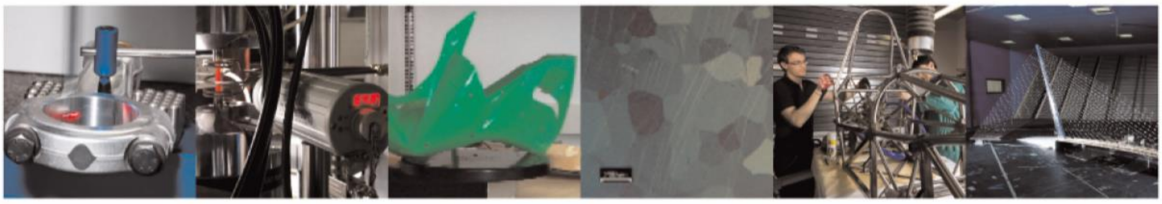

Selection of parameters in cost-tolerance functions: review and approach

Armillotta, A.

DIPARTIMENTO DI MECCANICA ■ POLITECNICO DI MILANO
via G. La Masa, 1 ■ 20156 Milano ■ EMAIL (PEC): pecmecc@cert.polimi.it
http://www.mecc.polimi.it

Selection of parameters in cost-tolerance functions: review and approach

Antonio Armillotta

Dipartimento di Meccanica, Politecnico di Milano
Via La Masa 1, 20156 Milano, Italy
E-mail: antonio.armillotta@polimi.it
Tel.: +39 02 23998296
ORCID: 0000-0003-2619-0669

Abstract

The paper deals with the cost-tolerance functions used in minimum-cost tolerance allocation on mechanical assemblies. The consistency of the cost-tolerance functions associated with different dimensions of a tolerance chain is a necessary condition for a correct optimization of tolerance values. This requires a careful selection of function parameters, in order to take into account all the variables influencing the cost, and thus to compare the costs of the different dimensions in a common scale.

The cost-tolerance functions proposed in the literature are reviewed, revealing that the parameters are often selected without explicit reference to empirical data or objective rules. The development of procedures for parameter selection has been addressed in several studies, which are also reviewed along with the needed cost data published in textbooks and reports. Finally, an original approach is proposed to build cost-tolerance functions for combined processes, for which fewer results or methods are available. The parameters of a reciprocal power function, chosen for the favorable balance between accuracy and ease of use, are expressed through an empirical relationship with some design variables; these include the nominal dimensions, and the type and size of the part features involved in the allocation problem. An application example shows that the proposed procedure results in optimal tolerance values consistent with common design criteria.

Keywords: tolerance synthesis, tolerance allocation, cost-tolerance model

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1 Introduction

Tolerance allocation is a widely studied problem in the design of manufactured products [1, 2]. In its basic formulation, it consists in optimizing the tolerances T_i for a set of dimensions X_i on individual parts of an assembly. The dimensions form a tolerance chain, i.e. they are mathematically related to an assembly dimension Y (a distance between features of different parts), whose variation is to be controlled within a tolerance T_Y for functional reasons.

The optimal allocation of tolerances is usually associated with the minimum manufacturing cost. On precision parts, dimensions are created through machining processes such as turning, milling or grinding. A tighter tolerance T_i on a dimension incurs a higher cost C_i because the machining process requires more expensive equipment and tooling, slower operations due to reduced cutting depths and feed rates, more frequent replacement of cutting tools, and more expensive inspection procedures with possible increase in scrap rates. This relationship is expressed through a cost-tolerance function $C_i(T_i)$ associated with each dimension. The objective function of the allocation problem is the sum of the cost-tolerance functions for the dimensions of a tolerance chain.

Several types of cost-tolerance functions have been used in solving allocation problems. Each has a different formulation (power law, exponential, polynomial, etc.), and includes a set of parameters. The choice of the function type is a compromise between various criteria: a complex function is more accurate with respect to real cost data, while a simple one is easier to use in tolerance allocation. Once the function type has been chosen, the parameters must be evaluated by considering the required machining operations and the geometry of the parts (type and size of machined features). This task is nontrivial and prone to errors, also due to the limited availability of cost data published in a suitable form.

Extensive discussions on cost-tolerance functions are reported in some reviews on tolerance allocation [1-6], and in studies comparing different types of functions [7-11]. These sources pay the most attention to the choice of function type: they list the available functions, show how to use them in allocation, and compare their accuracy. Relatively less studied is the selection of parameters, which is often treated as a secondary issue with no expected relevance on allocation results; as a consequence, parameter values may not be fully consistent with some requirements of tolerance allocation, such as the need to estimate costs in a common scale for all the dimensions of a tolerance chain.

This paper deals with the selection of parameters in cost-tolerance function. The main objective is reviewing the results available in literature for this task: these include the requirements posed by tolerance allocation (Section 2), the available types of cost-tolerance functions (Section 3), the published data that can be exploited to relate costs to tolerances (Section 4), and the methods proposed for parameter selection (Section 5). The review shows that further efforts are needed to improve parameter consistency; one way of doing this is by expressing the parameters as functions of the variables that influence the production costs associated with individual part features. This suggestion, which represents the main critical contribution of the review, translates into the definition of a possible functional expression for the parameters of a widely used function (Section 6), and in the discussion of possible research developments to improve and deploy it (Section 7).

The review is limited to the basic formulation of the allocation problem, where tolerances are optimized at product design stage according to general knowledge of available manufacturing options. The detailed selection of manufacturing processes is regarded as a downstream task rather than being integrated in the optimization problem. The same is assumed about the calculation of manufacturing tolerances in individual process phases. The reference allocation problem only includes dimensional tolerances, with no explicit consideration of geometric tolerances that may play a key role in the design of precision machine parts.

2 General properties

A cost-tolerance function is continuous and strictly decreasing, i.e. a dimension can have any tolerance in a given range, and shrinking the tolerance always involves additional manufacturing costs. Different dimensions of a tolerance chain have different cost-tolerance functions; as shown in Fig. 1, the difference is in both the absolute cost values (height of the curve) and the rate of change of cost with tolerance (slope of the curve). The latter aspect is the only one that influences the result of an allocation problem. For example, if the assembly tolerance T_Y is simply the sum of two tolerances T_1 and T_2, starting from a tentative allocation one can look for a better solution by adding ΔT to one tolerance and subtracting the same amount to the other. It is clearly better to reduce tolerance T_1 where the curve has a smaller slope, because the cost of T_1 will increase less than the cost of T_2 will decrease. If the slopes vary continuously and no tolerance reaches its extreme values, the curves have the same slope at the optimal T_i values [12].

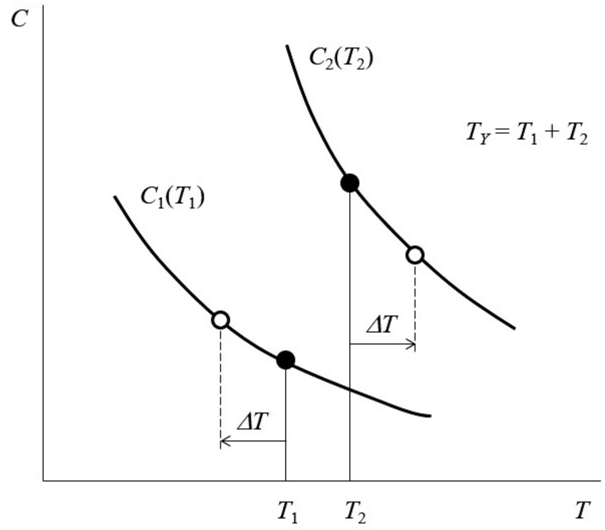

Fig. 1: Effect of the slope of cost-tolerance functions

In principle, any function with the above properties could be used for a numerical solution of the tolerance allocation problem. In many cases, however, an analytical solution is desirable so that interesting properties of the optimal tolerances could be highlighted. The easiest way to solve the problem analytically is the method of Lagrange multipliers [5], which requires the cost-tolerance functions to be differentiable, of simple form, and of the same type for all the dimensions. Therefore, the choice of the function is reduced to a few types, which offer a reasonable compromise between simplicity and accuracy.

The simplest case is when tolerances are optimized with given manufacturing processes. Under such assumption, a tighter tolerance will require some adaptation of the same process (e.g. reducing depth or feed in turning) without changing to a more accurate process (grinding). The tolerance ranges for the optimization have to be preselected, but a relatively simple cost-tolerance function can be accurate enough as it does not have to account for any discontinuity at process changes. The general form of the function is

$$C_i = C_{\mathrm{F}i} + C_{\mathrm{V}i}\left(T_i\right)$$

where $C_{\mathrm{F}i}$ is a ‘fixed’ cost independent of tolerance, and $C_{\mathrm{V}i}(T_i)$ is a ‘variable’ cost decreasing with tolerance. There is no full agreement about how different cost items are to be split into fixed and variable fractions. Most studies consider the operating cost (labor and equipment) as variable; in some cases, however, the variable fraction is assumed to be mostly related to quality costs: as the tolerance decreases, the cost increases slowly until the nonconforming rate is negligible, then it increases faster due to higher rates of scrap and rework. For an allocation with given processes, both views are probably valid: quality costs are likely to take the lion’s share of the variable cost below the natural tolerance of the process, while operating costs prevail at looser tolerances.

Another case is when the selection of the manufacturing process is part of the tolerance allocation problem: the solution is searched for in a wider tolerance range, and the chosen process will depend on the value of the optimal tolerance. Under this assumption, the parameters of the cost-tolerance function should change discontinuously at process changes, making the problem impossible to treat analytically. With some compromise on accuracy, this difficulty can be overcome by defining a single function spanning the whole tolerance range. The situation is illustrated in Fig. 2, where it is supposed that the tolerance range is covered by three different machining processes, each with its cost-tolerance function. Loose tolerances can be satisfied with an increasingly precise rough turning but, below a first break-even point, it is more economical to rough-turn with faster settings and add a finish-turning operation; a similar situation occurs at a second break-even point between finish turning and grinding. Considering the uncertainty on the shape of the curves, one can safely replace the three cost-tolerance functions with an envelope function, usually referred to as a cost-tolerance function for combined processes. The envelope curve will be of the same type as the three original ones, but will have a continuous curvature over the whole tolerance range. The curve could be determined by interpolating discrete data at midrange conditions for the different processes.

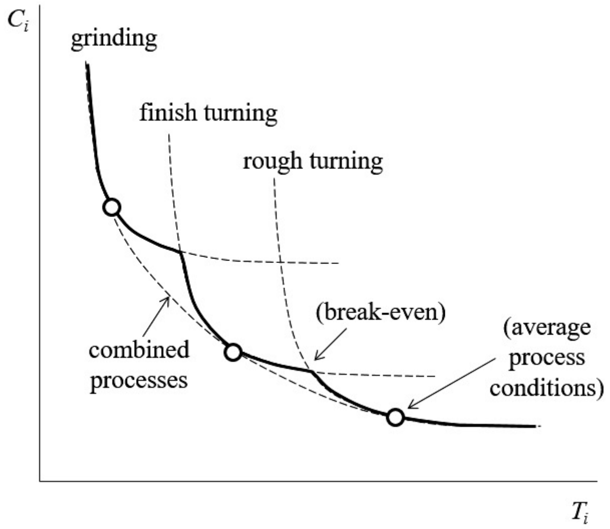

Fig. 2: Cost-tolerance function for combined processes

Regardless of the assumptions on manufacturing processes, the fixed cost has no influence on the slope of the curve and thus on the optimal allocation. Therefore, it can be ignored whenever an absolute estimate of the total cost is not required. All the effort to improve the accuracy of a cost-tolerance function should be made on the variable cost. A key consideration is that function C_{Vi} actually depends on other variables beside T_i. For example, the size of the toleranced feature has an influence on the machining time, and thus on the operating cost; factors with similar effects may include the machinability of the material and the production volume. These additional variables should be accounted for by the parameters of the cost-tolerance function to ensure consistency among the dimensions of a tolerance chain.

3 Cost-tolerance functions

The cost-tolerance functions proposed in literature are reviewed below. The comparison of the functions leaves out some aspects already addressed in detail elsewhere, such as the accuracy with respect to published cost data and the suitability to the different optimization methods used for tolerance allocation. Consistently with the objectives of the paper, attention is focused on the parameters of the functions, whose ranges of variation in available studies are discussed in order to assess their consistency and suggest possible directions for improvement. The current popularity of the different functions is also highlighted to help narrow the field in the development of new approaches to parameter selection.

The functions are classified according to the number of parameters in the cost equation; they include one parameter for the fixed cost, and one or more parameters for the variable cost. More parameters make a function more accurate in approximating actual cost data, but also more difficult to use, as analytical tolerance allocation is no longer guaranteed, and more data are needed for parameter estimation. The parameters have different values for each dimension X_i, although the subscript i will be omitted in the following equations.

3.1 Two-parameter functions

The first group includes three functions with limited accuracy, yet interesting due to their ability of provide especially simple and expressive analytical solutions to the allocation problem.

3.1.1 Linear

Proposed in [13], the function

$$C = a - bT$$

does not account for the variable rate of change that is a recurring pattern in published cost data. Therefore, it should be used only with given processes and within narrow tolerance ranges; a piecewise linear variant [14] has been proposed for combined processes. The function allows an analytical solution of the allocation problem by the Lagrange multiplier method: under the assumption of statistical stackup, optimal tolerances T_i are proportional to parameters b_i. However, the most convenient use of the function is with linear programming [13, 14] or in more complex methods where only an exceptionally simple function can provide an analytical solution [15, 16]. The parameters are usually set without explicit references, maybe from original cost data.

3.1.2 Reciprocal

The function

$$C = a + \frac{b}{T}$$

has been first used in [17]. In statistical allocation using Lagrange multipliers, it gets optimal tolerances T_i proportional to the power 1/3 of parameters b_i [1]. Its simplicity has allowed to use it with a diversity of methods, including fuzzy logic [18-21], robust design [22, 23], nonlinear 0-1 programming [24], mixed-integer linear programming [25], simulated annealing [8], and game theory [26]. The function has also been used with special assumptions on either the manufacturing system [27, 28] or the products, e.g. optical systems [29] and mechanisms [30]. The selection of parameters is explained in few cases [25, 31], where b seems to decrease with the precision of the process in a 1:4 ratio between minimum and maximum values.

3.1.3 Reciprocal squared

Proposed in [32], the function

$$C = a + \frac{b}{T^2}$$

is reportedly less accurate than the reciprocal function [3, 4]. It provides an analytical solution to the statistical allocation with Lagrange multipliers, with optimal tolerances T_i proportional to the power 1/4 of parameters b_i. The use cases are similar to the reciprocal function: allocation with neural networks [33], constraint networks [34, 35], and response surfaces [36]; with minimization of cost and quality loss [37-39] or nonconforming fraction [40]; for optical components [41], parts from different manufacturing plants [42], or selective assembly [43]. Again, b seems to decrease with increasing process precision in a ratio around 1:2 between minimum and maximum values [31, 42].

3.2 Three-parameter functions

The second group includes the two functions that are most frequently chosen due to their favorable balance of accuracy and ease of use.

3.2.1 Reciprocal power

The function

$$C = a + \frac{b}{T^k}$$

has been proposed in [44] and includes the reciprocal and reciprocal squared functions as special cases. Therefore, it is likely to ensure a better approximation of cost data, especially in the lower range of tolerances [31]. Yet it remains simple enough to get analytical solutions to the allocation problem: as reported in [1], statistical allocation by Lagrange multipliers results in optimal tolerances T_i proportional to the powers $1/(k_i+2)$ of products $k_i b_i$; more applications of the same method with different assumptions are reported in [6, 45, 46]. Other applications use different methods such as sequential quadratic programming [47-51], genetic algorithms [52-54], particle swarm optimization [55-57], migration algorithms [58], artificial bee colony [59], and Monte Carlo [60]. The function has also proved useful for dealing with complex mechanisms and assemblies [61-63] and with special formulations of the allocation problem, which include geometric tolerances [64], mean-shift stackup [65], optimization based on process capability [66], and the optimal partitioning of components for selective assembly [67, 68]. In [69], the function is extended to the case of multiple features machined on the same component: the fixed cost is counted only once for the part, while the variable costs are added together for the different features.

Tab. 1: Values of the parameters of the reciprocal power function from literature

Ref.	b	k
[31]	1.5 mill, 5 drill, 15 turn, 30-40 grind	0.15-0.2 grind, 0.5 drill/turn, 1.6 mill/bore/ream
[6]	0.15-0.3 drill, 0.15-0.6 grind, 0.2-0.7 bore, 0.25-0.5 turn, 0.3-0.7 mill	0.5 turn/mill/bore, 0.5-1 grind, 1-1.5 drill/hone/lap
[61]	-	0.4 mill, 0.7 grind, 1 lap
[64]	1.2-2 drill, 1.7-2.5 turn, 2.3-2.9 grind, 3.3-3.7 mill	0.25 mill, 0.45 turn, 0.5 grind, 0.6 drill
[70]	-	0.5 mill, 0.6 drill
[62]	1.2 ext. cylinder, 7.5-12.5 int. cylinder, 13-18 plane	0.6-0.8
[53]	0.3-0.5	0.45
[49]	0.01-0.07	0.7-0.9
[71]	7-15	1-4
[67, 72]	-	0.45
[47, 48, 60, 73]	-	1.7-3
[57]	-	1-2
[66]	-	1.8
[63]	-	1.3

Tab. 1 shows approximate values of the variable-cost parameters b and k collected from different studies. In some cases [6, 31, 49, 62, 64], the parameters are evaluated from original or previously published cost data. The following patterns can be recognized:

- k is dimensionless and therefore has comparable values among the different studies. It does not seem to vary with feature size, but may depend on production processes. It is usually in the 0.4-0.6 range, although higher values (1-2) are sometimes assumed to cover specific processes (drilling, milling) or particularly wide tolerance ranges.
- b is less consistent between the different references, because it depends on the unit of measurement adopted for the tolerance (here converted to mm where necessary), and on the absolute or relative scale of cost. However, it seems to depend on both the manufacturing process and the type of feature; in some cases it is also noted that b increases with the dimensions in a ratio around 1:2 between minimum and maximum values.

3.2.2 Exponential

Proposed in [74], the function

$$C = a + \frac{b}{e^{kT}}$$

has a smoother shape than the reciprocal functions, which is said to ensure a better accuracy in the higher range of tolerances [31]. However, an optimal statistical allocation by Lagrange multipliers can be found only with an iterative numerical procedure [75, 76]. Consequently, most applications of the functions use advanced allocation methods, such as genetic algorithms [62, 77-85], the bat algorithm [86], cuckoo search and particle swarm optimization [85], constraint networks [34, 35], scatter search [87, 88], pattern search [89], simulated annealing [90], a method based on Lambert W function [91, 92], and fuzzy methods [93]. The function has also been used to solve problems with special formulations, such as interrelated tolerance chains [94, 95], pre-selection of manufacturing processes [96], minimization of nonconforming fraction [97, 98], allocation of geometric tolerances [99-101], minimization of cost and quality loss [102-104], robust design [105], simultaneous allocation of process averages and tolerances [106-108], multistation systems [109], and allocation combined with scheduling [110].
Approximate values assumed in literature for the variable cost parameters b and k are listed in Tab. 2. Both are hardly comparable for the different studies, as they depend on tolerance units and cost scales. The ranges of the two parameters are very wide, and only few patterns can be recognized: b increases with machining precision and decreases with increasing dimensions; k increases with size, while the influence of the process seems to be inconsistent between the different studies.

3.3 Functions with more than three parameters

The functions of this last group have been used in very few cases, and their interest is mostly in evaluating possible accuracy improvements compared to simpler functions.

3.3.1 Michael-Siddall

In the function proposed in [111]

$$C = a + \frac{b}{T^{k_1} e^{k_2 T}}$$

the variable cost is the product of reciprocal power and exponential functions. This makes the function more accurate, but also too complex for an analytical allocation by Lagrange multipliers. The function has been used with other allocation methods such as nonlinear programming solved by Monte Carlo [112], genetic algorithms [113] and integrated Kriging meta-modeling [114]. In [113], the variable T is replaced by ($T - T_{\lim}$), where $T_{\lim}$ is the minimum tolerance allowable with a certain process.

Tab. 2: Values of the parameters of the exponential function from literature

Ref.	*b*	*k*
[75]	7 turn, 5 grind	500 turn, 450 grind
[79]	-	300 rough-turn, 800 finish-turn, 3000 grind, 9000 finish-grind
[110]	6.5 rough-turn, 8.4 semifinish-turn, 10 finish-turn, 12 grind	0.275 rough-turn, 0.25 semifinish-turn, 0.225 finish-turn, 0.2 grind
[99]	60 (10-30 geometric tol.)	1.5-2.5 (10-12 geometric tol.)
[88]	25-400 small, 4-9 large	3-17 small, 300-800 large
[4]	4.5-12	0.008-0.012
[78]	8-75	1.5-85
[31]	30-100	2.5-20
[89]	25-400	3-17
[85]	-	3-40
[94]	5-70	15-45
[104]	80-300	15-85
[92]	30-300	15-200
[106]	2-2500	30-300
[96]	200-300	40-100
[115]	0.2-15	50-4000
[86]	30-70	80-220
[84]	30-150	700-1400
[34]	2.5-4.2	1100-1400

3.3.2 Combined functions

In [31], the search for more accurate functions leads to the additive combination of simple functions. These include the combined reciprocal power and exponential function

$$C = a + \frac{b_1}{T^{k_1}} + \frac{b_2}{e^{k_2 T}}$$

and the combined linear and exponential function

$$C = a - b_1 T + \frac{b_2}{e^{kT}}$$

Both have been used for comparison to other functions with numerical resolution of the allocation problem using the constrained variable matrix method, apparently with little advantage.

3.3.3 Polynomial

Another family of functions proposed in [31] consists of polynomials of the type

$$C = a + b_1 T + b_2 T^2 + b_3 T^3 + \ldots$$

which have been used in the cubic (more common), fourth-order and fifth-order versions. They are unsuitable for analytical allocation, and have been mainly tested for the linear regression of cost data for geometric tolerances [116, 117], and for parts manufactured by unconventional machining [118].

3.3.4 Discrete

As an extreme case of a cost-tolerance function, some authors have proposed using discrete cost values for selected tolerance levels. They turn the allocation problem into an enumerative search, which has been carried out by various methods, such as dynamic programming [119], 0-1 integer programming [120, 121], branch and bound [122], and genetic algorithms [123, 124]. Discrete costs have also been used for a special formulation of the allocation problem for product families [125].

3.4 Overview

The above cited references cover a sample of 115 studies (35 of which in the last decade), where cost-tolerance functions have been used in optimal allocation problems. As shown in the usage statistics in Tab. 3, the three-parameter functions were chosen in 61% of cases (66% if only the most recent studies are considered), with a slight and increasing prevalence of the exponential function on the reciprocal power; as said before, their popularity seems to be justified by the search for a fair compromise between accuracy, analytical tractability and ease of parameter evaluation. The reciprocal and reciprocal squared functions were the preferred ones in earlier studies, but they have been attracting decreasing interest up to an overall share of 27% (20% in the most recent studies), with constant prevalence of the former probably due to the greater accuracy noticed in some reviews. The adoption of the remaining functions has been marginal, even if constant over time. These figures are in good agreement with previously published statistics [126].

Tab. 3: Breakdown of the applications of cost-tolerance functions

Function	*Overall*	*2010+*
Reciprocal	17%	14%
Reciprocal squared	10%	6%
Reciprocal power	25%	26%
Exponential	36%	40%
Others	12%	14%

The choice between the two most popular functions is not generally based on explicit reasons, although it is often mentioned that the reciprocal power and the exponential functions are respectively more accurate for tight (< 0.1 mm) and narrow tolerances; such a comparison could be relevant for allocation problems with given manufacturing processes, while the cost-tolerance functions for combined processes should cover the whole range of tolerance values with similar accuracy. Regarding mathematical simplicity, which is often emphasized as a major selection criterion, there seem to be no particular reasons for preferring the reciprocal power or the exponential: most of the allocation methods, with the main exception of Lagrange multipliers, do neither exploit properties of the derivative of the function nor seem to be sensitive to possible differences in the speed of evaluation of the function during search procedures.

For the present work, it is more interesting to compare the two functions according to the possible meaning of the parameters. Both functions have a factor b which implicitly carries the effects of all the variables that influence the manufacturing cost for a feature; a possible approach to make these effects explicit will be discussed below. For the k parameter, which determines the rate of change of the cost, it would be useful to identify properties that help predict the shape of the cost curve starting from partial data. From this point of view, considering only the variable cost ($C \equiv C_V$), it is easy to prove that:

- for the reciprocal power function, if $T_2 = 2T_1$, the ratio C_1/C_2 is equal to 2^k; from a known point (C, T) of the curve, therefore, k can be used to find the costs for tolerances $2T$, $4T$, etc. and $1/2T$, $1/4T$, etc.

- for the exponential function, the cost divided by its derivative with respect to tolerance (i.e. the intercept of the tangent to the curve on the tolerance axis) is equal to $-1/k$; therefore k can be used to find the tangent to the curve once the cost for a given tolerance is known.

In the above review of cost-tolerance functions, it should be noted that almost all applications are in the allocation of dimensional tolerances. Only few studies include geometric tolerances, again without agreement or discussion on which functions are to be used: the choices range from exponential [88, 99] to reciprocal power [64] to polynomial [116, 117], while the selection among discrete values was the preferred option in the earlier studies [123, 124].

4 Cost data

Quantitative relationships between cost and tolerance can be drawn from general knowledge on manufacturing processes. Depending on part specifications (shape, size, material, production volume, etc.), the best process chain can first be identified for any given tolerance; the choice is based on the natural tolerances of the different processes, which are charted in several textbooks [127-129]. For each phase of the process chain, the machining time can then be estimated by a few available methods [130-132]; the cost is finally calculated by applying an appropriate hourly rate to machining time.

Procedures of the above type have been used to prepare cost-tolerance data, which have been published in textbooks and research studies. It is often pointed out that such data are only a rough approximation of actual costs, which may depend on specific product types and production settings. It is likely that more accurate cost data are available at many companies, but are not published for confidentiality reasons.

An especially detailed dataset is provided in [133]. It includes many cost-tolerance graphs for single processes (drilling, milling, grinding, and some metal casting processes); the costs are expressed as percentage increments with respect to baseline tolerances for the different processes. A graph for combined processes is also provided for rotational parts.

Within an in-depth treatment of tolerance analysis and allocation, [14] reports some cost-tolerance curves for combined processes that had formerly been published in [134]. Each graph refers to a different feature type (planes, external and internal cylinders) on either rotational or prismatic parts. Although probably dating before the advent of CNC machining technology, the dataset is interesting as costs are expressed in a common scale, cover a wider tolerance range, and take into account expected nonconforming rates.

Other studies include cost data for combined processes in a common scale. In [31], costs of different machining processes (drilling, milling, turning, grinding) are derived from data previously published in [133, 135] and normalized with respect to unmachined castings. Data from [135] are reported in [4] without specifying the processes corresponding to the different tolerances. In [123], a graph provides relative costs to satisfy flatness tolerances with different processes (milling, planing, surface grinding).

A limitation of the above data is that the tolerances are not related to detailed specifications on machined features. Differently, [6] provides a chart where the relative cost is a function of a precision level corresponding to increasing tolerances for increasing dimensional ranges. The costs are normalized with respect to a minimum tolerance for each process, but can be easily recomputed in a common scale by applying a correction factor depending on the process. The data are taken from [136], and cover an especially large set of processes; in addition to the above mentioned ones, they also include boring, slotting, shaping, reaming, broaching, diamond turning and boring, lapping and polishing.

The datasets published in literature are heterogeneous by either purpose (types of features, machining processes, dimensions) and range (single or combined processes). They could be used for the selection of parameters in cost-tolerance functions, provided that a common variable is adopted to jointly consider tolerances and nominal dimensions. An obvious choice is the IT tolerance grade of the ISO system of limits and fits [137]; actually, the precision levels in [6] roughly match the tolerance grades between IT5 and IT13.

A cost expressed as a function of the tolerance grade can easily be converted to a function of tolerance T_i and nominal dimension X_i, considering that the IT grade is an approximate function of the $T_i / X_i^{1/3}$ ratio.

After such normalization, some common patterns can be recognized in the different datasets:

- for an individual process, the cost varies within narrow limits (about 1:2 ratio between extreme values) as long as the tolerance is so large to cause negligible scrap rates; over all the processes applicable to the same type of feature, the cost range is much wider (ratios between 6:1 to 10:1);
- for each feature type, the cost increases as the tolerance grade decreases (about 1:6 ratio between IT13 and IT5); the relationship is nonlinear but much smoother than what would be obtained if the cost were referred to absolute tolerance;
- for the same tolerance grade, the cost strongly depends on both part type (prismatic parts cost more to machine than rotational ones) and feature type (external cylinders, internal cylinders, and planes in increasing order of cost).

5 Methods for parameter selection

As shown in Section 3, there is some discrepancy among the values of the parameters assumed in different studies for cost-tolerance functions. They are often set without explicit references to equations, charts, or rules of thumb. This makes it difficult to compare the results of different allocation methods. Therefore, efforts have been made to develop parameter selection methods that could take into account the relevant influencing factors and improve the consistency among functions relating to different part features.

Some studies use linear regression to estimate the parameters from cost data such as those mentioned in Section 4. In [31], the parameters of the main cost-tolerance functions are estimated for many manufacturing processes, and the accuracy in approximating cost data is compared for the different function types. Similar results are presented in [118] for cubic polynomial functions. It is shown that higher-order polynomials are more accurate, but may lead to polynomial wiggle; in [138, 139], this issue is avoided by defining a modified function called extended spline, along with a regression procedure to estimate its parameters.

The approximation of cost data is more difficult with cost-tolerance functions for combined processes. As already shown in Fig. 2, trying to merge different cost-tolerance curves into one would result in an irregularly shaped curve that could not be treated analytically. To avoid that, the slope-based method proposed in [115] calculates discrete points of the combined curve from exponential functions for the different processes, then it interpolates the calculated points linearly. In an example, this procedure builds a cost-tolerance curve with a fairly regular shape along the whole tolerance range.

Approximation methods alternative to regression have also been used. In [140], a neural network replaces the cost-tolerance function for a specific process, demonstrating better accuracy than conventional functions due to training on a large set of cost data. In [141], a neural network for combined processes is designed to estimate the cost as a function of tolerance and additional influencing factors (machine tools, machining methods, process orders, process time, cutting tools, inspection instruments, skilled operators); these are evaluated by weighted scores to get a single influence coefficient. In [142], the parameters of cost-tolerance functions for single processes are estimated from the manufacturing difficulty of the feature with respect to the process, calculated using a method based on fuzzy logic. In [71], an equation based on process constraints (machine tools, workers, machining and assembly accuracy) is iteratively solved with the Newton's method to calculate the coefficients of a reciprocal squared function, thus obtaining a more complex model depending on a multiplicity of variables (variable coefficients reciprocal squared).

The cost data available in literature cover a small part of the possible combinations of influencing factors (product type, feature type, manufacturing process etc.). In real cases, the objective of an overarching cost-tolerance function might have to be resized into a context-related procedure where specific assumptions could be taken into account. For this purpose, the method proposed in [143] builds cost-tolerance functions for alternative process sequences from elementary functions related to low-level operations (roughing, semi-finishing, finishing); the minimum-cost alternative for each tolerance is then selected as a point of the combined cost-tolerance curve. Process sequences and elementary functions should be easier to collect and maintain at a manufacturing company. With a similar objective, the method proposed in [49-51] associates a

cost-tolerance function with a five-digit code (tolerance elements) that takes into account the shape and size of the part. Cost data are retrieved from a process database, where a company could assess the relative difficulty in satisfying given tolerances when machining different types of part features.
Quality costs are better estimated by a statistical approach. The method proposed in [126, 144] assumes that the cost-tolerance relationship is only related to the different incidence of scrap and rework. The additional cost is calculated from the nonconforming rate in a specific process, which is estimated from different statistical distributions. The method achieves similar relationships between costs and IT tolerance grades as those discussed in Section 3. An enhanced statistical approach could also consider the impact of inspection costs, whose relationship with tolerance has been discussed in [145].
The lack of consistent parameter estimations for cost-tolerance functions has suggested alternative approaches to cost-based allocation. In [146-148] the cost data are obtained by a procedure deriving from activity-based costing, where the activities are associated to machining and quality control stages. The CTSA (cost-tolerance sensitivity analysis) method, proposed in [149-151], replaces the cost-tolerance function with a sensitivity curve, which is built from the judgement of designers and manufacturing engineers (each giving advise on what would change if the tolerance increased or decreased). The Taguchi tolerancing equation [152] allocates tolerances considering the quality costs related to deviations on the assembly dimension, and the engineering costs of scrapping or reworking individual parts.

6 Proposed procedure

The above review suggests the need to ensure consistency of cost-tolerance functions for the dimensions involved in an allocation problem. After choosing the function type, the parameters of the function should be estimated considering the main influence factors, such as the nominal dimensions and the size and type of the features. This is critical in order to compare the costs of different features in a common scale.
This section describes a procedure for the selection of parameters in a cost-tolerance function for combined processes. It uses available cost data to find empirical relations between the parameters and appropriate design variables.

6.1 Development

The procedure is based on the reciprocal power function, which will be limited to the variable cost, i.e. $C \equiv C_V(T)$. The cost will be expressed in a relative scale, common to all the features of a tolerance chain. The factor b will be expressed as a function of three variables: the nominal dimension, the size and the type of the feature. The exponent k will be assumed to be constant for combined processes, while the literature shows that it may depend on the manufacturing processes when they are preselected for tolerance allocation.
A first assumption is made about the types of costs to be considered in the function. The combined cost-tolerance curve is an envelope of the process-related curves along the tolerance range. Each process is likely to be the minimum-cost option among alternative processes only at tolerances exceeding its natural tolerance, where low levels of scrap and rework are expected. Therefore, only machining costs will be considered in the combined cost-tolerance function, while quality costs would obviously be relevant when optimizing tolerances with given manufacturing processes. For a machined part, the fixed costs include the setup of the machining processes (fixturing, tooling, workpiece loading and unloading) as well as the roughing operations that determine the shape of the part, while the variable costs are only related to the finishing operations.
The machining cost corresponding to a given tolerance depends on the nominal dimension. As noted in Section 4, a relationship can be extablished between the variable cost and the IT tolerance grade. This can be approximated by a reciprocal power law:

$$C_0 = \frac{p}{g^q} \tag{1}$$

where C_0 is the average relative cost for a given nominal dimension, and g is the tolerance grade corresponding to the tolerance for that dimension. For the grades between IT5 and IT13, several datasets can be used to estimate the coefficients p and q. They are expressed in different scales and based on different assumptions (often not explicitly stated). Some of them were analyzed and recalculated in order to match costs to tolerance grades. The following sources yielded similar patterns for the cost-grade relationship:

- the graphs provided by Bjørke [14], which list relative costs in a common scale for different types of features; although the tolerance is expressed in absolute terms, the reported values seem to correspond to very small nominal dimensions;
- the dataset provided by Chase [6], which charts relative costs in a common scale for different tolerances grades (although different from the ISO classification) and manufacturing processes; the effect of the latter was averaged over the different process choices available for each grade;
- the dataset provided by Wu [4], which includes relative costs in an absolute scale of tolerances, apparently corresponding to midrange nominal dimensions;
- the graph proposed by Trucks [133] for rotational parts, which includes relative costs depending on the manufacturing processes and thus easily matched to tolerance grades.

The datasets were normalized so as to get the same cost ranges, thus making them fully comparable in a common scale. In the double logarithmic graph of Fig. 3, the four normalized datasets are merged into a single dataset; the regression line corresponds to coefficient values $p = 91$ and $q = 2.16$ (with $R^2 = 0.94$).
In the ISO system, the tolerance in mm is a multiple of the standard tolerance factor I in µm:

$$T = n \cdot I/1000 \tag{2}$$

where the factor n depends on the IT tolerance grade by a discrete exponential law. For the purpose of this work, it is better to approximate that relationship by a continuous power law:

$$n = r \cdot g^s \tag{3}$$

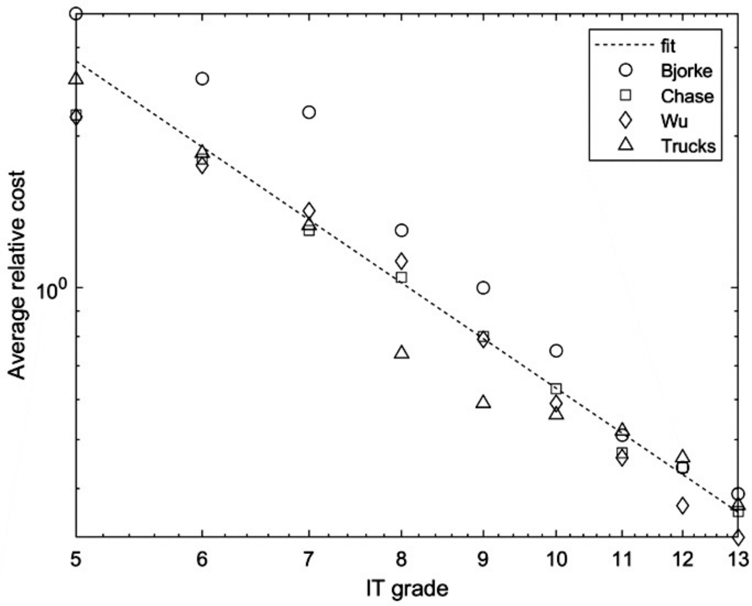

Fig. 3: Machining cost as a function of the IT tolerance grade

The double logarithmic graph in Fig. 4 shows the discrete values of n from [137]; the regression line is affected by a moderate lack of fit due to the choice of the statistical model, yet has a good correlation ($R^2 = 0.97$) and gives the coefficient values $r = 0.0114$ and $s = 3.80$.

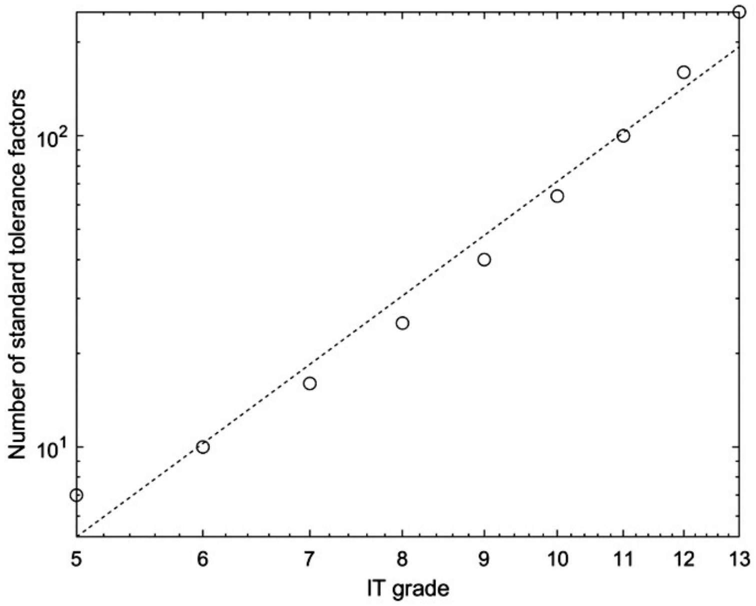

Fig. 4: Approximation of the proportionality coefficients of IT tolerance grades

Again from [137], the standard tolerance factor depends on the nominal dimension X in mm:

$$I = 0.45 \cdot X^{1/3} + 0.001 \cdot X \qquad (4)$$

The second term, related to measurement uncertainty, can be neglected for $X < 500$ mm. Rearranging equations (2-4), the tolerance grade is expressed as a function of the tolerance and the nominal dimension:

$$g = \left(\frac{1000 \cdot T}{0.45 \cdot r \cdot X^{1/3}} \right)^{1/s}$$

hence equation (1) gives

$$C_0 = p \cdot \left(\frac{0.45 \cdot r \cdot X^{1/3}}{1000 \cdot T} \right)^{q/s} = \frac{\beta \cdot X^{k/3}}{T^k} \qquad (5)$$

where $k = q/s$ and

$$\beta = p \cdot \left(\frac{0.45 \cdot r}{1000} \right)^{q/s}$$

The approximate values calculated for the coefficients of the function are $\beta = 1.0 \cdot 10^{-3}$ and $k = 0.55$.
Some corrections have still to be made on C_0 so that all the dimensions in a tolerance chain could be expressed in a common scale. For a given nominal dimension, the variable cost is expected to depend on the size and type of the feature. A simple way of estimating the time required by finishing operations (here related to the variable cost) is described in [131]: the surface area to be machined is multiplied by a removal rate depending on the type of feature (i.e. on the machining process). Accordingly, equation (5) is modified with two correction factors:

$$C = f_A \cdot f_F \cdot \frac{\beta \cdot X^{k/3}}{T^k}$$

where f_A is equal to the machined area in cm², and f_F has the approximate values listed in Tab. 4, calculated from the removal rates for different types of feature.
Should the allocation problem involve parts made of different materials, a third factor f_M related to the machinability of the material could also be considered. With base 1 for mild steel, data from [131] would suggest higher f_M values for difficult-to-machine materials (e.g. 2 for stainless steels), or lower ones for materials allowing high-speed machining (e.g. 0.3 for aluminum alloys).

Tab. 4: Values of the feature-related factor in the relative cost equation

Feature type	f_F
Outside features of rotational parts	0.75
Inside features of rotational parts	1.25
Cylindrical features of prismatic parts	1.5
Position of planes of prismatic parts (higher for smaller features)	0.5-1
Non-cylindrical features of prismatic parts (higher for smaller features)	1-1.5

6.2 Application

The use of the proposed cost-tolerance function will be demonstrated on the example illustrated in Fig. 5. The fork 1 and the link 2 are hinged by means of the spindle 3, which is axially restrained by the pin 4 (press-fit in a radial hole in the spindle) and the washer 5. The drawings show the dimensions X_1, ... X_9 of the part features involved in assembly relationships. All the parts are assumed to be made of the same material.
The dimensions are linked by a few tolerance chains corresponding to clearances that must be controlled for functional reasons. Tolerances T_1, ... T_9 are to be allocated by minimizing the total manufacturing cost for the dimensions involved in each tolerance chain.

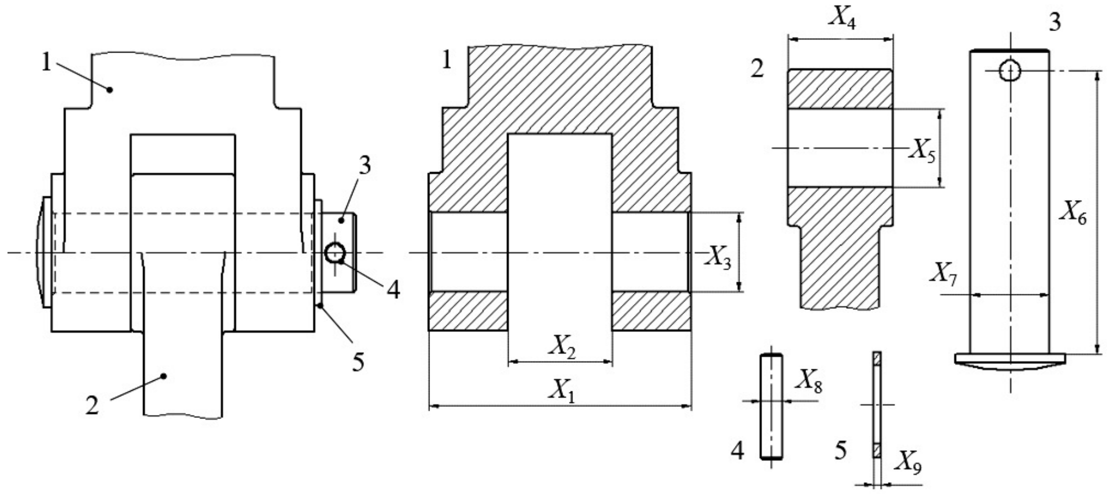

Fig. 5: Example of tolerance allocation

The cost-tolerance functions for the dimensions X_i have the expression

$$C_i = \frac{b_i}{T_i^k}, \quad b_i = f_{Ai} \cdot f_{Fi} \cdot \beta \cdot X_i^{k/3}$$

Tab. 5 lists the data needed for the estimation of factors b_i for the considered dimensions.

Tab. 5: Parameters of the cost-tolerance functions for the example

i	X_i [mm]	f_{Ai}	f_{Fi}	b_i
1	100	45	1.25	0.131
2	40	60	1.25	0.147
3	30	60	1.5	0.168
4	40	45	1.25	0.111
5	30	40	1.5	0.112
6	108	20	1.25	0.059
7	30	100	0.75	0.140
8	8	10	0.75	0.011
9	3	12	0.75	0.011

The tolerance chains have the following equations:

$$Y_1 = X_2 - X_4$$

$$Y_2 = X_3 - X_7$$

$$Y_3 = X_5 - X_7$$

$$Y_4 = -X_1 + X_6 - 0.5X_8 - X_9$$

where Y_1 is the side clearance of the link in the fork, Y_2 is the clearance between the spindle and the bore in the fork, Y_3 is the clearance between the spindle and the bore in the link, and Y_4 is the axial clearance of the spindle (i.e. the distance between the pin and the washer mating the side of the fork).
In each of these tolerance chains, a statistical stackup of tolerances is assumed according to the RSS (root sum square) equation:

$$T_Y^2 = f_Y \cdot \sum_i s_i^2 T_i^2$$

where T_Y is the tolerance specified on an assembly dimension Y, the subscript i is limited to the part dimensions involved in the tolerance chain, and s_i is the sensitivity of Y with respect to each dimension X_i. In the equation, the RSS stackup is corrected with an inflation factor $f_Y > 1$ to consider possible departures from the assumptions of statistical tolerancing [153]. Applying the Lagrange multiplier method of tolerance allocation, it results that the optimal tolerances are proportional to a factor F_i depending on the sensitivity and on the parameter of the cost-tolerance function:

$$T_i \propto F_i = \left(b_i / |s_i|\right)^{\frac{1}{k+2}}$$

Once F_i is known for each dimension of the tolerance chain, the individual tolerances can be calculated from the assembly tolerance:

$$T_i = \frac{F_i}{\sqrt{\sum_i s_i^2 F_i^2}} T_Y$$

Tab. 6 details the above calculations for the four tolerance chains. The assembly tolerances T_Y were set considering the different types of functional requirements (0.04 mm for cylindrical fits, 0.08 mm for prismatic fits, and 0.12 mm for axial clearances). The inflation factor was set to the customary value of $f_Y = 1.5$. The tolerances T_i are the results of unconstrained optimizations, and should be approximated to standard fits or tolerances of stock parts. If this further step is done, the resulting tolerances appear in reasonable agreement with design practice. Specifically:

- The prismatic fit between the fork and the link has tolerances 0.040 mm on the hole X_2 and 0.036 on the shaft X_4. For a nominal width of 40 mm, the closest standard fit is IT8/IT7 (0.039/0.025 mm). Both features can be machined at the required tolerance grades by side milling cutters on a horizontal milling machine.
- The cylindrical fit between the spindle and the fork has tolerances 0.020 mm on the hole X_3 and 0.018 mm on the shaft X_7, which may be approximated to an IT7/IT6 fit (0.021/0.013 mm) for a nominal diameter of 30 mm. These are likely to require reaming of the hole on a milling machine and finish turning of the shaft on a CNC lathe.
- In the cylindrical fit between the spindle and the link, the tolerance of 0.020 mm on the shaft X_7 is superseded by the tighter tolerance (0.018 mm) resulting from the previous requirement. If this is approximated to IT6, the 0.018-mm tolerance on the hole X_5 can again be narrowed to IT7 (0.021 mm), ending up in similar machining choices.
- Among the dimensions involved in the axial clearance of the spindle, the diameter X_8 of the pin has a tolerance of 0.030 mm, which is closely approximated by standard 8-mm dowel pins (ISO h8 or 0.022 mm). The 3-mm thickness X_9 of the washer has a tolerance of 0.023 mm, corresponding to IT9 (0.025 mm) and consistent with accurate face turning and parting-off operations on a CNC lathe if a custom size is required. The 100-mm width X_1 of the fork has a tolerance of 0.061 mm, which correspond to IT8 (0.054 mm) and is easily machined as previously suggested for the link. The position X_6 of the hole in the spindle has a tolerance of 0.044 mm, which can be standardized to IT8 grade (0.035 mm) on the nominal distance of 108 mm; the hole will have to be drilled and then reamed at the final diameter.

Tab. 6: Optimal tolerance allocation for the example

Chain	T_Y [mm]	X_i	s_i	F_i	T_i [mm]
Y_1	0.08	X_2	1	0.47	0.040
		X_4	1	0.42	0.036
Y_2	0.04	X_3	1	0.50	0.020
		X_7	1	0.46	0.018
Y_3	0.04	X_5	1	0.42	0.018
		X_7	1	0.46	(0.020)
Y_4	0.12	X_1	1	0.45	0.061
		X_6	1	0.33	0.044
		X_8	0.5	0.22	0.030
		X_9	1	0.17	0.023

7 Conclusions

The paper reviews the cost-tolerance functions for tolerance allocation problems. Differently from previous reviews on the same topic, attention is focused on the parameters of the functions, with the aim of highlighting a possible inconsistency in the way they are chosen for the different dimensions of the same tolerance chain. To shed light on this issue, information is also provided on related problems, such as the collection of cost data and the estimation of function parameters. As a supplement to the review, a possible approach has been suggested for a consistent estimation of the parameters for a particular type of function.

Tolerance allocation problems can be solved with many different cost-tolerance functions. Simplicity has traditionally driven the choice with the aim to get analytical solutions, with the three-parameter functions (reciprocal power, exponential) increasingly preferred to the two-parameter ones (linear, reciprocal, reciprocal squared) due to their better accuracy with respect to empirical cost data. The search for numerical solutions using advanced methods has created space for more complex functions (Michael-Siddall,

combined, polynomial); their applications have been limited, however, probably due to the larger amount of cost data needed for parameter estimation.
Even more than the choice of the function type, the selection of parameters is critical for a correct use of cost-tolerance functions. In literature, this problem is usually considered as an afterthought compared to methodological aspects relating to allocation; this carries the risk of setting the parameters inconsistently for the different dimensions of a tolerance chain. To avoid consequent inconsistencies in tolerance allocations, the parameters should be calculated by carefully considering all design variables having an influence of manufacturing and quality costs.
An obstacle to the development of procedures for the calculation of parameters is the limited availability of data relating costs and tolerances. Published data derive from different assumptions, and may not be updated to the current level of evolution of machining processes. More data should be collected by different possible sources: they include experimental tests, structured judgements of engineers, and cost estimation models at various levels of detail and accuracy.
As an example of how such a calculation procedure could be designed, the paper has proposed a novel formulation of the reciprocal power cost-tolerance function for combined processes. The parameters of the function are estimated from currently available cost data, cost estimation models, and basic concepts of tolerance standards. One of the parameters, which is mostly responsible for the scaling of costs, is expressed as an empirical function of design variables: they include the nominal dimension, the surface area and the geometric type of the feature. The procedure can be used for all the dimensions of a tolerance chain to get comparable cost-tolerance functions in a common scale. As verified on an example, it helps obtain allocation results in accordance to sound design criteria without apparent distortions in the influence of the variables. The proposed approach may have a potential if applied to specific production contexts, where accurate and consistent cost data could be easily collected and maintained.
Among the future developments of the proposed approach is the determination of cost parameters for the allocation of geometric tolerances. The problem could be treated as an extension of the one addressed here when dealing with one-dimensional tolerance chains, i.e. with all dimensions and geometric characteristics along the same direction. Under such assumption, a conversion to equivalent dimensional tolerances is customary done to solve stackup analysis problems (see e.g. [154]). Therefore, the dependence on the nominal dimension expressed in the proposed equation would be easily extended to the tolerance types associated to basic or toleranced dimensions (position, profile), while it should be substantially revised for other types (orientation). Two- or three-dimensional tolerance chains may require different approaches, as the more complex procedures needed for stackup analysis would make the choice of the cost-tolerance function insensitive to such properties as simplicity and analytical tractability.
Cost-tolerance functions are also used in allocation problems with given manufacturing processes, as well as in advanced formulations with concurrent selection of manufacturing routes from alternative process sequences. In this different and more complex context, it will be equally interesting to analyze the requirements for function parameters, identifying possible margins for improvement.

Compliance with ethical standards

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References

1. Singh PK, Jain PK, Jain SC (2009) Important issues in tolerance design of mechanical assemblies, Part 2: tolerance synthesis. Proc IMechE Part B J Eng Manuf 223:1249-1287
2. Karmakar S, Maiti J (2012) A review on dimensional tolerance synthesis: paradigm shift from product to process. Assem Autom 32:373-388

3. Chase KW, Greenwood WH (1988) Design issues in mechanical tolerance analysis. Manuf Rev 1:50-59
4. Wu Z, ElMaraghy WH, ElMaraghy HA (1998) Evaluation of cost-tolerance algorithms for design tolerance analysis and synthesis. Manuf Rev 1:168-179
5. Chase KW, Greenwood WH, Loosli BG, Hauglund LF (1990) Least cost tolerance allocation for mechanical assemblies with automated process selection. Manuf Rev 3:49-59
6. Chase KW (1999) Minimum-cost tolerance allocation. In Drake PJ (ed) Dimensioning and tolerancing handbook. McGraw-Hill, New York
7. Chase KW, Parkinson AR (1991) A survey of research in the application of tolerance analysis to the design of mechanical assemblies. Res Eng Des 3:23-37
8. Ashiagbor A, Liu HC, Nnaji BO (1998) Tolerance control and propagation for the product assembly modeler. Int J Prod Res 36:75-93
9. Jefferson TR, Scott CH (2001) Quality tolerancing and conjugate duality. Ann Oper Res 105:185-200
10. Ghie W (2009) Functional requirement cost for product using Jacobian-Torsor model. Int CIRP Conf Comput Aid Tolerancing, Annecy, France
11. Sahani AK, Jain PK, Sharma SC, Bajpai JK (2014) Design verification through tolerance stack up analysis of mechanical assembly and least cost tolerance allocation. Procedia Mater Sci 6:284-295
12. Latta LW (1963) Least-cost tolerancing. Prod Eng 16:111-113
13. Edel DH, Auer TB (1964) Determine the least cost combination for tolerance accumulation in a drive shaft seal assembly. General Motors Eng J, Fourth quarter, pp 37-38
14. Bjørke Ø (1989) Computer-aided tolerancing. ASME Press, New York
15. Govindaluri MS, Shin S, Cho BR (2004) Tolerance optimization using the Lambert W function: an empirical approach. Int J Prod Res 42:3235-3251
16. Shin S, Govindaluri MS, Cho BR (2005) Integrating the Lambert W function to a tolerance optimization problem. Qual Reliab Eng Int 21:795-808
17. Bandler JW (1974) Optimization of design tolerances using nonlinear programming. J Optim Theory Appl 14:99-114
18. Dupinet E, Balazinski M, Czogala E (1996) Tolerance allocation based on fuzzy logic and simulated annealing. J Intell Manuf 7:487-497
19. Srikanth K, Liou FW, Balakrishnan SN (2001) Integrated approach for assembly tolerance analysis. Int J Prod Res 39:1517-1535
20. Ji S, Li X, Ma Y, Cai H (2000) Optimal tolerance allocation based on fuzzy comprehensive evaluation and genetic algorithm. Int J Adv Manuf Technol 16:461-468
21. Ji S., Li X., Du R., "Tolerance synthesis using second-order fuzzy comprehensive evaluation and genetic algorithm", Int. J., Production Research, 38 (2000), 3471-3483
22. Choi HGR, Park MH, Salisbury E (2000) Optimal tolerance allocation with loss functions. Trans ASME J Manuf Sci Eng 122:529-535
23. Yu KT, Shen CY (2009) Combining tolerance design and monitoring process capability in a design-manufacturing integration procedure. Proc IMechE Part B J Eng Manuf 223:1389-1394
24. Robles N, Roy U (2004) Optimal tolerance allocation and process-sequence selection incorporating manufacturing capacities and quality issues. J Manuf Sys 23:127-133
25. Roy U, Fang YC (1997) Optimal tolerance re-allocation for the generative process sequence. IIE Trans 29:37-44
26. Zheng C, Jin S, Lai X, Yu K (2011) Assembly tolerance allocation using a coalitional game method. Eng Optim 43:763-778
27. Chen Y, Ding Y, Jin J, Ceglarek D (2006) Integration of process-oriented tolerancing and maintenance planning in design of multistation manufacturing processes. IEEE Trans Autom Sci Eng 3:440-453
28. Geetha K, Ravindran D, Siva Kumar MS, Islam MN (2015) Concurrent tolerance allocation and scheduling for complex assemblies. Robot Comput Integr Manuf 35:84-95
29. Lee JH, Gweon DG, Jin DK, Youn HK, Jeong J (2006) Tolerance allocation and auto alignment of focusing unit for near field recording system. Int J Adv Manuf Technol 29:1041-1049
30. Arenbeck H, Missoum S, Basudhar A, Nikravesh P (2010) Reliability-based optimal design and tolerancing for multibody systems using explicit design space decomposition. Trans ASME J Mech Des 132:021010
31. Dong Z, Hu W, Xue D (1994) New production cost-tolerance models for tolerance synthesis. Trans ASME J Eng Ind 116:199-206
32. Spotts MF (1973) Allocation of tolerances to minimize cost of assembly. Trans ASME J Eng Ind 95:762-764

33. Kopardekar P, Anand S (1995) Tolerance allocation using neural networks. Int J Adv Manuf Technol 10:269-276
34. Yang CC, Naikan VNA (2003) Optimum tolerance design for complex assemblies using hierarchical interval constraint networks. Comput Ind Eng 45:511-543
35. Yang CC, Naikan VNA (2003) Optimum design of component tolerances of assemblies using constraint networks. Int J Prod Econ 84:149-163
36. Jordaan JP, Ungerer CP (2002) Optimization of design tolerances through response surface approximations. Trans ASME J Manuf Sci and Eng 124:762-767
37. Cheng BW, Maghsoodloo S (1995) Optimization of mechanical assembly tolerances by incorporating Taguchi's quality loss function. J Manuf Sys 14:264-276
38. Chou CY, Chang CL (2001) Minimum-loss assembly tolerance allocation by considering product degradation and time value of money. Int J Adv Manuf Technol 17:139-146
39. Hsieh KL (2006) The study of cost-tolerance model by incorporating process capability index into product lifecycle cost. Int J Adv Manuf Technol 28:638-642
40. Wei CC (1997) Allocating tolerances to minimize cost of nonconforming assembly. Assem Autom 17:303-306
41. Hsueh CC, Lin PD, Sasian J (2010) Worst-case-based methodology for tolerance analysis and tolerance allocation of optical systems. Appl Opt 49:6179-6188
42. Tseng YJ, Huang FY (2009) A multi-plant tolerance allocation model for products manufactured in a multi-plant collaborative manufacturing environment. Int J Prod Res 47:733-749
43. Chen MS (1996) Optimising tolerance allocation for mechanical components correlated by selective assembly. Int J Adv Manuf Technol 12:349-355
44. Sutherland GH, Roth B (1975) Mechanism design: accounting for manufacturing tolerances and costs in function generating problems. Trans ASME J Eng Ind 97:283-286
45. Chase KW (1999) Tolerance allocation methods for designers. ADCATS Rep 99-6
46. Kumar MS, Kannan SM, Jayabalan V (2007) Construction of closed-form equations and graphical representation for optimal tolerance allocation. Int J Prod Res 45:1449-1468
47. Li CC, Kao C, Chen SP (1998) Robust tolerance allocation using stochastic programming. Eng Optim 30:333-350
48. Kao C, Li CC, Chen SP (2000) Tolerance allocation via simulation embedded sequential quadratic programming. Int J Prod Res 38:4345-4355
49. Dimitrellou SC, Diplaris SC, Sfantsikopoulos MM (2007) Cost-competent tolerancing in CAD. Int J Adv Manuf Technol 35:519-526
50. Dimitrellou SC, Diplaris SC, Sfantsikopoulos MM (2008) Tolerance elements: an alternative approach for cost optimum tolerance transfer. J Eng Des 19:173-184
51. Dimitrellou SC, Kaisarlis JG, Diplaris SC, Sfantsikopoulos MM (2009) A cost optimal tolerance synthesis methodology for design and manufacturing integration. Int CIRP Conf Comput Aid Tolerancing, Annecy, France
52. Iannuzzi MP, Sandgren E (1995) Tolerance optimization using genetic algorithms: benchmarking with manual analysis. Int CIRP Sem Comput Aid Tolerancing, Tokyo, pp 219-234
53. Prabhaharan G, Asokan P, Ramesh P, Rajendran S (2004) Genetic-algorithm-based optimal tolerance allocation using a least-cost model. Int J Adv Manuf Technol 24:647-660
54. Sivakumar K, Balamurugan C, Ramabalan S (2010) Evolutionary sensitivity-based conceptual design and tolerance allocation for mechanical assemblies. Int J Adv Manuf Technol 48:307-324
55. Zahara E, Kao YT (2009) A hybridized approach to optimal tolerance synthesis of clutch assembly. Int J Adv Manuf Technol 40:1118-1124
56. Forouraghi B (2009) Optimal tolerance allocation using a multiobjective particle swarm optimizer. Int J Adv Manuf Technol 44:710-724
57. Walter MSJ, Spruegel TC, Wartzack S (2015) Least cost tolerance allocation for systems with time-variant deviations. Procedia CIRP 27:1-9
58. Coelho LS (2009) Self-organizing migration algorithm applied to machining allocation of clutch assembly. Math Comput Simul 80:427-435
59. Kumar DV, Ravindran D, Lenin N, Kumar MS (2019) Tolerance allocation of complex assembly with nominal dimension selection using artificial bee colony algorithm. Proc IMechE Part C J Mech Eng Sci 233:18-38
60. Lin CY, Huang WH, Jeng MC, Doong JL (1997) Study of an assembly tolerance allocation model based on Monte Carlo Simulation. J Mater Process Technol 70:9-16
61. Barraja M, Vallance RR (2005) Tolerancing kinematic coupling. Precis Eng 29:101-112

62. Huang YM, Shiau CS (2006) Optimal tolerance allocation for a sliding vane compressor. Trans ASME J Mech Des 128:98-107
63. Ledoux Y, Teissandier D, Sebastian P (2016) Global optimisation of functional requirements and tolerance allocations based on designer preference modelling. J Eng Des 27:591-612
64. Kumaravel P, Anand S, Ullas U, Mohanram PV (2007) Cost optimization of process tolerance allocation: a tree based approach. Int J Adv Manuf Technol 34:703-713
65. Di Stefano P (2003) Tolerance analysis and synthesis using the mean shift model. Proc IMechE Part C J Mech Eng Sci 217:149-159
66. Gao Y, Huang M (2003) Optimal process tolerance balancing based on process capabilities. Int J Adv Manuf Technol 21:501-507
67. Kumar MS, Kannan SM (2007) Optimum manufacturing tolerance to selective assembly technique for different assembly specifications by using genetic algorithm. Int J Adv Manuf Technol 32:591-598
68. Kumar MS, Kannan SM, Jayabalan V (2007) A new algorithm for minimizing surplus parts in selective assembly by using genetic algorithm. Int J Prod Res 45:4793-4822
69. Heling B, Aschenbrenner A, Walter MSJ, Wartzack S (2016) On connected tolerances in statistical cost-optimization of assemblies with interrelated dimension chains. Procedia CIRP 43:262-272
70. Subramanyan N, Ragu K, Yadav DR, Mohanram PV (2008) Comparative study of tolerance on cost for various metal-removal processes. Int J Prod Res 46:1769-1786
71. Wang G, Yang Y, Wang W, Si-Chao LV (2016) Variable coefficients reciprocal squared model based on multi-constraints of aircraft assembly tolerance allocation. Int J Adv Manuf Technol 82:227-234
72. Rao SS, Wu W (2005) Optimum tolerance allocation in mechanical assemblies using an interval method. Eng Optim 37:237-257
73. Lee WJ, Woo TC (1990) Tolerances: their analysis and synthesis. Trans ASME J Eng Ind 112:113-121
74. Speckhart FH (1972) Calculation of tolerance based on a minimum cost approach. Trans ASME J Eng Ind 94:447-453
75. Patel AM (1980) Computer-aided assignment of manufacturing tolerances. ACM Des Autom Conf, Minneapolis MN, pp 129-133
76. Cheng KM, Tsai JC (2013) Optimal statistical tolerance allocation for reciprocal exponential cost-tolerance function. Proc IMechE Part B J Eng Manuf 227:650-656
77. Chen TC, Fischer GW (2000) A GA-based search method for the tolerance allocation problem. Artif Intell Eng 14:133-141
78. Ming XG, Mak KL (2001) Intelligent approaches to tolerance allocation and manufacturing operations selection in process planning. J Mater Process Technol 117:75-83
79. Singh PK, Jain SC, Jain PK (2004) A genetic algorithm based solution to optimum tolerance synthesis of mechanical assemblies with alternate manufacturing processes: benchmarking with the exhaustive search method using the Lagrange multiplier. Proc IMechE Part B J Eng Manuf 218:765-778
80. Singh PK, Jain PK, Jain SC (2004) A genetic algorithm based solution to optimal tolerance synthesis of mechanical assemblies with alternative manufacturing processes: focus on complex tolerancing problems. Int J Prod Res 42:5185-5215
81. Singh PK, Jain SC, Jain PK (2005) Advanced optimal tolerance design of mechanical assemblies with interrelated dimension chains and process precision limits. Comput Ind 56:179-194
82. Singh PK, Jain SC, Jain PK (2005) Comparative study of genetic algorithm and simulated annealing for optimal tolerance design formulated with discrete and continuous variables. Proc IMechE Part B J Eng Manuf 219:735-760
83. Singh PK, Jain PK, Jain SC (2008) Optimal tolerance design of mechanical assemblies for economical manufacturing in the presence of alternative machines: a genetic algorithm-based hybrid methodology. Proc IMechE Part B J Eng Manuf 222:591-604
84. Sivakumar K, Balamurugan C, Ramabalan S (2011) Concurrent multi-objective tolerance allocation of mechanical assemblies considering alternative manufacturing process selection. Int J Adv Manuf Technol 53:711-732
85. Zeng W, Rao Y, Wang P (2017) An effective strategy for improving the precision and computational efficiency of statistical tolerance optimization. Int J Adv Manuf Technol 92:1933-1944
86. Kumar LR, Padmanaban KP, Kumar SG, Balamurugan C (2016) Design and optimization of concurrent tolerance in mechanical assemblies using bat algorithm. J Mech Sci Technol 30:2601-2614

87. Krishna AG, Rao KM (2006) Simultaneous optimal selection of design and manufcturing tolerances with different stack-up conditions using scatter search. Int J Adv Manuf Technol 30:328-333
88. Prabhaharan G, Ramesh P, Asokan P (2007) Concurrent optimization of assembly tolerances for quality with position control using scatter search approach. Int J Prod Res 45:4959-4988
89. Kumar RS, Alagumurthi N, Ramesh R (2009) Calculation of total cost, tolerance based on Taguchi's asymmetric quality loss function approach. Amer J Eng Appl Sci 2:628-634
90. Zhang C, Wang HP, Li JK (1992) Simultaneous optimization of design and manufacturing tolerances with process (machine) selection. Ann CIRP 41(1):569-572
91. Cheng KM, Tsai JC (2012) Optimal statistical tolerance allocation based on the Lambert W function. Int CIRP Conf Comput Aided Tolerancing, Huddersfield UK, CAT-74
92. Kumar LR, Padmanaban KP, Balamurugan C (2016) Least cost-tolerance allocation based on Lagrange multiplier. Concurr Eng Res Appl 24:164-177
93. Wang ML, Liu XT, Wang YS, Wang XL, Gua H, Xing YF (2016) Research on assembly tolerance allocation and quality control based on fuzzy reliability. Proc IMechE Part C J Mech Eng Sci 230:3755-3766
94. Peng HP, Jiang XQ, Liu XJ (2008) Concurrent optimal allocation of design and process tolerances for mechanical assemblies with interrelated dimensions chains. Int J Prod Res 46:6963-6979
95. Peng HP, Jiang XQ, Xu ZG, Liu XJ (2008) Optimal tolerance design for products with correlated characteristics by considering the present worth of quality loss. Int J Adv Manuf Technol 39:1-8
96. Kumar MS, Kannan SM, Jayabalan V (2009) A new algorithm for optimum tolerance allocation of complex assemblies with alternative process selection. Int J Adv Manuf Technol 40:819-836
97. Wei CC (1997) Designing process tolerances to minimize loss of nonconforming parts. IEEE Trans Compon Packag Manuf Technol 20:312-316
98. Liu PH, Wei CC (2000) Concurrent optimization of process tolerances and manufacturing loss. Int J Comput Integr Manuf 13:517-521
99. Hu J, Xiong G (2005) Dimensional and geometric tolerance design based on constraints. Int J Adv Manuf Technol 26:1099-1108
100. Ghali M, Tlija M, Aifaoui N, Pairel E (2017) A CAD method for tolerance allocation considering manufacturing difficulty based on FMECA tool. Int J Adv Manuf Technol 91:2435-2446
101. Ghali M, Tlija M, Aifaoui N (2018) Optimal tolerance allocation based on difficulty matrix using FMECA tool. Procedia CIRP 70:132-137
102. Liu SG, Jin Q, Liu C, Xie RJ (2013) Analytical method for optimal component tolerances based on manufacturing cost and quality loss. Proc IMechE Part B J Eng Manuf 227:1484-1491
103. Liu S, Jin Q, Dong Y, Wang Y (2017) A closed-form method for statistical tolerance allocation considering quality loss and different kinds of manufacturing cost functions. Int J Adv Manuf Technol 93:2801-2811
104. Zong Y, Mao J (2015) Tolerance optimization design based on the manufacturing costs of assembly quality. Procedia CIRP 27:324-329
105. Hung TC, Chuan KY (2013) Multi-objective design and tolerance allocation for single- and multi-level systems. J Intell Manuf 24:559-573
106. Jeang A, Chung CP, Hsieh CK (2007) Simultaneous process mean and process tolerance determination with asymmetrical loss function. Int J Adv Manuf Technol 31:694-704
107. Jeang A, Chen T, Li HC, Liang F (2007) Simultaneous process mean and process tolerance determination with adjustment and compensation for precision manufacturing process. Int J Adv Manuf Technol 33:1159-1172
108. Jeang A, Chung CP (2008) Process mean, process tolerance, and use time determination for product life application under deteriorating process. Int J Adv Manuf Technol 36:97-113
109. Li Z, Kokkolaras M, Papalambros P, Hu SJ (2008) Product and process tolerance allocation in multistation compliant assembly using analytical target cascading. Trans ASME J Mech Des 130:091701
110. Altarazi SA (2011) An optimization tool for operational tolerances allocation, work in process inventory minimization, and machine assignment in a discrete part manufacturing environment. Int J Adv Manuf Technol 55:1069-1078
111. Michael W, Siddall JN (1981) The optimal tolerance assignment with less than full acceptance. Trans ASME J Mech Des 103:855-860
112. Choudhary AK, Singh KA, Tiwari MK (2006) A statistical tolerancing approach for design of synchronized supply chains. Robot Comput Integr Manuf 22:315-321

113. Andolfatto L, Thiébaut F, Lartigue C., Douilly M. (2014) Quality- and cost-driven assembly technique selection and geometrical tolerance allocation for mechanical structure assembly. J Manuf Sys 33:103-115
114. Khodaygan S (2019) Meta-model based multi-objective optimisation method for computer-aided tolerance design of compliant assemblies. Int J Comput Integr Manuf 32:27-42
115. Nagarwala MY, Pulat PS, Raman S (1995) A slope-based method for least cost tolerance allocation. Concurr Eng Res Appl 3:319-328
116. Hu J, Xiong G (2005) Concurrent design of a geometric parameter and tolerance for assembly and cost. Int J Prod Res 43:267-293
117. Hu J, Peng Y (2007) Tolerance modelling and robust deign for concurrent engineering. Proc IMechE Part C J Mech Eng Sci 221:455-465
118. Yeo SH, Ngoi BKA, Poh LS, Hang C (1997) Cost-tolerance relationships for non-traditional machining processes. Int J Adv Manuf Technol 13:35-41
119. Moy WA (1964) Assignment of tolerances by dynamic programming. Prod Eng 21:215-218
120. Ostwald PF, Huang J (1977) A method for optimal tolerance selection. Trans ASME J Eng Ind 99:558-565
121. Feng CX, Kusiak A (1997) Robust tolerance design with the integer programming approach. Trans ASME J Manuf Sci Eng 119:603-610
122. Lee WJ, Woo TC (1989) Optimum selection of discrete tolerances. Trans ASME J Mech Transm Autom Des 111:243–251
123. Kanai S, Onozuka M. Takahashi H (1995) Optimal tolerance synthesis by genetic algorithm under the machining and assembling constraints. Int CIRP Sem Comput Aid Tolerancing, Tokyo, pp 235-250
124. Nassef AO, ElMaraghy HA (1997) Allocation of geometric tolerances: new criterion and methodology. Ann CIRP 46(1):101-106
125. Lööf J, Söderberg R (2012) Discrete tolerance allocation for product families. Eng Optim 44:75-85
126. Sanz-Lobera A, Gomez E, Pérez J, Sevilla L (2016) A proposal of cost-tolerance models directly collected from the manufacturing process. Int J Prod Res 54:4584-4598
127. Conway HG (1966) Engineering tolerances. Pitman, London
128. Fortini ET (1967) Dimensioning for interchangeable manufacturing. Industrial Press, New York
129. Bralla JG (1999) Design for manufacturability handbook. McGraw-Hill, Boston
130. Creese RC, Adithan M, Pabla BS (1992) Estimating and costing for the metal manufacturing industries. Marcel Dekker, New York
131. Boothroyd G, Dewhurst P, Knight WA (2011) Product design for manufacture and assembly. CRC Press, Boca Raton
132. Swift KG, Booker JD (2013) Manufacturing process selection handbook. Elsevier, Amsterdam
133. Trucks HE (1987) Design for economical production. Society of Manufacturing Engineers, Dearborn MI
134. Peat AP (1968) Cost reduction charts for designers and production engineers. The Machinery Publishing Co, London
135. Dieter GE (1983) Engineering design: a materials and processing approach. McGraw-Hill, New York
136. Jamieson (1982) Introduction to quality control. Reston, Paramus NJ
137. ISO 286-1 (2010) Geometrical product specifications (GPS), ISO code system for tolerances on linear sizes, Part 1: Basis of tolerances, deviations and fits, International Organization for Standardization, Geneva, Switzerland
138. Yeo SH, Ngoi BKA, Chen H (1996) A cost-tolerance model for process sequence optimisation. Int J Adv Manuf Technol 12:423-431
139. Yeo SH, Ngoi BKA, Chen H (1998) Process sequence optimization based on a new cost-tolerance model. J Intell Manuf 9:29-37
140. Lin ZC, Chang DY (2002) Cost-tolerance analysis model based on a neural networks method. Int J Prod Res 40:1429-1452
141. Cao Y, Zhang H, Mao J, Yang J (2010) Novel cost-tolerance model based on fuzzy neural networks. Proc IMechE Part B J Eng Manuf 224:1757-1765
142. Wang Y, Zhai W, Yang L, Wu W, Ji S, Ma Y (2007) Study on the tolerance allocation optimization by fuzzy-set weight-center evaluation method. Int J Adv Manuf Technol 33:317-322
143. Dong Z, Wang GG (1998) Automated cost modeling for tolerance synthesis using manufacturing process data, knowledge reasoning and optimization. In ElMaraghy HA (ed) Geometric design tolerancing: theories, standards and applications. Chapman & Hall, London, pp 282-293

144. Sanz-Lobera A, Sebastian MA, Pérez JM (2010) New cost-tolerance model for mechanical part design. Int J Adv Manuf Technol 51:421-430
145. Moroni G, Petrò S, Tolio T (2011) Early cost estimation for tolerance verification. CIRP Ann Manuf Technol 60:195-198
146. Etienne A, Dantan JY, Siadat A, Martin P (2007) Cost estimation for tolerance allocation. Int CIRP Conf Comput Aid Tolerancing, Erlangen, Germany
147. Etienne A, Dantan JY, Siadat A, Martin P (2009) Activity-based tolerance allocation (ABTA): driving tolerance synthesis by evaluating its global cost. Int J Prod Res 47:4971-4989
148. Miramadi S, Etienne A, Hassan A, Dantan JY, Siadat A (2012) Cost estimation method for variation management. Int CIRP Conf Comput Aid Tolerancing, Huddersfield, UK
149. Gerth RJ, Klonaris P, Pfeiffer T (1999) Cost tolerance sensitivity analysis for concurrent engineering design support. Int CIRP Conf Comput Aid Tolerancing, Twente, Netherlands, pp 313-323
150. Gerth RJ, Pfeiffer T (2000) Minimum cost tolerancing under uncertain cost estimates. IIE Trans 32:493-503
151. Gerth RJ, Pfeiffer T, Oehme O (2000) Early cost tolerance sensitivity analysis for inspection planning. Hum Factor Ergon Manuf 10:309-329
152. Creveling CM (1997) Tolerance design: a handbook for developing optimal specifications. Addison-Wesley, Reading MA
153. Evans DH (1975) Statistical tolerancing: the state of the art, Part III: shifts and drifts. J Qual Technol 7(2):72-76
154. Fischer BR (2004) Mechanical tolerance stackup and analysis. Marcel Dekker, New York NY