Original Article

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#### Abstract

This article discusses the meaning and function of the act of measuring ancient Egyptian architecture in the present-day context, in which the advent of digital culture has allowed the accumulation of extremely precise and accurate data. Our expectations on our modern measurements may lead us to select the ancient data through a filter that does not correspond to the ancient perspective, thus affecting the validity of the results. In order to disentangle past and current perspectives, it may be useful to discuss two aspects: the difference between ancient measures obtained from calculations and observations, and the meaning of precision and accuracy in modern and ancient times. A reconstruction of the planning and building process of ancient monuments is likely to take a successful path only if we are willing to look at the evidence from a slightly different perspective, in which numbers become part of a larger and more complex operation.


يناقش هذا المقال معنى ووظيفة القياس في العمارة المصرية القديمة في سياقها الحالي، وقد أسهم ظهور الثقنية الرقمية في جمع بيانات مضبوطة ودقققة للغاية. قد تقودنا نوقعانتا بشأن قياسانتا الحديثة إلى اختيار البيانات القديمة من خلا مرشح لا يتوافق مع المنظور القديم، مما يؤثر على صحة النتائج. ومن أجل الفصل بين وجهات النظر القديمة والحالية، قد يكون من المفيد مناقثة جانبين: الفرق بين المقاييس القديمة التي تم الحصول عليها من الحسابات والملاحظات، ومعنيي الضّبط والدقة في العصور الحديثةٍ والقديمة. من المحتمل أن نأخذ عملية إعادة بناء وتخطبط الآثار آلقديمة مسارًا ناجحًا فقط إذا كنا على استعداد للانظر إلىى الأدلة من منظور مختلف ولو قليلاً ، بحيث تصبح الأرقام جزءًا من عملية أكبر وأكثر تعقيدًا.

## Keywords

Ancient Egyptian architecture, ancient Egyptian mathematics, archaeometry, astronomical orientation, architectural measurements

## Introduction

This article contains some reflections on the act of measuring ancient Egyptian architecture, as well as on its meaning and function. It contains a series of thoughts in writing and does not claim to represent a set of guidelines; it is rather meant as an occasion to assess the situation and hopefully trigger further discussions at this interesting time, when Egyptology is entering the era of archaeometry, in which measuring (anything measurable) opens the way to hitherto unknown directions of research. Measuring archaeological remains is not a new operation, of course, but new survey techniques are now greatly improving the precision and the accuracy that can be attained. We are certainly gaining a wealth of fresh information, but the issue is not the amount of data that we manage to accumulate, but rather what we make of it, that is, how we interpret and use it. ${ }^{1}$

Just when measuring has become the foundation of such a great portion of archaeological research, it may be worth assessing the specific situation of ancient Egyptian architecture, looking back to what has been done from a fresh vantage point to reflect on future strategies and developments. This complex operation cannot be achieved by one author

[^0]on one occasion, as it requires the scientific community to elaborate a shared approach. Moreover, as the multidisciplinary character of archaeological research increases, it is also necessary to establish a shared vocabulary among different specialists beyond the traditional field of Egyptology: concepts that may sound obvious to some, might be new to others. Building bridges among disciplines means sometimes appearing redundant to the specialists, but the creation of a common platform to later operate in an efficient way is a necessary operation.

This article will start with a discussion on the need to keep a balanced approach to the objects that we measure today and avoid confusing modern and ancient units of measurements, methods and mentalities. It will then focus on the opportunity to differentiate between observations and calculations, in order to avoid unnecessary overinterpretations of the remains. The third and final part of the article will attempt to sketch a coherent picture in which both our modern language and reference points, as well as what we have learned of the ancient approach can find their place and provide the foundations for further theoretical investigations and for practical applications.

[^1]
## What do we measure?

## Conditiones sine quibus non

When we take measurements, we collect three types of data: information that the ancient Egyptians possessed (first) and information that they did not possess, including both information of which they were not aware (second) and knowledge that we wrongly attribute to them (third). This distinction is true in all fields, including architecture: to the first group may belong the dimensions of a building; to the second, the chemical composition of the stone that was used to build it; to the third, the symbolic use of specific numbers or geometric figures in its layout.

Concerning the first point, measuring the dimensions of a building contains in itself a crucial bifurcation, depending on the unit of measurement that we choose. If we use modern units of measurement to communicate amongst ourselves the results (a fully legitimate operation, of course), we must bear in mind that we are not speaking the same mathematical language as the ancient Egyptians, and therefore that further elaborations of the modern data are unlikely to have a historical validity. If, instead, we adopt the ancient Egyptian units of measurement, we have the chance to study and evaluate the ancient buildings from the historically correct point of view. ${ }^{2}$ This, in itself, is not enough to ensure the validity of the analysis, as the latter might be skewed by our expectation that mathematics and calculations played a role that did not correspond to the reality, as we shall see below.

Concerning the second point, measuring the chemical composition of the stones used in a building provides information that was not directly known to the ancient Egyptians. And yet it can indirectly contribute to our knowledge of that building, as it may reveal the provenance of those stone blocks, a piece of information that was indeed known to the ancient builders. In such a case, the adoption of modern units of measurement does not affect the validity of the results in historical terms, as the investigation follows a path that runs separate from and in parallel to the ancient activities.

The third point concerns all the cases in which historically inaccurate geometric constructions were adopted to explain the layout of buildings; among others, a glaring case is the use of the Golden Section. ${ }^{3}$ As all these theories are based on modern mathematical concepts, a useful method to check their historical validity is to re-formulate them in ancient units of measurements: in most cases, they simply no longer work. ${ }^{4}$

These distinctions may sound like obvious statements to surveyors, and yet if historians fail to see these differences

[^2]they might take the wrong turn in one of the subsequent junctions along the path of research. When dealing with the remains of ancient buildings, we move along a dangerous edge, where several factors can derail our research. First of all, in the absence of a precise survey, no reliable geometrical analysis of a building is possible. In this respect, it may be important to bring to the attention of Egyptologists the distinction that surveyors make between precision and accuracy: a measurement is precise if, taken more than once, gives the same (or an extremely similar) result; a measurement is accurate if its result is close to the absolute value of the quantity that is being measured. Reliable surveys are expected to be both precise and accurate to an acceptable degree. The definition of this degree depends on the scale at which we intend to deal with the object that is being surveyed and that will be later represented: for a small object we might need a sub-millimetric definition, for large ones that are expected to be represented at a scale based on metres (for instance a building), an approximation of 1 cm may be considered extremely satisfactory.

Therefore, the first step in a reliable geometrical study of a building is to perform (or access the results of) a reliable architectural survey. The second is to ensure that we choose the correct unit of measurement. The third is to be able to keep facts and expectations separated. This combination of factors can be found only in a tiny minority of cases among ancient Egyptian monuments. Over time, the overall accuracy of available surveys has been growing thanks to the efforts of specialised surveyors and to the introduction of increasingly precise instruments. The recent spread of satellite images and of digital surveys (photogrammetry and laser scanner), that are inherently more precise than handmade surveys, will increase the number of cases in which at least the first step is achieved, both at a large and a small scale. The fulfilment also of the second and third conditions depends on our attitude, that is, on how we handle the available data.

## Guidelines vs. grids

A reliable survey may be a necessary condition for a geometrical study, but it is not sufficient, as it does not speak by itself. The moment in which we start looking for patterns is crucial, as the success of this operation depends on our expectations more than we are probably ready to admit (as will be discussed below). Generally, we look at plans (flat representations of a three-dimensional object) and start drawing lines across them, expecting to discover connections. Diagonals, alignments, triangles, and square grids are often dragged into the picture, and they might even seem to work. ${ }^{5}$ Too often, however, we tend to forget that the (most important) numbers used to trace a courtyard were, after all, its length and its breadth.

The length of the entire building, in particular, should be handled with care. In the countless cases of ancient Egyptian monumental buildings that were built in progressive stages along their main axes, any study of a plan should be carried out bearing in mind this aspect and avoiding automatically

[^3]superimposing the same geometric pattern to the entire plan. Extensions might be planned as direct geometrical emanations of the original core, but could also depend upon or take into account other considerations or constraints, such as the availability of space or financial resources. The evolution of every multi-phase building, small or large, might have followed a different path, ${ }^{6}$ and trying to establish a general rule is likely to represent a pointless effort.

Another issue that can easily produce a methodological mistake is the fact that, in order to transmit information on a building, nowadays we rely almost entirely on architectural drawings, in particular on ground plans. This was not the case in ancient Egypt: ancient Egyptian architectural drawings consisted of syncretic representations that included the outline of the building on the ground, as well as doors, columns and other vertical elements which were represented as seen from the front (and not from above); these drawings were not to scale, and the dimensions were written out beside each architectural element, if deemed necessary. ${ }^{7}$ Therefore, our modern method to analyse buildings mainly through their ground plans highlights only some aspects of the ancient planning process, with the double risk of giving an unnecessary importance to certain details and, at the same time, missing out part of the picture.

This is certainly the case of geometric patterns supposedly underlying the buildings' plans. If complex patterns rarely survive close historical and geometrical scrutiny, simpler patterns may appear more reliable; this first impression, however, may be equally misleading. Square grids, for instance, have been often used to explain the layout of ancient Egyptian buildings; ${ }^{8}$ once more, however, whenever this suggestion is systematically tested, the results are not fully convincing.

Before plunging into a specific discussion on this subject, it may be useful to make a distinction between the modularity due to the adoption of square grids (which will be discussed below) and to the use of modular constructional elements (which will not be discussed here); in the first case, the use of modules would reflect on the layout, in the second case more directly on the dimensions of the spaces. Mudbrick architecture is inherently modular; the dimensions of the mudbricks themselves have been carefully noted and

[^4]analysed, ${ }^{9}$ together with the various bonding techniques that produce walls of different thicknesses, ${ }^{10}$ but to my knowledge if and how the dimensions of the mudbricks affected the dimensions of the rooms (that is, not of the solid parts but of the voids) still remains to be properly investigated. It is interesting to note that, instead, throughout most of its history ancient Egyptian stone architecture was not based on modular elements, with the exception of the Amarna talatats. In that case, however, the modularity of the stone blocks appears to have played a role in the quarrying phase, rather than in the building process, as during the constructions the blocks were then chopped to smaller pieces to fit the needs ${ }^{11}$. They were indeed used as mudbricks: as standardised constructional elements to be easily produced and handled, but not necessarily to produce modular spaces.

Going back to the supposed use of square grids, apart from the cases in which unlikely modules have been suggested (e.g. Badawy's values of $1.75,2.6,94+1 / 2$ and 104 cubits $^{12}$ ), in general the main problem is that their use would make sense if the project was drawn in advance to a small scale, and was then transposed onto the ground to a larger scale. This is exactly how the ancient Egyptians used square grids in two-dimensional representations of people and objects from the Middle Kingdom onwards, as abundantly witnessed by a wealth of archaeological remains (Old Kingdom and First Intermediate Period artists used guidelines). ${ }^{13}$

If we move to statuary, the potential of square grids rapidly diminishes: a square grid incised on the surface of a block would make sense only at the very beginning, in case the proportions of the statue to be sculpted had to be enlarged or reduced starting from another model, and to establish the overall symmetry and structure of the figure. ${ }^{14}$ Square grids incised or drawn on the uncut block would work to this purpose only on relatively flat and regular surfaces; in any case, they would quickly disappear as soon as the first layer of stone was removed by the sculptor. A similar result (and thus the rest of the process) could be achieved by marking the stone block with the vertical axis of the proposed statue and all the various horizontal subdivisions. These lines, too, would materially disappear, but with two main differences: first of all, whilst a grid is related to the figure mainly in an abstract way, the original position of guidelines would be

[^5]progressively and automatically enhanced by the removal of stone and the appearance of the corresponding physical features. Secondly, whilst reticular grids would become very difficult to use once the originally flat surface gave way to the undulating surface of the statue, simple guidelines could be easily re-painted over and over on it, and would eventually shape the final object. ${ }^{15}$

Moving to architecture, we should make a distinction between architectural elements and whole buildings. Concerning the former, the sketch of a Hathor-headed capital on a square grid found in a quarry at Gebel Abu al-Foda ${ }^{16}$ suggests that its proportions were kept under control thanks to this system; although this find dates to the Roman Period, ${ }^{17}$ it is in line with, and may well reflect, an earlier practice. Concerning entire buildings, instead, the theoretical use of square grids finds no confirmation in practice. There is no evidence that ancient Egyptian architectural drawings were based on square grids, and thus that the builders were expected to proportionally enlarge on the ground geometric figures established in advance by using this method. ${ }^{18}$ Moreover, it is difficult to envisage a practical reason as to why one should draw a square grid on the ground before laying out a building that was not expected to consist of square spaces. As with statuary, it makes far more sense to draw the main axis, to establish right angles, and to mark the position of transversal lines, that is, the length and breadth of the various spaces that were expected to be built along it. As noted above, the simplest way to establish the dimensions of a rectangular space is just to establish its length and breadth, that is, to outline the general area(s) corresponding to parts of the building to be constructed. ${ }^{19}$

A clear example of the adoption of this process is offered by the foundation layers of the temples at Amarna. The overall outline and the position of the main elements of the Great Aten Temple were marked upon the spread of gypsum concrete that acted as the foundation layer, first using black or red ink, and then by cutting shallow grooves along these lines: the overall plan of the building was probably transposed from a preliminary sketch or drawing onto the gypsum surface thanks to a number of orthogonal lines, marking the position of the main architectural elements. ${ }^{20}$ These lines marked axes and defined rectangular spaces;

[^6]there is no evidence of the presence of a sequence of regular squares. The possibility of superimposing a 20 -cubit grid to the plan of the Small Aten Temple, which appears to fit with the dimensions of the three (rectangular) enclosures, ${ }^{21}$ highlights the fact that these dimensions were multiples of 20 (10, or 5) cubits, but does not imply that a square grid was really used to design or lay out the building itself.

Therefore, establishing the main axis of the building, and thus its total length, is likely to have been the first operation to be carried out, then followed by the definition of the breadth and of the internal subdivisions. This sequence might correspond to the two operations listed in the texts describing the Foundation Ceremonies: the $p \underline{d} \check{s} s r$ and the whe wzwst, respectively translated as the 'stretching of the cord' and the 'unravelling of the cord'. The first might correspond to the act of establishing the axis of the building; the second operation, interpreted by Badawy as the 'spreading of the plan net', might indeed refer to the process of marking on the ground the extent and position of the various spaces that composed the building. ${ }^{22}$ As already noted, however, there is no need to assimilate the plan net to a sequence of regular squares.

Finally, it is worth noting that square grids do not even appear in the layout of the 'gridded' settlements of Lahun, Amarna and Deir el-Medina: the internal subdivisions are rectangular in shape. ${ }^{23}$

In conclusion, square grids were certainly used in twodimensional representations from the Middle Kingdom onwards, and may have been used in sculpture (statues and architectural elements) whenever precise proportions had to be enlarged or reduced from existing models. The earlier system of guidelines (parallel and orthogonal, but not necessarily spaced in a regular way as grids), instead, is likely to have represented the main method to establish axis and dimensions in architecture, and probably also in statuary. Therefore, square grids superimposed to architectural plans should be used with care. A one-cubit grid can be extremely useful for detecting dimensional patterns, ${ }^{24}$ but should be treated and considered as a tool to perform an

[^7]investigation, not as a result of it; seemingly, square grids with other side-lengths might indeed help us to reveal patterns between length and breadth of spaces, but there is no reason to assume that they were actually used by the ancient builders.

## Experiencing vs. representing

Another important point to bear in mind is that a building is much more than its ground plan. The perception that we have of a building depends not merely on its dimensions, but rather on its proportions (that is, on the relationship among the three dimensions, length, breadth and height) and on the overall architectural composition, comprehensive of colour and texture of the walls, illumination and combination of volumes and openings. ${ }^{25}$ Representing a building as a plan certainly helps to grasp its dimensions, distribution and organisation, but two-dimensional plans do not satisfactorily convey information on how the space was actually perceived and experienced.

Architects and builders imagine spaces and volumes in advance and may use drawings to define and transmit aesthetic and technical information relating to their ideas. Even more efficient may be three-dimensional models, that were built in a wide variety of materials until the advent of the digital era, in which they are created and manipulated on screen. ${ }^{26}$ Virtual (re)constructions are spreading also within archaeology, forcing archaeologists to focus not only on length and breadth of the excavated spaces, but also on their height and three-dimensional shape.

Admittedly, archaeological remains are often unable to provide enough information on the elevation of a building; in these cases, both physical and virtual reconstructions are based on a combination of interpretations and assumptions. ${ }^{27}$ The advantage of a virtual reconstruction over a physical one is that the former can be easily modified; actually, more than one virtual reconstruction may be suggested, depending on the interpretations and assumptions that are constructed on the available data. Physical reconstructions certainly have the advantage of protecting the remains and of making them accessible to visitors, but take on themselves the responsibility to convey a fixed volumetric and spatial idea, ${ }^{28}$ that cannot be easily modified on the basis of fresh information that may later become available.

[^8]Reconstructing the actual volume of a badly-preserved ancient building is a difficult enterprise that requires an attention for the third dimension equal, if not superior, to the one dedicated to the plan, as the final perception of the building will depend on their combination. Also in this case, measuring the actual remains in a precise and accurate way represents the first, fundamental step. In the case of an absence of hard data, the extent of the assumptions that allow the reconstructions should be clearly stated: it is important to highlight the edge along which our measurements end, and our interpretation starts. ${ }^{29}$

## Calculations and observations

## Observing the horizon

Our widespread modern ambition to measure everything and our expectation that the resulting figures would be useful (or even meaningful) may lead us to overinterpreting the archaeological remains. It may be useful underlining another obvious distinction that, however, may have deep implications: not all figures handled by the ancient builders were the result of calculations, as some were obtained through observations. ${ }^{30}$ Calculations are here meant as mathematical operations performed in advance (that are later expected to find a correspondence in the reality); observations, instead, establish a direct match (a comparison) among elements. The volume of a pyramid, for instance, was calculated in advance; its orientation, instead, was established by observing the sky. ${ }^{31}$

Whilst pyramids were invariably oriented towards the four cardinal points, various elements appear to have determined the orientation of temples, including their physical relation with the Nile and/or their alignment with celestial bodies. ${ }^{32}$ It is not always easy to distinguish these factors; even when an association is established (e.g. between a building and a specific astronomical event), it may be difficult to attach a meaning to it. A clear example of this difficulty is the interpretation of the solar alignment of some temples: twice a year, at sunrise, the sunrays reach the innermost cell of all the temples laid out with their axes pointing within the arc progressively designed, along the horizon, by the rising sun. The most famous cases are Abu Simbel and the two Aten temples at Amarna, the ancient Akhet-aten ('the Horizon of Aten'), for which two slightly different explanations have been suggested.

At Abu Simbel (rock cut), the sun reached the innermost cell of the main temple around 22 October and 20 February.

[^9]The main temple might have materialised a unique coincidence, that is, the re-alignment, after 1500 years, of the beginning of the civil calendar with the solar year; under Ramses II, the two dates in which the phenomenon took place corresponded to the beginning of the calendar seasons of shemu and peret, symmetrically placed one before and one after the winter solstice. ${ }^{33}$ Both the Great and the Small Aten Temples of Amarna (built up) appear instead to have been laid out and designed in order to be aligned towards the rising sun around 12 October and 28 February. This case might have been slightly different from Abu Simbel: the February date for the Amarna temples might correspond to the day in which the city was founded, whilst its October counterpart could be just consequential. ${ }^{34}$ No specific studies have been carried out so far on the other temples in which this phenomenon can be observed and it is therefore difficult to draw any general conclusion on this subject. ${ }^{35}$

In some cases, alignments could also be established on the basis of how sun and landscape combined. The image of the horizon, embodied in the hieroglyphic sign akhet, might have been materialised both at Giza and at Amarna. In the first case, the sun sets between the Khufu and Khafra pyramids around the summer solstice. ${ }^{36}$ In the second case, the Small Aten Temple pointed towards the Royal Wadi, where the king prepared his tomb; the line of the horizon in that direction runs high along the profile of the escarpment, and then drops in correspondence of the wadi: twice a year, in the days in which the sun rose from that hollow, both the landscape and the pylons, aligned with one another, resembled the akhet. ${ }^{37}$

In general, Magli describes three patterns of astronomical orientation that can be detected in ancient Egyptian architecture: meridian (aligned along a north-south direction and linked to the importance of the circumpolar stars, immortal as they never set under the horizon, first adopted in the Old Kingdom cemetery of Saqqara); solstitial (aligned to the winter solstice, typical of New Kingdom temples of the Theban area); and inter-meridian (the most elusive alignment, with the buildings' axes pointing towards the intermediate directions between cardinal points, specific to the cemetery of Abydos, from the Early Dynastic Period onwards). ${ }^{38}$ All of these types of

[^10]orientation, plus the biannual solar alignments mentioned above, are based on direct observations of the sky (by marking on the ground the direction of the rising sun on a specific date, or by projecting on the ground the direction of the north celestial pole using sighting instruments) and do not necessarily request any type of mathematical calculation in advance.

## Visual vs. calculated alignments

Ancient Egyptian astronomical texts appear in the Middle Kingdom, whilst the first astronomical representations date to the New Kingdom. ${ }^{39}$ The extant ancient Egyptian mathematical texts dating to the Middle and New Kingdoms do not contain any specific example of astronomical calculations; this may be due to several reasons. As they are all school texts for scribes, ${ }^{40}$ a first explanation is that normal scribes were not expected to perform this type of activity; this would imply that someone else was in charge of this area of knowledge. A second explanation is that astronomical knowledge was acquired at a later stage of a scribal career, and perhaps corresponded to an area of knowledge that was restricted to sacred and religious affairs. A third explanation is that dealing with astronomical matters did not request a specific mathematical training, beyond what a normal scribe learned and applied to fields and distribution of ratios.

Embracing the first and second explanations means assuming that crucial evidence was lost, thus embarking in the dangerous field of untested and untestable conclusions. Choosing a more sceptical approach, as the third interpretation, involves the risk of challenging and downplaying the importance of astronomical observations. In both cases, caution is recommended. In the first century BC, writing about the education that the sons of the Egyptian priests received, Diodorus Siculus wrote that 'geometry and arithmetic are given special attention. [...] Arithmetic is serviceable with reference to the business affairs connected with making a living and also in applying the principles of geometry, and likewise is of no small assistance to students of astrology as well' (Diod. 1.81-3). This description does not rule out any of the three interpretations listed above, but as it dates to 1600 years after the Rhind Mathematical Papyrus, it is difficult to establish its weight when dealing with previous historical periods. ${ }^{41}$

The $p \underline{d}$ šsr, the 'stretching of the cord' of the Foundation Ceremonies, is likely to correspond to, or at least include,

[^11]the action of establishing the main axis of the building and marking it on the ground; in case the building was aligned on the movements of sun and stars, this operation was evidently preceded by astronomical observations. ${ }^{42}$ The alignments based on the appearance and disappearance of the sun from and behind the horizon would depend on the local orography and might be significantly influenced by the shape of the line marking the horizon itself (as for the Small Aten Temple). There is no need to suggest the involvement of complex calculations, meant to anticipate an event that had to be observed and recorded locally.

Marking the direction of the rising sun on the ground is a relatively simple operation. Quarrying the mountain behind it and turning it into Abu Simbel was a great artistic and architectural enterprise; from a technical point of view, however, once the direction of the sun in the due day(s) had been established, the axis of the temple had to simply follow that line, and be double-checked twice a year. In comparison with the size of the central space across which the sunlight had to penetrate, millimetric variations of the axis were irrelevant. Extra care had to placed only in the deepest room, in order to ensure that the size of the opening and the exact position of the statues ensured that the figure of Ptah remained in the darkness.

Different is the case of the sloping shafts of the pyramid of Khufu, that are said to point towards the culmination of specific stars, ${ }^{43}$ for two reasons: because they were not quarried, but progressively built into the solid masonry of the monument; and because, as a consequence, their construction would constantly require an intermediate step between the astronomical observation and the construction, that is, a measure to be transferred. This could be achieved in two ways: by establishing a geometric or an arithmetic model. In the first case, the height of a specific star on the horizon could be crystallised into a right-angled triangle, embodying the horizontal distance of two sighting rods and the difference in height between them; ${ }^{44}$ in the second case, a numeric value (that is, a precise measurement) should be attributed to the elements of the triangle.

The problem with the second solution, and partly also with the first, is that numeric values that would describe the culmination of a star ought to be extremely precise. The smallest unit of measurement attested in ancient Egyptian architectural documents (be they drawings or texts) is the finger, corresponding to $c .1 .8 \mathrm{~cm}$ (corresponding to an increment in the slope of $1.5^{\circ}$ ), which would not be sufficiently small; ${ }^{45}$ one should either revert to smaller fractions of the finger (attested on measuring rods) or to approximated measures. The first solution would imply keeping control of measurements below the centimetre for dozens of metres over the years; the second solution would undermine the expected precision. ${ }^{46}$ The problem lies just here: explaining the diagonal shafts as canals pointing to specific

[^12]stars implies the implementation of extremely precise architectural measures, which are in turn justified by the necessity to point to specific stars. It is difficult to escape this circular argument. Unfortunately, the diagonal shafts of the pyramid of Khufu are unique and the lack of parallels prevents a wider comparative study that might provide additional evidence and break the deadlock.

In conclusion, when investigating the issue of the precision that was meant to be attained in the construction, or in specific parts of the construction, the real problem is what we today expect to correspond to our modern (and disjoined) concepts of precision and accuracy. This means that we must question the role that we attribute to mathematics in the building process.

## A matter of roles

## How much?

What was the true role of mathematics in arts and architecture? In order to answer this question, we should first ask what we mean by mathematics. ${ }^{47}$ At a basic level, the use of numbers and geometric figures is (and has always been) pervasive; the use of complex concepts and calculations, instead, is a different story that, in the absence of clear evidence supporting a conscious use, may generate significant misunderstandings. In general, it is unnecessary to assume that every time a square was drawn on the ground, the author consciously referred to, or was even aware, of all the implications that we acknowledge and attribute today to $\sqrt{ } 2$; the same applies to the relationship between a circle and $\pi$. A square and a circle are simple geometric figures, and as all geometric figures they can be handled as simple shapes, by adopting a geometrical approach without necessarily involving arithmetical considerations. ${ }^{48}$ An example of this approach is Thales of Miletus, who probably proved his theorems by physically folding geometric figures. ${ }^{49}$

Perhaps the question to ponder is slightly different: to which extent did the ancient Egyptians measure their artistic and architectural endeavours? And what was the role of these measurements?

There is a significant percentage of architectural and artistic operations that do not require calculations, but just sheer practical skills and experience: for instance achieving perfect joints between stone blocks (the numeric value of their dimensions is irrelevant), setting up a stable portico (weighting the blocks was never an option), deciding the distance between vertical supports (Egyptian stone structures

[^13]are generally oversized, to remain on the safe side). Of course monumental buildings were planned in advance, as the necessary construction materials had to be retrieved; ${ }^{50}$ therefore, calculating in advance the volume of stone to be piled up and/or quarried out must have represented one of the first actions envisaged by the builders. But how much of the same building was calculated in advance after this initial step remains to be established.

## Wahrheit, Gerechtigkeit, Weltordnung

In the modern architectural practice, we expect to calculate in advance even the tiniest joint between architectural elements; we attribute a significant value to the accuracy that we manage to achieve and tend to extend this concept to ancient architectures. However, at least in the case of ancient Egypt, this approach may be misleading: if we spread backwards our modern expectations, our interpretation is unlikely to hit the target.

First of all, we should set aside the modern distinction between precision and accuracy, and acknowledge that in ancient Egypt they belonged to the overarching concept of $m 3^{`} . t$, meaning truth, right and world order ('Wahrheit, Gerechtigkeit, Weltordnung' in Assmann's words ${ }^{51}$ ), summarised as 'justice.'

References to the general concept of 'precision' are widespread in ancient Egyptian literary texts. Words such as $m h 3$ (measured, balanced), mtr/mty (exact, correct in comparison with an external element) and ${ }^{〔} q$ ? (precise, straight) convey not only a general idea of accuracy, but also, and more specifically, of the accuracy of weighing operations. ${ }^{52}$ A good example, that includes all these elements, is the sentence 'your tongue is measured ( $m h_{3}$ ) and your lips are more precise ( ${ }^{〔} q^{3}$ ) than the exact (mty) plummet of Thoth. ${ }^{53}$

Weighing, in its wide meaning of comparison between two items, represents the first and foremost comparative operation (Thoth's plummet docet), and may have a symbolic value in itself, unrelated to the physical aspect. The ancient Egyptian Weighing of the Soul is a moral judgement: the actual comparison is not between the physical weight of the heart and that of the feather, but between the heaviness of the sins and the lightness of the cosmic order. ${ }^{54}$

In linear measurements, instead, one quantity (the unit of measurement) acts as intermediary between the object and the person taking the measurement. This is the moment in which numbers might become (or simply appear to become) meaningful - provided that we can find an agreement on what it means to be meaningful.

[^14]In the pharaonic period, the level of detail of measurements appears to represent an important evaluating factor, but in a different way in comparison with today. Nowadays in the scientific realm, we pride ourselves on being able to measure physical objects down to a nanometric scale. We tend to apply a different evaluation system to the study of the past, instead, where we generally attribute a value to 'simple' numbers: a room of an ancient Egyptian building is considered by us to be well built if its internal dimensions are found to be, for instance, 5 by 8 cubits. The ancient Egyptians, instead, appeared to prize the care that was devoted to record its physical substance: a well-built room would be described as being, for instance, 5 cubits, 3 palms and 2 fingers wide, and 8 cubits, 2 palms and 1 finger long.

An example of this attitude is the plan of the tomb of Ramses IV drawn on papyrus (Museo Egizio, Torino cat. 1885), that represents a final survey carried out after the burial of the king (the coffin is shown in its final position, and the doors are closed and locked). The detailed dimensions of the rooms are expressed in cubits, palms and fingers, and do not appear to correspond to any type of pattern: this combination seems to suggest that their accuracy was meant to testify not the adherence to a previous model, but rather the care and attention that had been placed in recording the completed work. ${ }^{55}$

The overall impression is that part of the planning and building process of monuments was envisaged in advance by the officers in charge, but that then a substantial part of the practical implementation of the project was entrusted to the actual builders and workmen, including the final dimensions of several architectural elements. ${ }^{56}$ Therefore, our modern distinction between the architects who decide the dimensions of every single component of a building (and who therefore have full control of them) and the workmen who materialise the drawings that they receive is not always and fully applicable to ancient Egyptian architecture. ${ }^{57}$ In the case of pyramids, the dimensions were clearly

[^15]chosen in advance by the king or the officers in charge, and the workmen followed the instructions for all the years that it took to complete the monument; ${ }^{58}$ in the case of rock-cut tombs, instead, the initial generic plan acted as a guide over the years, and the work was carried out without necessarily sticking to the initial figures, ${ }^{59}$ in the case of major temples and palaces, we may assume that the overall dimensions were established before the beginning of the building works, but we cannot be sure that also the smallest architectural details were planned in advance. ${ }^{60}$

## Recording the quality

Therefore, in pharaonic Egypt the concepts of precision and accuracy in the building operations are unlikely to merely correspond to preconstituted numeric values. They are more likely to refer to the concepts of attention to the details and care in their execution: the ancient Egyptians focussed their attention on the quality of the operations, rather than on their numeric results. In this scenario, the precision of the measuring operation (meant as attention and care), and not necessarily the resulting figures, becomes the real 'metre' to establish the value of the object. The resulting figure is also important, of course, but more in terms of adherence to the overarching moral concept of order, than in terms of purely numeric value.

Measuring in an exact way meant being just. It is worth reporting how Gillings commented upon the equal division of nine loaves among ten men (Problem 6 of the Rhind Mathematical Papyrus):

While the modern answer is that each man gets $9 / 10$ of a loaf, this division requires that the last man must get $1 / 10+1 / 10+$ $1 / 10+1 / 10+1 / 10+1 / 10+1 / 10+1 / 10+1 / 10$ of a loaf $[\ldots]$. The Egyptians would have none of such solution. The answer given in the RMP is that every man gets exactly the same number of pieces and exactly the same-sized pieces, namely $2 / 3,1 / 5$ and $1 / 30$ each [...] One advantage of this division was that not only was justice done, but justice also appeared to have been done. ${ }^{61}$

Whether or not the actual subdivision of loaves was performed in this way, in the official version the formal equality of the portions guaranteed the substantial fairness of the operation. Precision implied accuracy, and vice versa, as they blended and were incorporated into the action of careful and detailed recording, that in turn ensured moral and social order. ${ }^{62}$ Practicing

[^16](and demonstrating) carefulness was the target; the resulting figures were important by-products, but probably not the focus of the whole process.

## Conclusions

How does the above translate into practice? Clearly, producing reliable surveys (that is, creating reliable intermediaries between the physical building and our desktops) represents a first, fundamental step. Then it would be important, in our analyses, to keep separated and yet connected the investigations carried out on what the ancient Egyptian knew and those on what they did not know. It is not a matter of trying to abandon our point of view to embrace that of people who lived long ago (provided that this is even possible), but rather to work at both levels in parallel, ${ }^{63}$ in a constant exchange of information between the two realms; in other, simplified words, we measure in metres and degrees, but if we reconstruct a monument, we ought to do it in cubits. Finally, we should avoid overinterpretations due to our desire to find overarching abstract rules.

Rules did exist, of course, but were probably more similar to guidelines than to formulae. In two-dimensional representations, human bodies were generally drawn in the same specific, conventional way, and yet the variety of scenes, poses and subjects is endless; the same happened with buildings. Specific artistic conventions rarely produced the same results: they had a numerical component, but did not follow strict mathematical rules; they acted more like words, to be combined to express different concepts, within limits which were wider and more flexible than those imposed by numbers. Today we tend to somehow sacralise the numeric results of our surveys; the ancient Egyptians, instead, appear to perceive numbers as parts of a wider and far more important operation, aiming at achieving global balance and justice. Measuring (that is, establishing a comparison) had a moral function, and as such had a symbolic meaning, but lay its foundations in the material realm of the objects, rather than in the immaterial realm of numbers. After all, the weight of the heart of the dead was not expressed as a number: it was directly and physically compared with the quintessence of rightness.

Calculations imply a previous idea and a mathematical process leading to results appearing before the object is actually built or crafted; observations are instead performed on things as they already are. The first operation corresponds to measuring ex ante, the second to measuring ex post. Ex post measurements describe reality better than any expectation, are performed in a specific moment and a specific place, and do not necessarily have to justify their existence within a broader frame. ${ }^{64}$ Lewis, for instance, labelled Borchardt's

[^17]suggestion that the Egyptians levelled the course of the Nile for 1200 km from the sea to the First Cataract to establish a datum for the nilometres as 'not only forbiddingly daunting but quite unnecessary', as 'the zero point on each town's nilometre would be established from observations on low water recorded locally over many years. ${ }^{.65}$

The final suggestion is therefore not to confuse, in our modern analyses of ancient Egyptian arts and architecture, ex ante and ex post measurements. They do not have to be mutually exclusive; on the contrary, they probably coexisted as parts of the same comprehensive workflow, which included not only the initial project (pre-existing the building) and the final record of the results (once all the decisions had been taken and implemented), but also a series of intermediate, progressive steps. Establishing the overall size, orientation and style of the decoration of a temple was in the hands of kings, high-ranking officers and priests, whilst building it was in the hands of builders and skilled workmen who took crucial and important decisions as the construction process unfolded. Some aspects were calculated in advance, others on the spot; some depended on general rules, others on local characteristics; numbers were involved in all these phases, in different ways and with different functions. The final result represented a huge, joint effort, resulting from the interplay of theoretical, practical, symbolic and casual aspects; focussing only on one of them would miss the point and not do justice to the ancient enterprise. What was celebrated at the end was the care (precision, accuracy, attention, balance) that had been poured into the entire operation, not the numbers for their own sake. We should find a way to do the same.

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[^0]:    ${ }^{1}$ C. Greco, 'The Biography of Objects', Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci., XLII-2/W11 (2019), 5-10, <https://doi. org/10.5194/isprs-archives-XLII-2-W11-5-2019> accessed 27.05.2020.

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    ${ }^{3}$ Rossi, Architecture and Mathematics, 23-56.
    ${ }^{4}$ C. Rossi and C. A. Tout, 'Were the Fibonacci Series and the Golden Section known in ancient Egypt?', Historia Mathematica 29:2 (2002), 101-13.

[^3]:    ${ }^{5}$ Rossi, Architecture and Mathematics, part I.

[^4]:    ${ }^{6}$ E.g. W. M. F. Petrie, Researches in Sinai (New York, 1906) on Serabith al-Khadim; W. F. M Beex and M. J. Raven, 'The Architecture', in M. J. Raven and R. van Walsem (eds), The Tomb of Meryneith at Saqqara (PALMA 10; Turnhout, 2014), chapter 3 on the tomb of Meryneith; E. Blyth, Karnak, Evolution of a Temple (London, 2006) on Karnak.
    ${ }^{7}$ Rossi, Architecture and Mathematics, 96-113.
    ${ }^{8}$ E.g. A. Badawy, Ancient Egyptian Architectural Design. A Study of the Harmonic System (Near Eastern Studies 4; Berkeley, 1965); K. G. Siegler, Kalabscha, Architektur und Baugeschichte des Tempels (AVDAIK 1; Berlin, 1970); Z. Wysocki, 'The Result of Research, Architectonic Studies and of Protective Work over the Northern Portico of the Middle Courtyard in the Hatshepsut Temple at Deir el-Bahari', MDAIK 40 (1984), 329-49; Beex and Raven, in Raven and van Walsem (eds), Tomb of Meryneith; in general, see D. Arnold, The Encyclopedia of Ancient Egyptian Architecture (Princeton, 2003), 177-8.

[^5]:    ${ }^{9}$ B. J. Kemp, 'Soil (including mud-brick architecture)', in P. T. Nicholson and I. Shaw (eds), Ancient Egyptian Materials and Technology (Cambridge, 2000), 78-103.
    ${ }^{10}$ A. J. Spencer, Brick Architecture in Ancient Egypt (Warminster, 1979). On a later case-study, see C. Rossi and F. Fiorillo, 'A metrological study of the Late Roman Fort of Umm al-Dabadib, Kharga Oasis (Egypt)', Nexus Network Journal 20:2 (2018), 373-91, $<10.1007 / \mathrm{s} 00004-018-0388-6>$ accessed 27.05.2020.
    ${ }^{11}$ Kemp, personal communication.
    ${ }^{12}$ Badawy, Architectural Design, 21.
    ${ }^{13}$ E. Iversen, Canon and Proportion in Egyptian Art (London, 1955); G. Robins, Proportion and Style in Ancient Egyptian Art (London, 1994); H. S. Smith and H. M. Stewart, 'The Gurob Shrine Papyrus', JEA 70 (1984), 54-64; J. Legon, 'The Cubit and the Egyptian Canon of Art', DE 35 (1996), 61-76.
    ${ }^{14}$ Cf. C. Riggs, Ancient Egyptian Art and Architecture: A Very Short Introduction (Oxford, 2014), 50-1.

[^6]:    ${ }^{15}$ E.g. A. Bednarski, 'Life After Amarna: the post-excavation history of JE 59286', in S. Ikram and A. Dodson (eds), Beyond the Horizon: Studies in Egyptian Art, Archaeology and History in Honour of Barry J. Kemp (Cairo, 2009), 1-8.
    ${ }^{16}$ W. M. F. Petrie, A Season in Egypt (London, 1887), 33.
    ${ }^{17}$ Cf. the introduction of module in the Hellenistic Period according to Arnold, Encyclopedia, 177.
    ${ }^{18}$ Rossi, Architecture and Mathematics, 12-28.
    ${ }^{19}$ It may be noted that this procedure also fits the construction of rock-cut temples and tombs: the first direction to be quarried was the depth, followed by breadth and height. These cases brought to extreme the characteristic of this process, that literarily started from the inside and expanded outwards and was not (as it could not be) physically superimposed from the outside.
    ${ }^{20}$ J. D. S. Pendelbury, The City of Akhenaten, III (London, 1951),
    6; B. J. Kemp and M. Bertram, 'Great Aten Temple Report on Recent Work (Autumn 2017, Spring and Autumn 2018)',

[^7]:    <http://www.amarnaproject.com/pages/recent_projects/excavation/ great_aten_temple/> accessed 27.05.2020.
    ${ }^{21}$ See Spence's hypothesis in B. J. Kemp and P. Rose, 'Proportionality in Mind and Space in Ancient Egypt', CAJ, 1:1 (1991), 103-29, fig. 4 [doi:10.1017/S0959774300000275](doi:10.1017/S0959774300000275) accessed 30.07.2020.
    ${ }^{22}$ Badawy, Architectural Design; A. Badawy 'Philological Evidence about Methods of Construction in Ancient Egypt,' ASAE 54 (1957), 51-74.
    ${ }^{23}$ W. M. F. Petrie, Illahun, Gurob, Hawara 1889-90 (London, 1891), pl. XIV; T. E. Peet and C. L. Woolley, The City of Akhenaten, part I, Excavations of 1921 and 1922 at El-'Amarneh (London, 1923), pl. XVI; G. Castel and D. Meeks, Deir el-Médineh 1970 (FIFAO 12; Cairo, 1980), plan 1. The same applies to the Late Roman settlements of the Kharga Oasis, see Rossi and Fiorillo, Nexus Journal 20:2, 379-81. In general, a gridded pattern is a simple and basic requirement to better organize the available space. Cf. B. J. Kemp, 'Bricks and metaphor (Ancient city design)', CAJ 10 (2000), 335.
    ${ }^{24}$ E.g. Di. Arnold and Do. Arnold, Der Tempel Qasr el-Sagha (AVDAIK 27; Mainz, 1979); also Kemp and Rose, CAJ 1:1.

[^8]:    ${ }^{25}$ Cf. P. Zignani, 'Light and Function: An Approach to the Concept of Space in Pharaonic Architecture', in P. I. Schneider and U. Wulf-Rheidt (eds), Licht - Konzepte in der vormodernen Architektur (Diskussionen zur Archäologischen Bauforschung 10; Regensburg, 2011), 59-70.
    ${ }^{26}$ N. Dunn, Architectural modelmaking (London, 2014).
    ${ }^{27}$ A problem that, of course, concerns the entire realm of historical studies, cf. A. Momigliano, Storia e storiografia antica (Bologna, 1987), 15-24.
    ${ }^{28}$ E.g. N. Warner, 'Protecting a Cemetery in Saqqara: Site Works 1975-2009', Conservation and Management of Archaeological Sites 11:2 (2009), 98-132; see also C. Rossi, 'Aristotle’s Mirror: Combining Digital and Material Culture', Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci. XLII-2/W11 (2019), 1025-9, <doi.org/10.5194/isprs-archives-XLII-2-W11-1025-2019> accessed 27.05.2020; Rossi, Architecture and Mathematics, 54-6.

[^9]:    ${ }^{29}$ E.g. C. Rossi, 'Immaterial Data and Material Culture; Surveying and Modelling the New Kingdom Necropolis of Saqqara', Saqqara Newsletter 17 (2019) 61-71.
    ${ }^{30}$ Cf. C. Rossi, 'Egyptian Architecture and Mathematics', in B. Sriraman (ed.), Handbook of the Mathematics of the Arts and Sciences (2018), 8 <https://doi.org/10.1007/978-3-319-70658$0 \_57-1>$ accessed 27.05.2020.
    ${ }^{31}$ Rossi, Architecture and Mathematics, part III; G. Magli, Architecture, Astronomy and Sacred Landscape in Ancient Egypt (Cambridge, 2013).
    ${ }^{32}$ J. A. Belmonte and M. Shaltout, In search of cosmic order, selected essays on Egyptian archaeoastronomy (Cairo, 2009).

[^10]:    ${ }^{33}$ Magli, Architecture, Astronomy, 223-4. The small lateral chapter dedicated to Ra-Horakhti, instead, was aligned towards the sunrise at winter solstice.
    ${ }^{34}$ Magli, Architecture, Astronomy, 212-13.
    ${ }^{35}$ For instance, it cannot be excluded that some temples aligned eastward pointed towards specific stars which crossed the sun's path, and not to the sun itself (Magli, personal communication). Also, it is possible that the important element to be taken into account was the number of days to and from the winter solstice, and not the dates themselves. In the case of Abu Simbel, this interval is 70 days, whilst at Amarna is 60 days.
    ${ }^{36}$ Magli, Architecture, Astronomy, 95-8.
    ${ }^{37}$ B. J. Kemp and P. Docherty, 'The solar observation and offering platform at the front of the Great Aten Temple' (2019), <http:// www.amarnaproject.com/documents/pdf/Solar-Observation-and-Offering-Platform.pdf> accessed 27.05.2020.
    ${ }^{38}$ Magli, Architecture, Astronomy, 18. See also G. Magli, 'From Abydos to the Valley of the Kings and Amarna: The Conception of

[^11]:    Royal Funerary Landscapes in the New Kingdom', Mediterranean Archaeology and Archaeometry 11:2 (2011), 23-36.
    ${ }^{39}$ S. Symons, 'Challenges of interpreting Egyptian astronomical texts', in A. Imhausen and T. Pommerening (eds), Translating Writings of Early Scholars in the Ancient Near East, Egypt, Rome, and Greece. Methodological Aspects with Examples (Berlin, 2016), 379-401.
    ${ }^{40}$ A. Imhausen, Mathematics in Ancient Egypt: A contextual history (Princeton, 2016); see also R. J. Williams, 'Scribal Training in Ancient Egypt', JAOS 92:2 (1972), 214-21.
    ${ }^{41}$ On the role of mathematical knowledge in antiquity, see S . Cuomo, Pappus of Alexandria and the Mathematics of Late Antiquity (Cambridge, 2000), 9-56.

[^12]:    ${ }^{42}$ Rossi, Architecture and Mathematics, 148.
    ${ }^{43}$ Magli, Architecture, Astronomy, 80.
    ${ }^{44}$ Magli, personal communication.
    ${ }^{45}$ Cf. Magli, Architecture, Astronomy, 81.
    ${ }^{46}$ Cf. J. J. Wall, 'The Star Alignment Hypothesis for the Great Pyramid Shafts', Journal for the History of Astronomy 38:2

[^13]:    (2007), 199-206; Magli (Architecture, Astronomy, 80-1) maintains that the function of the shafts was symbolic.
    ${ }^{47}$ J. Stedall, The History of Mathematics. A Very Short Introduction (Oxford, 2012); Rossi, in Sriraman (ed.), Handbook of Mathematics.
    ${ }^{48}$ Cf. Kemp, CAJ 10, 335.
    ${ }^{49}$ B. A. Rosenfeld, A History of Non-Euclidean Geometry. Evolution of the Concept of a Geometric Space (Studies in the History of Mathematics and Physical Sciences 12; New York, 1988), 110; see also M. Friedman, A History of Folding in Mathematics: Mathematizing the Margins (Science Network, Historical Studies 59; Berlin, 2018).

[^14]:    ${ }^{50}$ Rossi, in Sriraman (ed.), Handbook of Mathematics, 5-6.
    ${ }^{51}$ J. Assmann, Ma'at: Gerechtigkeit und Unsterblichkeit im alten Ägypten (München, 1990).
    ${ }^{52}$ Mathieu, personal communication.
    ${ }^{53}$ A. K. Kitchen, Ramesside Inscriptions II (London, 1996), 335, 15-16; cf. P. Prisse 16, 1; and B. Mathieu, 'L'Enseignement de Ptahhotep', in AAVV, Visions d'Égypte. Émile Prisse d'Avennes (1807-1879) (Paris, 2011), 62-85 n. 126.
    ${ }^{54}$ Cf. Iamblichus, In Nicom. 17.6 ff , for a late antique version of the same an analogy between balance and justice.

[^15]:    ${ }^{55}$ Different is the evidence provided by the Ptolemaic Building Texts from the temples of Edfu and Dendera, published by S. Cauville and D. Devauchelle, 'Les mesures réelles du temple d'Edfou,' BIFAO 84 (1984), 23-34; and S. Cauville, 'Les inscriptions dédicatoires du temple d'Hathor à Dendera,' BIFAO 90 (1990), 83-114. In those cases, the internal dimensions of the rooms do correspond to figures belonging to a pattern. The latter is difficult to reconstruct because only a selection of dimensions was recorded on walls, but clearly in these cases the numbers were all related with one another; numeric patterns can also be identified in the dimensions of types of statues listed in the crypts of Dendera, as illustrated in F. Hoffmann, 'Measuring Egyptian Statues,' in J. M. Steele and A. Imhausen (eds), Under One Sky. Astronomy and Mathematics in the Ancient Near East (Münster, 2002), 109-19. Whether or not all of these figures had a specific meaning beyond their being related is another matter, that still awaits an in-depth investigation (Rossi, Architecture and Mathematics, 166-73.)
    ${ }^{56}$ Rossi, Architecture and Mathematics, part II; B. J. Kemp, Ancient Egypt. Anatomy of a Civilization (3rd edn; London, 2018), 194-6.
    ${ }^{57}$ Kemp, Ancient Egypt, 158-9; Rossi, in Sriraman (ed.), Handbook of Mathematics, 3-6.

[^16]:    ${ }^{58}$ M. Lehner, The Complete Pyramids (London, 1997), 221.
    ${ }^{59}$ Rossi, Architecture and Mathematics, 139-47.
    ${ }^{60}$ Cf. Kemp, Ancient Egypt, 196.
    ${ }^{61}$ R. J. Gillings, Mathematics in the Time of the Pharaohs (New York, 1972), 105. It may be added that adopting such a system would also help highlighting at a glance the differences among shares to be given to simple workmen and to foremen, who were entitled to larger portions (cf. Gillings, Mathematics, 124-7).
    ${ }^{62}$ Cf. M. Ezzamel, 'Order and accounting as a performative ritual: Evidence from ancient Egypt', Accounting, Organizations and Society 34:3-4 (2009), 348-80; <https://doi.org/10.1016/j. aos.2008.07.004> accessed 27.05.2020.

[^17]:    ${ }^{63}$ Cf. Kemp, Ancient Egypt, 1-11.
    ${ }^{64}$ Broadly speaking, to the category of ex-post measurements might also belong ancient Egyptian onomastica: these apparently plain lists of items ('certainly there was never written a book more tedious and less inspired than the Onomasticon of Amenope', A. H. Gardiner, Ancient Egyptian Onomastica I (Oxford, 1947), 24-5) might have played more than one role, including recording (and indirectly exhibiting) the existence of large numbers of elements,

[^18]:    aiding memorisation by categorising them, as well as acting as reference. A similar role as the one played by the table texts included in the mathematical papyri (Williams, JAOS 92:2, 219).
    ${ }^{65}$ M. J. T. Lewis, Surveying Instruments of Greece and Rome (Cambridge, 2001), 15.

