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Method for Measurement Uncertainty Evaluation of Cylindricity Error Based on Good Point Set

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Abstract

Measurement uncertainty is one of the most important concepts in geometrical product specification (GPS). Monte Carlo method (MCM) is widely used to evaluate uncertainty in many cases. However, the main drawback of this method is that it is necessary to generate very large samples. In this paper, a Quasi-Monte Carlo method based on good point set is proposed for the evaluation of measurement uncertainty of cylindricity error. The proposed method is tested in comparison with GUM method and Monte Carlo method, which shows the proposed method has the advantages of less computation time and better precision.

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Keywords: Uncertainty evaluation; Quasi-Monte Carlo method; Cylindricity

1. Introduction

According to New Generation Geometrical Product Specification, not only the measurement result needs to be given, but also the uncertainty of the measurement result should be evaluated. The measurement error always exits so that the measurement result is not equal to the true value but shows a statistical distribution around the true value. The uncertainty is a non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand [1]. Uncertainty can be quantified by a standard deviation called standard uncertainty (or a specified multiple of it) or the half-width of an interval which corresponds a stated coverage probability. The "Guide to the express of uncertainty in measurement (GUM)"[2] is an internationally accepted master document of uncertainty evaluation, which provides a framework for assessing uncertainty based on the law of propagation of uncertainty requiring the use of a first-order Taylor series expansion. However, when measurement model is strongly non-linear, the GUM method is usually inadequate because its use of linear approximation [3]. So GUM S1 [4] has been proposed which is implemented based on Monte Carlo method. ISO 15530-4 [5] specifies requirements for the application of uncertainty evaluating software to measurements with CMMs and gives informative descriptions of simulation techniques used for evaluating task-specific measurement uncertainty.

At the same time, cylindricity error is one of typical geometric error, which is an important quality index of mechanical parts [6]. It describes the variation between a measured cylindrical surface and its ideal cylindrical surface [7] and impacts the performance of the precision products including working precision, connection strength, motion stability, abrasive resistance and working life [8]. So it is of significance to conduct the study on evaluation for cylindricity error and corresponding measurement uncertainty.

There are many algorithms for cylindricity error evaluation including computation geometry method [9, 10], Particle Swarm Optimization method[6, 11], support vector machine learning [12], genetic algorithm [13] and so on. However, these

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methods are only used to get cylindricity error but can't get the uncertainty of cylindricity error. Many researchers have studied the evaluation of uncertainty in coordinate metrology. Mao et al. [14] developed a method for uncertainty evaluation of cylindricity errors based on the uncertainty theory in new generation GPS. Zhao et al. [15] proposed a method for estimation of uncertainty propagation in verification operators of cylindricity errors based on the new generation GPS. Ren [16] presents a task specific uncertainty analysis method for ultra-precision freeform surface when the measured data are extracted from a specific surface with specific sampling strategy. Wen et al. [17] proposed an Adaptive Monte Carlo method for uncertainty of cylindricity error in which the number of trials can be controlled by the numerical results. Chifai Cheung et al. [18] proposed an uncertainty analysis model for freeform surface incorporating the effect of CMM, sampling plan and evaluation method. Du et al. [19] presented an uncertainty evaluation method for CMM measurement based on the error ellipse theory and Monte Carlo simulation method. Zhang et al. [20] developed a Monte Carlo method to estimate the uncertainty of the fitted position, shape and form error metric. Though the above uncertainty evaluation methods consider different factors, form errors, measurement models and applications, the basis of above methods can be classified into 2 kinds: GUM method and Monte Carlo method. GUM method is usually inadequate when the measurement model is strongly non-linear because of its use of linear approximation while Monte Carlo method can be applied in different occasions. The main drawback of Monte Carlo method is that it is necessary to generate very large samples to assure calculation accuracy. In this paper, in order to shorten the computation time and improve the precision, a Quasi-Monte Carlo method based on GP set is proposed.

The rest of the paper is organized as follows: Section 2 established the measurement model and introduced GUM method and Monte Carlo method in brief. A Quasi-Monte Carlo method based on good point set for uncertainty evaluation of cylindricity error was proposed in Section 3. Section 4 compared the proposed method with GUM method and Monte Carlo method. Finally, some conclusions were given in Section 5.

2. Establishment of measurement model

The solution of the minimum zone cylinder problem requires identification of two concentric cylinders that include all the measured points with the minimum difference of their radii, as shown in Fig.1. The two concentric cylinders defined as minimum zone reference cylinders according to ISO standards are determined by their centric axis, which can be represented as Eq. (1) in Euclidean space.

$$\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n} \tag{1}$$

Let $p=l/n$, $q=m/n$, $c=0$, then Eq. (1) can also be represented as

$$\frac{x-a}{p}=\frac{y-b}{q}=z \tag{2}$$

The centric axis represented by Eq. (2) passes through the point $\mathrm{M}(a,b,0)$ and its direction vector is $\vec{\mathrm{V}}\,(p,q,1)$. Suppose that the measured point is $P_i(x_i,y_i,z_i)$, The distance d_i from measured point $P_i(x_i,y_i,z_i)$ to the centric axis can be represented as Eq. (3) with approximation [11].

$$d_i=\sqrt{[x_i-(pz_i+a)]^2+[y_i-(qz_i+b)]^2} \tag{3}$$

Then cylindricity can be obtained by solving Eq.(4).

$$\min\varepsilon=\max_{1\le i\le n} d_i(a,b,p,q)-\min_{1\le i\le n} d_i(a,b,p,q) \tag{4}$$

Eq. (4) can be transformed into a nonlinear constrained optimization by introducing two variables as shown in Eq. (5).

$$\begin{gathered}\min\varepsilon=\varepsilon_1-\varepsilon_2\\ s.t.\ \varepsilon_2\le d_i(a,b,p,q)\le\varepsilon_1\\ i=1,2,\dots,N\end{gathered} \tag{5}$$

Various methods have been proposed to solve the above nonlinear optimization problem. In this paper, the method combining sequential quadratic programming technique and the trust region method based on interior point techniques proposed in reference [21] is used to solve Eq.(5).

Assuming that the two peak and valley points for minimum zone cylindricity error are (x_p,y_p,z_p) and (x_v,y_v,z_v), Eq.(5) can be represented as Eq.(6).

$$\begin{aligned}g=\ &\sqrt{\left[x_p-\left(pz_p+a\right)\right]^2+\left[y_p-\left(qz_p+b\right)\right]^2}\\ &-\sqrt{[x_v-(pz_v+a)]^2+[y_v-(qz_v+b)]^2}\end{aligned} \tag{6}$$

Where g represents the cylindricity error.

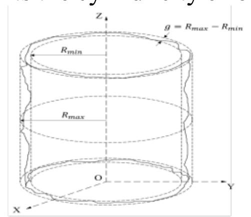

Fig. 1. minimum zone cylindricity error evaluation

According Eq.(6), the uncertainty of cylindricity error g can be attained through the uncertainty of the variable in Eq.(6), that is $x_p,y_p,z_p,x_v,y_v,z_v,a,b,p,q$. For GUM method, the uncertainty of cylindricity error can be attained by Eq.(7).

$$\begin{aligned}u^2(g)=\ &\left(\frac{\partial g}{\partial x_p}u_{x_p}\right)^2+\left(\frac{\partial g}{\partial y_p}u_{y_p}\right)^2+\left(\frac{\partial g}{\partial z_p}u_{z_p}\right)^2\\ &+\left(\frac{\partial g}{\partial x_v}u_{x_v}\right)^2+\left(\frac{\partial g}{\partial y_v}u_{y_v}\right)^2+\left(\frac{\partial g}{\partial z_v}u_{z_v}\right)^2\\ &+\left(\frac{\partial g}{\partial a}u_a\right)^2+\left(\frac{\partial g}{\partial b}u_b\right)^2+\left(\frac{\partial g}{\partial p}u_p\right)^2+\left(\frac{\partial g}{\partial q}u_q\right)^2\\ &+2\frac{\partial g}{\partial a}\frac{\partial g}{\partial b}\rho_{ab}u_{ab}+2\frac{\partial g}{\partial a}\frac{\partial g}{\partial p}\rho_{ap}u_{ap}+2\frac{\partial g}{\partial a}\frac{\partial g}{\partial q}\rho_{aq}u_{aq}\\ &+2\frac{\partial g}{\partial b}\frac{\partial g}{\partial p}\rho_{bp}u_{bp}+2\frac{\partial g}{\partial b}\frac{\partial g}{\partial q}\rho_{bq}u_{bq}+2\frac{\partial g}{\partial p}\frac{\partial g}{\partial q}\rho_{pq}u_{pq}\end{aligned} \tag{7}$$

Where $\frac{\partial f}{\partial x_p},\frac{\partial f}{\partial y_p},\dots,\frac{\partial f}{\partial q}$ can be calculated by Eq.(6). $u_{x_p},u_{y_p},\dots,u_{z_v}$ can be attained by analysing the uncertainty of measurement result of each point. The parameters a, b, p, q are regarded as statistical vector r, the covariance matrix of vector can be calculated by sampling points many times using the same sampling strategy. The uncertainty $u_a, u_b, u_p, u_q, \dots, u_{pq}$ can be obtained through the covariance matrix of vector as Eq.(8).

$$cov(r) = \begin{bmatrix} u_a^2 & u_{ab} & u_{ap} & u_{aq} \\ u_{ab} & u_b^2 & u_{bp} & u_{bq} \\ u_{ap} & u_{bp} & u_p^2 & u_{pq} \\ u_{aq} & u_{bq} & u_{pq} & u_q^2 \end{bmatrix} \tag{8}$$

For Monte Carlo method, variables $x_p, y_p, z_p, x_v, y_v, z_v$ are assigned to follow normal distribution and the statistical vector r constituted by a, b, p, q is assigned to follow multivariate normal distribution where the expectation and covariance matrix are obtained by statistical method. The variables $x_p, y_p, z_p, x_v, y_v, z_v, a, b, p, q$ are repeatedly sampling from assigned probability distributions, then many results of cylindricity error g can be obtained by Eq.(6) and the uncertainty of cylindricity error is the standard deviation of the results.

GUM method is usually not suitable when the measurement model is strongly nonlinear because of its usage of linear approximation. The Monte Carlo method is applicable for strongly nonlinear measurement model, however, it needs very large samples so the computation time is extensive. To reduce the computation time, a Quasi-Monte Carlo method based on good point set for uncertainty evaluation is proposed.

3. Uncertainty evaluation of cylindricity error

3.1 Definitions and theorems

$F(x)$ is a continuous multivariate distribution function in R^S and n is a given integer. The focus of this part is how to find n points $x_1, \dots, x_n$ in R^S so that they form a good representation for $F(x)$. Some concepts are given as follows.

Definition 1. Let $P = \{x_k, k = 1, \dots, n\}$ be a point set in R^S and its empirical distribution is:

$$F_n(x) = \frac{1}{n} \sum_{i=1}^{n} I\{x_i < x\} \tag{9}$$

Where $I\{x_i \leq x\}$ is the indicator function of $x_i \leq x$:

$$I\{x_i \leq x\} = \begin{cases} 1, when\, x_i \leq x \\ 0, when\, x_i > x \end{cases} \tag{10}$$

Then

$$D_F(n, P) = \sup_{x \in R^S} |F_n(x) - F(x)| \tag{11}$$

is the F-discrepancy of P with respect to $F(x)$.

Obviously, the F-discrepancy is a measure of representation of P with respect to $F(x)$.

Definition 2. For a given $F(x)$ and n, a point set $P^* = \{x_k^*, k = 1, \dots, n\}$, if $D_F(n, P^*) = minD_F(n, P)$, then the point set P^* is called the set of optimum rep-points with respect to $F(x)$.

The following theorem shows that for a given continue univariate function, the set of optimum rep-points always exits.

Theorem 1. Let $F(x)$ be a continuous univariate function, $F^{-1}(x)$ is its inverse function, then point set $\left\{F^{-1}\left(\frac{2i-1}{2n}\right), i = 1, \dots, n\right\}$ with F-discrepancy $\frac{1}{2n}$ is the set of optimum rep-points with respect to $F(x)$.

Theorem 1 is proved in reference [22]. When $F(x)$ is continue multivariate function, it is hard to find a point set with minimum F-discrepancy. Hence, we need to find a set of points with a low F-discrepancy.

A hypothesis in number theory states that for every given set of $n(\geq 2)$ points and every $s \geq 2$, we have

$$D_F(n, P) \geq c(s) n^{-1} \log^{s-1} n \tag{12}$$

Where $c(s)$ is a positive integer depending on s. Thus, if we find a point set P_n with n points so that the order of $D(n, P_n)$ is near to the right-hand side of Eq.(12), then P_n can be regarded as a set of representative points of $F(x)$.

Definition 3. Let $\{P_n, n \in N\}$ be a sequence of points set with certain structure and N is the set of natural numbers. If

$$D_F(n, P_n) = o\left(n^{-\frac{1}{2}}\right) \quad n \to \infty \tag{13}$$

Then the point set P_n is called rep-points of $F(x)$. A set of rep-points of the uniform distribution on a bounded domain D is called an NT-net on D. Good point set is one of widely used NT-net on D, which is the method used in this paper.

For random number sequence P_n generated by Monte Carlo method, Chung [23] pointed out that

$$D_F(n, P_n) = o(n^{-\frac{1}{2}} (\log\log n)^{\frac{1}{2}}) \tag{14}$$

with probability 1.

By comparing Eq.(13) and Eq.(14), it is clear that NT method including good point set used in this paper is more effective than Monte Carlo method.

3.2 Good point set

Let point $\gamma = (\gamma_1, \dots, \gamma_s) \in C^s$, $\{\gamma\}$ represents fractional part of point γ, if the first n points of

$$P_n = \{(\{\gamma_1 k\}, \dots, \{\gamma_s k\}),\ k = 1,2, \dots\} \tag{15}$$

satisfies

$$D(n, P_n) \leq c(\gamma, \varepsilon) n^{\varepsilon - 1}, \mathrm{n} = 1,2, \dots \tag{16}$$

where $c(\gamma, \varepsilon)$ is a constant depending on γ and ε.

Then the point set P_n is a good point set and γ is a good point.

In practical application, these good points can be used:

(1) Let $p_1, \dots, p_s$ be s primes, take

$$\gamma = (\sqrt{p_1}, \dots, \sqrt{p_s}) \tag{17}$$

(2) Let p be a prime, $q = p^{\frac{1}{s+1}}$, take

$$\gamma = (q, q^2, \ldots, q^s) \quad (18)$$

(3) Let p be a prime which satisfies $p \geq 2s + 3$, take

$$\gamma = \left(\left\{2\cos\left(\frac{2\pi}{p}\right)\right\}, \left\{2\cos\left(\frac{4\pi}{p}\right)\right\}, \ldots, \left\{2\cos\left(\frac{2\pi s}{p}\right)\right\}\right) \quad (19)$$

3.3 Quasi-Monte Carlo method for uncertainty evaluation

The procedure of Quasi-Monte Carlo method for uncertainty evaluation of cylindricity error is described as follow:

Step 1. Generate a NT-net $\{c_k = (c_{k1}, \ldots, c_{ks}), k = 1, \ldots, n\}$ on C^s employing good point method.

There are 10 variables in right-hand side of Eq.(6), that is $x_p, y_p, z_p, x_v, y_v, z_v, a, b, p, q$. So let $s = 10, p = 23$. A good point $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_{10})$ can be generated according Eq.(19) and the good point set $P_n = \{(\{\gamma_1 k\}, \ldots, \{\gamma_{10} k\}),\ k = 1,2, \ldots n\}$ can be generated according Eq.(15). We represent the good point set P_n with a matrix M_{ij} where the column represents the index of point set P_n and the rows represents the dimension of good point, then the matrix M_{ij} can be gotten according to Eq.(15) and Eq.(19).

$$M_{ij} = \left\{\left\{2\cos\left(\frac{2\pi j}{p}\right)\right\} i\right\}, j = 1, \ldots, 10, i = 1, \ldots, n \quad (20)$$

Where $\{\}$ represents getting fractional part.

Step 2. Generate rep-point set based on inverse function sampling method.

Apply inverse function of the standard normal cdf on each element of matrix M_{ij} according to Eq.(21).

$$M'_{ij} = norminv(M_{ij}) \quad (21)$$

Suppose that the variables $x_p, y_p, z_p, x_v, y_v, z_v$ are independent mutually and follow normal distribution, rep-point set for $x_p, y_p, z_p, x_v, y_v, z_v$ can be generated by Eq.(22).

$$\begin{aligned} x_p &= \sigma_{xp} M'_{i1} + \mu_{xp} \\ y_p &= \sigma_{yp} M'_{i2} + \mu_{yp} \\ z_p &= \sigma_{zp} M'_{i3} + \mu_{zp} \\ x_v &= \sigma_{xv} M'_{i4} + \mu_{xv} \\ y_v &= \sigma_{yv} M'_{i5} + \mu_{yv} \\ z_v &= \sigma_{zv} M'_{i6} + \mu_{zv} \end{aligned} \quad (22)$$

Where σ_{xp} , σ_{yp} , σ_{zp} , σ_{xv} , σ_{yv} , σ_{zv} are standard deviations of $x_p, y_p, z_p, x_v, y_v, z_v$ and μ_{xp}, μ_{yp}, μ_{zp}, μ_{xv}, μ_{yv}, μ_{zv} are means of $x_p, y_p, z_p, x_v, y_v, z_v$.

Suppose that variables a, b, p, q are correlated and follows multivariate normal distribution with covariance matrix V. Rep-point set for a, b, p, q can be generated by Eq.(23).

$$\begin{bmatrix} a \\ b \\ p \\ q \end{bmatrix} = R^T \begin{bmatrix} M'_{i7} \\ M'_{i8} \\ M'_{i9} \\ M'_{i10} \end{bmatrix} + \begin{bmatrix} \mu_a \\ \mu_b \\ \mu_p \\ \mu_q \end{bmatrix} \quad (23)$$

Where R is Cholesky factor of covariance matrix V, i.e. the upper triangular matrix satisfies $V = R^T R$ and μ_a, μ_b, μ_p, μ_q are means of a, b, p, q.

Step 3. Evaluate uncertainty of cylindricity error.

For each point in rep-point set with n points, calculate cylindricity error g by Eq.(6), then n results can be obtained. Note n results as $g_i, i = 1, \ldots, n$, the standard uncertainty of cylindricity error can be obtained by calculating standard deviation of g_i.

$$u(g) = \sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left(g_i - \bar{g}\right)^2} \quad (24)$$

3.4 Validation of the proposed method

The proposed method is validated by comparing the results obtained with those given by the methodology proposed by Wen et al [17]. The input data are both Table 1 in reference [17]. The expanded uncertainty of GUM method (GUMM), Adaptive Monte Carlo method (AMCM) and Quasi-Monte Carlo method (QMCM) are listed in Table 1. Adaptive Monte Carlo method is proposed and validated in reference [17]. The result gotten by Quasi-Monte Carlo method is agreed to Adaptive Monte Carlo method, so the proposed method is correct.

Table 1. Comparison between GUMM, AMCM and QMCM

Methods	Sample Size	$g(mm)$	$U_p(mm)$
GUMM		0.0146	0.0052
AMCM	1.93×10^6	0.0146	0.0051
AMCM	1.98×10^6	0.0146	0.0051
QMCM	10^5	0.0146	0.0051

*g is cylindricity, U_p is expanded uncertainty with 95% confidence

4. Test and discussion

4.1 Test Condition

The test is based on dataset in reference [17]. The data is acquired on Miracle NC 454 CMM with Renishaw MH20i Probe Head produced by Qingdao Leader Metrology. Cylindrical part with diameter 36.2 mm and the height 60 mm is measured four times in the same environment condition.

Table 2. Parameters of sampling strategy

Parameters	Value
Diameter of workpiece	36.2 mm
Height of workpiece	60 mm
Number of sampling sections	5
Spacing interval between sections	11 mm
Sampling points per section	12

Roundness profile extraction strategy is employed. Sampling points are taken at a uniform angle from each cross section. 12 points per section are recorded and 5 uniformly spaced cross sections paralleling the XOY plane are sampled. Other parameters of the sampling strategy are listed in Table 2.

4.2 Result of cylindricity error

The cylindrical part is measured 4 times, 4 groups of measurement data are obtained. For each group of obtained measurement data, the cylindricity error based on minimum zone cylinder is calculated according the method in Section 2 and the result is listed in Table 3. The LSM results are also listed on the Table 3 to provide references.

Table 3. Evaluation results of four times measurement data

Para.	a	b	p	q	Cylindricity Error (mm)
(a) 1st measurement data					
SQP	-0.033318	0.012615	0.004768	-0.002024	0.014484
LSM	-0.033716	0.013776	0.004753	-0.002013	0.017161
(b) 2nd measurement data					
SQP	-0.034437	0.014746	0.004741	-0.001958	0.014659
LSM	-0.034833	0.014126	0.004770	-0.001968	0.016019
(c) 3rd measurement data					
SQP	-0.034672	0.015525	0.004784	-0.001986	0.014258
LSM	-0.035372	0.014239	0.004787	-0.001954	0.016757
(d) 4th measurement data					
SQP	-0.032696	0.013120	0.004737	-0.002026	0.014872
LSM	-0.033735	0.013924	0.004749	-0.002012	0.017030

Regard the parameters a, b, p, q as a statistics vector and its covariance matrix can be calculated. The uncertainty $u_a, u_b, u_p, u_q, \ldots, u_{pq}$ can be obtained through the covariance matrix of vector as Eq.(8) and the results is listed in Table 4.

Table 4. Uncertainty and correlative uncertainty values of a, b, p and q (unit: mm^2)

Uncertainty	u_a^2	u_b^2	u_p^2	u_q^2	u_{ab}
Value ($\times 10^{-7}$)	8.72020	18.57885	0.00498	0.01087	-11.48048

Uncertainty	u_{ap}	u_{aq}	u_{bp}	u_{bq}	u_{pq}
Value ($\times 10^{-7}$)	-0.10053	-0.26784	0.10448	0.36727	-0.00008

4.3 Comparison between Monte Carlo and Quasi-Monte Carlo method

Monte Carlo and Quasi-Monte Carlo method are applied to evaluate uncertainty of cylindricity. Different sample size is applied and the relationships between the compute results and sample size are shown in Fig.2 and Fig.3.

(1) Comparisons for Means.

The relationship between the mean and sample size obtained by Monte Carlo method and Quasi-Monte Carlo method for cylindricity error uncertainty evaluation is shown in Fig.2. According Fig.2, the range of mean obtained by Quasi-Monte Carlo method based on good point tends to be stable quickly with less fluctuation than Monte Carlo Method when the sample size increases. The variation of mean by Quasi-Monte Carlo method is almost stable when the number of samples is larger than 10000 while the result by Monte Carlo is not stable until the number of samples is larger than 6×10^6.

(2) Comparisons for standard deviation.

The relationship between the standard deviation and sample size obtained by Monte Carlo method and Quasi-Monte Carlo method for cylindricity error uncertainty evaluation is shown in Fig.3. According to Fig.3, when the number of samples is less than 3×10^4, the variation of standard deviation by Monte Carlo method and Quasi-Monte Carlo method are both large. However, when the number of samples is larger than 3×10^4, the variation of result by Quasi-Monte Carlo method is stable while the result by Monte Carlo method changes relatively large until the number of samples increase to 6×10^4. To acquire the same precision, more samples are needed for Monte Carlo method than Quasi-Monte Carlo method based on good point, which means that the Quasi-Monte Carlo method needs less computation time and has lower complexity.

The comparison between Monte Carlo and Quasi-Monte Carlo method based on good point for cylindricity error uncertainty evaluation shows that the latter needs less samples which means less computation time because of its better representative point set generated.

4.4 The result by GUM, Monte Carlo and Quasi-Monte Carlo method

The results obtained by GUM method (GUMM), Monte Carlo method (MCM) and Quasi-Monte Carlo method based

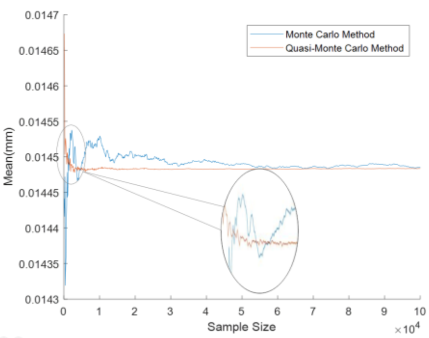

Fig.2. The relationships between the mean and sample size obtained by Monte Carlo and Quasi-Monte Carlo method

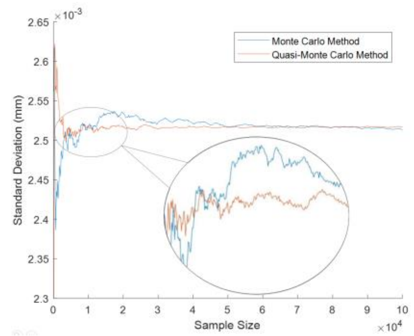

Fig.3. The relationship between the standard deviation and sample size obtained by Monte Carlo and Quasi-Monte Carlo method

on good point (QMCM) are listed in Table 5. According Table 5, Because the theoretical uncertainty can't be obtained, so we choose the result by Monte Carlo method with 10^7 samples as reference, which is relatively credible because of its largest samples. Though the computation time of GUM method is least, however, it has a largest deviation relative to reference in all the items listed in Table 5 for its use of linear approximation. The uncertainty by Quasi-Monte Carlo methods with 10^4, 2×10^4, 10^5 samples are closer to the reference than Monte Carlo method with the same number of samples, so the Quasi-Monte Carlo method based on good point can get an equal precision with less sample size.

Table 5. Uncertainty evaluation results

Methods	Sample Size	g(μm)	$\mathrm{U_p}$ (μm)	Computation time(s)
GUMM		14.48371	4.88105	0.0025
MCM	10^4	14.46857	4.89241	0.0674
MCM	2×10^4	14.47381	4.90571	0.1227
MCM	10^5	14.48539	4.92624	0.5791
MCM	10^6	14.48389	4.93697	5.6935
MCM	10^7	14.48350	4.93306	59.137
QMCM	10^4	14.48394	4.93185	0.0646
QMCM	2×10^4	14.48370	4.93512	0.1218
QMCM	10^5	14.48366	4.93333	0.6011

*g is cylindricity, U_p is expanded uncertainty with 95% confidence

5. Conclusion

In this paper, a Quasi-Monte Carlo method based on good point set for uncertainty evaluation for cylindricity error is proposed. The principle and procedure of proposed method is described. The proposed method is tested in comparison with GUM method and Monte Carlo method. The tests show that Quasi-Monte Carlo method needs less samples than Monte Carlo method and can attain higher precision.

The limitation of the proposed method is that it also needs to generate relative large samples when better precision is required, so in the future, the proposed method could be improved by find a more suitable rep-point set for function and could be applied into other complex situations. The proposed method can be extended to other geometric/dimensional tolerances by substituting corresponding measurement model.

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