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An adaptive tolerancing algorithm for freeform profiles

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Abstract

Manufacturing of freeform surfaces demands the development of tolerances based on the functionality of parts. Functionally critical portions of a surface can usually be described using the curvature of designed products. A uniform tolerance zone along the nominal freeform profile may not fulfill the functionality of the designed part, i.e. it allows the curvature to increase indefinitely inside the tolerance zone. This will affect greatly in the aerospace, automotive and other high value products. An adaptive tolerancing method based on the local curvature of a profile is presented to define smaller tolerance bands to reject non-functional products.

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Nomenclature

$\boldsymbol{x}$	geometric point
t	abstract parameter
$\boldsymbol{r}(t)$	parametric profile
$d(\boldsymbol{x})$	distance
$d_e(\boldsymbol{x})$	euclidean distance
T	tolerance
T_{max}	maximum tolerance
T_{min}	minimum tolerance
$k(t)$	curvature
k_{max}	maximum value of the curvature
k_{min}	minimum value of the curvature
$\boldsymbol{t}$	B-splines knot vector
$B_{i,j}(t)$	B-splines base
$\boldsymbol{p}_i$	B-splines control point

1. Introduction

Advanced manufacturing and digital metrology enable more complex geometrical products to be manufactured and measured, for example, geometrical products used in freeform optics, human-joint implants, interfaces in fluid-dynamics, MEMS and NEMS devices in nanotechnology applications. It edges a updated specification system that can cope with the increasing complexity of specifications. The current specification system, which was developed in the 1940's specifically for assembly of components, is based on tolerance zones.

Applying the traditional definition of a tolerance zone on such complex geometries is a difficult task. A tight tolerance zone can lead to high manufacturing costs due to high scarap rate for parts not meeting tolerance requirements, whereas a large tolerance zone may be inadequate for its functional requirements, resulting in failure of products. A typical example is the design of tolerance for turbine blades, where their geometrical variability will greatly impact on turbine performance (such as flow efficiency, flow pressure, dimensionless mass flow, etc.) [8].

A uniform tolerance zone defined on a nominal freeform profile not related to a datum system is defined in ISO 1101:2017 [3] and ASME Y14-5 [1]. It is limited by two lines enveloping a circle with a specified diameter, whose centre is situated on the designed profile. In this case the curvature of the tolerance zone can increase indefinitely inside the zone, and this may not fully apply to the intended functionality of the component.

A method based on the subdivision of the profile in different portion was proposed in Petitcuenot et al.[9]. The authors proposed to assign to each portion of the surface a different tolerance zone, according to the functionality of the zone. Their algorithm was designed for a profile of a turbine blade which requires tinier tolerances in specific portion of the surface. A complex profile is splitted in different sections according to its functionality and then each section is analysed independently.

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This leads to tolerance zone that is not continuous in the junction points.

Variable tolerance zone applied to complex surface manufactured by additive manufacturing was analysed in Moroni et al.[7]. The authors proposed a volumetric, voxel based, method ables to define the tolerance zone of surface with complex geometry. The algorithm is flexible and can be easily applied to a arbitrary complex surface.

In this paper, a new way of defining adaptive tolerance zone based on the local curvature of a profile is proposed. Some of the local curvatures of a turbine blade are of critical importance to turbine performance. For example, the stagger angle is closely related to mass flow rate and flow losses [4]. The aim of the proposed method is to design a tolerance zone that allows to the profile to vary in flat portions of the curve, while the tolerance should become smaller in the critical portion of the curve. It is designed specifically for freeform profiles such as turbine blades, blisk (bladed disk) and compressor blades.

The paper is organised as follow: in Section 2 the proposed algorithm is presented; in Section 3 two test cases of a simple profile and a turbine blade profile are presented and in Section 4 conclusions and future work are provided.

2. Tolerance zone definition

Let $\boldsymbol{r}(t) \in \mathbb{R}^3$ be a parametric curve on a plane, the computation of the tolerance zone correspond in finding all the points of the plane with distance less or equal to a certain tolerance value T.

To allow an adaptive tolerance zone, the distance of each point of the plane to the designed profile can be deformed taking into account a measure of the complexity of the curve. The distance should not change in the flat portion of the profile, but it should increase in the complex portions of the profile. The contour line of the initial tolerance value is now closer to the profile in the complex zones.

A quantity commonly used to describe the complexity of a curve is the curvature [2]. A redefinition of the distance of a generic point on the plane $\boldsymbol{x}$ to the designed profile can be therefore computed as

$$d(\boldsymbol{x}) = d_e(\boldsymbol{x}) - (T_{max} - T_{min}) \left(\frac{k(t) - k_{min}}{k_{max} - k_{min}} \right) \tag{1}$$

where $d_e(\boldsymbol{x})$ is the minimum euclidean distance between the point $\boldsymbol{x}$ and the curve $\boldsymbol{r}(t)$, T_{max} is the maximum value of the tolerance zone and T_{min} is the minimum allowable value. T_{max} correspond to the usual value of the tolerance T, while T_{min} is a value that has to be set by the designer, it corresponds to the minimum value of the tolerance zone for the analysed profile.

$k(t)$ is the curvature of the curve, which is used to measure the complexity of the curve, is computed as

$$k(t) = \frac{\|\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)\|}{\|\boldsymbol{r}'(t)\|^3} \tag{2}$$

where $\boldsymbol{r}'(t) = \frac{d\boldsymbol{r}(t)}{dt}$ and $\boldsymbol{r}''(t) = \frac{d^2\boldsymbol{r}(t)}{dt^2}$, k_{min} and k_{max} correspond to the minimum and the maximum curvature along the profile.

The tolerance zone can now be computed as all the points with $d(\boldsymbol{x})$ smaller or equal than T_{max}. With the proposed method the tolerance is constant in the flat portion of the profile and linearly decreases, according to the magnitude of the curvature, if the complexity of the profile increases.

T_{min} represents the value of the tolerance in the most complex part of the profile. The minimum value of the tolerance zone must the accurately chosen because if it is too small may leads to part impossible to manufacture. Let us assume that the profile is composed by the sections with different complexity, T_{min} can be determined applying the same method presented in Petitcuenot et al.[9]. With the proposed algorithm the tolerance zones of the two sections can be merged with continuity according to Equation 1.

The procedure adopted to approximate a profile, modify the distance function and compute the tolerance zone is now presented. The profile is first reconstructed starting from a set of parametrised points with a B-splines curve using the algorithm implemented in GoTools [12]. The available points are parametrised by computing the arc length. A B-splines curve is computed as the linear combination of the basis functions and the corresponding control points. A B-splines base function is defined by a non decreasing sequence of knots $\boldsymbol{t} = (t_i)_{i=1}^{n+r+1}$, the B-splines base $B_{i,r}(t) = B_{i,r,\boldsymbol{t}}(t)$ of degree $r \geqslant 0$ with support $[t_i, t_{i+1}]$ is defined by the De Boor's algorithm by [10]

$$B_{i,r}(t) = \frac{t - t_i}{t_{i+r-1} - t_i} B_{i,r-1}(t) + \frac{t_{i+r} - t}{t_{i+r} - t_{i+1}} B_{i+1,r-1}(t) \tag{3}$$

with

$$B_{i,0}(t) = \begin{cases} 1 & \text{if } t_i \leqslant t < t_{i+1} \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

A *r-th* degree B-splines curve is defined as

$$\boldsymbol{r}(t) = \sum_{i=0}^{n} B_{i,r}(t)\, \boldsymbol{p}_i \quad t_1 \leqslant t \leqslant t_{n+r+1} \tag{5}$$

where $\boldsymbol{p}_i \in \mathbb{R}^d$ are the control points. The B-splines curves were used because they can handle freeform profile and they can be combined with a desired degree of of continuity.

After the reconstruction of the profile the planar domain is divided into an uniform grid. The same method, using a voxels based approach, was proposed in [7] to compute the tolerance zone of complex additive manufactured surfaces. In order to compute the euclidean distance ($d_e(\boldsymbol{x})$), 1000 equally spaced points of the curve are first generated, for each point of the grid the euclidean distance to all the generated points is computed. The point of the curve with minimum distance is set as the starting point for the point to curve distance algorithm implemented in GoTools. This value is then distorted according to Equation 1 to compute $d(\boldsymbol{x})$.

The computation of the tolerance is performed using the marching squares algorithm [6] implemented in VTK [11]. The marching square is an algorithm used to extract a contour line from a scalar field defined on a plane. The output of the algorithm is a polygon that represents the adaptive tolerance zone.

The proposed method, with the modified distance computation and contour line extraction, is used instead of an offset method because the resolution of the final tolerance zone can be set through the spacing of the grid and the final profile has no self-intersection or cusps.

The B-spline reconstruction and the marching squared methods were used instead of the morphological filter approach because it is possible to achieve sub pixel resolution, allowing a smoother tolerance zone.

An important coefficient to set is the resolution spatial resolution of the grid (pixel size). If this value is too big the resulting tolerance zone may be jagged, but according to the simulations, setting an high value does not slow down the computation of the tolerance zone. A thumb rule may be setting the resolution using a Nyquist approach, *i.e.* the pixel size should be less or equal to half of T_{min}.

If the extracted profile is still too jagged, since it is oriented, a smoothing spline or a local regression technique can be applied to the points to smooth the tolerance profiles.

It should be observed that the discretisation method is independent respect to the continuity of the analysed profile. Let us assume that there are two or more profile, it is possible to compute $d(\boldsymbol{x})$ for each curve and then combine them together taking the pixel by pixel minimum. The tolerance zone of the union of two or more profiles is finally computed applying the marching squares algorithm.

3. Test cases

In this section an example with a simple profile is first analysed to explain how the proposed algorithm works and to analyse the difference respect to the constant tolerance zone. Then a portion of a profile from a turbine blade is analysed to show how an adaptive tolerance zone can be applied to a real profile.

3.1. Serpentine

A serpentine profile is a curve defined by the following equation

$$y = \frac{x}{x^2 + 1}. \tag{6}$$

The curve is a S shape curve that presents two change of concavity and it has two points of minimum curvature. In this example the value T_{max} was set to 0.4 mm, while T_{min} to 0.2 mm. Figure 1 shows the analysed profile along with the constant tolerance zone (blue line) and the proposed tolerance zone (red line). Setting two values of the tolerance, a small one in the two portions of the profile with high curvature and bigger one in the flat portions, would lead to three discontinuities in the tolerance zone. Using the proposed method the designer only needs to set the minimum and maximum values of the tolerance and the algorithm automatically compute the adaptive tolerance zone.

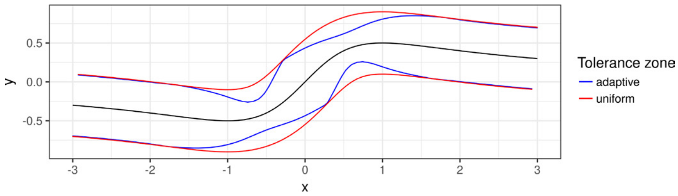

Fig. 1. Tolerance zone

Figures 2 and 3 show the analysed curve along with the tolerance zones and the distance field computed using the proposed method. It is possible to observe how the distance is lower where the curvature is higher.

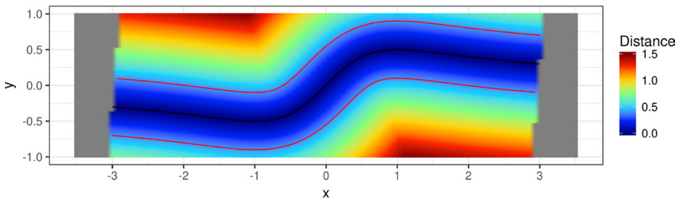

Fig. 2. Tolerance zone and distance field

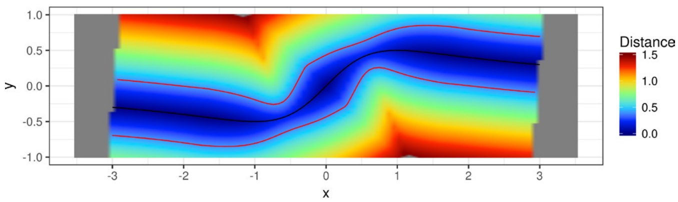

Fig. 3. Proposed tolerance zone and distance field

The effect of the discretisation can be observed in Figures 3 and 2. In this example there are 3 pixels between the nominal and the tolerance profiles, as it is possible to observe that the tolerance profiles are smooth.

3.2. Turbine blade profile

In this section a portion of a freeform turbine profile is analysed. A problem of the constant tolerance zone of the current standard is that the radius of curvature can arbitrarily decrease inside the tolerance zone, this can lead to product in tolerance but not functional. The idea behind the proposed method is that the tolerance should be constant in not complex part of the surface and smaller in critical portions. Figure 4 shows the portion of the analysed turbine blade. The profile was reconstructed from the point cloud using the curve approximation implemented in GoTools, setting a maximum reconstruction error of 0.001 mm. The maximum tolerance, T_{max}, was set to 0.06 mm, while the minimum, T_{min}, to 0.02 mm. The constant and the proposed method to compute the tolerance zone are reported in figure 4, while in figure 5 is reported a magnification of the portion of the blade with highest curvature.

The deformation of the distance can be observed in figure 6. The tolerance zone is constant where the curvature is null, or small, and it gradually decrease on the curved portion of the profile. This allows a better control of bottom part of the surface in a quality control scenario allowing to detect non functional products. It should be noted that if the nominal design is represented by the union of a profile and a circle the tolerance zone

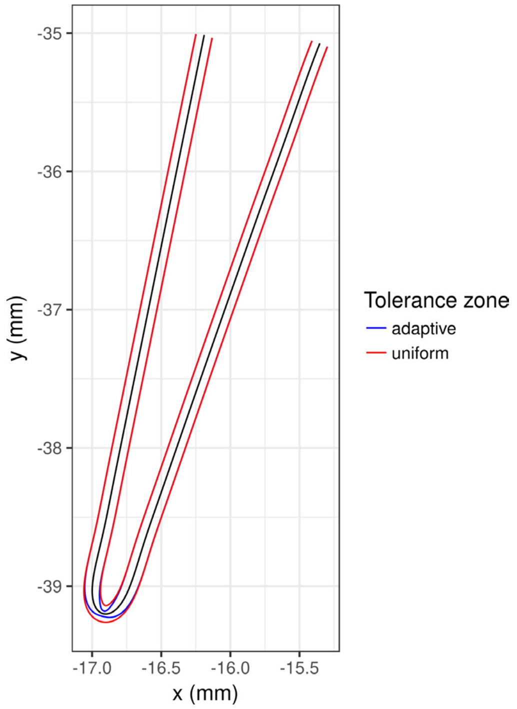

Fig. 4. Constant and proposed tolerance zone of a turbine blade portion

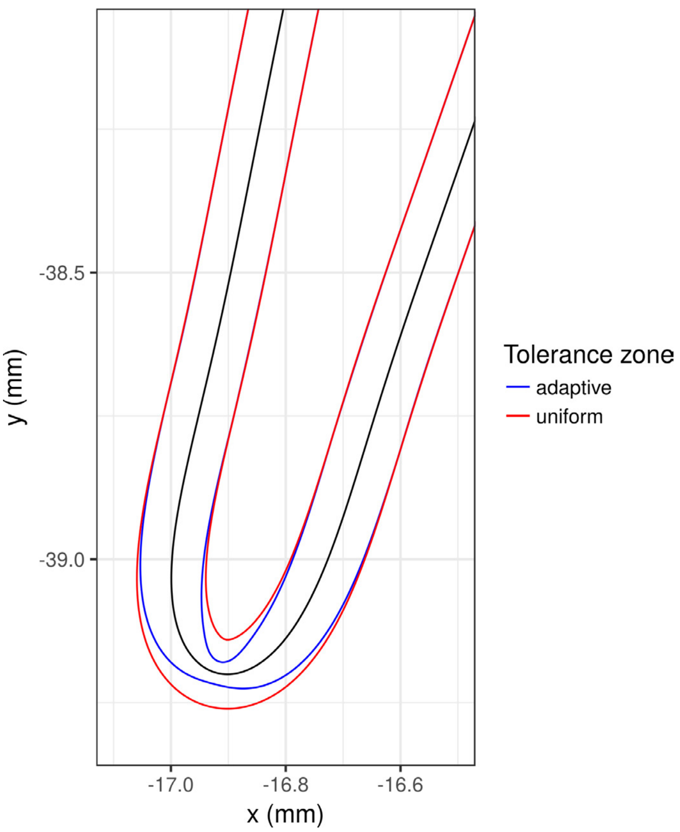

Fig. 5. Magnification of constant and proposed tolerance zone

will have a discontinuity. To allow a linearly decreasing tolerance zone the design of the part must have a C^2 continuity at the border of the two profiles.

To allow a larger tolerance zone without changing the curvature constraint, a hierarchical procedure can be designed. The adaptive tolerance zone can "follow" the profile up to the region of high curvature, it can then vary inside the constant tolerance band controlling the curvature of the final product.

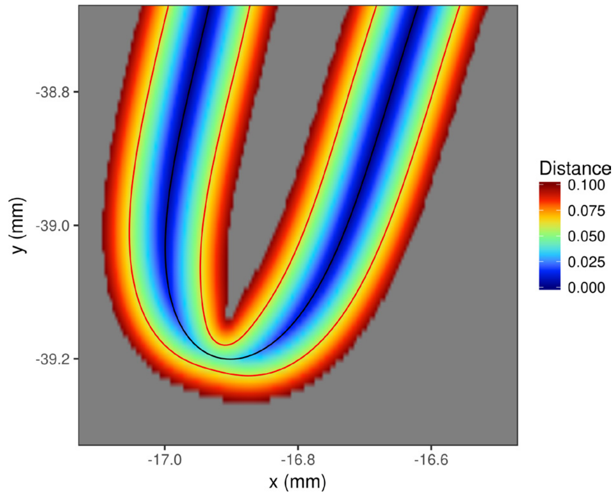

Fig. 6. Magnification of constant and proposed tolerance zone

4. Conclusions and future work

In this paper a method that uses the complexity of a freeform profile to change the tolerance zone was proposed. It shows that the proposed tolerance zone can combine small and large tolerancing controlling the size according to the complexity of the curve, defined through the curvature.
A future development could regards the integration of the proposed method to the one proposed in [9], i.e. defining different tolerance zones according to the functional portion of the surface and connected them continuously with an adaptive tolerance method. To check the tolerance of the whole product, The proposed method can also be extended to be able to process all types of tolerances of a complex product, especially for freeform surfaces. A possible extension to the surfaces can be done applying a voxel base method as in [7] to design a spatial grind. The final tolerance zone can be extracted as the iso surface of the scalar field defined on the volume using, for example, the marching cube algorithm [5].

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