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Computing clearances and deviations in over-constrained mechanisms

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Abstract

A manufactured over-constrained mechanism can't be assembled and can't be mobile without clearance in its joints. During the conceptual design phase, designers must specify the degree precision for the parts without proper quantitative decision-making tools. Those decisions have a great impact on the product's quality and cost. In this paper, clearance and deviation are defined for an assembly of rigid parts. A geometric model is presented to measure those values in a given mechanism using the TTRS model. Optimum values are computed by minimizing a weighted function of both clearance and deviation.

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Keywords: Clearance; Deviation; Over-constrained mechanism; Mobility; Conceptual design;

1. Introduction

An over-constrained assembly can only be assembled or mobile under strict geometrical conditions on its dimensions. Once manufactured, due to the imperfection on parts dimensions, those conditions can't be respected, as shown by Meng et al. [1]. To deal with this problem, clearances are generally added in the system's joints. These clearances impact the quality of the assembly thus creating deviation from the nominally designed assembly.

However, the effect of clearances on linkages have been studied widely. Frisoli et al. have studied the impact of clearances on the position of parallel manipulators [2]. Kumaraswamy et al. developed a generic method to study the impact of clearance using screw theory [3] and Cammarata developed a method to study clearance affected overconstrained mechanisms [4]. Most studies model joints using the actual surfaces of the parts. However, during the conceptual phase the mechanism only exists as minimal geometric elements and contacts in joints can't be represented. Currently, no efficient tools are available to help designers select correct values for those clearances during the conceptual design phase.

In this paper, clearances in imperfect joints and deviation of a manufactured assembly are defined. Clearance is a distance measured between two manufactured parts while deviation is a distance measured between a manufactured assembly and its nominal, virtual and unique equivalent. The deviation measured here is between two mechanism as opposed to a geometric deviation between a manufactured part and its nominal definition resulting from the manufacturing process. A geometrical model is presented to give formal expressions of those two measures using the TTRS (Technologically and Topologically Related Surfaces) [5] model. Then, a weighted function is minimized to find the relative position of the manufactured links with respect to the nominal ones. Values for clearances and deviations are also computed. Thanks to these results the minimum clearance needed in the assembly's joints is computed and the quality of the manufactured assembly is given. A simulation is then conducted to compute the maximum and average values for clearance and deviation. By sampling a high number of possible manufactured mechanisms, designers can choose clearances and tolerances that allow mobility or assemblability.

This paper will describe the model and representations for a generic closed loop linkage in section 2 and 3, gives definitions for clearance and deviation in section 4 and computes the

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values for clearance and deviation on an example in section 6 using the optimization described in section 5.

2. Geometrical model for a link

To represent a manufactured linkage, a model is first given for a nominal part in order to build a nominal assembly. Then, manufactured parts are represented as a variation around those nominal parts using a soft-gage specified by the designer.

2.1. Nominal link

2.1.1. Conceptual model of a link

In an assembly, parts are connected two by two by joints. In a joint, two functional surfaces are involved, each belonging to one of different part. Functional surfaces are represented by a TTRS and its characteristic MGRE (Minimum Geometric Reference Elements). Finally, each part of an assembly is modeled by an association of several TTRS.

For instance, a link between two revolute joints is given by four points defining the height and the axis of both physical revolute joints as shown in Fig. 1. Association of those geometrical elements defines a TTRS. The conceptual model of a point-axis/point-axis TTRS is given in Fig. 2.

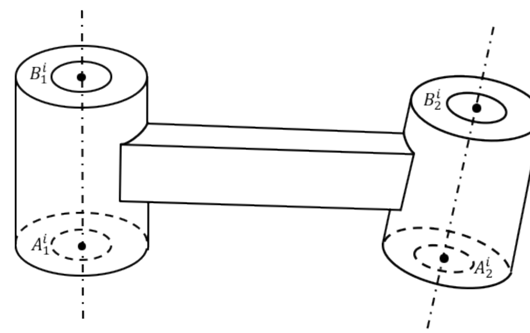

Fig. 1. A nominal link *i* between two revolute joints

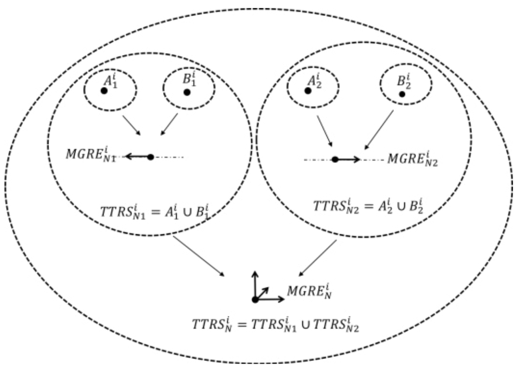

Fig. 2. TTRS model for a nominal link *i* between two revolute joints

A TTRS for a nominal link *i* between two revolute joints is defined by $TTRS_N^i$ given by the association of two TTRS involved in each joint: $TTRS_{N1}^i$ and $TTRS_{N2}^i$.

$$TTRS_N^i = TTRS_{N1}^i \cup TTRS_{N2}^i \qquad (1)$$

$TTRS_N^i$ is called a part-level TTRS while $TTRS_{N1}^i$ and $TTRS_{N2}^i$ are called joint-level TTRS.

2.1.2. Geometric construction of a link's MGRE

The four points of $TTRS_N^i$ named A_1^i, B_1^i, A_2^i and B_2^i define the two axes Δ_1^i, Δ_2^i of the two revolute joints as in Fig.3. Two points O_1^i and O_2^i are chosen as the respective center of the segments $[A_1^i, B_1^i]$ and $[A_2^i, B_2^i]$. The coordinates of each points are given in a local reference axes system $S_N^i = [O_1^i, \overrightarrow{x^i}, \overrightarrow{y^i}, \overrightarrow{z^i}]$ with $\overrightarrow{x^i}$ parallel to the common perpendicular between the two axes, $\overrightarrow{y^i}$ defined by the cross product of $\overrightarrow{z^i}$, $\overrightarrow{x^i}$ and $\overrightarrow{z^i}$ collinear to $\overrightarrow{A_1^i B_1^i}$.

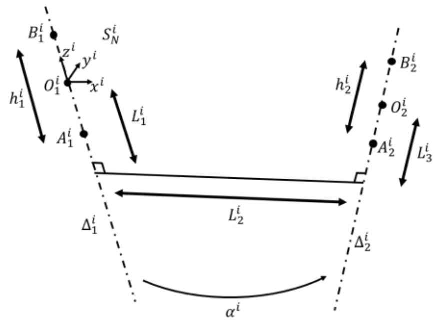

Fig. 3. Parametrization of a nominal link *i* between two revolute joints

A nominal link *i* is parametrized by the set of parameter $d_N^i = [L_1^i, L_2^i, L_3^i, \alpha^i, h_1^i, h_2^i]$ expressing the coordinates of points A_1^i, B_1^i, A_2^i and B_2^i in the reference frame S_N^i. Coordinates $L_1^i, L_2^i, L_3^i, \alpha^i$ are the Sheth-Uicker [6] [7] coordinates for a point-axis/point-axis TTRS.

$$A_1^i = \begin{bmatrix} 0 \\ 0 \\ -\frac{h_1^i}{2} \end{bmatrix} \quad B_1^i = \begin{bmatrix} 0 \\ 0 \\ \frac{h_1^i}{2} \end{bmatrix}$$

$$A_2^i = \begin{bmatrix} L_2^i \\ 0 \\ -L_1^i \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha^i) & -\sin(\alpha^i) \\ 0 & \sin(\alpha^i) & \cos(\alpha^i) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ L_3^i - \frac{h_2^i}{2} \end{bmatrix}$$

$$B_2^i = \begin{bmatrix} L_2^i \\ 0 \\ -L_1^i \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha^i) & -\sin(\alpha^i) \\ 0 & \sin(\alpha^i) & \cos(\alpha^i) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ L_3^i + \frac{h_2^i}{2} \end{bmatrix} \qquad (2)$$

2.2. Manufactured link

In this section, a model of a manufactured part is presented. The manufactured part is generated from the nominal part-level TTRS defined previously and a soft-gage specified by the designer.

For this study, the chosen soft-gages are defined as a cylindrical tolerance zone measured relatively to a reference point-axis. The manufactured part *i* is represented by a part-level TTRS named $TTRS_M^i$ which is composed of a joint-level

TTRS for every joint-level TTRS of the nominal part. The manufactured point-axis $MGRE_{M1}^{i}$ is coincident to the reference point-axis of the soft-gage and the manufactured point-axis $MGRE_{M2}^{i}$ is chosen in the tolerance zone of the soft-gage. The acceptable manufactured axis for a part *i* is defined by two points C_2^i and D_2^i expressed by four parameters $(r_1^i, r_2^i, \theta_1^i, \theta_2^i)$ defining two vectors perpendicular to Δ_2^i. Those points are given by the following cylindrical coordinates around the soft-gage axis expressed in the reference frame S_M^i:

$$C_2^i = O_2^i + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha^i) & -\sin(\alpha^i) \\ 0 & \sin(\alpha^i) & \cos(\alpha^i) \end{bmatrix} \cdot \begin{bmatrix} r_1^i \cos(\theta_1^i) \\ r_1^i \sin(\theta_1^i) \\ -\frac{h_2^i}{2} \end{bmatrix}$$
$$D_2^i = O_2^i + \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha^i) & -\sin(\alpha^i) \\ 0 & \sin(\alpha^i) & \cos(\alpha^i) \end{bmatrix} \cdot \begin{bmatrix} r_2^i \cos(\theta_2^i) \\ r_2^i \sin(\theta_2^i) \\ \frac{h_2^i}{2} \end{bmatrix} \quad (3)$$

With :

$0 \le r_{1,2}^i \le R^i$

$0 \le \theta_{1,2}^i \le 2\pi$

R^i being the radius of the chosen tolerance zone for the part *i*.

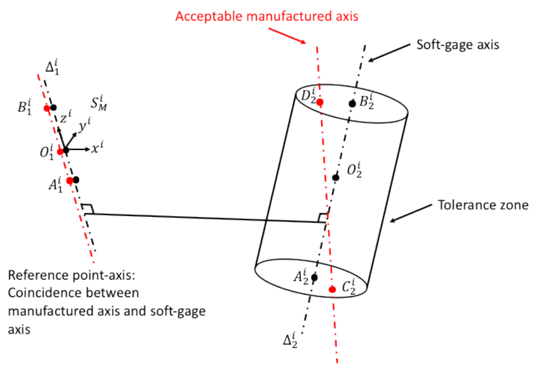

Fig. 4. Soft-gage and manufactured link *i*

For each nominal part, a manufactured link is generated using this model thus creating one manufactured part for each nominal one. The position of the manufactured MGRE in the soft-gage is defined by the set of parameters g_M^i.

$$g_M^i = \left[r_1^i, r_2^i, \theta_1^i, \theta_2^i, R^i \right] \quad (4)$$

A manufactured link is then parametrized by a set of parameters d_M^i:

$$d_M^i = d_N^i \lceil \; g_M^i \quad (5)$$

3. Geometrical model for a linkage

A nominal linkage is created by defining a list of joints between the parts.

3.1. Nominal linkage

3.1.1. Nominal joint

A nominal joint *i/j* is created by connection of two joint-level TTRS from two different parts. For instance, a joint between two point-axis MGRE from parts *i* and *j* using the model given in Fig. 5 is expressed as:

$$Joint^{i/j} = TTRS_{N2}^{i} \lceil \; TTRS_{N1}^{j} \quad (6)$$

Then, a list of positional parameters sets the relative positions $p_{i/j}$ between each part. For a revolute joint, the relative position between the two links is given by only one parameter $\beta_{i/j}$ defined by the angle between $\vec{x}_i$ and $\vec{x}_j$.

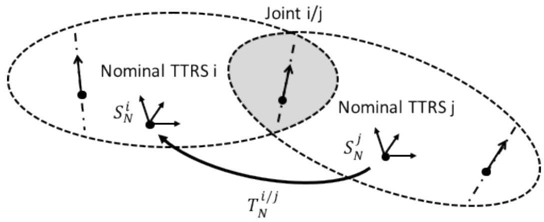

Fig. 5. Nominal joint between two TTRS

The relative position of the part *j* in regard of part *i* is then given by the joint *i/j* and the position $p_{i/j}$. This relative position is expressed by a spatial transformation $T_N^{i/j}$ such that:

$$S_N^i = T_N^{i/j}\left(S_N^j\right) \quad (7)$$

3.1.2. Nominal linkage

A closed-loop linkage is then created by a list of part-level TTRS and a list of joints. Each nominal assembly is then represented by a set of nominal dimensional d_N^i parameters and positional parameters $p_{i/j}$. For instance, the parameters for a linkage using only revolute joints are:

$$d_N^i = \left[L_1^i, L_2^i, L_3^i, \alpha^i, h_1^i, h_2^i \right]$$
$$p_{i/j} = [\beta_{i/j}] \quad (8)$$

In a closed single-loop linkage, the dependency between parameters $p_{i/j}$ is expressed by the following closing equation:

$$\subseteq T_N^{i/j} = I \quad (9)$$

3.2. Manufactured joint

A manufactured joint is built by positioning each manufactured link relatively to the nominal ones. Each manufactured MGRE is positioned relatively to the nominal MGRE using a spatial transformation T_M^i.

A manufactured joint between part *i* and *j* is entirely characterized by the manufactured dimensions d_M^i , d_M^j , nominal dimensions d_N^i, d_N^j, nominal positions $T_N^{i/j}$ and transformations T_M^i, T_M^j. Figure 6 shows an imperfect joint between two manufactured TTRS positioned relatively to their nominal equivalent.

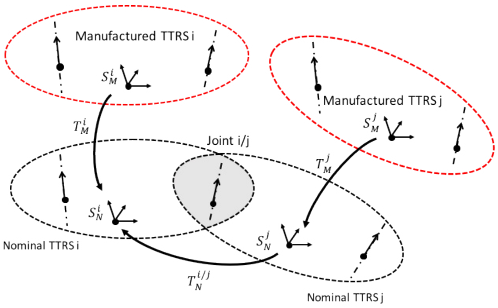

Fig. 6. Imperfect joint between two manufactured TTRS

A closed-loop linkage is then created by positioning a manufactured part-level TTRS for each corresponding nominal part of the nominal linkage.

4. Definition of clearance and deviation

To complete the conceptual model, two measures are defined to evaluate the functionality and the quality of the studied manufactured assembly. Minimization of those measures enables to compute the position of the manufactured linkage relatively to the nominal one.

4.1. Clearance

Local clearance is defined as a measure of a distance between two manufactured MGRE in a joint. In a revolute joint, this measure gives the designer the needed clearance between the axis of a hole and a pin in a joint. Clearance is measured between two reference geometries from two different joint-level TTRS sharing the same joint. Figure 6 shows a clearance measure between two MGRE from parts *i* and *j*.

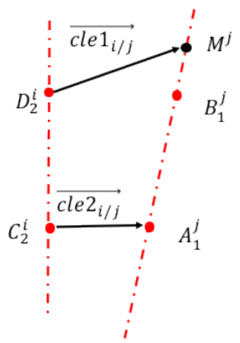

Fig. 7. Clearance between two manufactured axes

Local clearance is expressed by the function $L_{j/i}$ which characterizes an imperfect revolute joint between parts *i* and *j* as shown in Fig. 8.

$$L^{j/i} = \begin{bmatrix} T_M^i\left(D_2^i\right) - T_N^{i/j} * T_M^j\left(M^j\right) \\ T_M^i\left(C_2^i\right) - T_N^{i/j} * T_M^j\left(A_1^j\right) \end{bmatrix} = \begin{bmatrix} \overrightarrow{cle1_{j/i}} \\ \overrightarrow{cle2_{j/i}} \end{bmatrix} \quad (10)$$

M being a sliding point along the *(A₁, B₁)* axis. Similar function can be expressed for other types of joint.

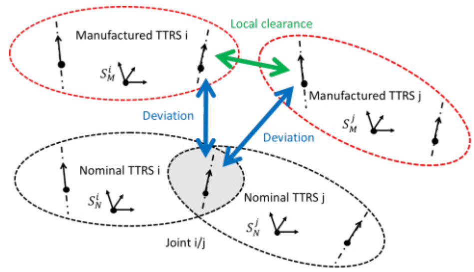

Fig. 8. Local clearance and local deviation in an imperfect joint

Local clearance is measured for each joint and a global clearance (CLE) is expressed as the norm of all local clearances of the linkage.

$$CLE = \frac{1}{2}\sum\left(\left\|\overrightarrow{cle1_{i/j}}\right\|^2 + \left\|\overrightarrow{cle2_{i/j}}\right\|^2\right) \quad (11)$$

4.2. Deviation

Local deviation is defined as a measure of a distance between a joint-level TTRS from a nominal part and its manufactured equivalent. This measure expresses the global deviation from a manufactured assembly relatively to a virtual nominal assembly. Fig. 8. shows the two deviations defined around a joint *i/j*.

Deviation uses the same function as clearance but between a nominal MGRE and a manufactured MGRE. The local deviation between $MGRE_{N2}^i$ and $MGRE_{M2}^i$ is given by:

$$L_2^i = \begin{bmatrix} A_2^i - T_M^i\left(C_2^i\right) \\ B_2^i - T_M^i\left(M^i\right) \end{bmatrix} = \begin{bmatrix} \overrightarrow{dev1_2^i} \\ \overrightarrow{dev2_2^i} \end{bmatrix} \quad (12)$$

A similar local deviation L_1^i is also expressed between $MGRE_{N1}^i$ and $MGRE_{M1}^i$ and gives the expressions of $\overrightarrow{dev1_1^i}$ and $\overrightarrow{dev2_1^i}$.

A deviation can be measured for every manufactured axis in the linkage (2 axes per links). The global deviation (DEV) can be expressed as the norm of the local deviations.

$$DEV = \frac{1}{4}\sum\left(\left\|\overrightarrow{dev1_1^i}\right\|^2 + \left\|\overrightarrow{dev2_1^i}\right\|^2 + \left\|\overrightarrow{dev1_2^i}\right\|^2 + \left\|\overrightarrow{dev2_2^i}\right\|^2\right) \quad (13)$$

5. Optimization and simulation

5.1. Optimization

An optimization method is used to compute the values of the transformation parameters T_M^i of every manufactured part relatively to the nominal linkage. The objective function is chosen as a combination of clearance and deviation and the resulting values for both measures are obtained after the computation. The objective function is defined by:

$$Obj = X.CLE + Y.DEV \tag{14}$$

X and Y are chosen by the designer as parameters as well as the nominal dimension d_N^i and the dimensions of the tolerance zones. The objective function can then be expressed as a function of the transformation variables T_M^i , the variables $(r_1^i, r_2^i, \theta_1^i, \theta_2^i)$ defining the position of the manufactured axes in the tolerance zones, and the current position of the nominal mechanism $p_{i/j}$. The optimization is then defined as:

$$\widehat{T_M^i} = \underset{T_M^i}{\arg\min}\left(Obj\left(T_M^i, r_1^i, r_2^i, \theta_1^i, \theta_2^i, p_{i/j}\right)\right) \tag{15}$$

Where $\widehat{T_M^i}$ are the values for each T_M^i minimizing the objective function.

5.2. Simulation

A high number of acceptable manufactured assemblies is generated to compute the minimum clearance values to guarantee the assemblability and/or mobility of the nominal linkage. The problem is optimized for each instance of manufactured assembly in order to determine the clearance and deviation values. For a mobile mechanism, the manufactured dimensions and the position of the nominal linkage can vary for each sample to compute the maximum values for clearance and deviation of any possible manufactured assembly. The resulting optimization is defined as:

$$\widehat{T_M^i}(k) = \underset{T_M^i}{\arg\min}\left(Obj\left(T_M^i, r_1^i(k), r_2^i(k), \theta_1^i(k), \theta_2^i(k), p_{i/j}(k)\right)\right)$$

For each instance k, a different manufactured assembly is positioned around another nominal position $p_{i/j}(k)$ as shown in Fig. 9.

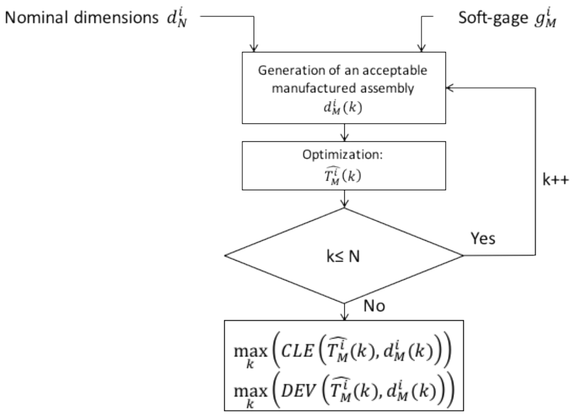

Fig. 9. Computation of maximum clearance and deviation

Global maximum values are then obtained for clearance and deviation.

6. Case study

6.1. 6R linkage – Industrial powder mixer

The studied mechanism is a mobile 6R linkage similar to a powder mixer shown in Fig. 10 and studied in [8] with nominal dimensions given in Table 1. Those dimensions are measured from a 3D CAD model shown in Fig. 10 using the Sheth-Uicker parametrization. The mechanism is built with 6 parts linked by revolute joints, the total number of variable for the optimizations is 36 (6 parameters positioning each manufactured parts).

Table 1. Nominal parameters for the studied linkage

Part	L_1 (mm)	L_2 (mm)	L_3 (mm)	α(rad)
1	0	200	0	0
2	45	0	0	π/2
3	0	115.5	0	π/2
4	0	115.5	0	π/2
5	0	115.5	0	π/2
6	0	0	-45	π/2

The radius of the tolerance zones is specified as 0.1mm. The software used is Maple 2017 with the Optimization package. The studied mechanism is mobile. Being in the conceptual design phase, no statistical model is known for the parts geometrical variations. The manufactured parts are generated using harmonic functions describing a uniform distribution for the parameters g_M^i similarly to Rameau et al. in [9] in order to cover all possible configurations.

$$\begin{aligned}
r_1^i(k) &= R^i e^{-\frac{1}{k}} \sin\left(\omega_{r1}^i k\right) \\
r_2^i(k) &= R^i e^{-\frac{1}{k}} \sin\left(\omega_{r2}^i k\right) \\
\alpha_1^i(k) &= 2\pi e^{-\frac{1}{k}} \sin\left(\omega_{\alpha 1}^i k\right) \\
\alpha_2^i(k) &= 2\pi e^{-\frac{1}{k}} \sin\left(\omega_{\alpha 2}^i k\right)
\end{aligned} \tag{16}$$

With:

$$\omega_{r1}^i = \frac{2\pi}{20+i};\ \omega_{r2}^i = \frac{2\pi}{26+i};\ \omega_{\alpha 1}^i = \frac{2\pi}{32+i};\ \omega_{\alpha 2}^i = \frac{2\pi}{38+i};$$

$$i \in [\![1;6]\!]$$

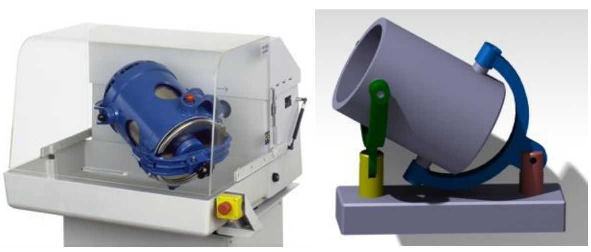

Fig. 10. Powder mixer and its CAD model

6.2. Results for the 6R linkage

Simulations are performed on N=1000 samples. X and Y values are set on 1 to give the same weight to clearance and deviation. The following figures show the different values for clearance and deviation for each instance *k*. Fig. 11 shows the 6 different values for local clearances in the linkage for each *k*, Fig. 12 shows the values for deviations for each sample and table 2 gives the maximum, average and standard deviation for both measures.

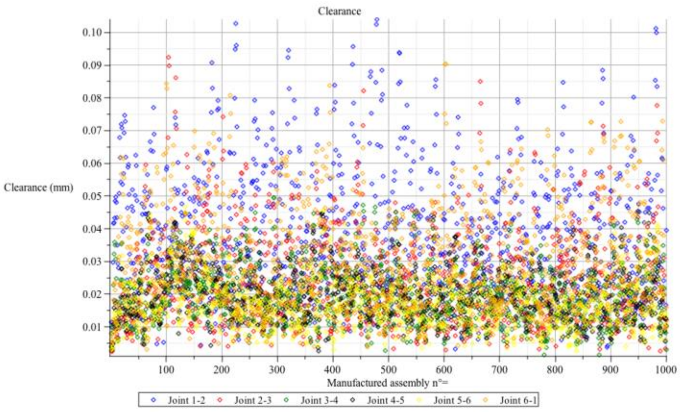

Fig. 11. Values of local clearances for the 6R powder mixer

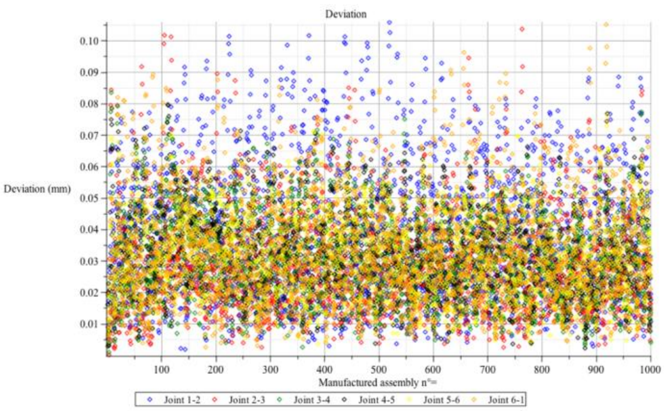

Fig.12. Values of local deviations for the powder mixer

Table 2. Numerical results for the powder mixer

	Max (mm)	Average (mm)	Standard deviation (mm)
Clearance	0.104	0.025	0.090
Deviation	0.106	0.033	0.096

The computed values for maximum local clearances give the minimum clearances to apply in each joint to guarantee the existence of every possible positions for any possible acceptable manufactured linkage. Optimization time is 3464s. This computation time is obtained on a 2015 MacBook Pro equipped with a 2.8GHz processor and 16Gb of RAM.

7. Conclusion

A geometrical and conceptual model has been developed to model a nominal linkage and its manufactured equivalent. The proposed model uses the TTRS theory. It allows the designer to study clearances and deviations during the conceptual design phase. From this model, clearance and deviation are defined and computed through a simulation to compute the maximum possible values. Results can then be used to determine the minimum clearance to apply to joint clearance to allow mobility or assemblability. Different objectives can be chosen by the designer; a weight can be assigned to both objectives depending of the application and a weight could also be applied to each independent joint.

This model will be applicable to any closed-loop over-constrained linkage by extending the definitions of clearance and deviation to other types of joints. The method developed to compute the values of clearance and deviation is also applicable to non-overconstrained linkage.

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