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Comparison of different methods for scrap rate estimation in sampling-based tolerance-cost-optimization

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Abstract

Tolerance-cost-optimizations help to find optimal tolerances for a given tolerance design, which achieve minimum total costs while satisfying the quality requirements. These requirements are commonly expressed as scrap rates in ppm-range.
This paper presents several methods for the scrap rate estimation in sampling-based tolerance-cost-optimizations. Moreover, the proposed approaches are applied to a typical case study and compared in terms of their required computation time and the obtained optimization results. The novelty of the contribution can be found in comparing different scrap rate estimation techniques for sampling-based tolerance-cost-optimizations and giving recommendations to researchers as well as practitioners.

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Keywords: tolerance-cost-optimization; scrap rate; Latin-Hypercube-Sampling

1. Introduction

In times of industry 4.0 and increased digitalization in manufacturing, geometrical variations management becomes even more important as an interdisciplinary process aiming at ensuring quality goals despite the presence of geometrical part deviations [1,2]. The resulting opportunities of the increasing digitalization facilitate the interconnection of product development, manufacturing, and quality management. Particularly, the availability of big data from all over the product-life-cycle opens versatile opportunities. In this context, the consideration of manufacturing costs in the geometry assurance process helps to decrease fabrication costs while maintaining the customers' confidence [3]. Intelligent methods, such as artificial neural networks, are adapted for creating proper cost-models for tolerance-cost-optimization [4]. Motivated by the increasing availability of process-specific data and suitable methods for handling and integrating them into product and process development, sampling-based tolerance-cost-optimization is coming more into industrial focus as a method which can help to identify the tolerance values for a given tolerance design to achieve minimum total costs and satisfy the quality requirements [3,5]. Thus, sampling-based tolerance-cost-optimization is based on sampling methods, such as Latin-Hypercube-Sampling, for analyzing the tolerance design. The quality requirements are typically expressed by the scrap rate which is measured in small ranges of parts-per-million (ppm) according to the quality standards of the Six Sigma Philosophy [6]. Thus, efficient scrap rate estimation techniques are indispensable for the tolerance-cost-optimization process.

Nomenclature

C	costs
iq	inequality condition
LSL / USL	lower / upper specification limit
n	sample size
t	tolerance
x_{ij}	machine selection parameter
X	dimension
Y	functional key characteristic (fkc)
$z, \hat{z}$	required scrap rate, estimated scrap rate
ρ	probability density function (pdf)
Φ	cumulative distribution function (cdf)

The aim of this contribution is to provide recommendations for researchers and practitioners regarding the application of suitable methods for the scrap rate estimation and the choice of sample sizes in sampling-based tolerance-cost-optimization. For this purpose, firstly the state of the art and related work are reflected in section 2. In order to emphasize the importance of suitable scrap rate estimation techniques, the general approach of sampling-based tolerance-cost-optimization is presented in

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section 3 and different methods for the scrap rate estimation are introduced. In section 4, a case study is used to compare the methods in terms of their required computational effort and the obtained results. Finally, section 5 summarizes the results.

2. State of the art and related work

In product design, the tolerance-cost-dilemma is well known: While loose tolerances allow cheap fabrication but lead to decreased product quality, tight tolerances may result in high-quality products but imply increased costs [7]. For more than five decades, numerous authors have been spending their efforts on developing different approaches to tolerance synthesis in order to solve this conflict [8]. At the beginning, graphical methods according to Peters [9] or Latta [10] as well as analytical approaches according to Wilde and Prentice [11] met the requirements. Driven by increasing computing power, especially optimization-based tolerance synthesis became more important in recent years [8]. Besides mathematical, deterministic optimization algorithms, such as nonlinear programming [7] or branch and bound method [12], the application of meta-heuristic algorithms, such as genetic algorithms [13] or particle-swarm-optimization [5], became popular for tolerance-cost-optimization in modern days [8]. In contrast to non-deterministic algorithms, metaheuristic optimizers cannot guarantee to find the optimum [14]. However, they compensate this drawback with their advantages: Due to their problem-independent proceeding, they are able to find the global optimum of nonlinear, multimodal functions without the need of gradients or derivatives [15]. Moreover, tolerance-cost-optimizations are not limited to one single objective function. Multi-objective optimizations are applied to consider several objectives at the same time, e. g. the Taguchi Loss and total manufacturing costs [15].

Concerning the evaluation of objective and constraint functions, the application of worst-case-, root-sum-square-, or estimated-mean-shift-approaches are quite common in tolerance-cost-optimizations [13]. In contrast to that, sampling-based methods are rarely used in tolerance-cost-optimizations compared to tolerance analysis. In this context, sampling-based methods are only used at times, such as Monte-Carlo-Simulation in [5], [16], or [17], while the use of Latin-Hypercube-Sampling (LHS) in the context of tolerance-cost-optimization is not known to the authors.

Nowadays, various methods for the tolerance representation, such as Deviation Domains [18], Skin Model Shapes [19,20], or vectorial models [21], are well known in theory as well as in practice. Because of its short computation times [22], the vectorial representation is very common in case of tolerance-cost-optimization, while also feature- or point-based representations in commercial CAT-software [3,23] are in focus of tolerance-cost-optimization. Moreover, systems in motion considering internal and external influences can be optimized [5].

Particularly for time-assuming tasks efficient and appropriate methods are needed for scrap rate estimation. In this regard, the choice of the adequate scrap rate estimation technique and of the sample size strongly influences the optimization process and results. However, within the scope of sampling-based tolerance-cost-optimization, this subject has not been discussed in detail so far.

3. Application of scrap rate estimation techniques in sampling-based tolerance-cost-optimization

First, this section describes the general idea of cost-driven sampling-based tolerance-cost-optimization, before three different scrap rate estimation methods are presented.

3.1. Background

Cost-driven tolerance-cost-optimization is an applicable method to achieve a cost-optimal tolerance design. In this regard, the optimization problem can be formulated as [5,8]:

$$\begin{aligned} \text{Minimize} \quad C_{\text{tot}}(\boldsymbol{t}) &= \textstyle\sum_{i=1}^{I}\sum_{j=1}^{J} x_{\text{ij}} \cdot C_{\text{ij}}(t_{\text{i}}), \\ \text{subject to :} \quad \hat{z}(\boldsymbol{t}) &= 1 - \textstyle\int_{LSL}^{USL} \rho(Y(\boldsymbol{t}))\,dx \leq z, \\ t_{\text{i}} &\in [t_{\text{i,min}}, t_{\text{i,max}}], \\ x_{\text{ij}} &\in \{0;1\}. \end{aligned} \tag{1}$$

The tolerances $\boldsymbol{t}$ are the design variables to be optimized. The nonlinear objective function determines the total costs C_{tot} which consist of the direct costs C_{i} for each machine j. The machine selection parameter x_{ij} decides whether a machine is chosen or not for providing the current tolerance t_{i} [8]. Thus, the tolerances are restricted by the machine or process limits $t_{\text{i,min}}$ and $t_{\text{i,max}}$. The estimated scrap rate $\hat{z}$ for each functional key characteristic (fkc) Y and the required scrap rate z form the nonlinear inequality conditions iq as $iq = \hat{z} - z$.

The process of tolerance-cost-optimization is shown in Fig. 1. Based on an initial set of tolerance values, the optimization algorithm iteratively tries to improve these tolerance values until a termination criterion is met. The set of tolerances $\boldsymbol{t} = [t_1, \ldots, t_I]^{\text{T}}$ is chosen by considering the results of the previous optimization runs. The total manufacturing costs are described by suitable **tolerance-cost-models**.

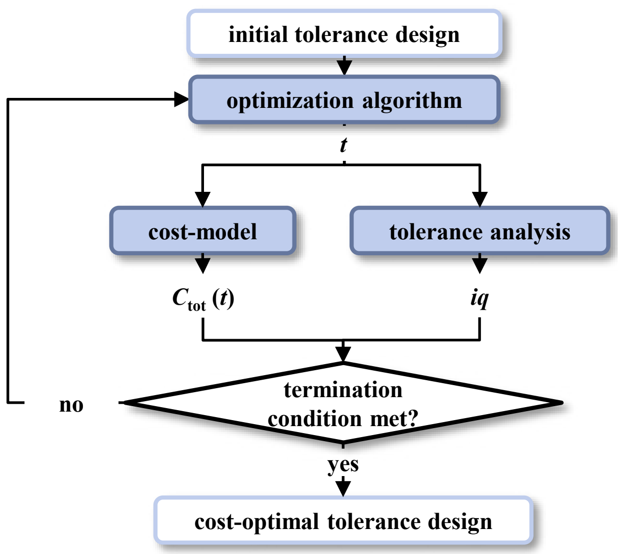

Fig. 1. Flowchart of cost-driven tolerance-cost-optimization.

The **tolerance analysis** is used to evaluate the effects of the input tolerances on the key characteristic of the assembly. Based thereon, the rejects per ppm of each key charactistic are estimated. The procedure is shown in Fig. 2. In a first step, the distribution of each dimension must be generated based on a

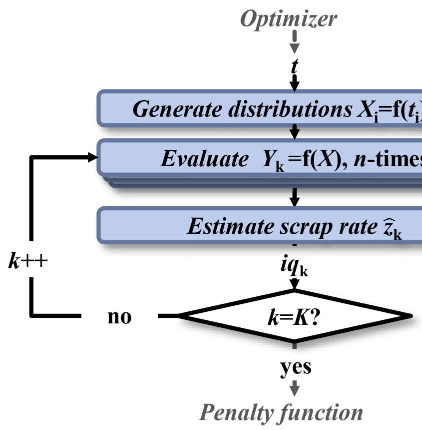

Fig. 2. Flowchart of tolerance analysis and scrap rate estimation.

given input tolerance t and a given distribution type. In the next step, the functional key characteristic Y_k is calculated n times according to the sample size and the scrap rate $\hat{z}_k$ is estimated from the resulting distribution. This loop must be repeated for each functional key characteristic. Population-based algorithms such as PSO or GA use more than one individual to explore the search space [14]. Thus, this procedure is iterated according to the number of individuals for each generation.

A common approach to consider the inequality conditions within the optimization process is the use of penalty functions. With the help of an adequate penalty term, the information about the objective as well as the inequalities iq are combined and serve as a basis for further optimization steps [24]. Differences in determining the inequalities influence the optimization process immensely. For this reason, different methods for scrap rate estimation are presented in the following.

3.2. Scrap rate estimation techniques

In modern manufacturing, scrap rates are commonly expressed as parts-per-million. According to eq. 1 the scrap rate is defined as the region outside the upper (USL) and lower (LSL) specification limits (see Fig. 3).

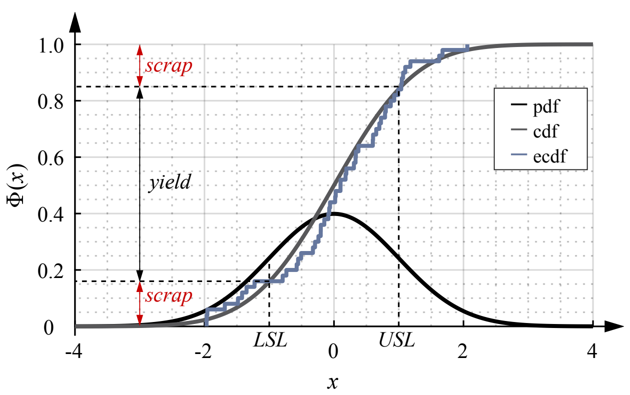

Fig. 3. Cumulative distribution function as basis for scrap rate estimation.

Thus, they define whether a product can be considered as functional or not. In order to identify the scrap rate for given simulation data, the cumulative frequency has to be determined at the specification limits. Mathematically spoken, the yield is calculated via the cumulative distribution function (cdf) Φ:

$$\hat{z} \;=\; 1 - yield = 1 - [\Phi(USL) - \Phi(LSL)]. \tag{2}$$

In general, the methods can be classified into two main groups, namely **parametric** and **non-parametric** estimation techniques (see Fig. 4).

If the distribution type of the key characteristic is known, the **cumulative distribution function of an estimated distribution** can be used. However, the assumption regarding the distribution type must be correct and proven. Otherwise, this can cause incorrect results. In this contribution, the cdf of a normal-distribution (cdf gauss) is considered. The mean and standard deviation of the resultant key characteristic distribution is required for this method.

In contrast to that, the proposed non-parametric methods are universally usable. Several approaches for the expression of the **empirical cumulative density function** (ecdf) can be found in the literature. The following formula is an easy way to determine the scrap rate in the context of tolerance analysis [25]:

$$\hat{z} \;=\; \frac{1}{n}\cdot\left[\sum_{i=1}^{n} 1_{\{Y_i<LSL\}} + \sum_{i=1}^{n} 1_{\{Y_i>USL\}}\right]. \tag{3}$$

The **kernel density estimation** is another non-parametric method to estimate the cdf (cdf kde). It employs a kernel density estimator to approximate the shape of the distribution [26]. Though different smoothing kernel types are presented in literature, the bandwidth of a kernel mainly influences the obtained results [26,27]. In the following, a Gaussian kernel is used.

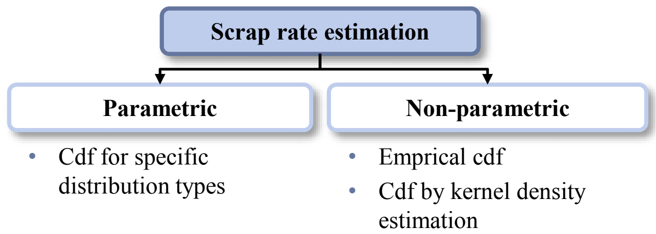

Fig. 4. Classification of methods for the scrap rate estimation.

4. Comparison of the scrap rate estimation techniques

In the following, the presented scrap rate estimation techniques are applied to a case study. The purpose of this study is to demonstrate the influence of the scrap rate estimation technique in combination with the chosen sample size on the results of a tolerance-cost-optimization.

4.1. Case Study

The case study is a typical problem from scientific literature for tolerance analysis as well as for synthesis as shown in Fig. 5. The gap between the left shaft shoulder and the sleeve is considered as a linear functional key characteristic [13]:

$$Y \;=\; -X_1 + X_2 - X_3 + X_4 - X_5 + X_6 - X_7 \tag{4}$$

First, the assembly is optimized with focus on minimum total costs. The fundamental process is known from Fig. 1. The tolerance-cost-function for each machine follows an exponential approach [28]:

$$C_{ij}(t_i) = a_{ij} + b_{ij}\cdot e^{-c_{ij}\cdot t_i} \tag{5}$$

The sleeve and the roller bearings are purchased parts with fixed tolerances and fixed costs. In order to reduce set-up costs,

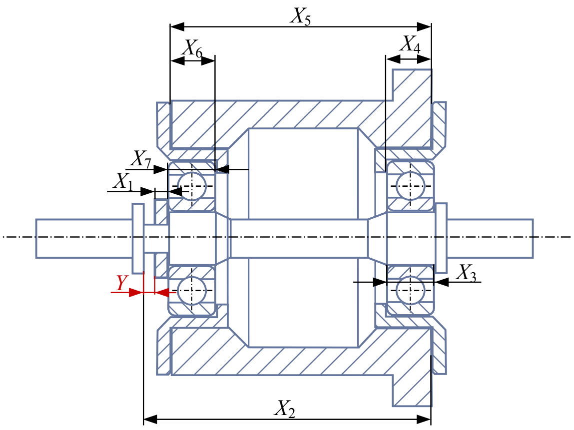

Fig. 5. Case study with a linear functional key characteristic acc. to [13].

the tolerances for X_4 and X_6 are equated: $t_4 = t_6$ [13]. The entire cost data is summarized in Table 1.

All tolerances are normally distributed with standard deviations $s_{\text{i}} = t_{\text{i}}/6$. The yield is limited by $LSL = 4.6169$ mm and $USL = 5.3831$ mm and a required scrap rate of $z = 63.37$ ppm ($\hat{=} \pm 4\sigma$ [6]) must be guaranteed. To consider discrete tolerance values, the genetic algorithm in integer-programming format is used. Based on these global settings and conditions, the optimization is repeated employing different scrap rate estimation techniques and for different sample sizes. Fig. 6 summarizes the results of the optimization runs.

In general, it can be noticed that the obtained cost minima C_{tot} for the different methods converge to a common value with increasing sample sizes. The divergence of C_{tot} for the estimation with gaussian cdf is low compared to the other methods. In contrast to the results for the kde, the empirical estimation is apparently more sensitive to the chosen sample size. Thus, the supposedly best result can be reached by using ecdf with a sample size of 10 000. The obtained tolerance design causes the lowest costs and the scrap rate $\hat{z} = 0$ ppm fulfills the requirements. For the other approaches a scrap rate near to the required value of $z = 63.37$ ppm is estimated for the different sample sizes. This effect can be explained as follows: The optimization algorithm tries to achieve the most cost-efficient design. It is known that tolerances and manufacturing costs are related hyperbolically. As a consequence, the optimization algorithm tries to select the highest tolerance values as possible

Table 1. Dimensions, boundaries and cost data for case study. [13]

Dimension	**Machine**	**Param. of cost function** a	b	c	t_{min}	t_{max}
$X_{01} = 10$		fixed tolerance, $C_1 = 5.00$			0.381	0.381
$X_{02} = 200$	1	5.34	66.43	2.738	0.001	0.5
	2	5.12	62.22	2.340	0.001	0.5
$X_{03}, X_{07} = 40$		fixed tolerance, $C_3 = 50.00$			0.063	0.063
$X_{04}, X_{0}6 = 37.5$	1	15.34	69.43	2.728	0.001	0.5
	2	15.12	65.22	2.340	0.001	0.5
	3	14.85	66.87	2.112	0.001	0.5
	4	500	70.62	2.985	0.001	0.5
$X_{05} = 180$	1	11.34	72.43	2.738	0.001	0.5
	2	11.12	68.22	2.340	0.001	0.5
	3	10.85	69.87	2.112	0.001	0.5
	4	500	73.62	2.985	0.001	0.5

and the scrap rate of the final design will be slightly less than the required one. However, the increment of the proposed ecdf approach is reciprocal to the sample size n. At $n = 10\,000$, for instance, one defect more or less causes a difference in scrap rate of 100 ppm. For this reason, the higher the sample sizes the more the estimated scrap rate $\hat{z}$ draws near the required scrap rate.

This example confirms that the influence of the selected scrap rate estimator and the sample size are remarkable. Hence, this issue must be studied in detail. In order to compare the different methods, suitable reference values are needed. It is known that the resulting variance of a linear fkc can be calculated as a superposition of the single variances and that the specification limits in turn can be defined according to a multiple of the resulting standard deviation [29]:

$$\begin{aligned} LSL &= Y_0 - u \cdot \sqrt{\textstyle\sum_{i=1}^{I} s_i^2}, \\ USL &= Y_0 + u \cdot \sqrt{\textstyle\sum_{i=1}^{I} s_i^2}. \end{aligned} \tag{6}$$

Given the normal distributions for the single dimensions with standard deviations $s_{\text{i}} = 0.2/6$, the specification limits for $z = 63.37$ ppm ($\pm 4\sigma$) are set up to $LSL = 4.6472$ and $USL = 5.3528$ according to eq. 6.

In order to compare the results of the scrap rate estimation techniques with these reference values, the tolerance analysis is repeated 2 000 times for different sample sizes. The average computation time τ_{mean} for each method in addition to the sample size is presented in Fig. 7. The difference between the estimated and the expected scrap rate $\Delta z = \hat{z} - z$ is used for comparison. The corresponding boxplots are shown in Fig. 8.

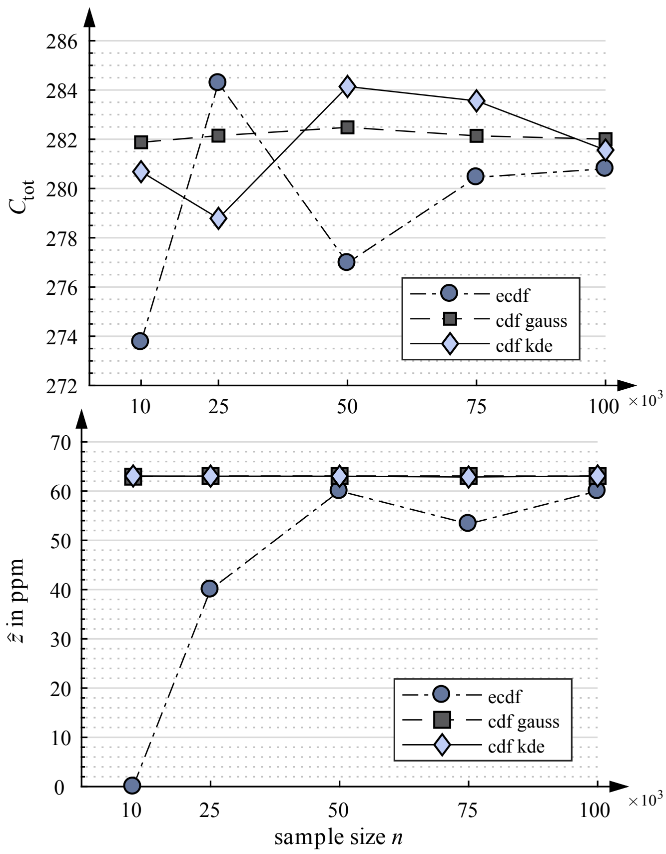

Fig. 6. Obtained cost minima and scrap rates for final tolerance designs.

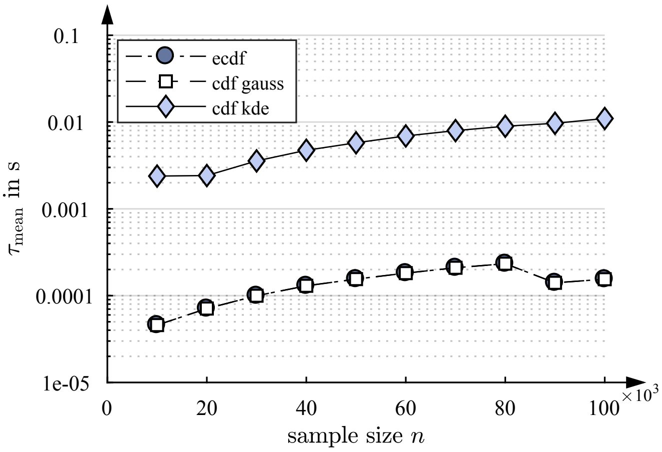

Fig. 7. Computational effort at different sample sizes.

4.2. Discussion of the results

The plots in Fig. 8 prove the previous assumption that both the sample size and the chosen method for the scrap rate estimation have great influences on the obtained results. It is plausible that with an increasing sample size the scrap rate converges to the reference values. For kde and ecdf this effect is clearly visible. As expected, the cdf for normal distribution delivers the best results. At a sample size of $n = 100\ 000$, the maximum error is $\max(\Delta z) = 8.8$ ppm and the 25%- and 75%-quantiles are $q_{0.25}$ ppm$(\Delta z) = -1.5$ ppm, $q_{0.75}(\Delta z) = 1.4$ ppm. Thus, a positive value signifies an over- and a negative one an underestimation. This can be considered as sufficiently precise. Furthermore, the plot shows that over- and underestimations approximately occur with the same frequency.

The results of the non-parametric estimation techniques show differences. Both methods show that low sample sizes are not suitable for estimating scrap rates in a range of few ppms. In the application of kde, the maximum errors are predominantly positive and overestimations prevail on an average. For $n = 100\ 000$, the results are as follows: $\max(\Delta z) = 107.4$ ppm, $q_{0.25}(\Delta z) = 9.6$ ppm, $q_{0.75}(\Delta z) = 20.4$ ppm at $n = 100\ 000$. Similar to the other methods, the maximum error and the values for the outliers for ecdf decrease with increasing numbers of samples. At $n = 100\ 000$ the results with $\max(\Delta z) = 106.3$ ppm, $q_{0.25}(\Delta z) = -13.4$ ppm, $q_{0.75}(\Delta z) = 16.6$ ppm are similar to the ones of kde. In comparison with the parametric estimation, higher sample sizes are needed for non-parametric approaches to achieve acceptable results.

In terms of the required computing times, ecdf and cdf with normal-distribution require less time for the estimation than the kernel density estimation. The extra expenditure of time caused by higher numbers of samples is comparatively low for the considered ranges. However, it can be noticed that the fkc must be evaluated according to the number of samples. For time-consuming tasks, such as time-invariant systems or other tolerance representations, the sample size has a stronger influence on the total computing time.

Overall, this study is conform to the results of the optimization process shown in Fig.6. The number of underestimations for ecdf and $n = 10\ 000$ explains the pretended cost minimum for the case study.

4.3. Recommendations

Based on the obtained results, the authors give the following recommendations:

- Provided the distribution type of the fkc is known, the determination of the scrap rate with the help of the distribution-dependent cdf provides good results. Due to short computing times, this approach is also usable for time-consuming sampling-based tolerance-optimizations. Subsequent to the optimization, the assumed distribution type must be proven by a suitable test, such as the Anderson-Darling-test for normal distribution.
- For unknown distribution types, non-parametric methods such as kde and ecdf should be used. The latter has particularly shown potential for tolerance-cost-optimization in terms of its comparatively low computing times.
- In general, the sample size has to be chosen according to the required scrap rate and the estimation method. Higher sample sizes can deliver more confident results, but affect the total computing time of tolerance-cost-optimization. Non-parametric estimators require larger sample sizes than parametric ones.

5. Conclusion and Outlook

Sampling-based tolerance-cost-optimization not only attracts attention in research but recently also in the industry. In this context, the choice of the scrap rate estimation technique and the sample size have a great impact on the optimization results. Within the optimization process, the scrap rate is set as inequality condition. Improper methods and too small sample sizes lead to over- and underestimations of the scrap rate. As a consequence, the optimized design cannot hold the functional requirements or does not reflect the cost optimum. For that reason, three different scrap rate estimation methods are introduced and discussed in this contribution. The comparative study shows that parametric estimation methods are very suitable for tolerance-cost optimization given the precondition that the assumption of a distribution for the key characteristic is tested by a suitable method. Otherwise, this approach leads to highly inaccurate results. If the distribution type is unknown, non-parametric methods are more convenient because they are universally applicable. In this regard, empirical approaches and the kernel density estimation show potential. However, especially for the use of non-parametric estimations, the choice of sample size has to be made advisedly in relation to the value of the required scrap rate and the applied estimation technique.

In conclusion, it must be noted that further research is needed. Alternative solution approaches such as data transformation by a suitable technique, e. g. the Johnson- or Box-Cox - transformation, or distribution systems like the Pearson-system should be considered. Moreover, further empirical approaches should be taken into account to handle the data in efficient ways.

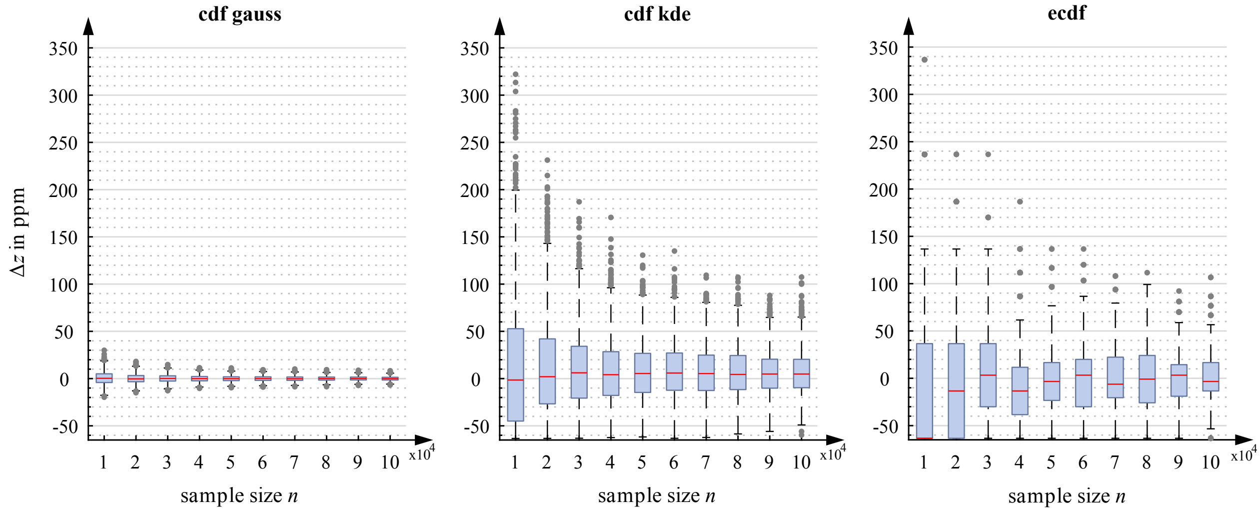

Fig. 8. Comparison of the results for different scrap rate estimation techniques and expected scrap rates.

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