State-of-the-art and challenges of railway and road vehicle dynamics with multibody dynamics approaches

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Abstract A review of the current use of multibody dynamics methods in the analysis of the dynamics of vehicles is given. Railway vehicle dynamics as well as road vehicle dynamics are considered, where for the latter the dynamics of cars and trucks and the dynamics of single-track vehicles, in particular motorcycles and bicycles, are reviewed. Commonalities and differences are shown, and open questions and challenges are given as directions for further research in this field.

Keywords Vehicle dynamics · Review · Railway vehicles · Road vehicles · Motorcycles · Bicycles

1 Introduction

This article gives an overview of the use of multibody system dynamics to vehicle dynamics. The special issue of Vehicle System Dynamics from 1993 [115] gives a survey and some benchmark results for the multibody system dynamics programs then in use, many of which have been further developed and are still current. The field of vehicle system dynamics as a whole was among the most transformed by the developments in multibody system dynamics. Since then new developments have been advanced and a new review of the state-

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of-the-art seems to be timely. Two recently updated books addressing the field of vehicle dynamics including aspects of multibody dynamics are [167, 232]. Here, not so much the formalisms and the software take the centre stage, but more the many different ways in which vehicle systems are modelled and analysed by methods from the field of multibody system dynamics. As there are relatively few reviews that cover the dynamics of all sorts of rail and road vehicles, this is an opportunity to address what they have in common and what are their differences. They have in common that their dynamics are strongly influenced by the contact of the wheels with the guiding and supporting surface. Furthermore, vehicles are more and more mechatronic systems, with the influence of control systems on their dynamics. On the other hand, single-track vehicles are strongly affected by the rider, both as a controller and as a mechanical part of the system, more so than cars, whereas the driver of a rail vehicle has a much smaller influence on the dynamics.

The subject is divided in railway vehicle dynamics and road vehicle dynamics. The latter is further subdivided into the dynamics of cars and trucks and the dynamics of single-track vehicles, which is split into the dynamics of motorcycles and the dynamics of bicycles.

2 Rail vehicle dynamics

In the same way as for other ground vehicles, the dynamics of rail vehicles can be categorized in:

– *vertical dynamics*, representing the vehicle's response to vertical gradients of the line, longitudinal level track irregularities and rail corrugation;
– *lateral dynamics*, involving the lateral movements of the vehicle involved with guidance, i.e. the ability of the vehicle to follow a straight or curved track, also considering the effect of track irregularities;
– *longitudinal dynamics*, describing traction/braking transients of the vehicles during their forward movement.

The study of vertical dynamics for rail vehicles is based on approaches similar to those in use for other ground vehicles, although rail vehicles for passenger service normally have two stages of suspension (primary and secondary) whilst other ground vehicles only have one. On the other hand, lateral and longitudinal dynamics of rail vehicles have distinctive features that make their study much different than for other ground vehicles. For lateral dynamics, the differences are mainly related to the peculiar geometry of wheels and rails, wheel conicity and the way in which rail vehicles establish guidance. These factors not only determine the way in which the railway vehicle negotiates a curve [73, 230, 231], but are also responsible for an instability phenomenon specific to rail vehicles known as hunting [110, 111, 230]. For longitudinal dynamics, the specificity of rail vehicles compared to road ones is represented by the fact that all vehicles in the same train set interact in longitudinal direction via draw gears and buffers. Therefore, the transmission of longitudinal forces during acceleration and braking manoeuvres may lead to complex interactions between vehicles in the same train set which may affect running safety and need to be accurately studied [50].

For the sake of brevity, in this paper we will mostly focus on the lateral dynamics of rail vehicles as this is the topic which has been the object of more extensive research effort. After a short outline of the historical development of theories and models, we will provide an overview of the approaches available to model wheel–rail contact forces, which are pivotal to the investigation of rail vehicle dynamics. Then, we will propose a categorization of problems and models in rail vehicle dynamics based on the frequency range of interest, and finally we will summarize some remaining open issues and trends in future research.

2.1 Historical perspective

The very first multibody model of rail vehicles was developed by F.W. Carter in 1916 to investigate the phenomenon of hunting. Carter's model from his 1926 publication only considered the lateral and yaw movements of a single bogie frame with two or more wheelsets rigidly constrained [230]. By the way, Carter was also the first researcher introducing the notion of creepage in wheel–rail contact and its relation to frictional forces. It was Matsudaira in 1952 who first introduced the flexibility of the primary suspensions: his work was one of the winners of the competition set up in 1954 by the Office for Research and Experiments (ORE) of the International Union of Railways for the best analysis of the hunting problem [110]. Matsudaira's model was then further refined by A.H. Wickens who considered in addition lateral suspension damping [86]. In 1969 Boocock introduced a model considering an entire vehicle with two bogies and flexible primary suspensions. Two coordinates, lateral displacement and yaw rotation, were considered for each one of the 7 bodies in the model, hence resulting in a total of 14 degrees of freedom [24]. Boocock's model considered a linear wheel–rail contact model and was substantially extended by Elkins and Gostling who in 1977 considered the effect of flange contact and the saturation of creep forces, resulting in the first non-linear analysis of quasi-static curving [74]. Among the studies cited above, Wickens was the first who made extensive use of both digital and analogue computation, but the study by Elkins and Gostling can be considered the first using a multibody computer program incorporating a 'modern' wheel–rail contact module.

In the 1970s and 1980s a number of multibody software specifically tailored for rail vehicle dynamics applications were developed, some notable examples being (in alphabetical order) GENSYS, MEDYNA, NUCARS, VAMPIRE, and VOCO. These codes included a refined wheel–rail contact module and often made use of some simplifying assumptions to reduce the computational effort of time domain simulations, e.g. small motion of the bodies with respect to one or more track-following references. A few years later, some general purpose multibody software such as ADAMS and SIMPACK started to integrate a wheel–rail contact module, so that they could be used in the context of rail vehicle dynamics. Most of the above mentioned software are still in use nowadays, although they have gone through extensive improvements and generalization of their scope of use. In 1998 a renowned benchmark exercise was championed by S. Iwnicki to compare the results of some of the software existing at the time [96], whereas in 2008 Shackleton and Iwnicki set up a benchmark of wheel–rail contact codes for railway vehicle simulation [202].

2.2 Wheel–rail contact models in a multibody dynamics context

Modelling wheel–rail contact forces to a sufficient degree of accuracy is certainly one major challenge with the multibody simulation of rail vehicle dynamics. Wheel–rail contact can be handled using unilateral constraints or via elastic force models in which the motion of the wheel relative to the rail is enforced by a stiff elastic contact between the two bodies [201]. Whatever approach is adopted, a wheel–rail module shall deal with three main tasks:

– locating the contact points on the wheel and rail surfaces (contact point search);
– defining the normal component of the contact force, either as a constraint force or as produced by the local deformation of the bodies;
– defining the tangential components of the contact force due to friction, known as *creep forces*.
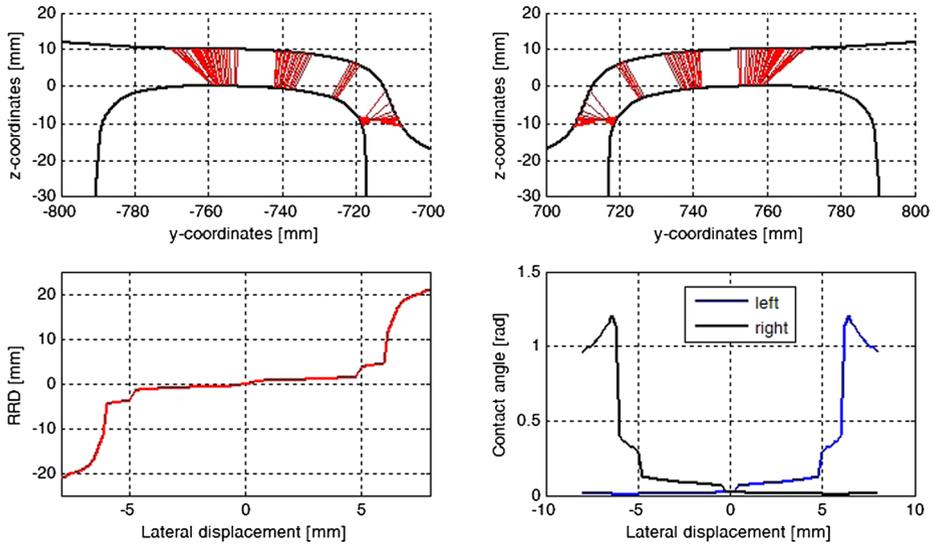
Fig. 1 (Top) Position of the wheel–rail contact points for different lateral shift of the wheelset relative to the rails; (bottom) rolling radius difference (RRD) and contact angle diagrams. The results shown are for ORE S1002 wheel profiles and UIC 60 rails with gauge 1435 mm and 1:40 inclination

The simplest strategy that can be used to perform the contact point search consists of considering the planar problem of the contact between two lines representing the transversal profiles of the wheels and rails, respectively. In this approach, the effects related to the yaw angle of the wheelset are neglected, which is an acceptable assumption as far as the vehicle travels in tangent track or in curves with relatively large radius (e.g. above 300 m). If rigid displacements of the profiles are assumed, only one contact point is found between each wheel and the corresponding rail. Under this assumption, jumps of the contact points may occur, which means that at some positions of the wheelset relative to the rails large changes in the location of the contact point occur for very small changes in the relative position of the wheelset to the track. As a consequence, jumps are also observed in the contact parameters used to compute wheel–rail forces, such as the rolling radius difference (RRD) and the contact angles, as shown in Fig. 1. To avoid these discontinuities, the effect of local deformability of the two profiles in the region close to the contact point is often introduced, resulting in a smoother transition of the contact point position and in the formation of multiple contacts between one wheel and the contacting rail [195]. Strategies are also available for solving the contact point search in 3-D, hence considering out-of-plane effects. A simple approach is represented by the 'apparent profile' method [28, 108] which assumes perfectly straight rails, whereas more general but possibly more computationally intensive methods are proposed in [128, 164, 165, 216].

Once the contact points are found, the magnitude and direction of the contact forces can be evaluated considering the local geometry of the profiles. This problem is usually dealt with in two stages, called the *normal problem* and the *tangential problem* [14, 104]. Solving the normal problem consists in evaluating the normal component of the contact force and the size and shape of the contact patch as a function of a kinematic approach of the wheel and rail surfaces in the normal direction. In some cases, the distribution of the normal pressure is also evaluated. The solution of the tangential problem consists in evaluating the components of the wheel–rail contact force in the tangent plane (frictional

forces). If the wheel and rail bodies can be assimilated to elastic semi-half spaces (and are made of the same material), the contact problem is said to be *quasi-identical* which means the normal and tangential problems are decoupled. In this case, the normal problem can be solved first and the solution will not depend on the frictional forces or stresses. Then, the tangential problem can be solved using as inputs the quantities obtained from the solution of the normal problem.

A frequently used approach for solving the normal problem is Hertz theory [14]: in this case the contacting surfaces are assumed to have constant curvature, an assumption that often represents an idealization of the actual contact condition. For this reason, more sophisticated approaches have been proposed to solve the normal problem under non-Hertzian assumptions. In decreasing order of complexity, the approaches available are: the Finite Element (FE) Method [145, 237], Algorithm CONTACT, originally proposed by Kalker [104] then improved by Vollebregt [224], approximated non-Hertzian methods such as the one due to Piotrowski and Kik [156] with extension by Liu and others [122], and the one proposed by Sichani et al. [211]. In the context of multibody simulation, the FE method is too demanding from the computational point of view to represent a viable approach; CONTACT is also quite computationally intensive although the improvement of its computational efficiency introduced by Vollebregt allow, al least in principle, the use of this method for vehicle dynamics simulations. At present, the most widely used approaches are approximated non-Hertzian methods or, alternatively, the so-called multi-Hertzian methods in which the non-Hertzian contact patch is approximated by multiple elliptic (Hertzian) contacts.

As far as the tangential problem is concerned, accurate solutions can be obtained using either the FE method or CONTACT, but again these are rarely used in dynamic simulations due to their computational cost. One popular approximate method, called FASTSIM [103, 104], relies on a simplified description of tangential flexibility in the contact region, similarly to the brush model in road–tyre contact. Alternatively, approximate methods can be based on a discretization of the contact patch in discrete strips [13, 155, 212]. When a Hertzian or multi-Hertzian solution of the normal problem is used, the tangential problem can be solved according to the linear theory due to Kalker [14, 104] and the saturation of frictional forces is introduced according to 'heuristic formulae' [160, 210] which are often capable of providing an acceptable accuracy and require a low computational effort. Finally, one suitable solution for MBS simulation is represented by the calculation of the creepage forces by interpolation of a book of tables. This approach was originally developed by British Railway Research for elliptic contacts and was recently extended to non-Hertzian contact patches [157, 158]. Recently, a contact model incorporating both a fast non-Hertzian method to solve the normal problem and a non-Hertzian book of tables has been proposed in [126].

2.3 Model of suspension components

A second major challenge with modelling railway vehicles is concerned with the representation of suspension components which are often characterized by a complex non-linear and frequency-dependent behaviour. A review of modelling issues and of typical models in use for suspension components in railway vehicles is provided in [32]. Particularly challenging is the modelling of hydraulic dampers, pneumatic suspensions and friction-based components.

Hydraulic dampers generally show a non-linear behaviour due to asymmetric behaviour in compression and extension and due to the presence of check valves. Their behaviour is

also frequency-dependent due to the flexibility effects in the damper and in the end mountings, so that the viscous damping effect is reduced at high frequency. Correctly modelling the frequency dependent behaviour is especially important for yaw dampers, as it may strongly affect the stability of the vehicle at high speeds. Hydraulic dampers are often modelled using a non-linear force–velocity relationship, defined either by discrete points or as an analytical function, whilst flexibility effects are represented by a serial stiffness, resulting in a model with at least one internal state variable. A discussion of modelling options for yaw dampers and their effect on the investigation of vehicle stability is presented in [3].

Pneumatic suspensions, known as air springs, are widely used in modern railway vehicles as they provide superior comfort compared to coil springs and also because they inherently provide carbody levelling. A complete pneumatic suspension is a complex system involving several components such as bellows, reservoirs, pipes, orifices, differential and levelling valves, emergency rubber springs, and therefore the modelling of the suspension in all its functioning modes can be particularly challenging. Equivalent mechanical models have been proposed such as that by Berg [21], but recently thermodynamic models are being increasingly used. These latter models provide a general and flexible representation of the pneumatic suspension [62, 146]. A discussion of alternative models for air springs and a procedure to define the parameters of the model based on laboratory tests is presented in [132].

Typical friction-based components used in railway vehicles include dry friction dampers, leaf springs and buffers. They are widely used in railway vehicles and especially in freight wagons as they are relatively inexpensive and capable of operating in poor maintenance conditions. Models for friction-based components are often based on a combination of dry friction elements with serial stiffness [215], but in some cases exponential expressions are used to give a better representation of the hysteresis cycle [101]. Present challenges with modelling these components are mainly concerned with modelling dry friction in 2-D and with considering the effect of dither, i.e. the effect on dry friction forces caused by high-frequency vibrations of the body [154].

Another very significant feature involved in the modelling of suspension components is the representation of joints. These can either be represented as simple kinematic joints or as contact joints introducing effects related to compliance and backlashes which may have significant effect on the vehicle's dynamic behaviour. A detailed model for a bushing with non-linear behaviour is presented in [7], while a general procedure to modelling joints with clearances and/or non-linear bushings in the context of multibody models is proposed in [6].

2.4 Problems and models in rail vehicle dynamics

We propose here a categorization of problems and models in rail vehicle dynamics based on the frequency range of interest. Due to the high axle load and to the large value of unsprung masses, when modelling rail vehicle dynamics, it is often required to consider the coupled dynamics of the vehicle and of the track, an effect referred to as *train–track interaction*. Therefore, the subsections below address not only the proper modelling of the vehicle but also the model of the track in different frequency ranges. It should be noted that different numerical integration schemes can be used for the multibody model of the vehicle and for the track model. This requires that a proper co-simulation procedure is implemented to ensure the synchronization of the time-domain solution for the two subsystems. A co-simulation approach for coupling a general multibody model of a rail vehicle and a finite element model of the track is presented in [10]. The reader is referred to [162] for a general overview of numerical simulation problems in railway vehicle dynamics, and to [77, 102, 200] for specific issues related to the modelling of body flexibility effects in railway vehicles.

2.4.1 Low frequency models

Rail vehicle models defined in a low frequency range (up to 20–30 Hz approximately) are generally used to investigate problems such as ride comfort [43, 75], vehicle stability [61, 97, 161] and curving behaviour [30, 97], including the formation of the so-called regular wear on the wheels [29, 98, 166]. These models generally make use of rigid body models for the wheelsets and bogie frames, whereas the consideration of carbody flexibility for ride comfort studies is the state-of-the-art in industry and integrated in the standard development processes. The track can be either considered as perfectly rigid or represented by means of a simplified sectional co-following model, i.e. a simple model based on a lumped parameter representation of the rails and sleepers, which is considered to move in the forward direction with the same speed as the vehicle [60]. In this frequency range, the type of analysis performed is mostly time domain simulation to consider non-linear effects in wheel–rail contact forces, large motion of bodies and non-linear behaviour of suspension components [32].

2.4.2 Intermediate frequency models

Multibody models defined in an intermediate frequency range (from few tens to some hundreds of Hz) are mostly concerned with damage effects and the long term behaviour of the rolling stock and of the track such as railhead corrugation [87, 221], rail squats [116], rolling contact fatigue [9] wheel out-of-roundness [99], metal fatigue and crack propagation in rails and wheelsets [20, 129]. These models consider the wheelsets as flexible bodies, whereas the sprung masses (bogie frames and car body) are usually represented as rigid bodies or not considered thanks to the de-coupling provided by the suspensions. An interesting investigation of the effect of bogie–frame flexibility in the intermediate frequency range is provided in [47]. Track flexibility is represented using detailed FE models, sometimes using modal condensation to reduce the number of degrees of freedom [147]. The equations of motion are often formulated as linear, assuming the motion of bodies to be a small perturbation of a steady-state motion and introducing a linearization of wheel–rail contact forces, although in some cases non-linear contact models are also used. The approaches used to solve the equations of motion are defined both in the time domain, by means of numerical time-step integration, or in the frequency domain using the Frequency Response Functions (FRF) of the vehicle and track evaluated at the wheel–rail contact point(s) [90, 217].

2.4.3 High frequency models

High frequency models for train–track interaction target a frequency range up to some kHz and are mainly concerned with rolling noise. They usually focus on the vibration of the wheelset and rails, represented as flexible bodies in most cases using FE schematizations [218]. One specific problem involved with modelling rail flexibility is represented by the need to consider a model of the rails having finite length. One way to solve this issue consists of defining the model of the rail in a moving reference [41, 81, 130]. Another option is to use non-reflecting boundary conditions for the finite length model of the track [234]. The motion of the sprung masses, i.e. bogies and carbody, is normally not considered under the assumption that the filtering effect of the suspensions decouples the motion of these masses at sufficiently high frequencies. Linear or simplified non-linear models of wheel–rail contact are used to reduce the computational effort. Solutions are often sought in the frequency domain, but in some cases time domain numerical simulation is also considered, allowing to consider non-linear effects such as the loss of contact between the wheel and the rail.

2.5 Open issues and research trends

We conclude this section by presenting a short overview of open issues and research trends in the field of multibody simulation of railway vehicles. In our opinion, these are mainly:

1. Dynamics of vehicles with mechatronic suspensions [31]. A particular challenge in this field is represented by the need to establish multi-physics simulation (often in the form of co-simulation) to consider the effect of actuator dynamics;
2. Use of hybrid simulation in railway vehicle dynamics. Hybrid simulation is an enhanced simulation method exploiting the interaction between a virtual simulation model describing one portion of the considered dynamic system while the rest of the system is represented by the physical hardware. The advantage, compared to standard numerical simulation, is that the modelling uncertainties related with the hardware (in the case of railway vehicles, e.g. non-linear behaviour of suspension components, wheel–rail contact) are eliminated, while retaining to some extent the advantages of numerical simulation with respect to testing. Exemplary attempt to use hybrid simulation in railway vehicle dynamics can be found in [51, 52];
3. Development of real-time models of railway vehicles [214] and of wheel–rail contact [25, 56], which are also pivotal to hybrid simulation;
4. Health monitoring of railway vehicles using multibody models [31, 117];
5. Use of multibody simulation for virtual certification and admission in service of railway vehicles [89, 163];
6. Use of multibody simulation for the calculation of design loads [193].
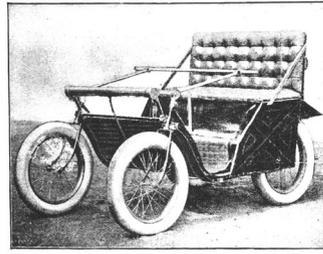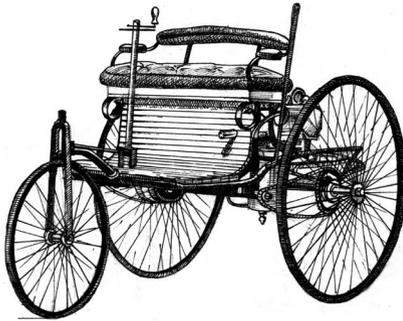
3 Road vehicle dynamics

The CISM course 'Vehicle Dynamics of Modern Passenger Cars' published in [124] provides a basic understanding of the dynamics of passenger cars including modelling aspects, tyre characteristics, optimization strategies, control systems and simulation techniques. However, road vehicle dynamics is not restricted to passenger cars. Agricultural vehicles, commercial vehicles, construction site vehicles and any kind of tracked vehicles can be driven on the road too. As passenger cars imply tyres as 'contact elements' to the road, focus is laid on tyre fitted vehicles, in particular on passenger cars and trucks which represent the most common types of vehicles on the road. The main challenge is to provide appropriate and valid models, usually multibody systems, which make it possible to enhance ride comfort and driving safety of this kind of road vehicles. When advanced driver-assistance systems or automated driving systems are developed, investigated, or enhanced, the roll-over hazard of passenger cars and trucks as well as potential stability problems of tractor trailer combinations must be taken into account, too.

3.1 History, objectives and requirements

Shortly after Karl Benz patented the first automobile in 1886, several companies put motor vehicles on the market, see Fig. 2. The layout of the first automobiles was quite strange. The wheels and the tyres, if any, looked like those used for bicycles, and the steering systems were quite unconventional and often not even designed properly.

Even if the speed of the vehicles is no longer an important issue, the main goal of the automotive industry to develop vehicles that combine (speed), safety, comfort and economy

Fig. 2 Drawing of the first automobile (patented by Karl Benz in 1886) and historic advertisement taken from a reprint of the Horseless Age [48]

has not changed since then. To achieve and constantly improve the (speed) safety and comfort in particular, the dynamics of the vehicles have to be studied very carefully. Starting with field tests and test-rigs, today computer simulations play an important part in the on-going improvement of the dynamics of vehicles. Since 1985, when the first driving simulator was opened, real-time simulations have become an important issue, too [70]. The multibody system approach forms the platform for all kind of vehicle models applied today [23]. However, a road vehicle model consists not only of the standard elements of a multibody system (rigid or flexible bodies, constraints, and massless force elements), but requires additional models for the driver, the load if not rigid or liquid, the combustion engine or an electric motor, respectively, the drive train if present at all, dynamic force elements for hydro-mounts, tyre models, as well as an appropriate road model.

3.2 Modelling aspects

The first 'handling model' or mistakenly called 'bicycle model' was published in 1940 by Riekert and Schunk [171]. It not really represents a multibody system because the whole vehicle was considered as one rigid body moving on a horizontal plane. Although simple and linear, it still serves as a basis for developing and enhancing control strategies. Today it usually is extended to a four wheel handling model [107] and/or supplemented by a rear wheel steering [40]. It even makes it possible to assess the cornering resistance of vehicles if geometric non-linearities are taken into account [175, 178].

Based on previous articles and handbooks, Manfred Mitschke published in 1972 the first book on the dynamics of road vehicles [139]. A hierarchical set of mainly linear multibody system models – from single-mass oscillators over several planar models right up to a three-dimensional model – is used here to analyse, assess, and improve the ride comfort of passenger cars.

Today, all kind of commercial multibody system packages, special-purpose packages, and 'home-made' multibody models are employed in research and industry. Multi-purpose multibody system packages like MSC.ADAMS, SIMPACK, RecurDyn, or DYMOLA provide a comfortable graphical user interface that allows the user to build up his/her own
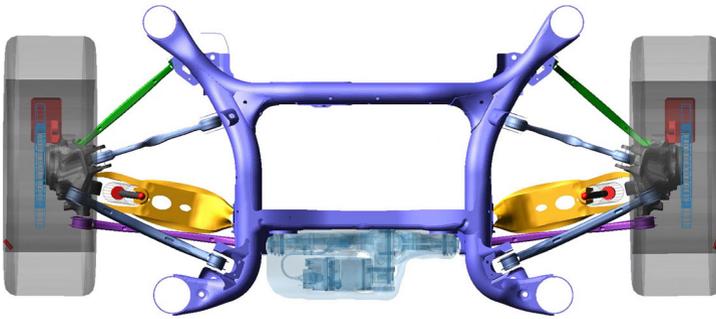

Fig. 3 Complex and detailed multibody model of a rear axle, taken from [190]

vehicle model, test it, perform simulations, and analyse the results. Usually very complex non-linear and three-dimensional vehicle models are generated, see Fig. 3. These models are then applied for handling and comfort analysis and often incorporate flexible bodies too. But complex models require a lot of data that are hardly available outside an automotive company. In addition, the validation of a flexible multibody systems approach requires at least the presence of a prototype [55], which is not available in the early development stage.

Software packages like CarSim, CarMaker, or veDYNA offer ready-to-use vehicle models, which may be used in driving simulators as well as for offline simulations or for software-in-the-loop (SIL) and hardware-in-the-loop (HIL) simulations. In general, fully non-linear and three-dimensional vehicle models are provided. To achieve real-time performance, the wheel/axle kinematics usually is approximated by lookup tables. However, the interface to third-party software tools is limited in general.

That is why appropriate 'home-made' multibody models are still in use. In particular, for real-time applications appropriate model simplifications that finally result in non-perfect multibody systems are possible [176]. Detailed instructions how to model, how to generate the equations of motion, and how to solve the differential equations numerically can be found in [173] and [194]. The dynamics of a tilting three-wheeled vehicle is discussed in [72]. The state-of-the-art in multibody system dynamics in general and in numerical methods applied to vehicle system dynamics are provided in [191] and [11]. The impact of liquid loads on the dynamics of road vehicles is demonstrated in [182] and [187].

3.3 Model components

In order to meet the standards of ready-to-use vehicle models offered by commercial providers, a fully non-linear and three-dimensional vehicle model is required for comfort and handling analysis. For practical reasons the overall vehicle model is often separated into different subsystems [168]. The components of a passenger car model that can be used to investigate handling and ride properties are shown in Fig. 4.

A generic vehicle model consists of the vehicle framework and subsystems for the steering system and the drive train. It must be supplemented by an appropriate model for the tyre–road interaction. The vehicle framework includes at least the module chassis and modules for the wheel/axle suspension systems. Most wheel/axle suspension systems can be described by typical multibody system elements such as rigid bodies, links, joints and force elements. Analytical models of some typical suspension systems can be found in [175] and [194]. However, relevant bushing compliances must often be taken into account for detailed
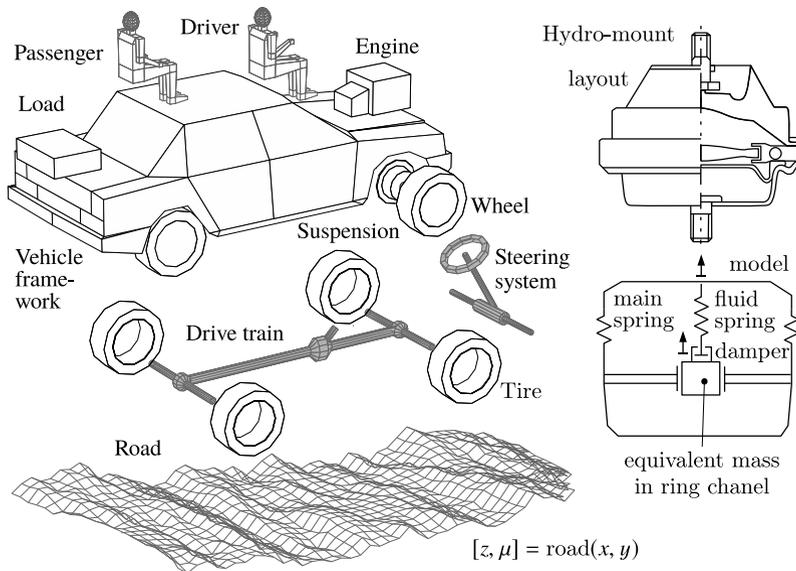
Fig. 4 Model structure of a typical passenger car including a road and a hydro-mount model

investigations of handling and ride properties [198]. That is why a leaf-spring suspension system that is still very popular on trucks demands for a special treatment [183]. The vehicle framework is optionally supplemented by modules for the load, an elastically suspended engine, and passenger/seat models. Sophisticated hydro-mounts that generate highly non-linear dynamic forces are commonly used as engine-mounts to overcome some problems arising from the conflict between mechanical and acoustic vibrations. In case of commercial vehicles or trucks, a flexible frame has to be taken into account. The torsional mode influences the handling properties of a truck in particular. It can be considered quite reasonable by a simple lumped mass system consisting of a front and rear part [172]. A more complex approach for modelling a flexible passenger car chassis is presented in [5]. Mechanical steering systems consisting at least of the steering wheel, a flexible steering shaft, and the steering box, are still in use. Today, the power assistance is usually provided by an electric motor. In addition, some manufacturers install overriding gears in order to provide a variable steer amplification and to open the door for steer by wire systems. Modelling concepts based on a multibody system approach can be found in [180]. A generic drive train model takes lockable differentials into account, and combines front-wheel, rear-wheel, and all-wheel drive [174]. The drive train is supplemented by a module describing the engine torque. It may be modelled quite simply by a first-order differential equation or by enhanced engine torque modules. Tyre forces and torques have a dominant influence on vehicle dynamics. Usually, semi-empirical tyre models are used for vehicle handling analysis. They combine a reasonable computer run-time performance with sufficient model accuracy. Complex tyre models are valid even for high frequencies and on really rough roads. But, they are computer time consuming and therefore used in special investigations only. The Tyre Model Performance Test (TMPT) [125] and the recent Tyre Colloquium [91] provide a lot of information about the efficiency and problems of tyre modelling and parameterization as well as the integration in standard multibody system program codes. The tyre provides the interface to the surface. Roads are characterized by roughness and friction properties. The open source

project OpenCRG [223] applies a curved regular grid to model arbitrary road profiles and represents a standard example for tyre/road interfaces. The dynamics of off-road vehicles requires appropriate surface models that take surface deformations into account. Although the vehicle model remains unchanged in general, the tyre/road or tyre/soft-soil interaction becomes extremely complicated now [188]. Hence, the subject of off-road vehicles is rather associated with the tyre/road dynamics than with road vehicle dynamics in general.

Besides that, a driver model that controls at least the steer input and the drive torque is needed to operate the vehicle. The part 'Advanced Chassis Control and Automated Driving' in [124] provides basic control strategies for active front and rear steering (AFS, ARS), steer-by-wire (SBW), direct yaw-moment control (DYC), adaptive cruise control systems (ACC), and lane keeping assist systems (LKAS). Complex driver models try to imitate the reactions of a real driver by taking the human-perception process, a preview concept, and a target path planning into account [140]. Usually, these models are validated via driving simulators where the real driver is part of the system. Some potentials of integrated control are discussed in [131] and [39, 40].

Like a chain, the quality of a vehicle model depends on its weakest part. Different simulation tasks, pure handling, pure comfort, or handling and comfort analysis, or even real-time applications, demand subsystems of different complexity. Providing subsystems of hierarchic model structure is a common practice to master the challenge of setting up overall vehicle models tailored to the simulation tasks. Alternatively, symbolic model reduction techniques can be applied to master this challenge, too [137]. In addition more complex models will require more parameters. In the early design stage, however, most of the parameters are not known. Lean or simplified vehicle models with a minimum set of parameters may be of advantage then.

3.4 Dynamics

The dynamics of a 'hand-made' vehicle multibody model can be described by a set of two sets of non-linear first order differential equations for the vehicle framework

$$\dot{y} = K(y)\,z \quad \text{and} \quad M(y)\,\dot{z} = q(y,\,z,\,s,\,u) \tag{1}$$
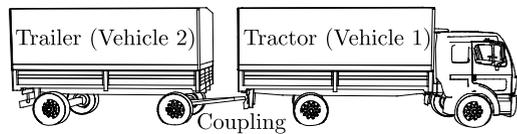
where the vector y collects the generalized coordinates of the vehicle, the kinematic matrix K defines generalized speeds which are arranged in the vector z, M names the mass matrix and q is the vector of generalized forces and torques applied to the vehicle. Non-trivial generalized velocities can be defined for three-dimensional vehicle models which in combination with Jourdain's principle generate more compact equations [173]. Although road vehicles exhibit a significant tree structure, a recursive formalism is not of advantage, because the number of bodies arranged in series (chassis, knuckle, wheel or chassis, subframe/axle, knuckle, wheel) is too short to compensate the overhead costs of a recursive formalism [191].

The vector of generalized forces q may depend on external inputs that are collected in the vector u and on additional states s that are required to describe dynamic force elements or tyres. Hence, an additional set of differential equations,

$$\dot{s} = f(y,\,z,\,s,\,u), \tag{2}$$

have to be taken into account, in general. These sets of differential equations can easily be transformed into one system of ordinary differential equations (ODE) that is usually written in the form of $\dot{x} = f(x, u)$ where u still describes the inputs and the vector x collects all

Fig. 5 Tractor trailer modelled by two coupled vehicle models

Trailer (Vehicle 2) Tractor (Vehicle 1)

Coupling

states of the vehicle model y, z, and s. Then, any standard ODE-solver can be applied to achieve a proper numerical solution.

A generic vehicle model may also describe a trailer or a semi-trailer, see Fig. 5. Then, vehicle (index 1) and trailer or semi-trailer (index 2) are described by coupled sets of differential algebraic equations (DAE)

$$\dot{y}_1 = K_1(y_1)\,z_1 \qquad \text{and} \quad M_1(y_1)\,\dot{z}_1 = q_1(y_1,\,z_1,\,s,\,u) + \lambda \left(\frac{\partial g}{\partial y_1}\right)^T K_1, \qquad (3)$$

$$\dot{y}_2 = K_2(y_2)\,z_2 \qquad \text{and} \quad M_2(y_2)\,\dot{z}_2 = q_2(y_2,\,z_2,\,s,\,u) + \lambda \left(\frac{\partial g}{\partial y_2}\right)^T K_2, \qquad (4)$$

$$0 = g(y_1,\,y_2) \qquad (5)$$

where the Lagrange multiplier λ describes the force coupling between vehicle 1 and 2 that is defined by the constraint equation (5). It turned out, that the Baumgarte stabilization [19] provides an appropriate approach in that particular case [79].

The state-of-the-art paper [11] discusses general aspects of the challenge how to solve the equations of motion in vehicle dynamics. A partly implicit solver that enables a real-time performance even for complex road vehicle models is presented in [181].

3.5 Simulation

Sophisticated vehicle models make all kinds of simulations possible. In particular, the performance of vehicles in the limit range can be studied without any risks and repeated with the same environment conditions over and over again. In addition, this can be done in the early production stage where no real vehicle or not even a prototype is available to perform field tests. The challenge is, to set up an appropriate simulation environment that provides easy to use interfaces to road and tyre models as well as control systems. Commercial software packages usually offer such a simulation environment which however is limited to specific interfaces. In practice, the code of 'hand-made' vehicle models is often embedded into Matlab/Simulink which provides access to standard control strategies.

Figure 6 shows the reaction of a tractor semi-trailer to a sudden tyre puncture at one rear wheel of the tractor. The tractor trailer combination becomes unstable and starts to jack-knife. To prevent a possible roll-over the tractor semi-trailer combination is equipped with supporting wheels. It could be proved in simulation and field test that a one-sided brake input at the wheel of the semi-trailer similar to the input of a common Electronic Stability Program (ESP) could avoid this unstable and dangerous reaction. The tractor semi-trailer was modelled in SIMPACK as a multibody system and the TMeasy tyre model was used to model the standard truck tyres as well as the punctured tyre. The tyre model TMeasy [177] requires no initialization process. That is why the tyre parameters can be changed at any time of the simulation from a standard to a punctured tyre. The sophisticated but quit simple contact calculation, incorporated in TMeasy [179], makes it even possible to simulate the instabilities of trucks running along track-grooves [94].
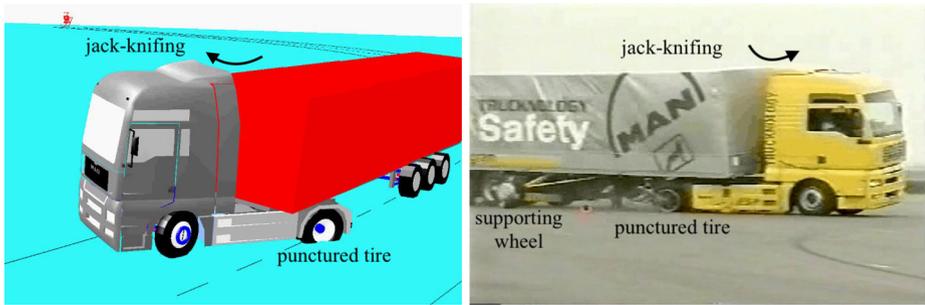
Fig. 6 Punctured tyre on a tractor trailer combination simulated with a multibody system and in a field test (courtesy of MAN)

Modern control devices that react also in critical driving situations, like ABS, ESP, or Steer-by-Wire (SBW), demand for sufficiently accurate vehicle models in order to develop and enhance this systems [228] and [40]. To master this challenge, commercial or 'hand-made' multibody system models are predestinated because they represent an excellent compromise between computation effort and accuracy.

4 Motorcycle dynamics

This section discusses the modelling of the dynamic behaviour of motorcycles with the aid of techniques from multibody system dynamics. As the emphasis is on the modelling of the complete system, the modelling of the components and control systems is only included in this connection. After hand-derived models, mainly models making use of symbolic computer programs are discussed. Then tilting narrow-track vehicles are discussed, and some indications for future research are given.

4.1 Hand-derived models

A fundamental model for a motorcycle which includes a realistic tyre force model needed to represent the three basic instabilities, capsize, weave and wobble, was published by Sharp in 1971 [203]. The capsize mode represents a steady increase of the roll and steering angle, a weave mode is an oscillatory mode of the motorcycle as a whole, and the wobble mode is an oscillatory mode of higher frequency that mainly affects the steering angle. This model was later extended to include frame flexibility [204], the suspension system [205], acceleration [206] and more advanced tyre models. A comprehensive model that could also handle cornering was developed by Koenen [112].

4.2 Computer generated symbolic models

These hand-derived models reached a level of complexity that could not be surpassed, and it was difficult to ascertain the correctness of the equations. A first attempt at the automatic derivation of the equations of motion in a symbolic form was made by Thomson and Rathgeber [219], who used the program NEWEUL developed by Kreuzer and Schiehlen [192] to obtain a model for the straight-running motorcycle with twelve degrees of freedom. The derivation of the equations took an excessive amount of computer time. Simulations with

the linearized equations agreed well with experimental results if the tyre parameters were tuned.

Later, Sharp and Limebeer [83, 208] built motorcycle models with the program AutoSim developed by Sayers [189]. Some of the hand-derived models were modelled with this software, where it was sometimes difficult to get an exact agreement. Their most complex model had 13 degrees of freedom: 12 for the motorcycle and 1 for a body lean angle of the rider. Some extensions were later made to include more refined tyre models and the kinematics of the rear suspension [209]. This advanced multibody model was used in several applications. The problem of the stability of a motorcycle under acceleration was revisited [119]. It was found that acceleration could stabilize the wobble mode, whereas deceleration could destabilize this mode. Another study considered the influence of road unevenness on a cornering motorcycle [120], which could induce severe steering oscillations. More recently, the burst oscillations of an accelerating racing motorcycle were investigated [80]. Moreover, the influence of using two alternative front suspension systems, the girder and the Hossack double wishbone systems, instead of the familiar telescopic front fork, was studied for several choices of the design parameters [46].
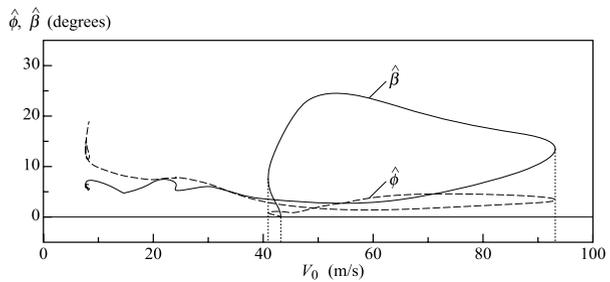
Lot and Da Lio [123] developed a package, MBSymba, with procedures for analysing multibody systems in Maple [42]. As the equations were derived in a symbolic form, linearized equations could directly be obtained by symbolic differentiation. The method was applied to derive Sharp's four-degree-of-freedom model [203] and also to derive the equations of motion for a model with eleven degrees of freedom, called *FastBike*, in [53], which includes a comprehensive tyre model. The formulation of the equations of motion was based on the natural coordinate approach developed by Garcia de Jalón [84]. The resulting differential-algebraic equations could be integrated by the implicit integration method DASSL [153]. More recently, a model including the rider with up to 29 degrees of freedom was developed along the same lines [54]. The additional degrees of freedom took into account the flexibility of the frame, the suspension systems and the wheels and the passive compliance of the rider.

A planar model of a motorcycle including the chain drive was presented in [1]. The natural coordinate approach was used to derive a system of differential-algebraic equations, which were numerically integrated by means of an augmented Lagrangian method. A nonlinear tyre model with possible lift-off of the tyre was included.

Meijaard and Popov [133] used the same symbolic software as Sharp and Limebeer, AutoSim, to derive a model with nine degrees of freedom based on the motorcycle model in [203], but with a tyre model that includes saturation of the forces at high slip values. The linearized equations were also derived symbolically, after which the model was exported in a Fortran program. Continuation software, AUTO [63], was used to find stationary and periodic motions and their bifurcations for the conditions of running straight ahead and for handling a curve with constant radius. Hopf bifurcations leading to wobble and weave and the resulting periodic solutions for both conditions were obtained. The analysis was later extended to a model with eleven degrees of freedom, including the suspension system [134]. Also the influence of drag and the rider's position, based on biomechanical data, were taken into account. Parameters derived from modern high-performance motorcycles and their tyres were used. The kind of solutions obtained for the amplitude of the roll angle and the steering angle depending on the forward velocity are shown in Fig. 7. The unperturbed motion becomes unstable at a nominal forward speed of about 43 m/s in a Hopf bifurcation. Besides that, there is a weave mode that is mostly unstable. The two modes merge at high speeds, where the distinction between wobble and weave disappears.

For the analysis of the stability during braking conditions, the same kind of modelling, including the linearization, was used in [135]. The concept of practical stability, which takes

Fig. 7 The amplitudes of the steering angle $\hat{\beta}$ (solid line) and the roll angle $\hat{\phi}$ (dashed line) for a motorcycle running straight ahead; after [134]

into account the expected size of the perturbations and the allowed size of the lateral motions, was worked out for this case.

4.3 Computer generated numerical models

Donida et al. [65] used the multibody library of Modelica to build an 11-degree-of-freedom motorcycle model. Huyge, Ambrósio and Pereira [8, 95] built the model with 11 degrees of freedom developed by Cossalter and Lot [53] with the aid of DAP3D, developed by Nikravesh [148], and added a 10-degree-of-freedom biomechanical model of the rider. Control could be added by invoking calls to a routine programmed in Matlab. Sequenzia et al. [199] developed an elaborate motorcycle model combined with a model for the rider. One of the bodies of the motorcycle, the chassis, was considered a flexible body, where the Craig–Bampton method was used to reduce the number of degrees of freedom. The program Adams was used to simulate the riding of a lap on the Monza circuit.

4.4 Narrow-track tilting vehicles

Narrow-track tilting vehicles show some similarities with motorcycles. Several configurations were proposed for these vehicles and multibody simulations were necessary to assess their dynamic properties. Bartaloni et al. [16] modelled a three-wheeled tilting vehicle with Adams/Motorcycle. The results compared well with those from an in-house model. Amati et al. [4] modelled another three-wheeled tilting vehicle with a narrow track. They used Matlab/Simulink/SimMechanics [233] to model the system with lumped masses and a lumped stiffness to include frame flexibility. This modelling environment has the advantage that it is fairly easy to include models from other physical domains, in particular controllers. The model had 16 degrees of freedom. The simulation results were compared with those from a similar model in Adams/Motorcycle and experimental results. The agreement was qualitatively satisfactory, but for a small steering wheel offset, the results showed marked differences between the models and also from the experiments. Edelmann et al. [72] analysed a fully tilting three-wheeled vehicle as an elaborate multibody system modelled with SIMPACK [186] and as a simplified analytical system similar to Sharp's motorcycle model [203]. The multibody model had 14 degrees of freedom, including one lumped flexibility of the front fork. The purpose of the simplified model was to design a control system to enhance the stability of the vehicle. The results from the two models were close for small lateral accelerations. Some simulations of manoeuvres were made and preliminary tests showed promising results.
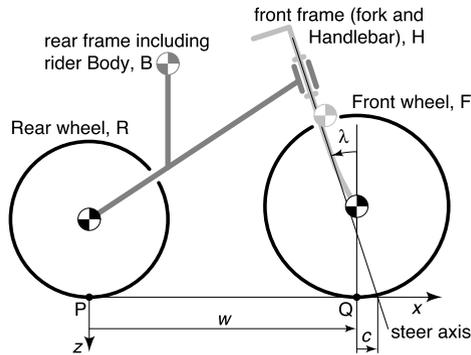
Fig. 8 Whipple–Carvallo bicycle model, which consists of four rigid bodies: a rear wheel R, a rear frame B with the rider body rigidly attached to it, a front frame H consisting of the handlebar and fork assembly, and a front wheel F, connected by three revolute joints. The two wheels make idealized knife-edge rolling point contact with the level ground. This minimal bicycle model has three degrees of freedom: forward speed, rear frame roll angle, and steer angle of the front frame relative to the rear frame

4.5 Concluding remarks and research directions

To conclude, it appears that the use of programs that partly or fully use symbolic methods to derive the equations of motion are preferred in the modelling of motorcycles. Flexibility is included in a lumped way and little direct use is made of modelling methods for flexible multibody systems. As symbolic methods become less efficient for larger models, the use of numerical methods is worth considering. Also flexible multibody models with distributed stiffness of flexible bodies, possibly with subsequent model order reduction methods, as in the case considered by Sequenzia et al. [199] form a viable future research direction. These kinds of models are widely used for railway vehicles, cars and trucks, so their application to motorcycle and bicycle models seems a straightforward extension.

Presently, in the design of advanced control systems for anti-lock braking systems, traction control and stability improvements, fairly simple hand-derived equations of motion are used. The application of more advanced multibody dynamics models is worth a further investigation.

5 Bicycle dynamics

The research in bicycle dynamics by means of multibody dynamics models started around 1899 with the work of the French mathematician Carvallo [38] and the Cambridge undergraduate Whipple [229]. Both derived, independently, the linearized equations of motion for a minimal bicycle model (see Fig. 8) by hand, using rigid-body dynamics, to show, what was already known in practice, that some bicycles could, if moving in the right speed range, balance themselves.
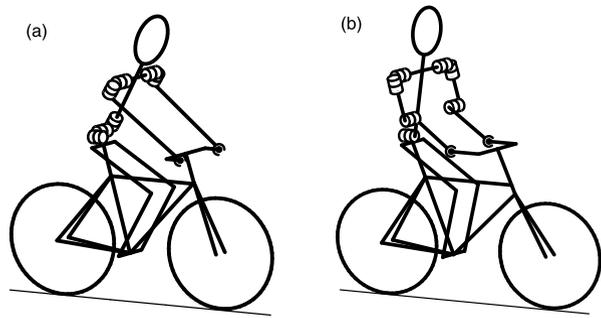
Then, in the 1980s with the development of multibody dynamics computer codes, among the first who reported the application of such a computer code to a model of a bicycle, for demonstration purpose, was Besseling et al. [22]. They used the computer code SPACAR [100], a code for dynamic analysis of flexible multibody systems, which includes idealized rolling contact. Through the years various other multibody codes have been applied to study

the motion of the bicycle. Limebeer and Sharp [208] derived a model to study motorcycle dynamics by means of the symbolic program AutoSim, which then later was applied to study bicycle dynamics [118]. Fisette et al. [82] used their code ROBOTRAN, a symbolic multibody program, as a teaching tool, such that the students can exploit and analyse the dynamics of real applications, and formulate and check hypotheses. Minaker and Rieveley [138] developed a method to automatic generation of linearized equations of motion for mechanical systems, suited to vehicle stability analysis, and implemented in GNU Octave. As an example they demonstrated the stability analysis of an uncontrolled bicycle. Escalona and Recuero [76], for educational purposes, derived equations of motion for an uncontrolled bicycle in a differential-algebraic equation (DAE) form, transformed it to an ordinary differential equation (ODE) form and linearized these equations about a steady motion. As an example, the non-linear equations were implemented in Matlab-Simulink to make an interactive real-time bicycle simulator. This work was then continued in 2018 by using the model for experimental validation by means of an instrumented bicycle [78]. Peterson [151] derived the equations of motion for the Whipple bicycle model by means of symbolic computing in the Python based SimPy language. The method was based on Kane's equations [105] and a general implementation of this method was made in a Python based module PyDy [152]. The model was then used for experimental validation of the uncontrolled dynamics by means of a robot bicycle built by themselves. Orsino [149] developed a method for modular modelling of multibody dynamics systems and demonstrated the method on the modelling of an uncontrolled bicycle. Bulsink et al. [33] developed a model of a bicycle–rider–environment in the ADAMS software to study the uncontrolled and controlled dynamics of such a system for the development of a more stable bicycle for the elderly [71]. Ali [2] developed a systematic approach to derive equations of motions for wheeled multibody systems and implemented this in Matlab. One of his examples is the application of the method to the Whipple bicycle model. Boyer et al. [27] presented a highly mathematical method, based on the so-called reduced dynamics [106], to derive the equations of motion for the Whipple bicycle model, in terms of a minimal set of coordinates and speeds.

5.1 Uncontrolled dynamics

The study of the uncontrolled dynamics of a bicycle, by means of computer code simulations, started around the 1970s with the work of Roland and Rice [170, 184] on the determination of the critical parameters associated with the dynamics and handling of a bicycle. They derived equations of motion by hand, included tyre models, and used the FORTRAN language to implement and simulate the motion. Results were also presented in graphic animations of the motion. After the seminal work of Carvallo [38] and Whipple [229], scores of people studied bicycle dynamics, either for a dissertation, a hobby or sometimes as part of a life's work on vehicles. Unfortunately, very few researchers used or compared their work with others, and as a result, most publications showed flaws or were incorrect. Therefore, in 2007 Meijaard et al. [136] presented a benchmark and review on the linearized dynamics equations for the balance and steer of a bicycle. At the same time, in close cooperation, Basu-Mandal et al. [18] presented a non-linear bicycle model and studied uncontrolled steady circular motions. Plöchl et al. [159] extended the Whipple model with frame flexibility, a passive rider model, and tyre models, to study the wobble or shimmy instability. This work was continued by Klinger et al. [109] who studied the wobble mode in racing bicycles, with rider hands on and off the handlebars. Tomiati et al. [220] included non-linear tyre behaviour to the Klinger model, to study non-linear wobble behaviour. The effect of a linear

Fig. 9 Two distinct bicycle models which include a leaned and steering rider: (**a**) rider with forward leaned body and stretched arms and (**b**) rider with upright body and flexed arms [197]

tyre model on the stability of the bicycle was presented by Souh [213]. He showed the unstable wobble mode in a linear model and verified his results on a non-linear dynamic model of the system. Dao and Chen [59] used system identification techniques to determine the dynamic properties of a non-linear bicycle model, and showed some limit cycle behaviour of that model. The effect of a passive rider on the lateral dynamics and controllability of a bicycle by steer and lateral upper body motions was investigated by Schwab et al. [197], see Fig. 9. They demonstrated that rider posture can destroy self-stability, and that unstable modes of bicycle–rider combinations have very good modal controllability for steer torque control but are marginally controllable by lateral upper body motions. The same effect of rider posture on the lateral stability on a somewhat different model of the rider–bicycle system was shown by Doria [66].

5.2 Controlled dynamics

Although some uncontrolled bicycles can balance themselves [114], most bicycle–rider systems are in need of rider control, either to balance at low or high speed, or to manoeuvre in the environment. Moreover, since the directional control of bicycles by means of steering is a so-called non-minimum phase system (you first have to steer to the left to turn to the right, and vice versa), the bicycle has attracted much attention from control oriented researchers. Åstrom et al. [12] reviewed the dynamics of bicycles from a control perspective, mainly for educational purposes. Von Wissel and Nikoukhah [225] developed a model to manoeuvre a bicycle around obstacles by a continuous feedback stabilizing controller and trajectory optimization. Sharp [207] applied optimal linear control theory to develop a linear preview stabilizing and tracking controller for a Whipple bicycle model. He demonstrated the effect of tight and loose control on the tracking error and control effort. Chen and Dao [44] developed a fuzzy type controller to balance and steer a bicycle into straight and circular motions. Dao et al. [58] further developed a sliding mode controller to stabilize the lateral motions of a Whipple type bicycle model at various forward speeds. A fuzzy controller, but now based on Lyapunov rules, to balance an unmanned bicycle was presented by Hashemnia et al. [92]. They demonstrated the robustness of the controller by applying the method to various different bicycle designs. To demonstrate the basic concept of lateral stability control in a bicycle, that is, to steer into the undesired fall [114], Mutsaerts [144] successfully built a small scale LEGO bicycle, controlled by a LEGO Mindstorms NXT implemented very simple steer-into-the-fall controller. Basso and Innocenti [17] further developed this Lego-Bike idea into an educational example of rapid prototyping in a Mindstorms/Simulink environment. Wang et al. [227] developed a non-linear analytical Whipple-type bicycle model using symbolic mathematics, to be applied in non-linear bicycle control. Chu and Chen [45] used
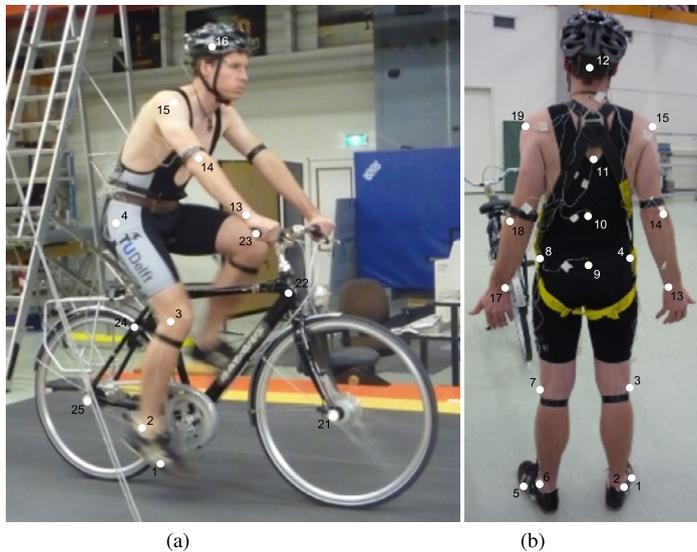
(a)   (b)

Fig. 10 (**a**) Rider 1 and the Batavus Stratos Deluxe bicycle with marker positions, riding on a large treadmill; (**b**) Body marker positions visible from the rear [142]

their rider–bicycle model with steer control and rider upper body lean control to develop a model predictive controller for lateral stability. García et al. [85] developed a novel retractile flywheel mounted on the rear frame to stabilize and manoeuvre a riderless bicycle in the environment. They developed a simulation model in the ADAMS software and demonstrated the feasibility of the system. Baquero-Suárez et al. [15] designed a two-stage observer based feedback controller for lateral stability. They built a prototype and demonstrated the effectiveness of the controller.

5.3 Rider control identification and handling

After most of the models for the uncontrolled dynamics of a bicycle had been established and validated, attention shifted to modelling and identification of rider control with a perspective on ease of control and handling. A comprehensive review of the literature on that subject was presented by Kooijman and Schwab [113] and by Schwab en Meijaard [196]. One approach is to add a complete musculoskeletal model of the rider to the bicycle model, as done, for instance, by Damsgaard et al. [57] by means of the Anybody Modelling System. This could then be used to analyse motion, loads, and motion coordination, either in inverse dynamics or forward dynamics analysis. In another approach, rider motions were identified and form the basis for constructing a rider model based on linear or non-linear feedback control. Moore et al. [142] used a fully instrumented bicycle–rider system and the method of principal component analysis to identify rider motion and showed that most of the rider control is done by steering. The instrumented bicycle–rider system with 32 Optotrack active markers, measuring the xyz locations at 100 Hz, is shown in Fig. 10. Moore [141] then continued with Hess [93] to model bicycle rider control based on well established aircraft pilot modelling from the 1960s, in order to predict handling qualities of the bicycle–rider system. With these models they were able to predict distinct variations in handling qualities for various forward speeds, and little variations due to bicycle–rider combinations. Cain and Perkins

[34] and Cain et al. [35] investigated the contribution of rider lean in the balance control of the bicycle, while riding an instrumented bicycle on rollers. They found that at high forward speeds, skilled riders, opposed to novice riders, showed superior balance by less steer control and more upper body lean control. Lipp et al. [121] analysed a Whipple type bicycle model with added rider posture control. They demonstrated, by means of linear stability analysis, that adding reaction time in the form of rider control delays shrinks the stability region, and that a time trial rider position also reduces the decreased stability, mainly due to the narrower handlebars.

5.4 Tyre modelling

Advanced simulations on the stability and handling of bicycles requires, among other things, detailed tyre–road interaction models. Although pneumatic tyre behaviour had been studied extensively for automotive industry [150], little attention had been given to the measuring and modelling of bicycle tyre behaviour. While somewhat similar in construction, the bicycle tyre distinguishes itself from the automotive tyre by a relatively long and slender, almost one-dimensional, contact patch. Roland and Massing [185] were among the first to measure the cornering and camber stiffness of bicycle tyres. The work was done at Calspan, contracted by the Schwinn bicycle company for a larger project on computer simulations of bicycle dynamics and handling. Cole and Khoo [49] measured cornering stiffness of bicycle tyres, on a back-to-back test apparatus, where the tyres were rolled on a common road surface. Dressel and Rahman [69] measured cornering and camber stiffness for a number of bicycle tyres by means of a custom-built low-cost towed device. Then Dressel [68] continued the work in Delft, on a large tyre testing drum, and was able to collect a large amount of tyre force data from 14 tyres, at three loading cases and three inflation pressures. An example of the results for one specific case are presented in Fig. 11. Dressel also made a start to capture the results in physics based tyre models. Doria et al. [67] measured the cornering and camber stiffness for four tyres at various loadings and inflation pressures on their rotating disk tyre tester. Unfortunately, some of the results for identical tyres and load cases do not comply with the results found by Dressel [68].

5.5 Structural aspects

Structural integrity, comfort and safety are topics which can also be addressed by analysis on multibody dynamics models of bicycles. Good and McPhee [88] analysed the effect of rear suspension on the in-plane dynamics of a mountain bicycle–rider system, with the help of the MapleSim symbolic language software. They clearly identified the oscillations induced in the system by the chain and suspension system. Waechter et al. [226] developed an in-plane multibody model for the dynamic simulation of bicycle suspension systems. With the model, they were able to predict fairly accurately the measured vibrations from various experimental setups. Redfield [169] developed an in-plane mountain bicycle and rider model, with the help of bondgraphs, to predict the forces and large motion behaviour of mountain bicycles on rough terrain. In order to predict braking dynamics from a safety perspective, Maier et al. [127] developed a multibody dynamics model for the in-plane motions of a bicycle with front suspension and rider, to study the effect of a braking dynamics assistance system. The model showed good agreement with experimental data from road tests.
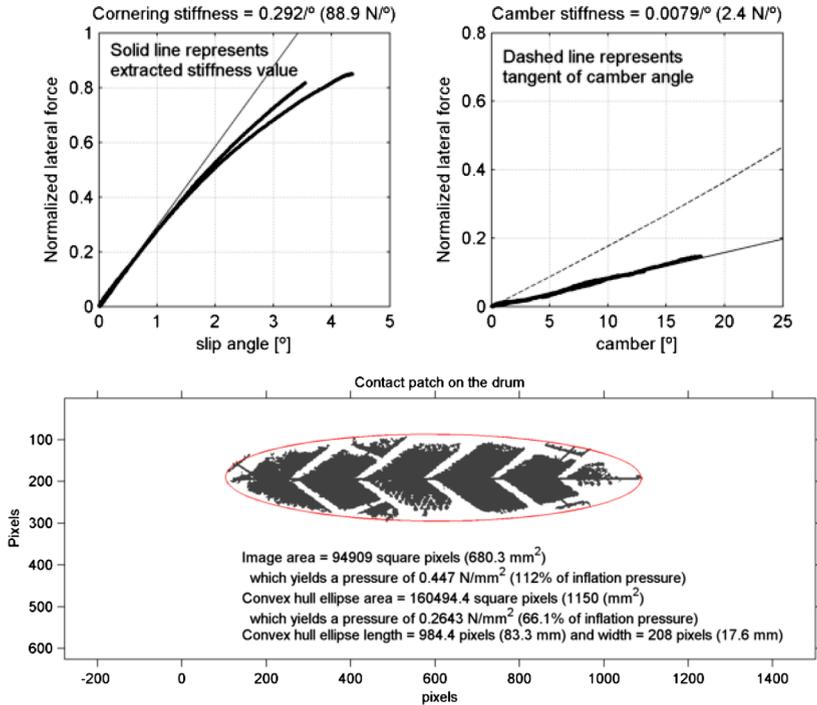
Fig. 11 Normalized cornering and camber stiffness as a function of the slip and the camber angle together with the footprint for the Vredestein Perfect Tour 37-622 bicycle tyre, at a normal load of 304 N and an inflation pressure of 4.14 bar [68]

5.6 Accident analysis

With increasing fatalities in cycling, accident reconstruction for bicycle crashes is an emerging topic. Mukherjee et al. [143] used a multibody model developed in MADYMO to investigate the correlation between throwing distance and impact speed, point of impact, and angle of approach, for various bicycle–car crash configurations. They demonstrated the obvious, namely that throwing distance increases when the impact velocity increases. However, variation in the angle of approach or point of contact showed significantly different results. Carter and Neal-Sturgess [37] studied the kinematics and specific injuries and points of contact involved in car–bicycle accidents by means of a multibody dynamics model built in MADYMO. Accidents were reconstructed from data obtained from real accidents, for which they found a good agreement of the contact points and resulting injuries. Bourdet et al. [26] reconstructed 24 well-documented bicyclists' head trauma incidents from real world accidents by means of a multibody model made in MADYMO. The accident data were obtained from the German In-Depth Accident Study (GIDAS) database. Results showed that the first head impact occurs more often on the lateral top of the head. Carollo et al. [36] used multibody simulations in visual Nastran on a multibody model of a cyclist and a car to investigate the crash behaviour. Among other things, they found differences in injury patterns between teenagers and adults.

Fig. 12 A 'bricycle': a vehicle which can transition between a bicycle and a tricycle [64]

5.7 Out-of-the-ordinary designs

Multibody models are also successfully used to study the dynamic behaviour of more out of the ordinary single track vehicles. Verlinden et al. [222] studied the rideability of a novel single track articulated vehicle, the anaconda. They used the EasyDyn software for modelling and showed, what was already demonstrated in practice, that with increasing articulation modules the rideability decreased. Dong et al. [64] analysed and built a model of a bicycle which cancels the effect of gravity by springy training wheels; the so-called bricycle, see Fig. 12. They demonstrated on various models and an experimental setup that such a bicycle in zero-gravity can easily be balanced, but is unable to turn. With the need for a safe environment for testing cycling behaviour, and for doing cycling balance and manoeuvring training, bicycle simulators are an emerging topic. Yin and Yin [235, 236] developed an interactive bicycle simulator on a moving base with visual feedback via a head mounted display, a haptic feedback on the handlebars, and realistic pedal resistance. For the underlying dynamic model, they developed a multibody dynamics model of the rider bicycle system. Initial result showed that the system is effective and showed realistic motions.

5.8 Conclusion

To conclude, multibody dynamics models, generated by computer codes, have paved the way for basically correct, more detailed, and in-depth dynamic analysis of bicycle dynamics and rider control. With the increased interest in rider safety, future directions of research are now focused on rider control identification and handling qualities.

6 Conclusions

After the current topics in the field of the application of multibody system dynamics to vehicles have been reviewed, it can be concluded that great progress has been made in building adequate models. The greatest challenges are still in the accurate modelling of components of the system: these often require an experimental determination of their characteristics. Especially the modelling of the contact with the rails and the road in all circumstances is complex. In railway vehicle dynamics, the accurate modelling and fast simulation of the wheel–rail contact is still a point of concern, as is tyre force modelling in road vehicles. For single-track vehicles, the influence of the rider as a physical component of the vehicle and as a controller on the dynamics is relatively large and needs further attention.

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