting is available. Also, methods for assessing the stability of generated data are still ambiguous and need more research.

Future work will mainly cover the establishment of a standardized validation process. At the moment, no standard methods for the validation routine are described. The standardized process should define the minimal number of data sets that must be used in a given test as well as guidelines to make statements about the acceptance or the rejection of the algorithm under test.

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