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Comparative Study of Tolerance Analysis Methods Applied to a Complex Assembly

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Abstract

Many tolerance analysis methods are in use today, both manual and software based. However, there is a lack of comparative studies on the results produced by these procedures. This paper describes the results of predictive studies on a complex mechanical assembly. The procedures include 1D min/max charts, 3D Monte Carlo statistical analysis, T-map based worst case and statistical analysis and VisVSA, a leading commercial system from Siemens. Both axial and radial stacks of a surrogate gearbox were considered. All analyses were driven by the same geometry, assembly and tolerance data produced by Siemens NX and VSA in the form of *pdo* files, which were translated into an Augmented Constraint Tolerance Feature (aCTF) format. This provided a uniform input to all procedures. Differences in the results can be categorized into three potential sources: variation types included or omitted, treatment of assembly constraints in mathematical formulations.

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Keywords: Tolerance analysis; Monte-Carlo simulation; T-maps; VisVSA - Tearmcenter

1. Introduction

Tolerance analysis methods range from manual 1D min/max charts [1][2] to sophisticated 3D mathematical models, such as T-Maps® [3][4][5] and deviations space modelling [6]. Commercial packages, such as VisVSA allow users to interactively model assembly stacks with point-to-point feature variations used in Monte Carlo simulations [7][8]. Tolerance analysis is done to determine the variation in a dependent dimension. This requires that the contributors to that variation be identified first, often referred to as the tolerance stack or loop. This can be done manually by examining the GD&T on drawings, or interactively on toleranced CAD models of parts in an assembly. We have devised a method to automate tolerance loop detection from a neutral data model, called an aCTF (augmented Constraint Tolerance Feature Graph) format, which is an augmented version of a previous format called the CTF [9]. This Data structure is a hierarchical tree structure which has the info of Features in the first part, constraints and tolerances of those features next, and points on the boundary of irregular planes or irregular surfaces along with the "to be analyzed dimension" last in the augmented part. By using the same aCTF file as input to different analyses procedures, we can compare the results on a common basis. In addition, the modelling errors are drastically reduced since the results from each method are compared for the same tolerance loop due to the automation of tolerance loop detection.

Past comparative studies have been based on the similarities and differences between the approaches used [10] or on very simple case studies, such as 2-pin-2-hole assemblies [11][12]. In contrast, this study is based on a complex assembly that consists of more than10 parts. This study was a collaborative effort between an aircraft engine manufacturer, a major CAE software vendor and two universities. The case study presented in this paper is a surrogate of the actual assembly used, due to proprietary restrictions. The surrogate model has similar parts and tolerances as the actual one from the aircraft engine. The

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same radial and axial clearances were analyzed with five different procedures: 1D manual and automated charts, 3D Monte Carlo, T-maps and VisVSA. Our end goal was to see how which of these results compare best to actual measured data (not included in this paper).

2. Tolerance Analysis Methods

2.1. Background

The purpose of Tolerance Analysis is to study the accumulation of variations on a geometric attribute of interest (dimension, location, orientation, etc.). The need for this arises because the analyzed dimension is not explicitly specified. Analysis can involve multiple parts in an assembly or variations on a single part, but the most common case is analysis of clearances in assemblies. All dimensions and tolerances that affect the clearance gap are called the contributors. A stack*,* also called a stack path, tolerance chain, datum flow chain or dimension loop, is a continuous of specified dimensions from one feature of interest to another (start & end of stack). It is obtained by traversing a series of known dimensions.

One analysis approach used in industry today is a manual procedure called Min/Max Tolerance Charting; it is consistent with ASME 14.5/ISO standards [13][14], but limited to 1-D worst case analysis only. Another popular approach is to use a commercial package. A review of the underlying method used in VisVSA can be found in our prior paper [15]. Dimensional, geometric and assembly relations are converted to point-to-point constraints whose variations are simulated based on probability distributions of input variables. It may be noted that current tolerance standards are based on tolerance zones, not point-to-point variations. Emerging methods are attempting to address the challenge to build a math model of geometric variations that is consistent with already existing tolerance standards and capable of supporting comprehensive 3D analysis of stack-up conditions. Table 1 compares the scope of the methods considered in this study.

Table 1. Scope of Tolerance Analysis Methods.

Analysis Method	Dimension	Type	Automation
Tolerance charts	1-D	Worst case	Manual*
Feature based Monte Carlo	3-D	+Statistical	Automated
T-maps	3-D	+Statistical	Automated
CATs with abstract feature (point to point Monte Carlo)	3-D	+Statistical	Interactive

** conventional charts are manually constructed; ASU has developed an automated version as part of the ASU GDT Testbed*

2.2. Automatic Loop Detection

In order to create fully automated software that can do tolerance analysis, we need to automate the extraction of the tolerance loops. This also makes the input model provided to different tolerance analysis methods consistent. Our past work on "Directions of Control (DoC)" [16] creates related feature graphs; all features related by linear dimensions in the same direction are placed sequentially in a single branch of a graph; branches are related by orientation dimensions to form the full DoC graph (DCG) for each part in an assembly. To extract a loop, the path starts on one side of the analysed dimension and follows the part DCG until it encounters a mating part, in which case, the path jumps to the DCG of the mating part. This is repeated until the path returns to the start feature. Based on this concept, Haghighi et al [17] developed an algorithm that could traverse through the assembly model, and find the loops (tolerance stacks) that connect the initial node (one side of the gap) to the end node (the other side of the gap) in the assembly graph. There can be one or more loops extracted for each analysed dimension (Parallel loops and stack-ups).

2.3. MIN/MAX Tolerance Charts

Tolerance analysis using 1D charts, also known as min/max chart, is done to determine the maximum and minimum permissible values, of the gap dimension or orientation, resulting from the limits specified on the contributors. This is a manual bookkeeping procedure for 1-D stack calculation [1][2]. The analyst typically works with engineering drawings and interprets the Y14.5 symbols. A 1D coordinate system is set up with the origin at the left side of the unknown dimension (of an axial stack or lower end for radial stack), with positive direction to the right and negative to the left. The rationale for this convention is that if the gap comes out to be positive it is a clearance and if negative, it is an interference (for assemblies); for part level analysis, a negative value means that the feature disappears. All tolerances encountered in traversing the stack are accounted for by rules that are specific to each class. The chart contains two main columns in which a value and a sign are entered for each tolerance contained in the stack, based on these rules. Learning and following all these class specific rules is tedious and error prone. To alleviate this difficulty, we implemented an automated 1D Min/Max charting system in conjunction with auto loop detection. From the assembly and DoC graphs of parts in an assembly, the system can automatically find the loops for a given dependent dimension (e.g., a clearance) and apply charting tolerance rules to create 1D charts. Procedures for handling each GDT class encountered in the stack have been codified. Both manual and automated charting procedures were applied to the case studies presented in this paper.

2.4. Commercial CATS (VisVsa)

VisVSA facilitates statistical analysis of dimensional variations in parts and assemblies, based on Monte Carlo simulation. Tolerance specific entities and attributes are interactively extracted from CAD models. Feature attributes are varied within the specified tolerance range, and user-defined statistical distributions are used in simulation runs to determine the contributors, the extent of contributions, sensitivities, and statistical distribution of the analysed part dimension or assembly clearance/interference.

The basic approach adopted is to import the geometry from CAD into CATS and have the user interactively create an abstracted "feature model" for each analysed dimension. VisVSA imports geometry from CAD system via its file format (*.jt), which is also an ISO standard [18]. Abstraction of the geometry and selected dimensions is done in terms of points, lines, planes and distances, angles between them. For example, a cylindrical feature will be represented by "point + vector (axis) + radius + length" and a planar feature will be represented by "point + vector (normal) + width + length". These abstract objects can be created independently or from actual CAD geometry. Then the dimension of interest to be analysed is defined in terms of Point Coordinate, Point-to-Point, Point-to-Line, Point-to-Plane, Gap/Flush, Angle, Maximum or Minimum Virtual Clearance.

Creating the abstract feature model requires considerable skill and experience to get valid results. For a specific dimension of interest, it is not obvious which type of measurement to choose of all the types of measurement options. For example, if one wants to analyse the distance between two parallel planar features, then one can choose a point-to-point or point-to-plane measurement. The results are sensitive to the location of the points on the target plane.

VisVSA classifies tolerances specification into four groups and each group includes a number of tolerance types:

1. Size - equal bilateral, unequal bilateral and unilateral. Size tolerance can be applied to features-of-size only.
2. Location - position, surface profile, total runout & +/-.
3. Orientation - perpendicularity, parallelism, angularity & total runout.
4. Form - flatness, surface profile, straightness & cylindricity.

There are certain pre-set rules for assigning tolerances to features. For example, it is necessary to specify the datums before assigning them to a particular tolerance feature frame. This creates a problem in cases of tandem datums used in some runout specifications, because VisVSA consider it self-referencing. A few other observations are given below.

- Some tolerance types will not be available for the feature, depending upon the feature type. For instance, flatness is not available for a pin feature (ends).
- Diameter modifier and material modifier (MMC, LMC) may be added to a tolerance depending on the feature type, tolerance type and the types of the datum features.
- A material modifier can only be specified to a feature of size for a straightness, positional, or orientation tolerance.
- For a specific feature, the user can apply at most, one tolerance type from each group, even if the DRFs are different.

The type of results obtained from *VisVSA* can include statistical distribution (nominal, mean, standard deviation), contributors and corresponding contribution percentage. *VisVSA* uses Monte Carlo simulation to predict the amounts of variation [7][8]. For the amounts of variation, all of the toleranced dimensions in the model are assigned a statistical distribution (Gaussian, by default). Monte Carlo simulation then chooses one value from each distribution (based on the parameters of the distribution) to create a unique sample of each component. Once the tolerance model is created, feature attributes (e.g., feature location, orientation, and shape parameters) are varied within their corresponding tolerance ranges according to the user-defined or default statistical distributions (e.g., normal, uniform, triangular, exponential, Pearson, gamma, Weibull, and Laplace distributions). Then all the assembling constraints are sequentially and/or simultaneously satisfied, via certain constraint solving techniques, and finally the measurements are computed. This "feature variation constraint satisfaction measurement computation" process is repeated in each of the simulation runs to determine the contributors, the extent of contributions, sensitivities, and statistical distribution of the analysed part dimension or assembly clearance/interference. As can be seen, *VisVSA* uses point-based analysis. It might not guarantee that the relative position among the simulated points satisfies the tolerance specification enforced on the feature to which these sample points belong. It is not clear to outside parties how *VisVSA* deals with tolerance refinement relation during tolerance analysis. For example, it is unknown how to represent a floating form tolerance zone inside an orientation or location tolerance zone. Besides, the capability and accuracy of the solver become an issue as every solver has its own limitations.

2.5. Feature & Zone based simulation

As opposed to a point-to-point statistical analysis based on Monte Carlo simulation for tolerance analysis, we have implemented a feature and tolerance zone based Monte Carlo simulation procedure. The 3D parametric variation for tolerance analysis is divided into three phases pre-processing, simulation and post-processing. The inputs to the tolerance analysis system are the stacks extracted by the loop detection module. The first process in the pre-processing module extracts the directions in which it is important to measure the analysed dimension. For multidimensional loops, clearances should be measured in multiple directions since in a multidimensional stack the transformation accumulation will have components in different vector directions. These measurement vector directions are simply the directions in which the dimensions of stack exist. The second function in pre-processor decomposes an assembly level stack to part level stacks in order to simplify computation of total accumulation. Next, the datum flow is extracted as variations are processed with respect to datum flow pattern. It is also important to follow the datum flow in carrying out variations so that the instantaneous bonus from a datum can be computed beforehand and included in the target feature tolerance zone. The stacks are decomposed into part level stacks in pre-processing block, and is then passed by assembly simulation to part level simulation one by one. Part level simulation generates a single instance of variation on each feature in the stack. The resulting transformations of varied features are combined with respect to the datum flow to compute final accumulated transformations for the start and end feature. Each instance also generates the related mating envelop for all FOS in a stack and may be used for stack up if the size dimension participates in the stack. These instantaneous transformed locations and related mating envelopes of the start and end features of the part level stacks are combined by assembly simulation to get overall transformation in the assembly level. Finally, the clearances can be determined with respect to the measurement directions. Once enough (acceptable accuracy and random perturbation) samples of clearances are collected they can be used for estimating the probability distribution function (PDF) for analysed dimension.

2.6. T-Maps®

Various researchers have developed models to represent the allowable variation in the toleranced feature by associating variables to each degree of freedom of the feature within the tolerance zone. The relationship of these variables can be easily modelled by representing the relationship geometrically or by equations. Different researchers [19][20][21] have represented geometrically the relationship of the allowable degrees of freedom of the tolerance feature within the tolerance zone. Not many other models except the ASU T-Map model proposed in [21] or Polytope based Tolerance Analysis [22] have recommendations for floating tolerance-zones or for developing stack-up relations in an assembly.

A T-Map is a hypothetical Euclidean point-space, the size and shape of which reflects all variational possibilities for a toleranced feature (target feature). It is the range of points resulting from a one-to-one mapping based on the variational possibilities of a feature, within its tolerance-zone, to the Euclidean point-space [21]. These variations are determined by the tolerances that are specified for controlling size, position, orientation, etc., of the feature. The T-Map® for any combination of tolerances on a feature is constructed from a basis-simplex and described with areal coordinates. If the mapping is done for n-types of variation of a feature, a T-Map will be created in n-dimensions; the basis-simplex will also be of n-dimensions. Consider a cross-section ABCD (Fig 1). The upper and lower extreme positions, AB and CD can be used to map to basis points σ 1, σ 2 and the extreme angle CB to a third basis point σ 3. Now every line in ABCD can be represented by a corresponding point in the basis triangle σ 1, σ 2, and σ 3 (Fig 1.b). To represent max rotation in the opposite direction we have a complementary point on the other side (Fig 1.c). To represent all cross-sections, this double triangle can be rotated to get a 3D figure called the T-map for size of the round bar (Fig 1.d.). Any end plane of the round that satisfies the size tolerance will be represented by σ, uniquely determined by the linear combination

$$\sigma = \lambda_1\sigma_1 + \lambda_2\sigma_2 + \lambda_3\sigma_3$$

The three λi can be normalized by setting λ1 + λ2 + λ3 = 1. It is only necessary for affine geometry that the positions for basis-points to be independent.

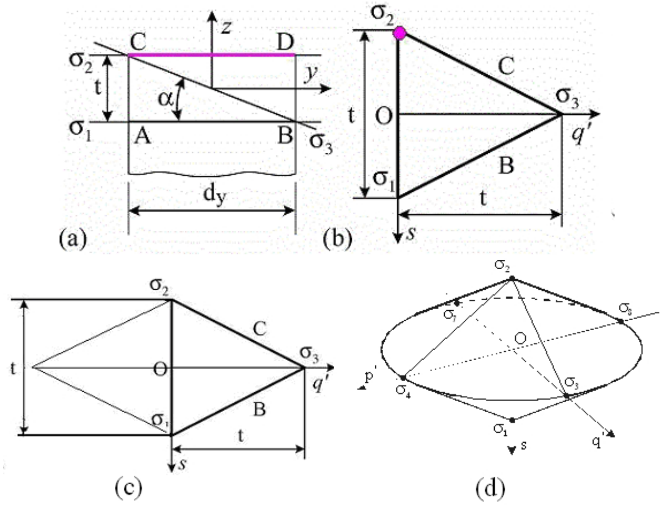


Fig. 1. Generation of T-map for size tolerance of a round bar [21].

The dimensions of this T-Map® depend on the tolerances and the dimensions of the toleranced plane. Rotational variations of the plane are represented along (σ_1 - σ_3), or (σ_2 - σ_3) direction translational variations of the plane are represented along σ1-σ2 of the T-Map. Planes σ *i* in the tolerance zone correspond to points σ *i* in the T-Map. Thus, this T-Map® represents all 3D variations of the toleranced plane in Figure 1 (d). If an orientation tolerance (e.g., parallelism) is specified on the end plane, its effect will restrict the rotational direction, i.e. the T-map will be truncated along σ3 which gives the map shown in Figure 2 for combined size and orientation.

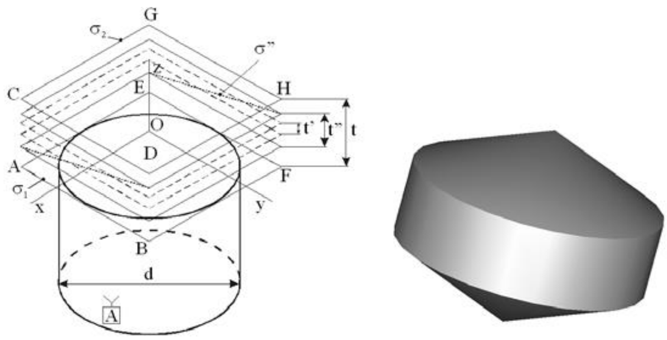


Fig. 2. Modification of size T-map to include orientation.

3. Comparative Study

In addition to worst-case results, designers are interested in statistical results: Sensitivity and Percent Contribution of each contributor and Acceptance Rates. Sensitivity is the first derivative of gap size with respect to each contributor and Percent Contribution is an indication of the proportion of the variance of the gap size attributed to each contributor. Acceptance rates determine the percentage of assemblies in which a given analyzed dimension is within the acceptable upper and lower values. In our comparative study, we are looking at only the top-level results from each procedure: worst-case min, max values, arithmetic and statistical mean values and standard deviation. All contributors are assumed to be normally distributed. Four analysis methods were compared with the same model.

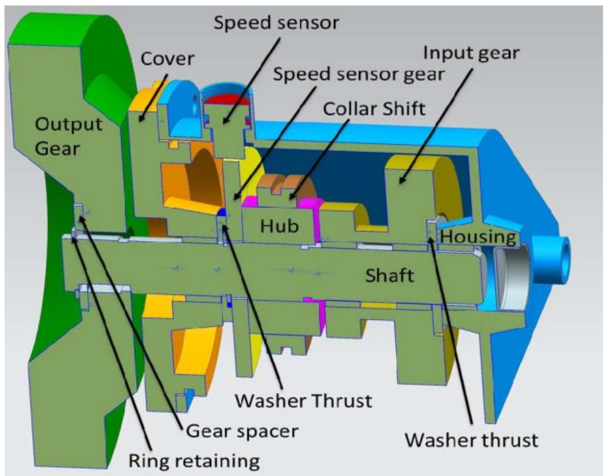


Fig. 3. 3D model and Cross section view with part names

3.1. Case study assembly model:

The gearbox model, shown in figure 3, along with the GD&T specifications was created in Siemens NX for this study. In this model, we selected one radial gap and one axial gap for analysis as specified in figures 4a and 4b. The axial gap is between the hub and the input gear, and the radial gap is between the speed sensor and the gear that it uses to read the speed from. The reason for selecting these measurements is that in spite of having relatively simple geometry, it has wide variety of features and tolerances, involved in each stack. This variety of contributors will help identify the differences between various tolerance analysis methods.

3.2. Automatic Loop detection of the gearbox assembly

Part and assembly CAD models were created in NX, then transferred to TeamCenter. Features, GD&T and Assembly mating conditions) for each analyzed clearance were defined interactively in VisVSA. Operations (We saved this "process document" in PDO format for analysis with VisVSA. The model was then saved in PDO summary format which is textual tab delimited format that includes all features and tolerance information to be translated to aCTF format needed by our non-VSA applications (1D chart, 3D Monte Carlo and T-maps).

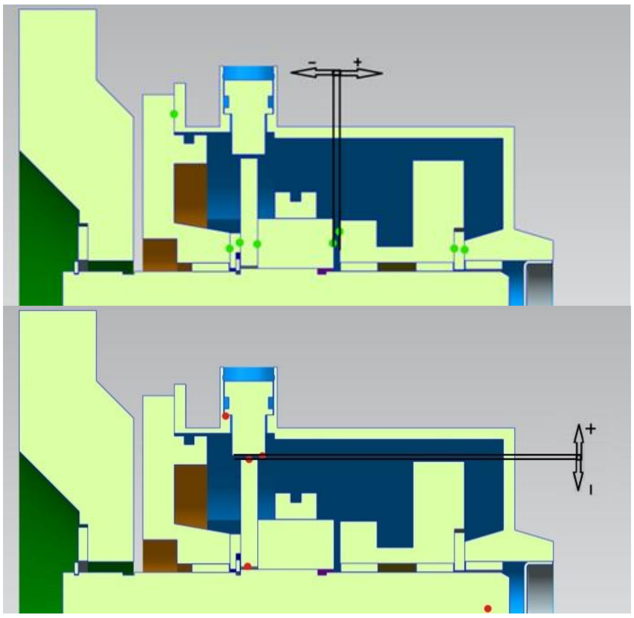


Fig. 4. (a) Axial stack for the gap between the hub and input gear; (b) Radial stack: The gap between the speed sensor and gears of the gear speed sensor

After feeding the aCTF to loop detection module, we found two loops: one axial loop (Loop No. 1) and one radial loop (loop No. 2) which are stored in textual format listing the name of contributors as shown in figure 5.

```
------Loop detection time statistics-----
          2.271seconds
----------------------------------------
Loop 1 of 2:          Loop 2 of 2:
     (Plane11)             (Hole20)
     (Plane12)             (pin22)
     (Plane15)             (Hole19)
     (Plane16)             (Hole18)
     (Plane22)             (Plane28)
     (Plane20)             (Plane43)
     (Plane30)             (Plane60)
     (Plane45)             (Plane 59)
     (Plane49)             (Pin11)
     (Plane48)             (Hole6)
     (Plane56)             (Pin1)
     (Plane54)             (Pin0)
     (Plane13)
     (Plane14)
```

Fig. 5. Output of the loop detection module

3.3. Gearbox analysis results:

We tested all tolerance analysis methods discussed earlier on the surrogate gearbox. Each tolerance analysis method uses the tolerances on contributing features, as well as the distance or the mating condition between them available in aCTF for analysis. Our 1D chart outputs a separate file corresponding to each loop. The two charts, one for axial and one for radial loops, are not presented here due to limitation of space. In the same time, the feature based Monte-Carlo analysis and T-map based Tolerance analyses are executed in our testbed, the results of which are summarized in the following two tables, along with the results we have derived from VisVSA.

4. Discussion

It is clear in the table that the manual and automated 1D chart have similar results for the axial stack while they are different for the radial. As per our investigation, the reason for this incompatibility only in the radial gap comes from a well-known problem in tolerances analysis of cylindrical mating features; and that is the difference between the nominal and the extreme possible location of the mating FOS. In the worst-case analysis, the analyser has considered the maximum clearance by moving the pin in the furthers extreme location, while the automated system, has done analysis based on the extracted loop and the mating condition and location of features as translated from the geometry in the minimum scenario.

Table 2: Comparative results for the axial stack

Analysis Method	Worst case (max / min - Mean)	Stat. Mean	Stat. std. deviation
1D chart (manual)	2.6/1.6 – 2.1	-	-
1D chart (auto)	2.6/1.6 – 2.1	-	-
3D Monte Carlo	2.2141/1.8882	2.074	0.035
3D VSA	3.6173/1.1623 (wc*)	2.0656	0.1736
	2.898/1.449 (6 σ)		
3D T - map	2.786/1.414-2.1 (wc)	1.9869	0.0724
	2.3256/1.8744-2.1 (6 σ)		

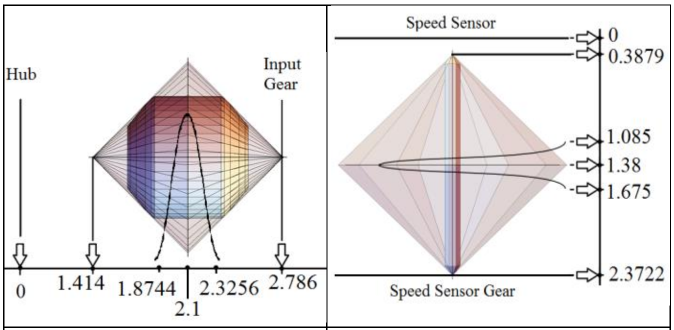
Table 3: Comparative results for the radial stack

Analysis Method	Worst case (max / min - Mean)	Stat. Mean	Stat. std. deviation
1D chart (manual)	1.975/0.725 - 1.35	-	-
1D chart (auto)	1.72/1.01 – 1.36	-	-
3D Monte Carlo	1.75/0.84	1.12	0.064
3D VSA	1.8235/0.9058 (wc)	1.3805	0.128
	1.78/1.003 (6 σ)		
3D T - map	2.3722/0.3879-1.38 (wc)	1.38	0.0982
	1.675/1.085-1.38 (6 σ)		

* wc: worst case

The 3D Monte Carlo results are based on 4000 simulations. In Monte Carlo Simulation, since nominal variation is assumed in our analysis, thus to get closer Min/Max values very large number of Simulations are required. The main drawback of this method is that to get accurate estimates, it is necessary to generate very large samples and this is computationally expensive. Also, if the distributions of the independent variables change or shift, the whole analysis must be redone, as there is no way of adjusting the existing results. In addition, the automated feature based MC has a different mean for radial stack, which after investigations, we found it is due to the fact that the measurement is specified between a planar feature and a FOS inducing an error by the amount of the gap. Because the mating condition specified between the plan and the axis of the FOS has a distance less than the radius of the cylinder. Thus, we have to fix the error regarding plan to FOS measurements.

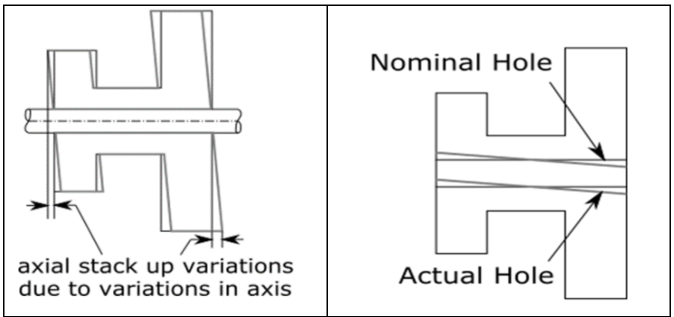
Figure 6 illustrates Tolerance analysis results produced using T-maps. T-map analysis considers every possible variation in features in stack loop altogether and therefore gives elaborate results. Consider parts named 'Input Gear' and 'Shaft' in figure 3(b). Input Gear is supported on Shaft through two roller bearings. Tolerance over axis of Input gear allows some tilt in axis as shown in figure 7(b). When Input gear with tilted axis is mounted on the shaft as per figure 7(a) effective axial length of Input gear increases. T-map analysis considers every possible contributor and hence results in Tables 2 & 3 show a larger range of variations for T-map analysis.



a) axial gap	b) radial gap

Fig. 6. 2D X-section of T-map analysis showing Max/Min & statistical results

The difference in max values in 1D and 3D analyses comes from the former not including the effect of two orientation variations. The VisVSA results presented in these tables are not the first results we achieved, but the one which made most sense after tweaking with different settings, most importantly “weights” that seems to play an important role in VisVSA point based Monte-Carlo simulation.



a) tilted axis and shaft	b) Variation in axis of Input gear.

Fig. 7. Assembly of input gear

To conclude, in this paper we have summarized the results of tolerance analysis on a complex assembly using several methods. Tolerance charting, can offer a quick tolerance check and relatively accurate results when the contributions in the other directions (rather than the analysis direction) are at the negligible level. The automated charting method along with the Monte Carlo analysis, are good tools for initial investigation of the assembly. But the Monte Carlo simulation, due to its limitation of modelling constraints while doing the simulation which is the core basis for this statistical model, it is not the most reliable method for manufacturability analysis. VisVSA uses a point based analysis method, which does not conform to true 3D tolerance zones and geometric variations. The results from VisVSA are prone to the knowledge and expertise of the user. The T-map method is still in beta testing and is currently not available in any commercial packages. However, T-Map can model all the 3D variations of a feature and can precisely model the interaction of these variations.

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References

[1] Krulikowski A. Tolerance Stacks - A Self-Study Course, Effective Training Inc. Westland, MI, 2002.
[2] Wade OR. Tolerance Control in Design and Manufacturing. New York: Industrial Press, 1967.
[3] Ameta G, Davidson JK, Shah JJ. The effects of different specifications on the tolerance-maps for an angled face. ASME 2004 International design engineering technical conferences and computers and information in engineering conference 2004 (pp. 303-311).
[4] Ameta G, Davidson JK, Shah JJ. Tolerance-maps applied to a point-line cluster of features. Journal of Mechanical Design. 2007;129(8):782-92.
[5] Ameta G, Davidson JK, Shah JJ. Using tolerance-maps to generate frequency distributions of clearance for pin-hole assemblies. ASME 2006 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference; 2006; 249-259.
[6] Ameta G, Serge S, Giordano M. Comparison of Spatial Math Models for Tolerance Analysis: Tolerance-Maps, Deviation Domain, and TTRS. J Comput Inf Sci Eng; 2011;11(2):021004.
[7] SIEMENS, VisVSA Solutions Training Manual, Version 1.3. .
[8] SIEMENS, VisVSA online help, version 4.0. .
[9] Ramnath S. 2016. Automating Fixture Setups Based on Point Cloud Data & CAD Model. Arizona State University; 2016.
[10] Shen Z, Ameta G, Shah JJ, Davidson JK. A Comparative Study of Tolerance Analysis Methods. J Comput Inf Sci Eng; 2005;5(3):247-56.
[11] Shen Z, Shah JJ, Davidson JK. Simulation-based tolerance and assemblability analyses of assemblies with multiple pin/hole floating mating conditions. ASME 2005 International Design Engineering Technical and Computers and Information in Engineering 2005;429-440.
[12] Zhuang L. Analysis of Mounting Tolerance based on VisVSA. Automob Technol; 2011;1:57-61.
[13] ASME Y14.5-2009. Mathematical definition of dimensioning and tolerancing principles. The American Society of Mechanical Engineers; New York, 2009.
[14] ISO 1101:1983. Technical drawings—Geometrical tolerancing—Tolerances of form, orientation, location and run-out—Generalities, definitions, symbols, indication on drawings. International Organization for Standardization; Geneva, 1983.
[15] Shen Z,. Tolerance analysis with EDS/VisVSA. J Comput Inf Sci Eng; 2003;3:95–99.
[16] Mohan P, Haghighi P, Shah JJ, Davidson J K. Automatic detection of directions of dimensional control in mechanical parts. ASME Manufacturing Science Engineering Conference; 2014.
[17] Haghighi P, Mohan P, Shah JJ, Davidson JK. Automatic Detection and Extraction of Tolerance Stacks in Mechanical Assemblies. ASME 2014 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference; 2014.
[18] ISO 14306: Industrial automation systems and integration – JT file format specification for 3D visualization. www.iso.org; 2012.
[19] Whitney DE, Gilbert OL, Jastrzebski M. Representation of geometric variations using matrix transforms for statistical tolerance analysis in assemblies, Res Eng Des; 1994;6(4):191–210.
[20] Teissandier D, Delos V, Couetard Y. Operations on Polytopes: Application to Tolerance Analysis, 6th CIRP Seminar on CAT, Enschede; 1999.
[21] Davidson JK, Mujezinovic A, Shah JJ. A New Mathematical Model for Geometric Tolerances as Applied to Round Faces. J Mech Des; 2002;124(4): 609–622.
[22] Arroyave-Tobón S, Teissandier D, Delos V. Tolerance analysis with polytopes in HV-description. J Comput Inf Sci Eng; 2017;17