Accurate Synthesis of Extracted-Pole Filters by Topology Transformations

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***Abstract*—This paper presents a novel, simpler and more accurate synthesis method for inline extracted-pole low-pass prototype filters. At the present time this topology is only synthesized by using section extraction or optimization methods, known to suffer from accuracy problems, especially for high order filters. The method described in this paper relies on the robustness and accuracy of the coupling-matrix synthesis methods by using it as a first step. The second step is instead a transformation to obtain the desired inline extracted-pole topology. Three synthesis examples are presented to validate the novel synthesis procedure.**

***Index Terms*—Elliptic filters, extracted-pole synthesis, topology transformation.**

I. Introduction

With the advent of different 5G solutions and the massive deployment of IoT, the need for highly selective filters also arises. With size as a constraint in many applications, the introduction of several finite transmission zeros (TZ) is necessary. This conversely often leads to intricate topologies and therefore more complicated implementation and tuning. One possible solution to overcome this issue is the inline extracted-pole topology, which is a simple topology that allows implementing even fully canonical filters [1-3]. The current synthesis methods for this topology are based on section extraction [4-5], which are known to suffer from accuracy issues. This is because the very nature of the method: round-off errors are accumulated in polynomials that are computed after each extraction. This is a product of the finite representation of polynomials, usually with a 64-bit double-precision standard. According to [5], the section extraction procedure for this topology works fine until the order of the polynomials (64-bit precision) is about 15. In [6] an optimization-based method tried to overcome this, however, the computation times and the uncertainty of the results increased jointly to the filter order. In [7-9] different synthesis methods for mixed topologies are presented. However, the extracted-pole blocks are still obtained with section extractions, furthermore they are constrained to be grouped and placed at ports and not arbitrarily like Fig. 1.

On the other hand, it is well-known the robustness in terms of accuracy of the coupling matrix (CM) synthesis methods for cross-coupled topologies, like the one described in [10]. The novel synthesis method described here, takes advantage of this by using the CM synthesis as a first step, and then performing a transformation from cross-coupled to extracted-pole topology. This transformation is fully analytically and uses the triplets extraction procedure described in [11] together with a delta-to-star (D2S) transformation [12-14], allowing the dangling block producing the TZ to be arbitrarily placed in the circuit.

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II. Synthesis Methodology

Consider an n-order inline extracted-pole low-pass prototype filter with transmission zeros TZs placed at $[\infty_1, ..., f_A^{(1)}, ..., \infty_B, ..., f_C^{(2)}, ..., \infty_D, ..., f_x^{(i)} ..., f_E^{(k)}, ..., \infty_n]$ rad/s. The capital subscripts denote the sorting for the n TZs (finite and infinite) in the inline extracted-pole circuit, proceeding from the source port S toward the load port L. Superscripts of finite TZs instead number and sort only the k finite TZs, each at frequency $f_x^{(i)}$ as shown in Fig. 1. When $k = n$ the filter is fully canonical (without TZs at ∞), and the subscripts and superscripts coincide ($x = i$).

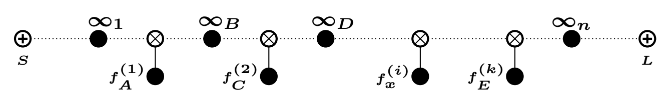


Fig. 1. Inline extracted-pole topology. Black circles: unit capacitance in parallel to frequency-independent (F-I) susceptances (resonating nodes: RN). Circles with (x) are F-I susceptances (non-RN: NRN), and with (+) are unit conductance (source: S, load: L). Lines are admittance inverters (couplings).

With this notation, the procedure starts with the synthesis of a canonical cross-coupled configuration (the transversal prototype [10] can be used, although every other configurations are allowed). The synthesis result is represented by the $n+2$ coupling matrix M $(S, 1, 2, ..., n, L)$ and capacitance matrix of the same dimension $C = \text{diag}(0, 1, 1, ..., 1, 0)$. Then, the synthesized prototype is transformed into the Wheel form [15] by means of the algorithm detailed in [11], obtaining the configuration shown in Fig. 2a. Note that matrix C is not altered by the conversion into the Wheel form.

If the finite TZ f_n^k is required to be located in the last RN n of the circuit, close to L (e.g. fully canonical filter), a preliminary step is necessary before applying the wheel algorithm: a fake RN is added before L to the initially synthesized prototype, whose order becomes $n+1$ (more details on Sec. II A). The resulting wheel form for these cases is depicted in Fig. 2b.

Once the Wheel form has been generated, matrices M_{wheel} and C are ready to be converted to the extracted-pole topology

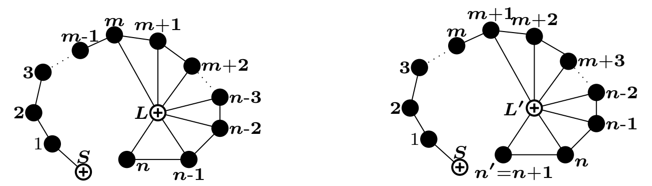


Fig. 2. (a) Wheel topology ($m = n - k$). (b) Wheel topology if the artifice of Sec II A is applied before. Black circles are RN. White circles are S, L. $L' = 1/(1\text{-}jw)$: check Fig. 4a. Black lines represent admittance inverters.

by using the algorithm depicted in Fig 3a. It is based on the triplet extraction method of [11].

The algorithm tries to extract the i-th finite TZ $f_x^{(i)}$ in the x-th RN of the filter; to this purpose, n-x matrix rotations are required plus one D2S transformation as sketched in Fig. 3b. The first rotation with pivot $[n, n\text{-}1]$ and angle $\theta_{i,0}$ creates the coupling between RNs n and n-2 ensuring that the triplet $[n, n\text{-}1, n\text{-}2]$ contains $f_x^{(i)}$. The second rotation ($\theta_{i,1}$) remove the previously created coupling, and create a new one between RNs n-1 and n-3, now the triplet $[n\text{-}1, n\text{-}2, n\text{-}3]$ contains $f_x^{(i)}$. In the same way, subsequent rotations push $f_x^{(i)}$ (triplet) towards the RN x. Note that the rotation with angle $\theta_{i,n\text{-}m\text{-}1}$ ($m = n\text{-}k\text{-}i\text{+}1$) also removes the coupling between the RN m and L. Note also that after the last rotation with angle $\theta_{i,n\text{-}x\text{-}1}$, the triplet $[x\text{+}1, x, t]$ contains f_x^i; node t can be the RN x-1, S or the NRN of an extracted-pole block created in a previous extraction. This configuration, shown in Figs. 3b-c is then converted into extracted-pole by the D2S conversion detailed in (1).

$$
\begin{aligned}
B_1' = B_1\text{-}\frac{M_{12}\,M_{13}}{M_{23}}, B_2' = B_2\text{-}\frac{M_{12}\,M_{23}}{M_{13}}, B_3' = B_3\text{-}\frac{M_{13}\,M_{23}}{M_{12}},\\
M_1 = M_{12}\text{+}M_{13}\text{+}\frac{M_{12}\,M_{13}}{M_{23}},\;\; M_2 = M_{12}\text{+}M_{23}\text{+}\frac{M_{12}\,M_{23}}{M_{13}},\\
M_3 = M_{13}\text{+}M_{23}\text{+}\frac{M_{13}\,M_{23}}{M_{12}},\;\; b_1 = \text{-}M_1\text{-}M_2\text{-}M_3.
\end{aligned}
\tag{1}
$$

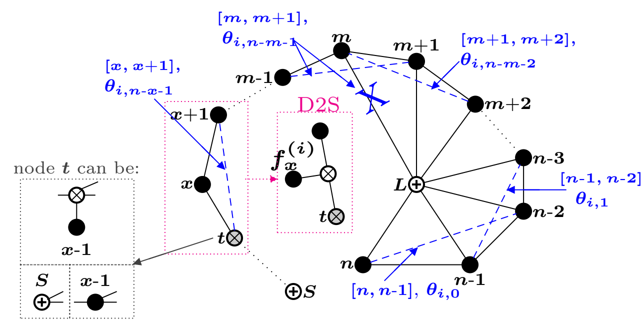
This procedure has to be done for each of the k finite TZs. Note that, even if a NRN is added after each D2S transformation, these nodes are never used as pivot of matrix rotations. Consequently, the matrix C can be discarded during the described procedure and properly re-defined at the end by assigning 1 to the RNs and 0 to NRNs. After the extraction of the last finite TZ we got the inline extracted-pole topology.

A. Artifice for cases when the TZ in position n is finite: f_n^k

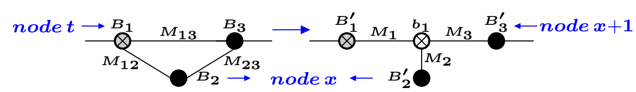
The described procedure works properly in all cases that do not exhibit a finite TZ extracted in last position (before L). For this reason, when a finite TZ must be extracted in last position (e.g. fully canonical filter), an additional step is required before the generation of the Wheel form. The trick consists in adding a fake RN (i.e. a unit capacitance) at the output of the initially synthesized canonical prototype. In order to preserve the response, L is modified as well, becoming frequency-dependent (L' in Fig. 4a). Note that the combination fake RN-unit inverter-L' is equivalent to the original L, so the prototype response is not affected by the fake node introduction.

Input: $M = M_{wheel}$, C, $[f_A^{(1)}, f_C^{(2)}, ..., f_x^{(i)}, ..., f_E^{(k)}]$
for $i = 1$ to k **do**
  for $j = 0$ to $n - x - 1$ **do**
    if $j = 0$ **then**
      $\theta_{i,0} = \arctan\left(M_{n-1,n} / \left(f_x^{(i)} + M_{n,n}\right)\right)$
    else
      $\theta_{i,j} = \arctan\left(M_{n-j-1,n-j+1} / M_{n-j,n-j+1}\right)$
    end if
    $pivot_j = [n - j - 1, n - j]$
    $M = R_{pivot_j,\theta_{i,j}}\; M\; R^{-1}_{pivot_j,\theta_{i,j}}$
  end for
  $[M, C] =$ **Delta2Star** (M, C, x)
end for
return $M_{ext-pol} = M$, $C_{ext-pol} = C$

(a)



(b)



(c)

Fig. 3. (a). Algorithm for transformation of a n-order filter with k finite TZ from Wheel form to inline extracted-pole topology. (b) Graphic representation for the extraction of the i-th finite TZ $f_x^{(i)}$ in the x-th RN of the circuit. Black circles are RNs. White circles with (x) are NRNs, with (+) are S or L. Silver crossed circles may be RNs, NRNs or S. Black lines represent couplings. Blue dashed lines are the couplings created after the corresponding rotation with $pivot_j = [n\text{-}j\text{-}1, n\text{-}j]$ and $\theta_{i,j}$. Each of these blue inverters is annihilated in the subsequent rotation. Rotation with $[m, m\text{+}1]$ and $\theta_{i,n\text{-}m\text{-}1}$ ($m = n\text{-}k\text{-}i\text{+}1$) also annihilate the inverter between nodes m and L (big blue X). After the last rotation with $[x, x\text{+}1]$ and $\theta_{i,n\text{-}x\text{-}1}$, a delta-to-star (D2S) conversion is done to extract $f_x^{(i)}$. (c) D2S Transformation, relationships are detailed in (1).

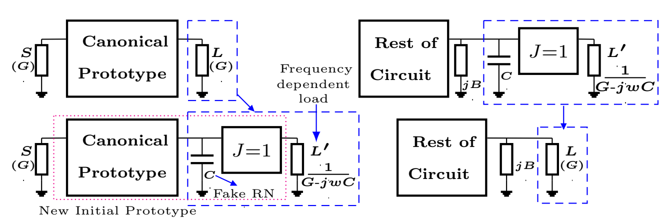
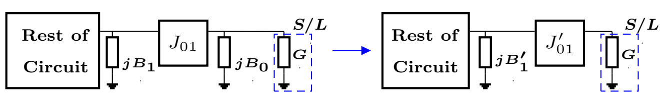


Fig. 4. Artifice when a finite TZ is extracted in last position. (a) A fake RN is added at output of the canonical prototype, close to L (position n+1). In order not to affect the response, a frequency-dependent load L' is introduced, coupled to the fake RN through a unitary inverter. Blocks inside the dashed rectangles are equivalents. The Wheel form is generated starting with the modified prototype inside dotted rectangle. (b) Back conversion after having applied the algorithms. C, G are unitary capacitance and conductance respectively. B is a susceptance. w is the normalized frequency.

We can now transform the prototype with the added fake RN into the Wheel form (Fig. 2b) and then apply the procedure for generating the extracted pole topology. In the resulting extracted-pole circuit, the fake RN (now, including a frequency independent susceptance B) is coupled to L' by means of a unitary inverter. As observed before, this blocks can be replaced with the original frequency independent and unitary L (in parallel with B) as shown in Fig. 4b, and the final form of the extracted-pole topology is finally obtained (Fig. 1).

B. Dealing with susceptances at Source or Load

As a result of the previous procedures, there could be a frequency-independent susceptance B_0 in parallel with the conductance at S or L as shown in Fig. 5 left. This happens when there is a finite TZ in the RN 1 (close to S) or n (close to L). Relationships of Fig. 5 can be applied to get rid of B_0 and leave only the conductance (Fig. 5 right). Note that these formulas basically apply a phase shift of $\phi = \arctan(B_0/G)$ in S21, this is usually not relevant.



$$B'_1 = B_1 - \frac{B_0\, J_{01}^2}{G^2 + B_0^2}, \qquad J'_{01} = J_{01}\, G \sqrt{\frac{1}{G^2 + B_0^2}}.$$

Fig. 5. Removing susceptance B_0 from S/L. G is unitary conductance.

III. Numerical Validation

The first example is the synthesis of a filter with 20 dB of return loss (RL) and TZs at $[\infty$, -1.889, 1.1512, 1.702] rad/s. The result of all the TZs extractions are illustrated in Fig. 6. Intermediate steps are also sketched for the first two.

In the second example, we show the synthesis of a dual-band filter with: passbands ([-1, -0.7249], $[0.8433, 1]$); order (2+4); RL: 20 dB; TZs: $[1.0125, 1.0286, 1.1142, \infty, \infty, \infty]$. The ideal (polynomial) response of this filter exhibits a very sharp transition at the end of the second passband. Consequently, even small errors (like those produced by round-off), may impact on the synthesis results. In fact, the synthesis performed with a in-house developed software based on the traditional method [4], fails for this particular filter. Using instead the novel method, the result of the synthesis is shown in Fig. 7.

In the last example, we show the feasibility of the novel method for successfully synthesizing high-order filters that cannot be obtained with traditional methods. A stopband filter of order 15 with 15 TZs (fully canonical as shown in Fig. 8a) and 14 reflection zeros is considered. With the method in [4], the synthesized extracted-pole circuit produces the response in Fig. 8b. As it is evident, the round-off errors have destroyed the ideal response. Using instead the novel procedure, the obtained response, shown in Fig. 8c, is coincident with the ideal one (equi-ripple in both stopband and passbands).

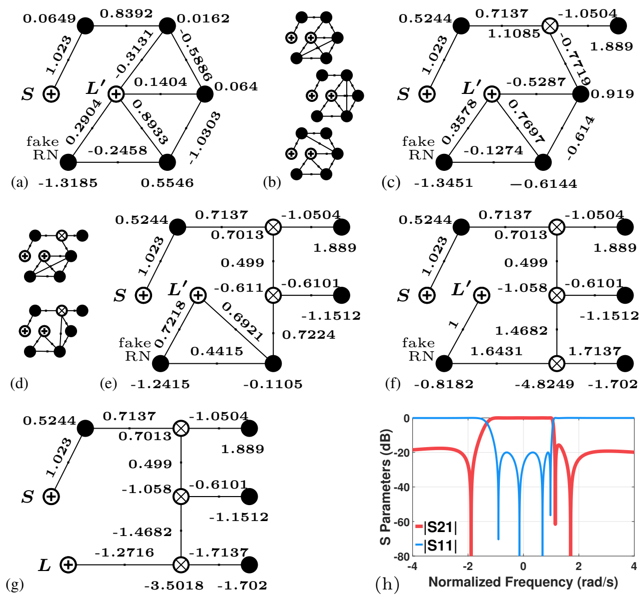


Fig. 6. (a) Wheel form (fake RN added). Extractions of $f_2^{(1)}$, $f_3^{(2)}$, $f_4^{(3)}$ are shown from (a)-(c), (c)-(e) and (e)-(f) respectively. From (f)-(g): the block fake RN-unit inverter-L' was replaced with L (Fig. 4b), then susceptance B_0 at L was removed (Fig. 5). (g) Extracted-pole topology. (h): Response of (g).

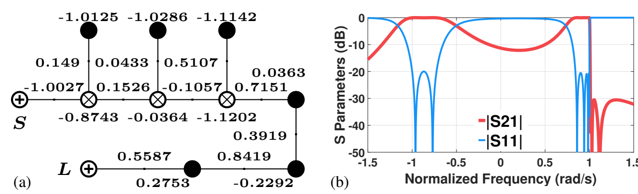


Fig. 7. Dual-band filter synthesized with the new procedure (a), response (b).

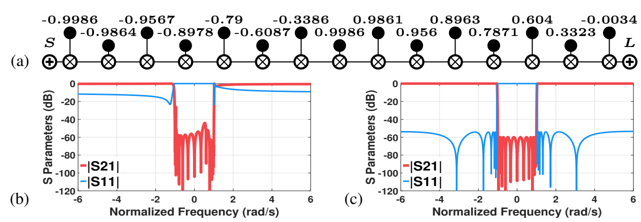


Fig. 8. (a) Fully canonical stop-band filter circuit (novel method). Scattering parameters computed by using (b) classical, (c) novel synthesis method.

IV. Conclusions

This paper describes a novel, simpler and more accurate synthesis method for inline extracted-pole low-pass prototype filters. The method relies on the robustness in terms of accuracy of the coupling-matrix synthesis methods for cross-coupling topologies by using it as a first step. It then performs a transformation to obtain the desired inline extracted-pole topology. The method is fully analytical and allows the prescribed location of the TZs in the circuit. Three examples were reported. The first shows an outline of the synthesis method. The second is a six order dual-band filter, whose synthesis fails when using traditional methods. The last example is a stopband fully canonical filter of order 15, with 14 reflection zeros. In this last case, the synthesized circuit with classical methods exhibits a response strongly degraded by round-off errors while the novel method produces a filter whose response coincides with the ideal polynomial response.

REFERENCES

[1] N. Mohottige, O. Glubokov and D. Budimir, “Ultra Compact Inline E-Plane Waveguide Extracted Pole Bandpass Filters,” in *IEEE Microwave and Wireless Components Letters,* vol. 23, no. 8, pp. 409-411, Aug. 2013.

[2] G. Macchiarella, S. Tamiazzo and V. Verri, “A design methodology for fully canonic NRN filters in coaxial technology,” *2017 IEEE MTT-S International Microwave Symposium (IMS),* Honolulu, HI, 2017, pp. 748-751.

[3] O. Glubokov and D. Budimir, “Extraction of Generalized Coupling Coefficients for Inline Extracted Pole Filters With Nonresonating Nodes,” in *IEEE Transactions on Microwave Theory and Techniques*, vol. 59, no. 12, pp. 3023-3029, Dec. 2011, doi: 10.1109/TMTT.2011.2168967.

[4] S. Amari, G. Macchiarella, “Synthesis of In-Line Filters With Arbitrarily Placed Attenuation Poles by Using Non-Resonating Nodes,” in *IEEE Transactions on Microwave Theory and Techniques*, vol. 52, n.10, pp. 3075-3081, Oct. 2005.

[5] S. Tamiazzo and G. Macchiarella, “Synthesis of Cross-Coupled Prototype Filters Including Resonant and Non-Resonant Nodes,” in *IEEE Transactions on Microwave Theory and Techniques*, vol. 63, no. 10, pp. 3408-3415, Oct. 2015.

[6] K. Wang, “Synthesis of Microwave Filters with Non-Resonating Nodes Using Optimization, ” M.S. Thesis, Department of Electronics, Information and Bioengineering, Polytechnic of Milan, Milan, 2013.

[7] P. Zhao and K. Wu, “A direct synthesis approach of bandpass filters with extracted-poles,” 2013 Asia-Pacific Microwave Conference Proceedings (APMC), Seoul, 2013, pp. 25-27.

[8] Y. He, G. Wang and L. Sun, “Direct Matrix Synthesis Approach for Narrowband Mixed Topology Filters,” in *IEEE Microwave and Wireless Components Letters*, vol. 26, no. 5, pp. 301-303, May 2016.

[9] Y. He, G. Wang, X. Song and L. Sun, “A Coupling Matrix and Admittance Function Synthesis for Mixed Topology Filters,” in *IEEE Transactions on Microwave Theory and Techniques*, vol. 64, no. 12, pp. 4444-4454, Dec. 2016.

[10] R.J. Cameron “Advanced Coupling Matrix Synthesis Techniques for Microwave Filters,” in *IEEE Transactions on Microwave Theory and Techniques,* vol. 51, no. 1, pp. 1693-1698, Jan. 2003.

[11] S. Tamiazzo and G. Macchiarella, “An analytical technique for the synthesis of cascaded N-tuplets cross-coupled resonators microwave filters using matrix rotations,” in *IEEE Transactions on Microwave Theory and Techniques,* vol. 53, no. 5, pp. 1693-1698, May 2005.

[12] M. Oldoni, G. Macchiarella, and F. Seyfert, *Synthesis and Modelling Techniques for Microwave Filters and Diplexers: Advances in Analytical Methods with Applications to Design and Tuning*, 02 2014.

[13] M. van Lier and R. Otten, “Planarization by transformation,” *IEEE Transactions on Circuit Theory*, vol. 20, no. 2, pp. 169-171, March 1973.

[14] S. Bedrosian, "Converse of the Star-Mesh Transformation," *IEEE Transactions on Circuit Theory*, vol. 8, no. 4, pp. 491-493, December 1961.

[15] H. C. Bell, “Canonical Asymmetric Coupled-Resonator Filters,” in *IEEE Transactions on Microwave Theory and Techniques,* vol. 30, no. 9, pp. 1335-1340, Sep. 1982.