

Rejoinder

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I would like to thank Dr. Parker, Prof. Tsung and Mr. Li for their insightful and thought-provoking discussions. I also heartily thank them for the so nice words they spent on the line of research I have been developing with my colleagues and collaborators. Their appreciation for the work presented is very encouraging and highly motivating.

I very much appreciate the new and different perspectives added to what I have described by Prof. Tsung and Mr. Li. The discussants also reviewed some complementary methods, such as profile monitoring, other types of formulation of spatial and spatio-temporal dependencies, and generalizations to tensor data. As I briefly mentioned in Section 5 of the paper, the methods I presented have been recently extended to spatio-temporal data [see Bernardi et al., 2017, Arnone et al., 2019]. Such an extension could also serve as a basis for profile monitoring techniques; exploring this research direction indeed appears highly interesting. In terms of statistical process monitoring of complex data, I would also like to cite the fascinating presentation made at the Stu Hunter Research Conference 2019 by Prof. Enrique Del Castillo, and described in del Castillo and Zhao [2020]. It would as well be very interesting to combine the differential regularizations I described in the paper with those mentioned by Prof. Tsung and Mr. Li. Finally, the extension of the proposed methods towards tensor data is highly fascinating. I welcome this suggestion made by the discussants, that would even further broaden the applicability of the methods.

I take the point made by Dr. Parker, about the required prerequisite knowledge to implement the methods I propose: Dr. Parker commented that the prerequisites appear quite extensive, and this may

pose difficulties for the broad adoption and utilization of the methods by practitioners. I do not intend to deny this point. While the application of the proposed methods in standard settings requires no more expertise than classical techniques, these methods deploy their full potential in complex data analysis problems; and to tackle these complex problems, a multidisciplinary approach is indeed necessary. In fact, I fully concur with Dr. Parker that, if we want to face the many challenges posed by the analysis of big and complex data, we must significantly strengthen collaborations among different disciplines, including statistics, applied mathematics, computer science, as well as problem-specific domain disciplines. This crucial aspect is also emphasized by many of the contributions to the special issue on *The role of Statistics in the era of big data* [Sangalli, 2018].

A fundamental issue to be considered in this respect is the education of the next generations of graduates. In particular, concerning statistics, I believe we must foster undergraduate and graduate programs that give students not only a strong statistical background, but also the competencies needed to efficiently interact with fellows mathematicians, computer scientists and domain experts, thus enabling those collaborations that are essentials for advancing our ability to analyze complex data.

Let me conclude by heartily thanking the organizers of the 2019 Stu Hunter Conference, Prof. Bianca Colosimo, Prof. Allison Jones-Farmer, Dr. Ross Sparks and Prof. David Steinberg. I have been deeply honored by their invitation. I would also like to thank the discussants at the conference, Prof. Fugee Tsung and Prof. Ernst Wit, the moderator Prof. Peter Parker, and all the audience, who greatly contributed during the discussion session: it has been for me

invaluable to have our research discussed and commented upon from so many different perspectives, and to receive many stimulating and enriching suggestions.

References

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