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A study of contact state for non-ideal planar surface assembly

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Abstract

In assembly process, there are various contact states because of the non-ideal surface morphology, which will cause contact stability problems. In this paper, a method of analyzing the contact state and stability of non-ideal planar surface is proposed for the contact stability problem. Based on the representation model of non-ideal planar surface, a Gradual posture adjustment method is developed for the contact points searching. The area ratio coefficient is incorporated to evaluate the assembly stability caused by the contact state of non-ideal planar surfaces. To deal with the poor stability problem such as local contact, new contact point search strategy is conducted to obtain more stable contact state. An example of analyzing the assembly contact state and stability of the non-ideal planar surfaces machined by end milling processes is provided to show the effectiveness of the method.

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Keywords: Non-ideal planar surface; contact state; contact stability; gradual posture adjustment method; area ratio coefficient

1. Introduction

Due to the deviation caused by manufacturing errors in the manufacturing process, the manufactured surfaces are usually non-ideal surfaces [1]. The assembly contact state will change because of the deviation during the assembly process between two non-ideal surfaces. Contact stabilities of various contact states are different, that may cause the contact stability problem and furthermore influence the assembly performance.

In the researches on the non-ideal surface, Schleich et al. proposed a skin model method based on Geometrical Product Specification (GPS) to describe the uncertainty of the surface. At the early design stage, the surface geometric errors should be predicted by considering random deviation and system deviation [2]. Different approaches to the assembly simulation of Skin Model Shapes are proposed [3]. As for the researches on non-ideal surface deviation modelling, Cao decomposed the surface deviation into location deviation, direction deviation, and shape deviation, etc. A modelling method for multiscale deviations in respect to deviation principles was proposed [4].

One of the most critical factors affecting the assembly performances is the assembly contact state between the non-ideal surfaces. Many researches have been done in this field. Lee et al. researched on the planar surface assembly model and cylindrical surface assembly model, and method was presented to adjust the location and direction of surface for the two models [5]. Zuo et al. studied the assembly contact problem of the planar surface with shape deviations. A method of gap surface was proposed to determine the assembly contact points [6]. Fei et al. presented a contact state prediction algorithm based on errors for convex polyhedron, which could predict the possible contact states of the assembly surfaces [7]. Greenwood et al. regarded rough surface as a set of spherical micro-bulges ruled by Gaussian distributions and proposed the GW Model for solving rough surface contact problem [8]. Based on GW Model, Rao et al. studied the effect of surface waviness. Specifically, the geometric shape of rough surfaces could be described by using random distribution model. Contact stiffness was calculated as well [9].

However, in the early researches, the major concern focused on the mathematical expression of non-ideal surface and deviation, rather than manufacturing errors which actually lead to deviation. The researches on assembly surface contact state had many assumptions and simplified the surface morphology

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of rough surfaces, which cannot fit the research on assembly and contact state of non-ideal surface caused by the manufacturing errors. The study in this paper is based on the non-ideal planar surface caused by manufacturing errors. The deviation model of the non-ideal surface is built and a homogeneous transformation model is established for position and posture adjustment. Then, the non-ideal planar surface contact state realization and analysis method is proposed, with which the non-ideal surfaces are assembled and contacted. And the contact stability of assembly contact state is analyzed and a new contact point search strategy is conducted to obtain more stable contact state.

2. Realization of non-ideal planar surface contact state

Assembly contact process of non-ideal planar surface will produce various contact states. The achieving of assembly contact state is the basis of contact stability analysis. For non-ideal planar surface, the formation of a stable contact state needs three non-collinear points to locate which can restrain a translational degree of freedom and two rotational degrees of freedom [10]. Therefore, the realization of non-ideal planar assembly surface contact state can be considered as the process to find the three contact points.

2.1. Non-ideal planar surface caused by the manufacturing errors modeling

The non-ideal surfaces express the geometric deviations caused by the manufacturing process, they are fluctuations based on the ideal surfaces [11]. Modeling for non-ideal surfaces is a prerequisite for non-ideal surface calculation and analysis.

The non-ideal surface is produced during the manufacturing process due to the manufacturing errors. There are many manufacturing errors that can cause different deviations. For example, the error from the fixture and the tool can cause the different deviations of the manufactured parts. According to the superposition principle, deviations caused by the main manufacturing errors are linearly superimposed on the ideal surface to form the non-ideal surface. Fig. 1 shows the model of our non-ideal planar surface caused by the manufacturing errors.

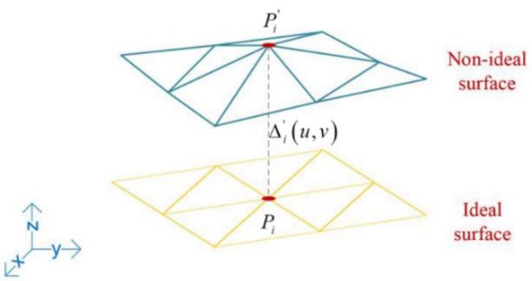


Fig. 1. Non-ideal planar surface deviation model.

Let the point on the ideal planar surface be P_i, where i is one of all points. The coordinate of P_i in Cartesian coordinate system is (x_i, y_i, z_i). In the deviation coordinate system, $\{u,v\}$ is the relative coordinates of the point on the surface grid. The deviation caused by each manufacturing error is established by the model with $dev = f(u,v)$. Use the deviation superposition principle, the total deviation is:

$$\Delta_i(u,v) = \sum_{j=1}^{n} dev_j = \sum_{j=1}^{n} f_j(u,v) \quad (1)$$

Where n is the number of main errors, j is one of the all errors.

Through coordinate transformation, the deviation value of point P_i in Cartesian coordinate system is $\Delta_i'(u,v)$ .The deviation value is superimposed on the z value of ideal coordinate to obtain the point P_i' on the non-ideal planar surface, as shown in Fig. 1. The coordinates are as follow:

$$P_i' = \{x_i, y_i, z_i + \Delta_i'(u,v)\} \quad (2)$$

The point cloud model and the triangle facet model are used to model the non-ideal surface scattered data. The point cloud model organizes the scattered points according to the grid relative coordinates of the non-ideal surface sample points. The data structure of the point cloud model is simple and convenient for calculating.

On the basis of the point cloud model, the triangulation of the point cloud is carried out. The three points corresponding to the nearest relative coordinates $\{u,v\}$ are grouped into a triangle facet and whole point cloud is triangulated to construct the triangle facet model of non-ideal surface. The triangle facet model interpolates the gaps between the point clouds to improve the accuracy of calculation. Triangle facet model can enable the non-ideal surface point cloud to become continuous and represent the non-ideal surface geometry morphology better.

2.2. Non-ideal planar surface homogeneous transformation

The homogeneous transformation model is established for position and posture adjustment of non-ideal planar surface. Homogeneous coordinates can be used to achieve the transformation of points in three-dimensional space from one coordinate to another. For three-dimensional space points with coordinate as $p=(x,y,z)^T$, the homogeneous coordinates can be expressed as:

$$p = (x, y, z, 1)^T \quad (3)$$

The entire non-ideal planar surface can be represented by $4\times n$ homogeneous coordinate matrix composed of these homogeneous coordinates of points.

$$Surf = [p_1, p_2 \cdots p_i \cdots p_n] = \begin{bmatrix} x_1 & x_2 & \dots & x_i \dots x_n \\ y_1 & y_2 & \dots & y_i \dots y_n \\ z_1 & z_2 & \dots & z_i \dots z_n \\ 1 & 1 & \dots & 1 \dots 1 \end{bmatrix} \quad (4)$$

Where *Surf* is the non-ideal planar surface matrix.

The Homogeneous Transformation Matrix (HTM) is a matrix that describes the position and posture relationship between two rigid bodies in robot kinematics [12]. In this paper, the homogeneous transformation matrix is used to describe the spatial position change of non-ideal assembly surface relative to the datum surface. Let T be homogeneous transformation matrix. After homogeneous transformation, non-ideal surface $Surf_0$ get a new position $Surf_1$, the transformation formula can be written as:

$$Surf_1 = T \bullet Surf_0 \tag{5}$$

Homogeneous coordinates are used to transform the non-ideal assembly surface. Subsequently, establish translational and rotational HTM for surface adjustment.

The translation operation is an important adjustment of the assembly surface position. Translational transformation matrix is created for the translational motions as T_{trans} . A point in space and a ray starting from that point can form the point-ray axis. Rotating around the point-ray axis can adjust the local position and orientation of the surface. Rotational transformation matrix is created for rotating around the point-ray axis as R_{rot} .

During non-ideal surface assembly contact process, it takes m steps of operation, including translation and rotation. Each step produces a translational matrix or rotational matrix. The total adjustment matrix is T as:

$$T = T_1 \bullet T_2 \bullet ... T_i \bullet R_1 \bullet R_2 \bullet ... R_j \tag{6}$$

Where T_i is translation matrix and R_j is rotation matrix, the i or j is the number of translation or rotation steps and $i + j = m$.

2.3. Gradual posture adjustment method

On the basis of homogeneous coordinate representation of non-ideal planar surface and the establishment of homogeneous transformation matrix, the three contact points will be found in turn. Due to the irregular geometry of the non-ideal surface, the locations of the contact points are uncertain. The assembly surface is translated and rotated. Subsequently the assembly clearance is calculated to find three contact points.

Assembly clearance refers to the clearance between two assembly surfaces, which is the shortest distance from the point on the non-ideal surface to the datum surface [13]. Based on the non-ideal surface point cloud model and triangle facet model, all assembly clearances are calculated between the non-ideal surface and the datum surface. The point where the clearance value is *zero* is regard as the contact point.

The first contact point is found by translating the surface into contact with the nearest point. Calculate all assembly clearances between points on non-ideal planar surface to the datum surface. The point that has the minimum assembly clearance value is $P^1_{\min}$ and the corresponding point on the datum surface is $P^2_{\min}$. The non-ideal surface is translated to coincide that two points, then the first contact point is obtained. The adjustment matrix is T_{step1} .

The second and third contact point searching is based on the first contact point that has been determined. With the first contact point as the origin point of rotation, the non-ideal surface is rotated until contact the assembly surfaces. However, due to the irregular geometry of non-ideal surface, the locations of the second and third contact points are difficult to determine. In this paper, Gradual posture adjustment method is proposed to find the second and third contact points.

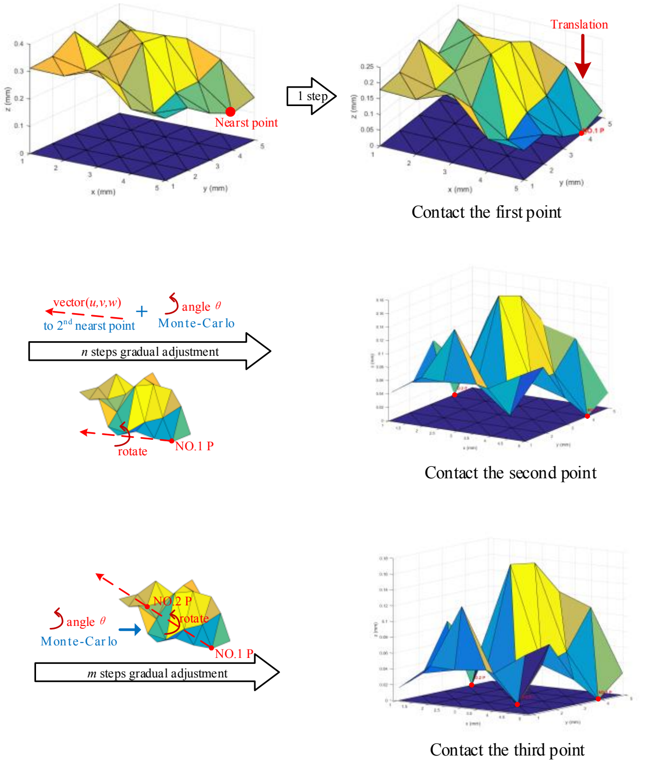


Fig. 2. Gradual posture adjustment method process

The adjustment matrix is used for rotating surface around the point-ray axis. The origin of axis is the first contact point P^1_{contact} , which is already determined. The unknowns of the rotation matrix are the ray direction $a = (u, v, w)$ and the angle θ of rotation. Therefore, the calculation of the rotation matrix becomes parameters of $\{u, v, w, \theta\}$ determination problem. Gradual posture adjustment method will divide the entire adjustment process into multi-steps. Each step should get the direction of rotation to determine the parameters $\{u_i, v_i, w_i\}$ according to a certain strategy. Deviations of non-ideal surface are minor, so the rotation angle is quite small. According to the measurement of the deviation value, the scope of rotation angle is determined. Subsequently the rotate angle θ_i is generated from the scope use the Monte-Carlo method. The parameters $\{u_i, v_i, w_i, \theta_i\}$ of each step can be used to determine the rotation matrix R_i . Finally, the total adjustment matrix is derived use formula (6).

The second contact point is calculated using Gradual posture adjustment method. After contact the first point $P^1_{contact}$, the assembly clearance between the assembly surfaces is calculated. Take the strategy of rotating toward closer point to determine the direction of rotation, $d_m^{\ 2}$ is the second minimum clearance value, the corresponding point on the non-ideal surface is P^1_m , the point on the datum surface is P^2_m . The ray direction is the cross direction of vector $\overline{P^1_{contact} P^1_m}$ and vector $\overline{P^1_{contact} P^2_m}$.

According to the statistical results, Monte-Carlo method is used to select the appropriate rotate angle. According to the experimental statistical summary, the surface is rotated every step with angle in [0,1] degrees randomly selected. The rotation matrix R_i is got to complete one step of surface rotation as $Surf_i = R_i \bullet Surf_{i-1}$, where $Surf_{i-1}$ is the surface of previous step.

The above operation is repeated until the second minimum clearance value is less than the given accuracy to find the second contact point $P^2_{contact}$. After n steps, the second adjustment matrix is calculated as:

$$R_{step2} = R_1 \bullet R_2 ... R_n \tag{7}$$

The calculation of the third contact point also uses Gradual posture adjustment method, and the process is similar to the calculation of the second contact point. But there are two differences, one is that the third minimum clearance value $d_m{}^3$ need to be calculated instead of $d_m{}^2$ and the other is that direction of rotation axis is vector $\overrightarrow{P^1_{contact}P^2_{contact}}$. Rotate the surface with rotation angle θ_i, and the rotation matrix is R_i. The process is repeated until $d_m{}^3$ is less than the given accuracy and the third contact point $P^3_{contact}$ is found. After m steps, the third adjustment matrix is calculated as:

$$R_{step3} = R_1 \bullet R_2 ... R_m \tag{8}$$

The Gradual posture adjustment method can realize the contact of two non-ideal surfaces caused by the manufacturing errors, obtain three contact points and each adjustment matrix. The total adjustment matrix is calculated as:

$$T = T_{step1} \bullet R_{step3} \bullet R_{step3} \tag{9}$$

The Gradual posture adjustment method is a simulation of the real assembly contact process, which realizes the contact of the non-ideal planar surface based on manufacturing errors directly through the adjustment of the surface posture and the control of the assembly clearance.

3. Contact state and contact stability analysis

3.1. Contact state analysis

After completing the assembly process of the non-ideal surfaces, a certain assembly contact state is formed. For non-ideal planar surface, different contact states are produced due to changes of surface morphology.

The clearance surface is a surface that reflects the distribution of clearances between the assembly surfaces [12]. The assembly clearance between the point on the non-ideal surface and the datum surface is *C*. The assembly clearance values and the grid coordinate (u, v) of points are combined to form a clearance surface, while the expression is:

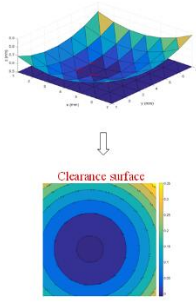
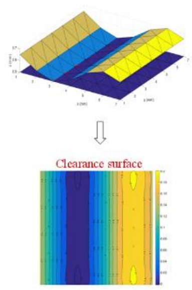
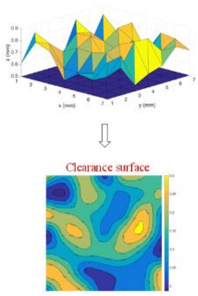
$$C = f(u,v) \tag{10}$$

The clearance surface is calculated, subsequently the clearance distribution between the assembly surfaces is reflected. Therefore, the assembly contact state can be expressed through the clearance surface. Different contact states correspond to different clearance surfaces.

In this paper, contact states of three kinds of non-ideal planar surface are studied including concave-convex surface, ripple surface and random surface. The three types of surfaces are assembled and contacted using Gradual posture adjustment method. The clearance surface is calculated and contour plotted. The contact states and clearance surface images are shown in table 1.

Three contact states are analyzed according to the clearance surfaces. The contact area of concave surface is mainly concentrated at the bottom of surface to form an aggregation area. The surface is raised around, so it is easy to tilt. The contact area of ripple surface is mainly concentrated at the concave parts of surface to form the strips. The two sides of the surface is raised, it may tilt toward both sides. The distribution of random surface contact area is relatively scattered. There are many contact areas at different positions, so the contact is more stable than others.

Table 1. Three types of non-ideal planar surface and clearance surfaces

Concave	Ripple	Random
		

3.2. Contact stability analysis

3.2.1. Area ratio coefficient

Due to the non-ideal planar surface, the actual contact area of different contact states only accounts for a fraction of the total surface area. The presence of local contact reduces assembly contact stability and directly affects the mechanical properties of the assembled product.

The area ratio coefficient is put forward in this paper to indicate the contact stability of a certain contact state. After three contact points are determined and two non-ideal surfaces are contacted, the triangular area covered by the three points is called the contact stability area. The area ratio coefficient is the ratio of the contact stability area to the total surface area.

The three contact points of non-ideal planar surface are $\{P^1_{contact}, P^2_{contact}, P^3_{contact}\}$, the three edges formed by three contact points are $\{P^1_{contact}P^2_{contact}, P^1_{contact}P^3_{contact}, P^2_{contact}P^3_{contact}\}$. According to the Heron's formula, the triangular area S_{con} covered by these three points is calculated as:

$$\begin{cases} L = (P^1_{contact}P^2_{contact} + P^1_{contact}P^3_{contact} + P^2_{contact}P^3_{contact}) / 2 \\ S_{con} = L(L - P^1_{contact}P^2_{contact})(L - P^1_{contact}P^3_{contact})(L - P^2_{contact}P^3_{contact}) \end{cases} \tag{11}$$

The total area of non-ideal surface is S_{all}. The area ratio coefficient κ is calculated as:

$$\kappa = \frac{S_{con}}{S_{all}} \qquad (12)$$

For non-ideal planar surface contact state, the larger area ratio coefficient value is, the more stable contact will be. Compared with the statistical distribution of the clearance [3], the area ratio coefficient can directly reflect the stability of non-ideal planar surface contact state by a single numerical value. Through the contact points obtained by the Gradual posture adjustment method, the contact stable area can be directly calculated to obtain the area ratio, and the calculation is convenient, which can be used as an index to evaluate the assembly quality.

3.2.2. The strategy of adjusting to the further point

Due to the surface morphology, the contact states of the concave and ripple non-ideal planar assembly surface will lead to contact stability problems. As mentioned before, the concave surface is easy to tilt around and the ripple surface is easy to tilt toward two sides.

In these surfaces, local contact appears during the assembly process because of surface morphology. The contact stability area covered by the three contact points is small and the area ratio is small, so the stability performs poorly.

In order to improve the contact stability of these non-ideal planar assembly surfaces, the adjustment strategy is changed based on the Gradual posture adjustment method. For the second and third contact point searching process, the direction of point-ray axis which is the $\{u_i, v_i, w_i\}$ parameters of each step need to be determined. The improved strategy is to adjust the surface toward the further point with larger clearance value. After contact the first point $P^1_{contact}$, the assembly clearance between the assembly surfaces is calculated, d_{max} is the maximum clearance value, the corresponding point on the non-ideal surface is P^1_{max}, and point on the datum surface is P^2_{max}. The rotation axis direction is the cross direction of vector $\overrightarrow{P^1_{contact}P^1_{max}}$ and vector $\overrightarrow{P^1_{contact}P^2_{max}}$. Select the appropriate angle θ_i to complete one step of surface rotation. The above operation is repeated until the second minimum clearance value is less than the given accuracy and the second contact point is found as $P^2_{contact}$.

The change for finding the third contact point is similar to the second one. Adjust the surface toward the direction of the point with maximum clearance value use appropriate angle. Repeat this operation until the third minimum clearance value is less than the given accuracy and the third contact point is found as $P^3_{contact}$.

Improved Gradual posture adjustment method adjusts the surface toward the further point, which can increase the local contact area of non-ideal planar surface and achieve a more stable contact state. For the surface such as ripple surface, the smaller clearance values are concentrated around the first contact point. Rotating the surface toward closer point to find the second and third contact point will cause the calculation to converge prematurely, then the local contact will be caused with poor contact stability. Rotating the surface toward farther point can avoid the calculation to converge prematurely and make the contact points cover more surface area, thus the contact is more stable.

4. Example of non-ideal planar surface contact state calculation and analysis

The assembly contact process of non-ideal planar surfaces caused by the manufacturing errors is studied in this paper. The contact states and contact stability are analyzed to validate of the algorithms and methods mentioned above.

End milling plays an important role in the plane forming process. The non-ideal planar surface data obtained by end milling simulation are used to the calculations in the paper. Four main errors, including spindle axis tilt of milling cutter, the straightness error in vertical plane of guide rail of milling machine, the cutting heat as well as the tool wear, are considered when using end milling to process the plane. The deviations caused by four errors are superposed to obtain the non-ideal planar surface of end milling simulation. The size of example surface is $9mm \times 11mm$. *120* points are extracted as sample points for non-ideal planar surface. On the basis of the non-ideal planar surface point cloud model and the triangle facet model, the Gradual posture adjustment method is used to realize the assembly contact state as follows:

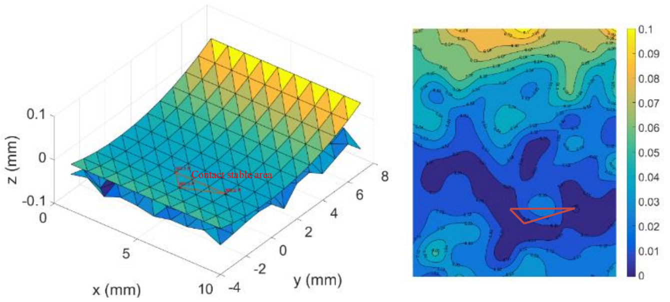


Fig. 3. Contact state of gradual posture adjustment method

The total adjustment matrix is calculated as:

$$T = \begin{bmatrix} 1.0000 & -0.0000 & -0.0000 & -0.0000 \\ 0.0000 & 1.0000 & 0.0009 & -0.0000 \\ -0.0000 & 0.0009 & 1.0000 & -0.0167 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Considering the non-ideal surface, different assembly contact states can be produced during the assembly process, resulting in different contact stability. The stability of the non-ideal surface contact state will affect the quality of the assembly. Therefore, the stability of the contact state should be evaluated by the area ratio coefficient.

The contact stability area is calculated as $S_{con} = 1.50mm^2$, the total area of the surface is calculated as $S_{all} = 99mm^2$, area ratio coefficient is calculated as:

$$\kappa = \frac{S_{con}}{S_{all}} = 0.0152$$

Improved Gradual posture adjustment method is used to adjust the surface toward further point. Assembly contact calculation on non-ideal plane surface is carried out the second time to update the contact state. The new assembly contact state is as follows:

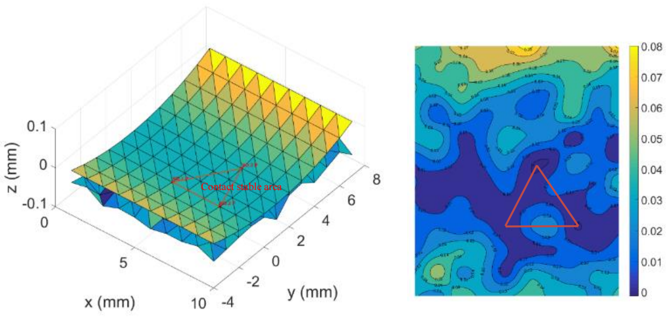


Fig. 4. Contact state of the improved gradual posture adjustment method

The total adjustment matrix is calculated as:

$$T = \begin{bmatrix} 1.0000 & -0.0000 & -0.0000 & -0.0000 \\ 0.0000 & 1.0000 & 0.0017 & -0.0000 \\ -0.0000 & -0.0017 & 1.0000 & -0.0193 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

The contact stability area is calculated as $S_{con}^{'} = 4.50mm^2$, the total area of the surface is calculated as $S_{all} = 99mm^2$, new area ratio coefficient is calculated as:

$$\kappa' = \frac{S_{con}^{'}}{S_{all}} = 0.0455$$

The new area ratio coefficient of contact state is larger than the old one, as $\kappa' > \kappa$. Therefore, the contact stability of the new contact state is better performed. The calculation results show that the improved Gradual posture adjustment method which adjust the surface toward further point can achieve more stable contact state for the non-ideal planar surface produced by the end milling process.

5. Conclusion

Contact states of non-ideal planar surface produced during the assembly process are studied in this paper. The method of contacting non-ideal planar surfaces, analyzing contact states and contact stability is put forward. Gradual posture adjustment method can determine the positions of three contact points. The area ratio coefficient can evaluate the contact stabilities of contact states, and the larger coefficient value is, the more stable contact is. For the assembly contact states of three different non-ideal planar surfaces, concave surface and ripple surface tend to tilt, and random surface has the better stability. Regarding to the surface with poor contact stability such as ripple surface, the Gradual posture adjustment method is improved to enlarge the contact stability area and increase the area ratio coefficient, which can achieve a more stable contact state. The method lays a foundation for assembly simulation calculation and assembly performance analysis.

Acknowledgements

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