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Automated procedure for the creation of finite element mesh: application to non-periodic historical masonry

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***Abstract* – This paper presents an automated procedure that enables the creation of a finite element mesh directly from the image file representing the rasterized sketch of a generic masonry element. This procedure goes under the name "pixel strategy" if a 2D finite element mesh is needed, where the elements are planar and rectangular; conversely, its extension in the 3D case is named "voxel strategy", and there the resulting finite elements are solid bricks. The finite element meshes so obtained are then used for extracting homogenized in-plane failure surfaces for historical masonry cells, which display a non-periodic arrangement of units. These surfaces are consistent with the expected results, and their shapes suggest that the behavior of such type of masonry may range between orthotropic (if bed mortar joints are clearly noticeable) and quasi-isotropic (if some units spread over two or more masonry layers).**

I. INTRODUCTION

In the past 30 years the technique known as homogenization has proved itself as one of the most reliable and effective tools for modelling the behavior of masonry. This technique is rooted on the basic idea of Representative Element of Volume (REV in short) that is the smallest core encircling all the physical and geometrical characteristics required for a comprehensive representation of the material. Two main modelling strategies are usually employed for masonry: macro-modelling and micro-modelling. The former considers masonry as a homogeneous material that is equivalent to that coming from the composition of units and mortar [1], whereas the latter models in a distinct way the two constituents [2][3], at times considering also the physical interfaces that separate them. It can be said that homogenization stands out as a satisfying in-between technique: as far as the critical issues of the two other strategies are concerned, it neither requires experimental tests to be performed, nor it needs the distinct modelling of units and mortar on a large scale (but only at a cell level, in the REV). However, the correct geometrical representation of the masonry bond is requested for homogenization, which becomes crucial when considering non-periodic masonry bonds that are not infrequently found in heritage buildings. In fact, both the creation of the actual masonry geometry for meshing purposes and the generation of a finite element mesh for a suitable representation of that geometry are two issues on their own and have been seldom addressed in the past. This paper presents two techniques for creating a FE mesh directly from the rasterized picture of a masonry element (panel, wall, pillar): one is based on the so-called "pixel strategy" and enables the creation of a 2D FE mesh, the other is its extension in the 3D case, this time based on a "voxel strategy". The meshes obtained with these procedures are then used in numerical applications that concern the extraction of homogenized in-plane failure surfaces, which act as macroscopic strength criteria for masonry REVs.

II. AUTOMATED STRATEGIES FOR CREATING FE MESH OF REAL MASONRY BONDS

A. *"Pixel strategy" for 2D FE mesh*

A fast, effective procedure for the generation of a finite element mesh directly from the rasterized image file of a real masonry element is presented in this section. A specific procedure called "pixel strategy" is devised, which is named in this way because any finite element is automatically created from one pixel of the considered rasterized image file. This is easily obtainable using the Image Processing Toolbox functions made available in the software Matlab [4], and needs to represent the greyscale or, better, the black-and-white sketch of the considered masonry bond. A dedicated Matlab script enables the actual generation of the finite element mesh. Taking as input only the real dimensions of the masonry element under consideration, the script first extracts the RGB triplet for each pixel, which are written into an $M \times N \times 3$ array, where M and N are the number of pixels along the vertical and horizontal directions of the image.

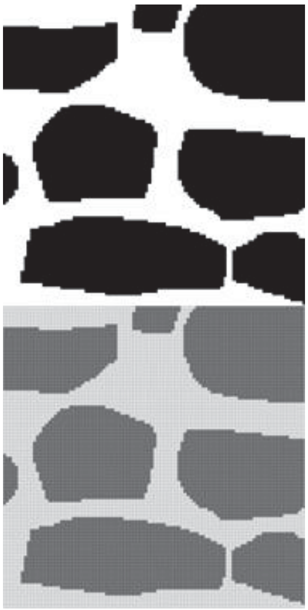

Fig. 1. Rasterized sketch of sample masonry panel (top); resulting 2D finite element mesh (bottom).

Second, an M×N matrix containing only the Red values of the RGB triplet is extracted from the bigger array, which is then used as a threshold for determining the physical nature of the pixel (i.e. if it pertains to mortar or to a unit). Each pixel is subsequently considered as the centroid of a planar, rectangular-shaped finite element, and the script provides it with a pair of XY coordinates that are aptly calculated from the input global dimensions; the center of the reference system is located at the centroid of the considered masonry element. Eventually, the XY coordinates of the four adjoining nodes of each finite elements are determined. Overall, three matrices are then created: one is the so-called "node matrix", containing the XY coordinates and ID number of each node (ordered from top to bottom first, and from left to right second); the second is the so-called "element matrix", containing the ID number of each finite element, the ID numbers of its four nodes (listed in a counterclockwise sense starting from the top-left corner), the XY coordinates of its centroid, and eventually its "material flag" that depends on the Red value of its RGB triplet. Finally, the third matrix is the so-called "macro element matrix": after an ID number is assigned to each unit ("macro element") in the masonry element here considered, for each macro element this matrix summarizes its ID number and the XY coordinates of its centroid. Fig. 1 shows an example of the resulting finite element mesh for a sample masonry panel after the application of the aforementioned procedure, compared to the original black-and-white rasterized source image of the panel itself.

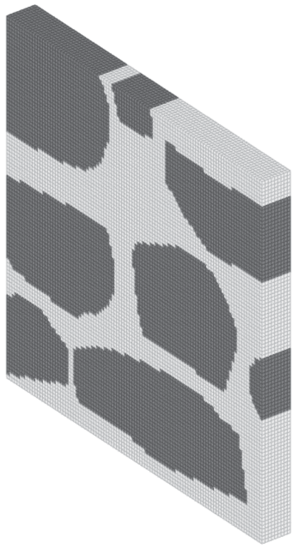

Fig. 2. Resulting 3D finite element mesh for the sample masonry panel of Fig. 1.

B. "Voxel strategy" for 3D FE mesh

As in the previous case, the 3D finite element mesh is created in Matlab from the rasterized image file of a masonry element. Here, the mesh generation is based on the so-called "voxel approach", which means that each 3D pixel (the "voxel") is to be transformed into a finite element. Voxels are once more provided with a "material flag" indicating if they belong to either a masonry unit or mortar, depending on the Red value of their related RGB triplet. Because of this, units and mortar must be denoted by clearly distinguishable colors in the source image, which again must be a simple black-and-white or greyscale sketch of the considered masonry element. Its overall dimensions are set as input by the user and exploited to determine the XYZ coordinates of the elements' centroid, according to a reference system originated at the center of the test-window. This reference system is a permutation of the one created in the 2D case: here, axis Y represents the horizontal axis of the test-window, axis Z represents the vertical axis, and axis X represents the transversal direction. Solid brick elements are then generated from the centroid's coordinates, and the number of elements over the transversal dimension is also to be input by the user. Three matrices are eventually created, one listing the node IDs and their coordinates (the "node matrix"), the second listing the finite element IDs, those of their 8 adjoining nodes, the material flag, and the coordinates of their centroid (the "element matrix"), and the third listing the macro element IDs and the coordinates of their centroid (the "macro element matrix"). It is worth noting that the transversal layout of the considered masonry element is here obtained by simply extruding the in-plane geometry. Fig. 2 shows an example of the 3D mesh resulting from the procedure for the sample test-window of Fig. 1.

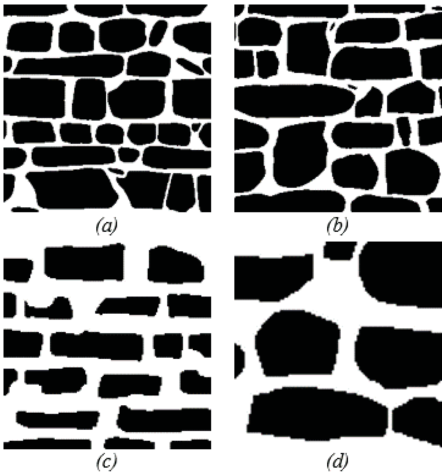

Fig. 3. Four masonry REVs used in the 2D numerical application.

III. HOMOGENIZED IN-PLANE FAILURE SURFACES

A. 2D FE mesh

The 2D FE mesh created through the procedure named "pixel strategy" is used as input in a separate Matlab script that is devoted to the determination of homogenized in-plane failure surfaces, which represent in-plane macroscopic strength criteria for masonry elements. They result from the solution of an upper bound limit analysis problem that includes a homogenization approach, formulated as a minimization problem (a sub-class of a linear programming problem) in standard form. The finite elements of the mesh are supposed to be rigid, therefore dissipation solely occurs across the interfaces of adjoining elements. A Mohr-Coulomb failure criterion with a cut-off in tension is used to address the velocity jumps across the interfaces between mortar elements and a unit and a mortar element. The equality constraints for the minimization problem come from the velocity jumps due to dissipation, the periodicity conditions on the sides of the considered masonry element (as required by the homogenization approach), and from the normalization of the dissipated external power (which is needed for finding a single solution in terms of deformed configuration at collapse). Four different masonry REVs are considered in this numerical application, and they are pictured in Fig. 3. The parameters of the Mohr-Coulomb criterion used in this application are listed in Table 1. Fig. 4 shows the homogenized in-plane failure surfaces in the tension-tension range for the four considered masonry REVs. It can be seen that most of them display a shape that suggests an orthotropic response under tensile loads, which is to be expected since the bed joints are clearly visible, despite a non-periodic arrangement of the units. Only case *(b)* displays a quasi-isotropic response, due to the presence of units that span two layers of masonry.

Table 1. Parameters of Mohr-Coulomb criterion for the 2D numerical application.

Cohesion [MPa]	Friction angle [°]	Tensile strength [MPa]
0.15	30	0.1

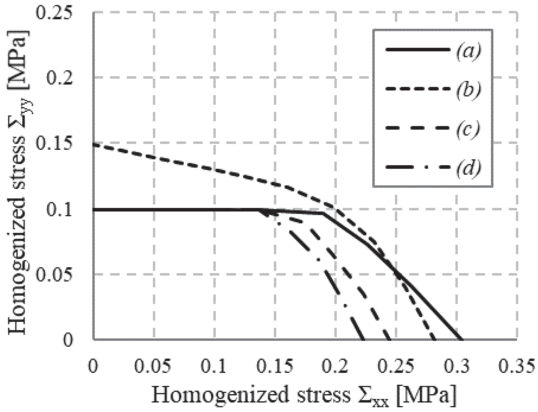

Fig. 4. Comparison among the homogenized in-plane failure surfaces for the four cases.

B. 3D FE mesh

Also the 3D FE mesh created through the procedure named "voxel strategy" is used as the basis for creating homogenized in-plane failure surfaces, which come as the results of another Matlab script containing a minimization problem. This is aptly modified to accept as input a 3D mesh instead of a 2D one. In this case, a Mohr-Coulomb failure criterion with a cut-off in both tension and compression is used to address the velocity jumps across the interfaces between mortar elements and a unit and a mortar element. The parameters used in this application are those listed in Table 1, with the only addition of a compressive strength equal to 1.5 MPa. Fig. 5 shows the 3D FE mesh used in this application, which represents case *(d)* of the previous section. Fig. 6 shows the full homogenized in-plane failure surface for the considered case. In the compression-compression range, the resulting homogenized failure surface is limited by the value 1.5 MPa, which is equal to the considered compressive strength; for composite in-plane load conditions, the considered masonry REV displays an increase in its strength, as expected in such a case. Fig. 7 shows the failure modes for 4 uniaxial load conditions, which are all consistent with the expectations. Horizontal tension (Fig. 7a) causes widespread cracks in the masonry test-window, while vertical tension (Fig. 7b) originates horizontal cracks. Horizontal compression (Fig. 7c) also originates horizontal cracks due to lateral expansion, whereas crushing of mortar is observed in case of vertical compression (Fig. 7d).

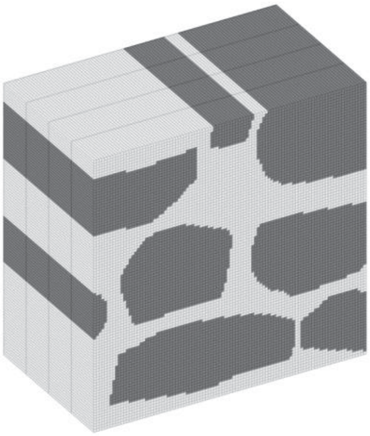

Fig. 5. 3D finite element mesh for case (d) of the previous section.

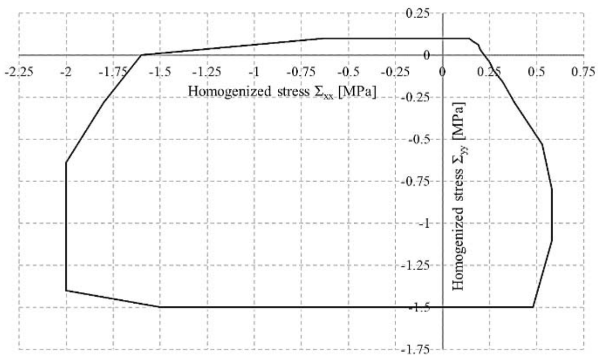

Fig. 6. Full homogenized in-plane failure surface for the considered masonry REV with 3D FE mesh

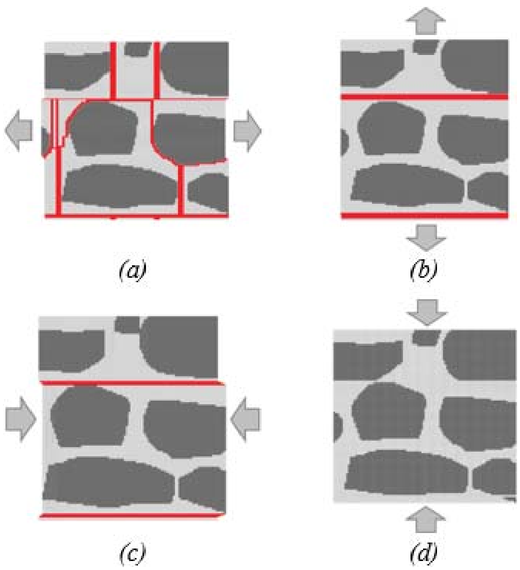

Fig. 7. Failure modes for four uniaxial load conditions applied to the 3D FE mesh.

IV. HOMOGENIZED OUT-OF-PLANE FAILURE SURFACES

Eventually, the 3D FE mesh created through the procedure named "voxel strategy" is used as the basis for creating homogenized out-of-plane failure surfaces, which come as the results of yet another Matlab script containing a minimization problem. Here, the goal is investigating the collapse behavior of non-periodic masonry bonds under out-of-plane loads such as seismic actions. Also in this case, a Mohr-Coulomb failure criterion with a cut-off in both tension and compression is used to address the velocity jumps across the interfaces between mortar elements and a unit and a mortar element. The parameters used in this application are once again those listed in Table 1 plus the compressive strength equal to 1.5 MPa. In this application, two 3D FE meshes are created for cases *(b)* and *(c)* of Fig. 3, whose depiction is here omitted for sake of brevity. The thickness of the created meshes is equal to 40 cm for case *(b)* and 15 cm for case *(c)*, respectively. Fig. 8 and Fig. 9 show the homogenized out-of-plane failure surfaces for the two considered cases: in fact, the aforementioned Matlab script enables the extraction of two distinct homogenized out-of-plane failure surfaces, one dealing with the flexural collapse behavior and defined in the M_{xx}-M_{yy} plane, the other dealing with the torsional collapse behavior and defined in the M_{xx}-M_{xy} plane. Namely, M_{xx} is the vertical bending strength, M_{yy} the horizontal bending strength, and M_{xy} the torsional strength. In both surfaces, the collapse moments are normalized with respect to a horizontal bending strength equal to $(f_t \cdot t^2)/2$. It is possible to observe that case *(c)* displays a greater resistance in terms of vertical and, to a lesser extent, torsional moments.

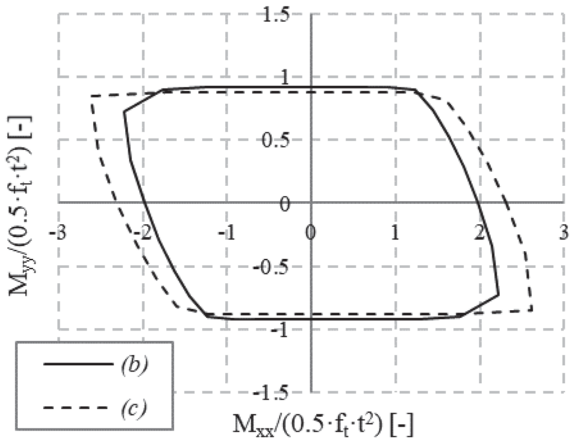

Fig. 8. Flexural homogenized out-of-plane failure surface for cases (b) and (c) of Fig. 3

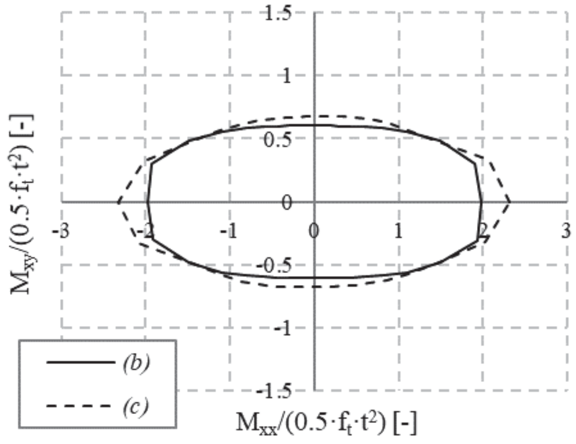

Fig. 9. Torsional homogenized out-of-plane failure surface for cases (b) and (c) of Fig. 3

Fig. 10 shows the failure modes related to case *(c)* extracted for three relevant out-of-plane load conditions, namely M_{xx}, M_{yy}, and M_{xy}. They are all consistent with the expectations for such a case, which represents a quasi-regular masonry bond. Specifically, the vertical bending moment M_{xx} (Fig. 10a) causes widespread cracks in the masonry test-window, while the horizontal bending moment M_{yy} (Fig. 10b) originates a single horizontal crack.

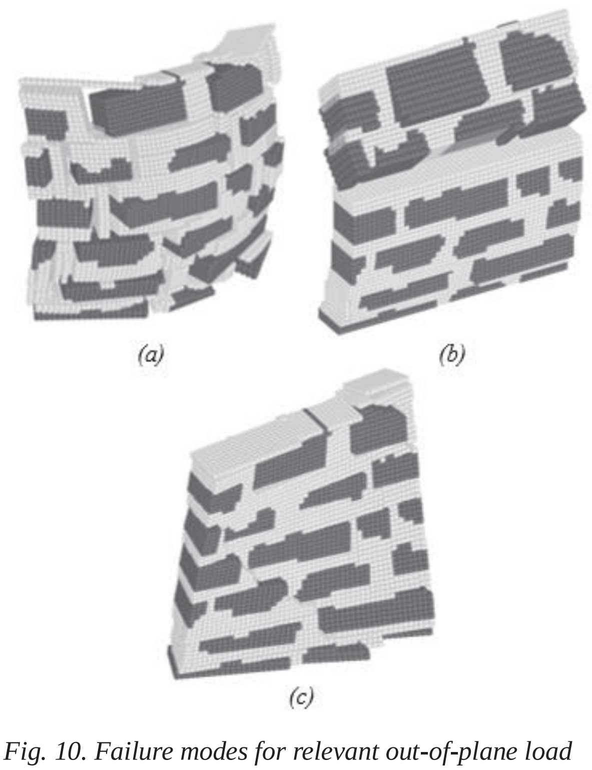

Fig. 10. Failure modes for relevant out-of-plane load conditions applied to case (c)

V. CONCLUSIONS

A fast, automated procedure for the generation of finite element mesh directly from the rasterized sketch of a generic masonry element is presented, which is particularly suitable for complex and irregular (non-periodic) masonry bonds that can be observed in heritage buildings or found in archaeological sites. Two procedures are set for the creation of the finite element mesh. One is named "pixel strategy" because it converts each pixel into a single finite element, then allowing the creation of a 2D FE mesh consisting of planar, rectangular finite elements. The other is called "voxel strategy" because it first transforms a 2D pixel into an analogous 3D entity called "voxel", which is then converted into a single finite element; eventually, it enables the creation of a 3D FE mesh consisting of solid brick elements. The 2D or 3D FE meshes can then be employed as input for several numerical applications that involve finite element analyses. In this case, the numerical application concerns the extraction of homogenized in- and out-of-plane failure surfaces of masonry elements, which basically represent macroscopic strength criteria for in- and out-of-plane load conditions, respectively. A series of homogenized in-plane failure surfaces are derived for 4 masonry REVs by using 2D FE meshes coming from the "pixel strategy", and for one of them by using a 3D FE mesh coming from the "voxel strategy". The shapes of the various surfaces indicate that non-periodic masonry displays a response that is orthotropic in case the bed joints are still distinctly visible, but it may become quasi-isotropic in presence of some units that span two (or more) masonry layers. Also, two different sets of homogenized out-of-plane failure surfaces are extracted for a couple of the masonry REVs investigated in terms of in-plane collapse behavior. Some relevant failure modes are also obtained for both the in- and out-of-plane cases. Future numerical applications will involve the use of 3D FE meshes of the out-of-plane behavior of non-periodic masonry multi-leaf walls.

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