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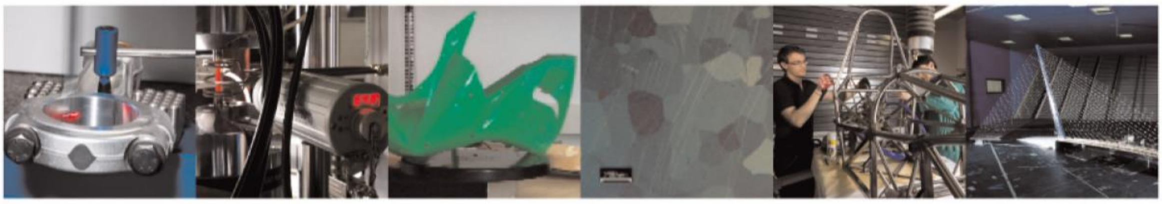

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Performance analysis of unreliable manufacturing systems with uncertain reliability parameters estimated from production data

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ABSTRACT
System engineering methods for the performance evaluation of manufacturing systems require the estimation of input parameters, which is either based on real data or experts' knowledge. In both cases, the input parameters are subjected to uncertainty. However, most of the systems engineering methods, which are based on analytical models assume that machine reliability parameters such as Mean Time to Failure and Mean Time to Repairs are precisely known. In order to overcome this limitation, this paper proposes an approach for the performance evaluation of unreliable manufacturing systems that considers uncertain machine reliability estimates. The method enables to calculate the distribution of the output performance, given the distribution of the input parameters' uncertainty. The evaluation procedure is based on the combined use of Bayesian estimation, probability density function discretization and existing decomposition-based techniques for analyzing transfer lines composed of unreliable machines and capacitated buffers. Experimental results obtained by using the method show that neglecting uncertainty in the input parameter estimates generates significant errors in the output performance measure estimation, thus making the subsequent system operation and reconfiguration decisions sub-performing. Finally, a real case study is presented to demonstrate the potential benefits of the proposed method for real industrial applications.

KEYWORDS
manufacturing systems; performance analysis; reliability parameters; uncertainty

1. Introduction, Motivations and Objectives

1.1. *Introduction*

In today's highly dynamic and competitive markets, the design, reconfiguration and continuous improvement of manufacturing systems that are capable of guaranteeing target production performance play a key role for the competitiveness of an enterprise (Koren et al. 2017). Thus, decision making in each of these phases needs to be supported with accurate and robust system engineering tools. The accuracy of these decisions is important for manufacturers in order to avoid potential performance deviations that can seriously undermine the effectiveness of the adopted decisions(Plehn et

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al. 2016). For this reason, manufacturers increasingly support their decision making by using state-of-the-art system engineering tools, which incorporate digital models of resources/machines. In order to ensure a realistic characterization of resource models, the parameters of the digital models need to be estimated from operational data and information gathered from the manufacturing system. However, most commonly applied methods assume the parameters of these digital models' to be deterministic values, for example by considering the mean values of the gathered data. This assumption neglects the related uncertainty and the amount of data used to estimate the model parameters. For example, the reliability of machines is usually modeled through the characterization of the Mean Time to Failure (MTTF) and the Mean Time to Repair (MTTR) of each failure type affecting the machine's efficiency. Although these are assumed to be the mean of statistical distributions, typically exponential or geometric distributions, their value is considered as known deterministically. But if they are estimated by gathering a sample size of 5 instead of 1000 failure observations, the resulting level of confidence on the mean values of input parameters is different. This also translates into different levels of confidence on the output performance measures estimated by the related models. As a consequence, effective reconfiguration and continuous improvement decisions of manufacturing systems strongly rely on the ability to carry out a sound performance analysis, by incorporating parameters' uncertainty within the digital model of the system (Lee et al. 2009).

In the presence of uncertain parameters, modeling incomplete knowledge and parameters' uncertainty as perfectly precise information leads to several fundamental risks for manufacturers. Firstly, if uncertainty in the input parameters estimation is excluded from the digital model there is an inherent risk that system level decisions fail to meet the target performances, in real settings (Park et al. 2010; McFarland et al. 2009). Secondly, if extra capacity is chosen as a strategy to guarantee a higher probability of achieving target performance under uncertainty, system over-sizing and resource capacity waste can be observed (Colledani et al. 2013). Both counter measures directly translate into additional costs and competitiveness loses for manufacturing companies. At the same time, input parameters uncertainty can be reduced by increasing data collection efforts at critical process stages. This effort also translates into additional costs for manufacturers, but contributes to obtain more precise output predictions. Therefore, in order to analyze the trade-off between available options, there is a need for system engineering tools that are capable of embedding the uncertainty of input parameters into the performance evaluation model (Wenzel et al. 2017).

The practical relevance of exploiting operational data and parameters' uncertainty in performance analysis is drawing more interest from manufacturers (Colledani et al. 2013; Nylund et al. 2012; Chryssolouris et al. 2009). Indeed, there is a growing trend and a wide spread implementation of technologies for supporting operational data acquisition, analysis and decision-making tools. These technologies are collectively referred to as Cyber-Physical Systems (CPS) (Lee et al. 2015), defined as the integration of computation and physical processes, in which embedded computers and networks monitor and control physical processes, usually with feedback loops where physical processes affect computations and vice versa. The operational data that is acquired from the physical manufacturing systems through such technologies can be used to generate the digital model and understand the behavior of manufacturing resources with the goal of making optimal decisions and continuous improvements. By embedding the digital model of manufacturing resources within CPS framework digital manufacturing tools can enable a high-fidelity virtual representation of the real system. In the state-of-the-art manufacturing practice, such virtual representations of individual resources are required to closely match their physical counterparts' behavior, namely their "digital twins". For this reason, digital manufacturing tools are continuously fed with process and system data from technical tests or directly gathered from the field. However, these "twins" are required to mirror both the physical behavior of resources and also the amount of data used to model them, thus quantifying the level of uncertainty they feature.

Different scenarios that aim to embed digital manufacturing tools for a data driven performance evaluation require specific CPS framework and information architectures. A framework for manufacturing systems with the goal of enabling continuous process improvement is proposed as in (Figure 1). In this scheme, the initial performance evaluation can begin by using existing data and information collected via sensors about the physical system and the associated input parameter uncertainties. This information is used to set up the first system models, and to predict the output performance and the related uncertainty. Based on the output analysis, the physical system can be adjusted considering both the improvement of target performance, and also to refine the accuracy of output performance precision. This requires a systematic data sampling plan which optimally targets the most relevant input parameters. This procedure can be continued in an iterative framework until a given confidence level is achieved as depicted in (Figure 1), or when the next batch of data is available. On the other hand, such analysis tools can assist to improve the estimation of the output performance uncertainty by optimally allocating (reconfiguring) supervisory/monitoring resources. The reconfigured supervisory plans aim to dedicate data acquisition efforts/resources on the most relevant data that are critical for improving performance estimation over less relevant data thereby saving key information system resources. These two sided reconfiguration decisions on both the manufacturing systems and supervisory systems are schematically shown in Figure 1. This approach allows the integration of different resource models, in the digital layer of the CPS framework to predict key performance indicators for manufacturing system continuous improvements. Overall industrial trends in the application of CPS, big data and digital manufacturing tools for different domains are discussed in (Lee et al. 2015) and (Assunção et al. 2015).

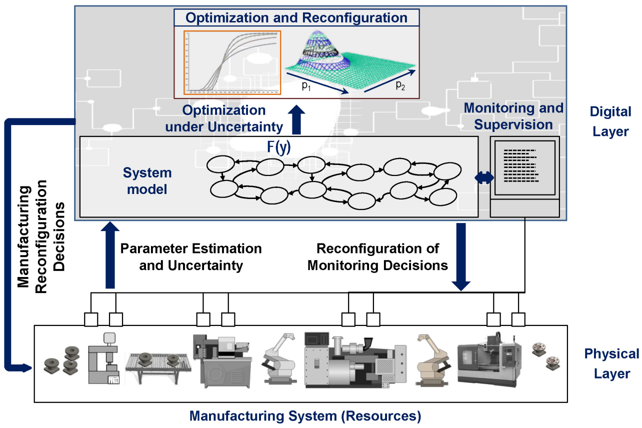

Figure 1. A performance evaluation framework for a continuous improvement of manufacturing systems

In this paper, a new approach for the evaluation of the output distributions of key performance measures, such as throughput and work in progress in multi-stage manufacturing systems, considering uncertain machine failure and repair parameters is proposed. The method is based on the combined use of Bayesian estimation and analytical techniques for analyzing transfer lines composed of unreliable machines and capacitated buffers. The two major research questions that this paper aims to answer are formulated as follows: "What is the error/deviation in the estimation of the system throughput observed if only the expected values of the estimated input parameters are considered, i.e., uncertainty is neglected?" and "What system decisions can be significantly affected by this error". The results show that internal uncertainty modifies the performance evaluation results and can significantly affect the related system reconfiguration decision, thus paving the way to the development of a new manufacturing system engineering theory for robust analysis of manufacturing systems under uncertainty.

1.2. *literature*

The advantages of performance evaluation methods based on online operational data, which reflects the actual behavior of the system and the associated uncertainty are discussed in the literature (Li et al. 2009; Lin et al. 2008; Ni et al. 2012). In spite of the industrial relevance of this issue, state-of-the-art manufacturing system engineering approaches do not considered this issue (Tolio et al. 2002; Colledani et al. 2013). Particular emphasis on the application of data collection for the analysis of machine and component level reliability parameters are highlighted in (Leondes 2002; Inman 1999). In these methods, the reliability of machines is modeled through the MTTF and the MTTR of each failure mode affecting the machine. Usually, the considered output performance measures are the average throughput and the average inventory levels of the system. Again, these are considered to be precise estimates, although they are strongly affected by the input parameters' uncertainty.

1.2.1. Analytical methods without considering input parameter uncertainty

Widely used manufacturing system engineering approaches are either based on exact and approximate analytical methods or simulation methods. Each of these approaches can present specific advantages and limitations. For example, simulation methods are suitable for modeling a wide range of manufacturing system architectures, provide accurate estimation of performance measures and the associated uncertainties (Jahangirian et al. 2010). However, creating simulation models of multi-stage manufacturing lines and the subsequent simulation time can be too high for fast decision making applications (Fowler et al. 2004), (Negahban et al. 2014). In such cases, approximate analytical methods represent a suitable alternative to simulation when estimating the performance of complex manufacturing systems in a short time. They are accurate and fast in the estimation of the main performance measures. Such early models dealt with the analysis of serial transfer lines (Gershwin 1987) and asynchronous flow lines (Burman 1995), with single failure mode machines. Later, these methods were extended to deal with more complex machine models, featuring multiple failure modes (Levantesi et al. 2003). Recently, methods for studying complex system layouts (Colledani et al. 2005) and different product variants (Colledani et al. 2015) have been proposed. These works have been foundation for the development of various decision support system for the design and management of complex manufacturing systems (Colledani et al. 2014), (Terkaj et al. 2009). Although these works bring significant advancement in this area, they do not take into account the uncertainty related to the estimation of input reliability parameters. Thus the impact of uncertainty on the output performance accuracy is neglected.

1.2.2. Performance evaluation approaches considering input parameter uncertainty

In the literature, there are performance evaluation approaches that aim to consider operational data and the related uncertainty. The first class of approaches rely on discrete event simulation (DES) (Negahban et al. 2014) and application of statistical techniques which are based on a gathered operational data. DES models can be built with the aim of grounding the subsequent analysis on an empirical data (Koh et al. 2006; Tako et al. 2012), thus relaxing some of the stringent assumptions in analytical methods. Such approaches are applied to study the performance of multi-stage manufacturing lines for the analysis of average throughput (Nandagawe et al. 2009), work in progress (Xu et al. 2012), buffer allocation (Amiri et al. 2012) and for the identification of bottlenecks (Li et al. 2009). However, manufacturing system simulation models are generally problem specific, therefore they need to be tailored or adapted by considering the specific characteristics of each problem. The second research area that attempts to consider uncertain parameters within analytical approaches uses Fuzzy Markov models which are capable to deal with uncertain transition rates and probabilities. Fuzzy Markov models are developed to overcome the deterministic parameter assumption by emphasizing on the uncertainty of transition probabilities from real data or limited information. Although many of these works do not focus on multi-stage manufacturing systems, they demonstrate the potentials of this approach on Markov based reliability models. For example, (Binh et al. 2006) demonstrated the application of fuzzy Markov in calculating reliability of power systems. (Kumar et al. 2009) calculated fuzzy reliability and fuzzy availability of the serial processing plant. (Liu et al. 2010) introduced a modified fuzzy multi-state system availability assessment approach to compute the system availability under the fuzzy user demand. (Lata et al. 2012) used the fuzzy Kolmogorov's differential equations evaluate fuzzy reliability system, the fuzzy Kolmogorov's differential equations are solved analytically for solving n^{th} order fuzzy linear differential equations. However to the authors' knowledge there are no generalized approaches to extend analytical manufacturing systems engineering tools to consider uncertain reliability parameters.

The paper is structured as follows. Section 3 introduces a generalized manufacturing system architecture and the related assumptions used to demonstrate the applicability of the proposed method. Section 4 describes the steps for uncertain parameters inference from observed data. Section 5 presents the entire procedures for performance evaluation by integrating the steps described in section 4 and the system introduced in section 3. Numerical results featuring new findings of selected experiments for different systems are reported in section 6. Finally a real case study and the application of the method for the analysis of a serial manufacturing lines is reported in section 7.

2. Nomenclature

The following notations are frequently used throughout the paper.

M_i	The i^{th} machine in a manufacturing line
B_i	The i^{th} buffer between machine $i-1$ and machine i
$MTTF$	Mean Time to Failure
$MTTR$	Mean Time to Repair
TTF	Time to Failure
TTR	Time to Repair
TH	Throughput
$n(i)$	The average buffer level B_i
p_j	The j_{th} failure probability of a given machine
r_j	The j_{th} repair probability of a given machine
$\hat{p}$	The mean value of probability of failure p
$\hat{r}$	The mean value of probability of repair r
$f_X(x)$	The cumulative distribution function of random parameter x
$B(\alpha, \beta)$	Beta distribution with parameters, α and β

3. System description

The applicability of the proposed method is not limited to a specific layout or configuration of a manufacturing system; however, in this paper transfer line systems are considered. Transfer lines are generally composed of multiple machines with a linear architecture, and they are quite common in real manufacturing environment. Manufacturing systems with this configuration are composed of K unreliable processing machines (represented in squares) and *K-1* storage buffer spaces (represented in circles) as shown in Figure 2. The machines (squares) perform operations on parts flowing in the system. Buffers (circles) have the role of decoupling the machines in the system. They can be either inventory storages or automated material handling systems that transport semi-finished materials between machines. The i^{th} machine and buffer are denoted with M_i and B_i (with $i = 1, ..., K-1, K$) respectively: B_i has capacity equal to N_i and it contains only pieces already worked by M_i. A generic M_i is blocked if the downstream B_i is full and it is starved if the upstream dedicated buffer B_{i-1} is empty.

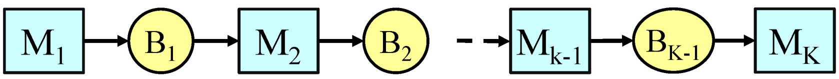

Figure 2. Representation of transfer lines

3.1. *Modeling Assumptions*

The following list of system modelling assumptions are adopted. The assumptions related to the TTF and TTR distributions type are selected to be consistent with the assumptions already made for the underlying analytical models used for performance evaluation. However, if a different type of distribution is considered in the underlying analytical models,then these assumptions can be changed accordingly, and the proposed method can be similarly applied for the rest of the procedure.

- Discrete and synchronous material flow model is considered.

- Machine processing times are deterministic and equal for all the machines in the system. This time unit is scaled to the processing time.
- Machine M_i can fail in F_i independent failure modes.
- Times to Failure ($TTF_{i,j}$) and Times to Repair ($TTR_{i,j}$) are geometrically distributed.
- Failures are Operational Dependent Failures (ODF), i.e. the machine can fail only if operational (not starved or blocked).
- Machine M_i fails in failure mode $j = 1,..,F_i$ with probability $p_{i,j}$ in a time unit. It is assumed the value of $p_{i,j}$ is not precisely known, therefore $p_{i,j}$ follows a probability density function $(pdf) f_P(p_{i,j})$. In case of availability of historical data on the machine behaviour, $p_{i,j}$ can be estimated from s observed samples of $TTF_{i,j}$, for example through Bayesian inference (next section).
- A failed machine, M_i is repaired in a time unit with probability $r_{i,j}$. It is also assumed that the value of $r_{i,j}$ is not precisely known, therefore $r_{i,j}$ follows a *pdf*, $f_R(r_{i,j})$. If sample data is available, the distribution of $r_{i,j}$ can be estimated as shown for $p_{i,j}$.

The performance measures of interest are the system throughput TH, i.e. the average number of parts produced by the system in a time unit, and the average level of each buffer in the system $n_i, i = 1,..,K-1$.

4. Reliability Parameter Inference

In many of the experimental cases considered in this paper, it is assumed that reliability parameters, i.e., failure probability p and repair probability r, are estimated from empirical data gathered on TTF and TTR respectively. Although the inference schema presented in this section considers a generic failure probability p, it is equally applicable to all the uncertain parameters that needs to be estimated in the system, i.e., $p_{i,j}$ and $r_{i,j}$.

The Bayesian inference method is used to model the state of knowledge about uncertain parameters based on observed data during a given time period. This technique is grounded on an iterative procedure to update a previous belief about an uncertain parameter, and then deriving a posterior distribution of the parameter by integrating new observations or evidence as they become available. Formally, if a prior distribution, $\pi(p)$ of the uncertain parameter exists, then the posterior density $\pi(p|TTF)$ can be updated after s new observations are made on TTFs. This procedure can be repeated periodically whenever new data is gathered about the parameter. Updating the prior into a posterior can be performed using Bayes formula as follows:

$$\pi(p|TTF) = \frac{\pi(p)\pi(TTF|p)}{\int_P \pi(p)\pi(TTF|p)dp} \quad (1)$$

In this analysis, conjugate priors are used for the characterization of unknown failure and repair parameters. Since in Section 3.1 TTFs are assumed to be geometrically distributed, then the prior distribution for the parameter p follows a Beta(α_p,β_p) distribution which can be expressed as (2).

Theoretically, the choice of different priors can be considered in a Bayesian approach. In this work, the authors limit the use of conjugate priors corresponding to a the geometric distribution. One of the main motivations to use conjugate priors is that they provide simplistic algebraic forms, particularly in a cyclic applications of Bayesian updating from new data. By using conjugate priors, the updating of the density function with new data to compute for the posterior has a similar form to the prior. In this particular case of inference on the parameter p from a geometrically distributed TTFs then conjugate prior for the parameter p is the two parameter Beta distribution. If the observed data follows a different distributions than the assumptions adopted in this paper, i.e., geometrically distributed failure and repair times assumed in (Section 3.1), the same approach can be followed for the parameter inference. However, in those cases the appropriate conjugate distributions needs to be selected. Tables with the list of conjugate priors and associated likelihood functions can be found in statistical references, such as (Gelman et al. 2014).

On the other hand, if the distribution type of the TTF and is uncertain or unknown, the subsequent uncertainty in the estimated parameter probability density function of p will be uncertain as well, but such considerations are beyond under the scope of this work.

$$\pi(p) = \frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1} \tag{2}$$

Where $B(\alpha, \beta)$ is the Euler integral of the first type. The likelihood after an observation of a vector of TTFs is:

$$\pi(TTF|p) = \prod_{i=1}^{s}(1-p)^{TTF_i-1}p = (1-p)^{\sum_{i=1}^{s} TTF_i - s}p^s \tag{3}$$

Substituting (2) and (3) in (1) and after some manipulations the posterior distribution can be expressed as:

$$\pi(p|TTF) = Beta(\alpha + s, \beta + \sum_{i=1}^{s} TTF_i - s) \tag{4}$$

This provides the new posterior distribution $p \sim$ Beta($\alpha_p^{'}$,$\beta_p^{'}$) after a sample of s new observations on TTFs, where $\alpha_p^{'} = \alpha_p$ + s and $\beta_p^{'} = \beta_p$ +($\sum TTF_i$-s).

At a given observation time, all the uncertain parameters can be characterized in the same way by using their conjugate prior distributions. Traditionally, most performance evaluation methods and approaches rely only on the mean value of these observations as input, i.e. the maximum likelihood given in (5). On the other hand, the proposed method uses the density function of the estimated input parameters (4) in the evaluation of output performance measures.

$$\hat{p} = argmax\{\pi(p|TTF)\} = E[\pi(p|TTF)] = \frac{\alpha^{'}}{\alpha^{'} + \beta^{'}} \tag{5}$$

5. Performance Evaluation Method description

This section presents a method for the evaluation of system performance distributions by using uncertain input parameters. However, other alternative solution methods can also be used depending on the complexity and size of the problem. Some of the possible approaches include exact analytical methods or numerical methods, such as Monte Carlo (MC) simulation and approximate integration techniques. Exact analytical methods derive the output distribution of the required performance measures in a closed form, given the distribution of the uncertain input parameters. Such approaches are more suited for systems where a simplified closed form solution is available. On the other hand, numerical solution techniques such as Monte Carlo method and discretization methods depend on sampling the values of $p_{i,j}$ and $r_{i,j}$ from the original *pdf*. These techniques too depend on an existing performance evaluation method, for example a decomposition-based approximate analytical method, to compute the output performance for any combination of the sampled parameters. By repeating this approach for each sample, the output performance distribution can be approximated. Regardless of the chosen solution method using uncertain parameters, the following main procedure in Figure 3 is proposed to be used iteratively until the desired degree of certainty is achieved on the output performance measures.

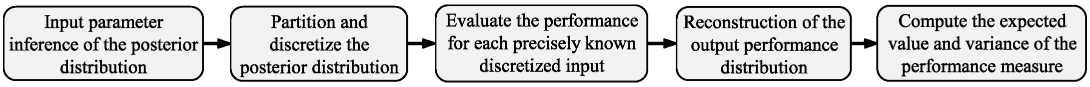

Figure 3. Main steps of the proposed performance evaluation technique

Once the distribution of the uncertain parameters is inferred as in Section 4, the evaluation method used in this paper is a numerical approach based on partitioning and discretization of the input distributions. This approach is far computationally efficient because it reduces the number of evaluations required compared to Monte Carlo approach which relies on random sampling. Exact approaches are only used in order to demonstrate important proofs on the characteristics of using uncertain input parameters.

5.1. *Partitioning and Discretization*

The partitioning and discretization method aims to transform the estimated density function of an input parameter into an equivalent probability mass function. This procedure requires the definition of a sufficient number of partitions T_u for a predetermined level of accuracy, for each uncertain parameter Q_u, with $u = 1, ..., U$. For consistency, all the uncertain parameters, $p_{i,j}$ and $r_{i,j}$, for each station M_i, are mapped in a convenient order to the generic parameter $Q_u, u = 1, ..., U$. The lower and upper limit $q_{u,min}$ and $q_{u,max}$ are determined to enclose an area approximately equal to 1 under the original *pdf*, $f_{Qu}(q_u)$. The partition width Δq_u with equal widths can be calculated as:

$$\Delta q_u = \frac{q_{u,max} - q_{u,min}}{T_u} \tag{6}$$

If x_0 corresponds to the lower limit $q_{u,min}$, and x_{T_u} corresponds to the upper limit $q_{u,max}$, each partition bound x_{t_u} is obtained as:

$$x_{t_u} = q_{u,min} + t_u.\Delta q_u, t_u = 1, ..., T_u \tag{7}$$

The area of each partition is represented as trapezoidal areas that approximately fit to the upper profile of the original density function. Therefore, the weight, w_{t_u}, of each partition and its corresponding centroid value, $\hat{q}_u(t_u)$, are then approximately computed as follows, for $t_u = 1, ..., T_u$:

$$w_{t_u} = \frac{f^{q_u}(x_{t_u-1}) + f^{q_u}(x_{t_u})}{2} \Delta q_u \tag{8}$$

$$\hat{q}_u(t_u) = x_{t_u-1} - \frac{\Delta q_u \Big(2.f^{q_u}(x_{t_u-1}) + f^{q_u}(x_{t_u}) \Big)}{3.f^{q_u}(x_{t_u}) + f^{q_u}(x_{t_u-1})} \Delta q_u \tag{9}$$

This procedure transforms the probability density function of each uncertain parameter into an equivalent probability mass function (pmf), as shown in Figure 4.

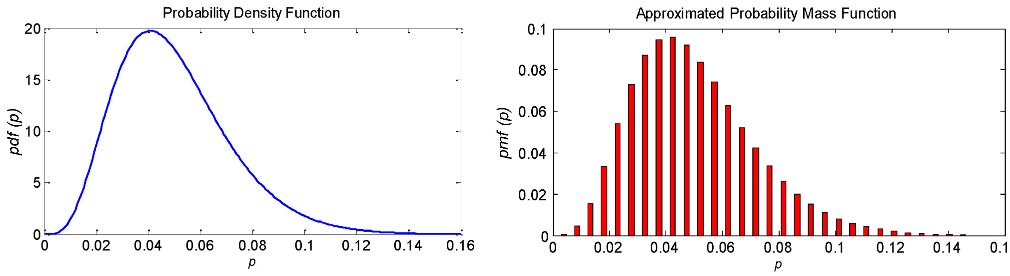

Figure 4. Probability density function of an uncertain parameter and its equivalent probability mass function.

After computing the probability mass function of all the U uncertain input parameters, then the joint probability mass function the U, $u = 1, ..., U$ independent parameters is derived as follows.

$$P(Q_1 = \hat{q}_1(t_1), Q_2 = \hat{q}_2(t_2), ..., Q_U = \hat{q}_U(t_U)) = \prod_{u=1}^{U} \hat{q}_u(t_u) \tag{10}$$

By considering each value corresponding to the joint probability mass function of all the U random variables, a set of $(T_u)^U$ experiments with precisely known inputs are generated. Then each of these experiments are evaluated using the method described in Section 5.2.

5.2. *Performance evaluation with precisely known input*

The individual experiments specified in the probability mass function involve the performance evaluation of the transfer line using a decomposition based approximate analytical method, to calculate the main performance measures under precisely known input settings. In this method, the original transfer line is decomposed into a set of *K-1* sub-systems *l(i)*, named building blocks. These are composed of two pseudo-machines, $M^u(i)$ and $M^d(i)$, and one buffer, $B(i)$. Building blocks are easy to solve because of their lower complexity compared to that of the original system. The coherence of building blocks is made possible by the definition of decomposition equations that establish proper relationships among them. According to the decomposition logic, all the interruptions of the material flow entering (leaving) the buffer $B(i)$ are modeled by the pseudo-machines $M^u(i)$ and $M^d(i)$, including starvation (blocking) events. For each building block performance measures, such as the average throughput of the building block ($TH(i)$) and the average buffer level ($n(i)$) are evaluated. The parameters of these pseudo-machines are iteratively updated by considering the performance of the neighboring sub-systems by decomposition equations, until convergence is met. The decomposition equations for our system assumptions are provided in Tolio et al. (2002). Upon convergence, the performance measures average throughput (TH), and average buffer level (n) of the system is computed as follows:

$$\begin{aligned} TH &= TH(i); \\ n &= \sum_{i=1}^{K-1} n(i) \end{aligned} \tag{11}$$

5.3. *Output performance distribution evaluation*

With the objective of reconstructing the overall performance measure distribution, the following procedure is adopted (here reported only for the average throughput). For each experiment featuring a specific combination of realizations $(t_1, t_2, ..., t_U)$ of the U input uncertain parameters, the throughput and the weight of the combination is computed as follows:

$$TH(t_1, t_2, ..., t_U) = TH(\hat{q}_1(t_1), \hat{q}_2(t_2), ..., \hat{q}_u(t_U)) \tag{12}$$

$$w(t_1, t_2, ..., t_U) = \prod_{u=1}^{U} w_{t_u} \tag{13}$$

By using these weights, the throughput distribution can be easily reconstructed. Moreover, interesting statistics can be computed from this distribution, such as the mean and the variance of the average throughput estimate. This technique can be generally applied to other performance measures and different performance analysis problems involving uncertain inputs, provided that a performance evaluation method for the precisely known inputs is available.

6. Numerical Results

This section presents main findings obtained using the proposed method. The first part focuses on the accuracy testing of the discretization approach against exact numerical solutions. The following sections investigate the important characteristics found on the output performance measurement by considering input uncertainty against traditional results which do not take this aspect into account.

6.1. *Accuracy testing*

The overall accuracy of the proposed method depends on the accuracy of the analytical method and the discretization technique, which can be considered as a controlled error. The error from the discretization process in turn depends on the number of partitions and the type of discretization technique used. For example, increasing the number of partitions decreases the error, which gives a higher accuracy; however this increases the computational time required for evaluating the distribution of output performance. Thus, several experiments are conducted in order to measure the impacts of different number of partitions on the accuracy of the method.

The input parameter ranges for the experiments are specified based on frequently observed machine reliability and maintenance data from previous case studies (Blischke et al. 2003) and from the literature (Gershwin 1994). The test parameters are randomly generated within the following ranges, with respect to their first two moments, i.e., mean and variance. The mean failure probabilities $E[p]$ are varied between 0.0001 and 0.25, with variance ranges varying between 10^{-7} and 0.05; mean repair probabilities $E[r]$ are varied between 0.01 and 0.75, with variance ranges between 10^{-6} and 0.08. To measure the errors arising from the proposed method, the output evaluation results are compared against exact integration (*Integr*) techniques by considering systems with different sizes. These experiments are repeated using different number of discretization partitions, to evaluate the output accuracy. The accuracy of the method (*Approx*) is measured and reported as:

$$\varepsilon_{method} = \frac{\theta_{Approx} - \theta_{Integr}}{\theta_{Integr}} \times 100\% \tag{14}$$

By considering the trade-off between the accuracy and the computation times, (Figure 5) the number of partitions is limited to 30, which gives a satisfactory accuracy of the output performance distribution in a short time. To demonstrate the accuracy of the proposed method with respect to the exact numerical techniques by considering isolated machines. The average of the absolute value of errors obtained on the first two moments of the sampled 500 experimental cases, i.e., $E[e]$ and $var[e]$ are reported in (Figure 5).

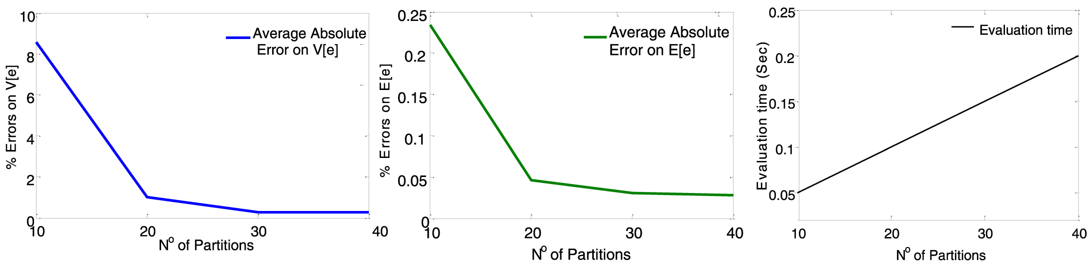

Figure 5. Average errors on E[e], var[e] and corresponding computation time

Under these experimental settings, the maximum error on the efficiency of isolated machine cases is 0.43% on the E[e], and 4.6% on the var[e]. In 95% of the tests the error is below 0.2% for E[e] and below 2.5% for var[e].

The same approach is extended to measure the accuracy of the method on buffered transfer lines lines. The inter-operational buffer capacities are varied between 3 and 30. The number of machines in the transfer line is varied from 2 to 10 machines and

500 cases are randomly generated and tested. The maximum error reported for all the test cases is 0.65% on the mean throughput estimate, and 5.2% on the variance of the throughput estimate. In 95% of the tests the error is below 0.25% for $E[\text{TH}]$ and below 3% for $var[\text{TH}]$.

6.2. *Impact of uncertainty on performance measurement*

This section investigates the impact of considering input parameters' uncertainty on the system performance evaluation results, in comparison to the traditional approaches which might lead to different conclusions. Two main impacts are investigated that has significant practical implications of considering uncertainty in input parameter estimation. The first impact focuses on the differences observed between the expected values of the estimated output performance obtained by using the proposed approach, against other approaches which consider only the expected value of the input parameters as a precise point estimate. The second aim of the proposed method is to provide a quantitative indications of the sources of uncertainties on the output performance measure. This information enables to devise strategies and actions that can improve estimation precision by properly targeting those parameters which highly contribute to output uncertainty.

6.2.1. Impact of input uncertainty on expected value of output

One of the important findings obtained by using the proposed method is, if parameters' uncertainty in the performance evaluation is ignored, a consistent error/deviation is observed on the expected value of the output performance measure. In this section, the evidences demonstrating this behavior, both on isolated machines and transfer line systems are provided, using mathematical proofs and numerical experiments. The direction of this deviation, i.e., overestimation or underestimation, and its magnitude are influenced by the type of system and the uncertain input parameters involved in the analysis.

Single Machine
A total of 1,500 experiments considering isolated machines are carried out, under three experimental conditions. In the first case, 500 experiments are conducted by considering only uncertain p (*Case 1*), and in the second case, additional 500 experiments are run by only considering r as the uncertain parameter (*Case 2*). The remaining 500 experiments are conducted by considering both uncertain p and r (*Case 3*).

In these experiments, the isolated efficiency, e of the machines is the output performance measure of interest. The input parameters are the failure and repair probabilities of the machines, p and r respectively. For each specific experimental condition the isolated efficiency of the machines is evaluated, and the following three specific behaviors have been identified:

Case-I. If p is the only uncertain parameter, and if only the mean value of p, i.e., $\hat{p}$ is considered as the input parameter to evaluate $E[\text{e}]$, then an underestimation error is always committed. This consistent deviation is demonstrated by using the proposed method on the isolated machine experiments. Additionally, a mathematical proof is given confirming the presence of a consistent underestimation. The magnitude of these underestimations are calculated according to the formula given in (14). Underestimation errors for sampled 50 cases are reported in Figure 6. The entire results from the

500 experiments showing these deviations errors are summarized into a histogram and is reported in Figure 7 (a). Finally, a simplified graphical interpretation of the this deviation is provided in Figure 8.

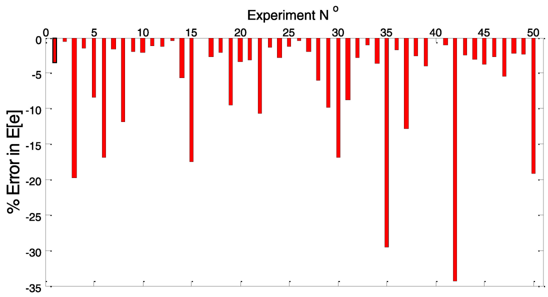

Figure 6. Underestimation errors on E[e] by ignoring uncertainty in p (Case-I).

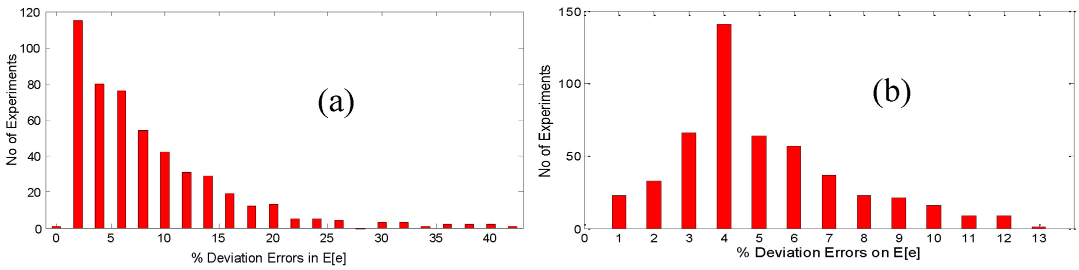

Figure 7. Frequency distributions of underestimation errors in E[e] by ignoring uncertainty in p (a) and overestimation errors in E[e] in r (b).

Furthermore, the existence of the consistent underestimation is proven by comparing the true expected value of E[e], using an exact mathematical solution (left hand side of (15)) against the traditional approach (right hand side of (15)). A mathematical verification of this behavior is given in the Appendix, Proof 1.

$$E[e] = \Gamma(\alpha_p, \beta_p)_2F1(1, \alpha_p, \alpha_p + \beta_p, -1/r) > \frac{r}{r + \hat{p}} \tag{15}$$

Where Γ is the incomplete Gamma function and ${}_2F1$ is a regularized Hypergeometric function; and $\hat{p}$ is given by (5).

A simple graphical interpretation of the overestimations committed on E[e] of an isolated machine when uncertainty in p is not considered, i.e., (Case-I) is given in Figure 8. This graph shows the isolated efficiency function e against p plotted in (B), while on the bottom-right (A) shows the density function of input p. The top-left graph (C) shows the projections of output performance under the two approaches (i.e., the traditional and the proposed approach). When uncertainty is not considered (traditional approach), the red line in (A) shows the mean value of $\hat{p}$ and then the function (B) is used to get a point estimate of the output performance on (C) indicated also by a red line. On the other hand, the proposed approach which considers uncertainty, takes the distribution of p on graph (A) and projects it to the density function of $f_E(e)$ on (C) in blue. From (C) it can be observed that the true expectation of E[e] is higher than the result obtained by only using the point estimate of $E[p]$. This error is consistent as long as the function in (B) is convex. Applying the same graphical approach to the Case-II, i.e., a consistent underestimation by considering uncertain r as a precisely known parameter can be verified.

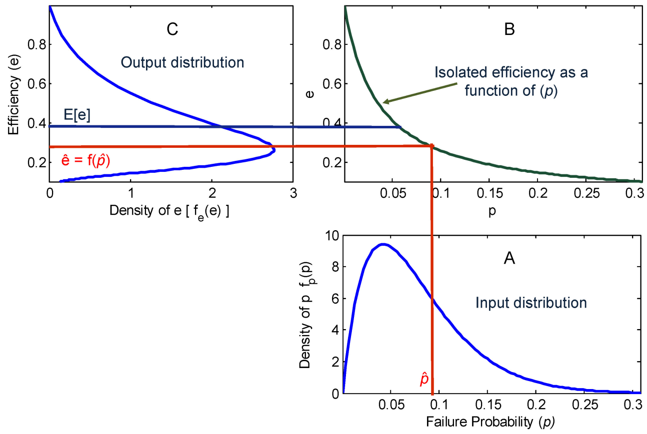

Figure 8. Underestimation errors using the traditional estimation on *E[e]* when p is uncertain (Case-I)

Case-II. In this case, p is assumed to be precisely known, and only r is the uncertain parameter. If the uncertainty in r is ignored, by considering only the mean value of r, i.e., $\hat{r}$ as a deterministic input to evaluate $E[\mathrm{e}]$, then an overestimation error is always caused. This property is proved similarly. Overestimation errors for sampled 50 cases are reported in (Figure 9). Histogram of the 500 experiments conducted against the deviation errors is reported in Figure 7 (b). In this case too the consistent overestimation is demonstrated by comparing the exact analytical solution of the proposed method against the traditional approach (16). A mathematical verification of this behavior is again given in the Appendix, Proof 2.

$$E[e] = 1/p(\alpha_r + \beta_r)\Gamma(\alpha_r + \beta_r)_2F1(1, \alpha_r + 1, \alpha_r + \beta_r + 1, -1/p) < \frac{\hat{r}}{\hat{r} + p} \tag{16}$$

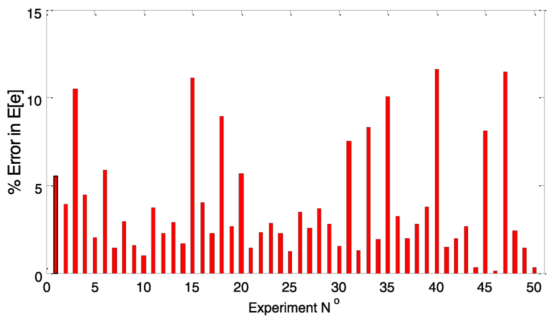

Figure 9. Overestimation errors on E[e] by ignoring uncertainty in r (Case-II).

The summary of all results in the histogram shows that the magnitude of these deviations can be significant when uncertainty in estimation is ignored. As it can be seen from Figure 7, errors greater than 5% on the expected average efficiency $E[\mathrm{e}]$ of the machines are reported in more than 50.5% of the cases. In certain cases, errors as high as 40.5% are recorded.

Case-III. When both p and r are uncertain, the expected throughput can be either overestimated or underestimated, depending on the input parameter settings. However, a consistent deviation is always found. This behavior is demonstrated using sampled numerical experiments in Figure 10. Particularly, this underlines the importance of considering uncertainty in parameters when the nature of the bias error cannot be generalized.

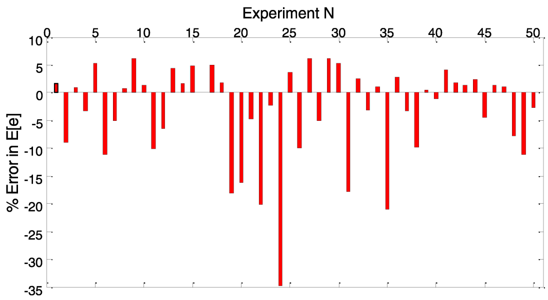

Figure 10. Performance estimation errors using the traditional estimation on E[e] with uncertain p and r

Multi-stage Line

Similarly, the output performance deviations which are exhibited in the cases of isolated machines are also observed in multi-stage manufacturing systems. In this section, experimental tests confirming this behaviour in buffered two-machine line systems are presented (Figure 11). Thus, the impacts of machine reliability parameters' uncertainty on the expected value of the average throughput E[TH] are studied. For this purpose, two sets of experiments are designed; the first set composed of 50 tests consider cases with uncertain failure probabilities of the two machines, i.e., (p_1 and p_2). The second set, which is composed of 50 tests considers uncertain repair probabilities (r_1 and r_2).

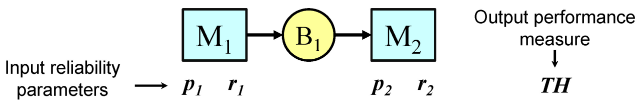

Figure 11. A buffered two-machine line system with unreliable machines

The parameters of the two-machine line systems, including failure and repair probabilities and buffer capacities, are chosen according to the experimental setup specified in Section 6.1. For the first set of experiments, with uncertain p_1 and p_2 the observed deviations are reported in Figure 12 (a), which shows a consistent underestimation of the expected TH. This behavior mirrors the effects observed in the Case-I of the isolated machine experiments. By considering the throughput (TH) of the two-machine line system as a function of the failure probabilities p_1 and p_2, a similar graphical interpretation provided in Figure 8 is also valid for the consistent bias.

In the second set of experiments, i.e., when the uncertainty in the estimation of the repair probabilities (r_1 and r_2) are ignored for both the upstream and downstream machines, there is a consistent overestimation as shown in Figure 12(b). These properties are also mathematically proven for buffered two-machines lines; however, the detailed mathematical proof is out of the scope of this paper.

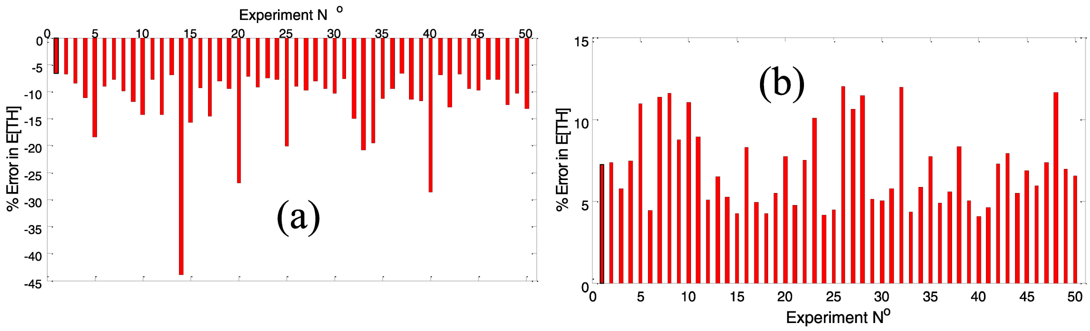

Figure 12. Underestimation errors on $E[\mathrm{TH}]$ when uncertainty in p_1 and p_2 are not considered (a) and Overestimation errors when uncertainty in r_1 and r_2 are ignored (b) in buffered two-machine lines.

6.2.2. Contribution of input uncertainty on output uncertainty

Experiments in the previous sections demonstrate the relevance of methods that consider input parameters' uncertainty to address potential deviations on the expected value of the output performance. However, the importance of similar methods is not only limited to address this deviation. These methods can also provide a quantitative measure of the uncertainty on the output performance measures. In statistics, the second moment of a parameter, i.e., variance is considered as an indicator for the magnitude of uncertainty, which can be used to build prediction confidence intervals. The variance is calculated by using the discretized probability mass function for the input parameters, and from the output distribution for the output performance measure. Furthermore, the relationship between input parameter uncertainty and output performance uncertainty, helps to understand the robustness of the analysis and to devise strategies that can improve the output estimation accuracy. This requires measuring the proportions of the uncertainty contributed by each input parameter on the output uncertainty.

Mathematically, it can be demonstrated that by reducing uncertainties in the input parameters reduces the output performance uncertainty. However, the degree of contribution by each input parameter uncertainty on the total output uncertainty is not investigated and it is the main interest of this section. In order to consider multiple parameters, multi-stage systems with two machine and ten machine lines are investigated. Here, a Taylor approximation technique is used to measure the individual contribution of uncertainty by each uncertain parameter q_u. This approximation technique evaluates the output performance uncertainty $var[TH]_{(q_u)}$ as a function of the input parameter $var[q_u]$. By considering each parameter q_u once at a time, the contribution by a given uncertain parameter q_u is evaluated by keeping the rest of the parameters at their expected value (17). The process can be repeated for all U input uncertain parameters. $var[TH]_{(q_u)}$ can be evaluated using the discretization approach introduced (6-13).

$$var[TH]_{(q_u)} = var[TH|q_1 = \hat{q}_1, ..., q_i = \hat{q}_i, ..., q_U = \hat{q}_U](\forall i = 1, ..., U \wedge i \neq u) \quad (17)$$

The total uncertainty of the output performance measure var[TH], can be approximated as the sum of individual contributions (18).

$$var[TH] = \sum_{u=1}^{U} var[TH]_{q_u} \quad (18)$$

The analysis provides an indication of the percentage of input uncertainty c_u, contributed by a parameter q_u on the total output performance uncertainty var[TH]. This relationship can be used to improve the precision of output performance estimation by devising uncertainty reduction strategies for input parameters with higher c_u on var[TH]. In the following, two experiments are presented highlighting the implication of this analysis on uncertainty reduction on output performance. The first experiment regards a buffered two-machine line, with two uncertain parameters featuring different levels of uncertainty. The second experiment analyzes, a buffered 10 machine line with identical uncertain parameters.

Two Machine Line

This experiment considers a buffered two-machine line (similar to the one depicted in Figure 11), with two uncertain parameters with different uncertainty level. The failure probabilities of the upstream machine p_1 and downstream machine p_2 are assumed to be uncertain, while repair probabilities r_1 and r_2 are considered as precisely known parameters. The goal is to evaluate the contribution of uncertainty by the two input parameters on the uncertainty of the output performance measure, i.e., TH. The inputs parameters used in the analysis are given in (Table 1).

Table 1. Input parameters for the considered two machine line

Machine	α_p	β_p	$\hat{p}$	var[p]	$\hat{r}$	$\hat{e}$	N
Upstream (M_1)	9	599	0.01480	2.395E-05	0.23	0.9395	27
Downstream (M_2)	3	896	0.00334	3.695E-06	0.02	0.8570	

From Table 1, the uncertainties of p_1 and p_2 measured in variance, has ratio of 6.5:1. Therefore, looking at the parameters independently, p_1 is highly uncertain compared to p_2. One interesting question to address is thus, "which of these two parameters should get more attention by data acquisition efforts to reduce the total uncertainty on the throughput TH?". The proposed method is applied and the total uncertainty of the average throughput of the two machines, and then the individual uncertainty contributions of the parameters are computed. The results are reported in Table 2.

Table 2. Output results from the analysis of the two machine line

E[TH]	var$[TH]_{(p_1)}$	var$[TH]_{(p_2)}$	Var[TH]
0.843269992	0.000135	0.003142	0.003277
Percentage contribution	4.11%	95.89%	100%

The results from 2 demonstrate that the percentage contribution by p_2 is 95.9%, while that of p_1 is 4.1%. Therefore more effort should go to reduce $var[p_2]$ instead of $var[p_1]$ for an improved estimation of TH. On the contrary, if only the uncertainty of the inputs parameters are independently observed, p_2 has smaller uncertainty, and thus it appears less critical than p_1. In this specific experiment, this counter intuitive behavior can be explained by considering the isolated efficiencies of M_1 and M_2 which are 0.9395 and 0.8570 respectively. Since M_2 is the bottleneck machine of the two-machine line system, the impact of parameters and the corresponding uncertainty associated to M_2 highly influence the output performance TH. This is the reason why the uncertainty of p_2 is the one that merits more data gathering effort to further improve the estimation accuracy of the TH. A first order differential analysis proves the importance of focusing on the uncertainty of the bottleneck machine parameters, however this proof is out of the scope of this paper.

Ten Machine Line

In this experiment, a transfer line system composed of 10 identical machines with the same speed and reliability parameters is analyzed. Each machine i is affected by only one type of failure mode p_i. The failure probabilities are identical and assumed to be uncertain, while the repair probabilities r_i are precisely known. The input parameters used for the experiment are reported in Table 3. Here, the aim is to investigate how these identical and equally uncertain parameters contribute to the uncertainty of the output performance measure TH.

Table 3. Input parameters for the 10 machine line

Machine	α_p	β_p	$\hat{p}$	var[p]	$\hat{r}$	N
i	5	355	0.0139	3.794E-05	0.25	10

By applying the proposed method, the following results are obtained on the uncertainty of the TH and the individual contribution of each p_i in Table 4.

Table 4. Contribution of uncertainty in the ten machine line

var[TH]$_{(pi)}$ Variance contribution by each failure probabilities										var[TH] Total uncertainty
p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	
3.0E-05	1.2E-05	2.3E-05	3.4E-05	5.1E-05	5.1E-05	3.4E-05	2.3E-05	1.2E-05	3.0E-06	2.47E-04
1.26%	4.92%	9.20%	13.96%	20.69%	20.69%	13.96%	9.20%	4.92%	1.26%	100%

The evaluation results show that higher contributions on the TH uncertainty arise from the machines that are located at the middle of the line (Figure 13), compared to the machines that are located at the beginning or at end of the line. Therefore, despite each parameter being identical, if the goal is to determine a data acquisition scheme that reduces the total uncertainty primary focus should go to the machines located at the middle. For instance, the ratio of uncertainty contribution of p_1 to p_5 is approximately (1:16), which implicates the relative proportion of estimation efforts.

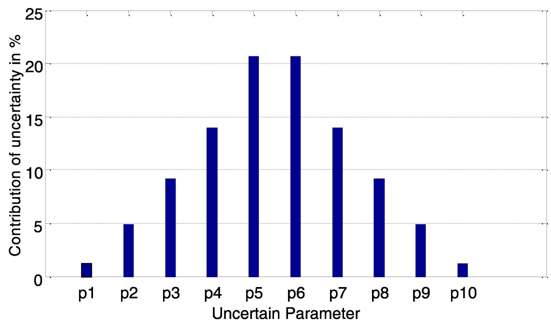

Figure 13. Contribution of uncertainty by the failure probabilities of each machine.

7. Case Study

A real manufacturing line for the production of engine blocks is studied to investigate the impacts of introducing parameter estimation uncertainty in the performance evaluation of a real manufacturing system (Colledani et al. 2010). This line is composed of 22 machines in series separated by intermediate buffers. The analysis focuses on investigating the impact of performance analysis using actual data collected about the 22 machines that are affected by a total of 144 failures. Nine months of data concerning failure and repair occurrences were collected, which is equivalent to an average duration of 10,000,000 cycle times. This raw data is summarized into equivalent input parameters which are approximated by a single unique failure mode for each machine. The approximated input parameters which are reported in Table 5.

Table 5. Machines and the approximated moments of input parameters of the SCANIA line

M ID	N	peq	req	$\sigma^2 peq$	$\sigma^2 req$	α_p	β_p	α_r	β_r
OP20		0.1130	0.1671	7.41E-07	1.10E-06	15287.29	120012.70	21229.64	105816.00
OP30	7	0.0903	0.1694	1.09E-06	2.05E-06	6790.76	68403.22	11629.26	57028.87
OP40	9	0.0743	0.1885	3.16E-07	8.01E-07	16184.52	201571.20	35978.15	154928.20
OP50	4	0.0979	0.1855	3.82E-07	7.24E-07	22644.06	208682.00	38750.53	170098.20
OP60	4	0.0978	0.1781	7.62E-07	1.39E-06	11313.91	104427.70	18778.67	86654.32
OP70	3	0.0287	0.1293	6.47E-07	2.92E-06	1236.75	41845.85	4995.21	33623.27
OP80	3	0.1068	0.1812	5.67E-07	9.62E-07	17972.56	150318.10	27949.34	126329.70
OP90	18	0.1182	0.1743	6.94E-07	1.02E-06	17753.28	132432.30	24517.32	116107.00
OP100	5	0.1003	0.1703	1.11E-06	1.89E-06	8145.71	73062.85	12755.71	62132.17
OP110	4	0.0812	0.1689	2.70E-06	5.61E-06	2245.40	25417.20	4225.81	20795.92
OP120	5	0.1031	0.1534	6.80E-06	1.01E-05	1402.42	12199.27	1969.41	10869.61
OP130	5	0.0604	0.1244	1.33E-05	2.75E-05	256.57	3994.02	492.83	3467.87
OP140	4	0.0483	0.1430	5.81E-07	1.72E-06	3829.89	75399.19	10205.56	61138.72
OP150	34	0.0607	0.1783	1.06E-06	3.11E-06	3272.11	50644.46	8411.01	38752.34
OP160	6	0.1067	0.1788	7.35E-07	1.23E-06	13830.08	115793.70	21305.01	97857.15
OP170	4	0.0547	0.1748	3.72E-06	1.19E-05	760.18	13136.91	2120.46	10011.03
OP180	3	0.0853	0.1919	2.79E-07	6.27E-07	23882.58	256054.50	47461.95	199851.60
OP190	5	0.1098	0.1756	8.39E-07	1.34E-06	12799.15	103749.30	18956.31	88975.23
OP200	3	0.0636	0.1347	9.18E-06	1.95E-05	412.05	6069.85	806.60	5183.11
OP210	3	0.0613	0.1472	1.19E-07	2.85E-07	29693.58	454705.20	64793.14	375257.60
OP220	3	0.0013	0.0367	2.63E-08	7.70E-07	59.66	47549.86	1685.24	44234.54
OP230	4	0.0688	0.1815	4.65E-07	1.23E-06	9462.80	128142.30	21948.50	99003.71

The average throughput of the line is evaluated and the results from the method are reported for a 95% confidence interval in Table 6. Results indicate that in spite of the significantly high amount of data used (9 months), the width corresponding to 95% confidence interval is approximately equal to 5% of the average throughput. Considering this fact the 2.4% difference between the actually observed production and the estimated is not a surprising outcome relative to the estimation uncertainty. On the other hand, if the estimation were to be based solely on expected values there is no quantitative way to measure the reliability of the results obtained from the analytical results. The second goal of this analysis is the contribution of uncertainty by each machine which leads to a targeted effort on data gathering and acquisition effort for an improved estimation. This reveals from the total of 22 machines only 4 machines has contributed for approximately 90% of the total uncertainty.

Table 6. Evaluation results of the 22 machine line

Data estimation	Traditional approach TH	Expected average throughput E[TH]	Uncertainty of throughput V[TH]	95% Confidence interval
0.37535	0.3668	0.3696	1.265E-05	0.3803-0.3625

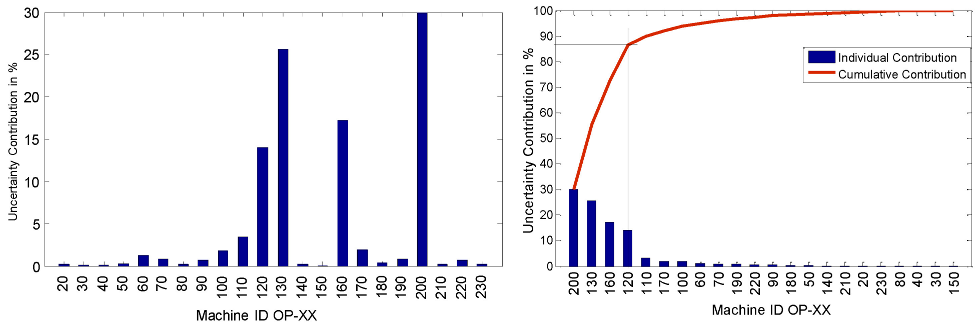

Figure 14. Individual and cumulative uncertainty contribution of Scania line.

8. Conclusion

A performance evaluation method for the analysis of manufacturing systems with uncertain parameter estimates is proposed. The method is proved to be accurate in estimating the distribution of important output performance measures. The analysis performed using the proposed method by considering uncertainty in manufacturing system analysis is compared in contrast with traditionally adopted assumptions. In this regard, if uncertainty in parameters estimates is neglected three fundamental categories of risks are observed; consistent underestimation, consistent overestimation and non-conclusive cases where either underestimation or overestimation might be committed. There is high risk associated in neglecting these errors since the magnitude of underestimation or overestimation in the expected value of the performance measure in some cases is shown to be as high as 40%.

Practical implications of considering uncertainty in operational manufacturing systems are highlighted. Decisions that are affected by considering input uncertainty are demonstrated using isolated machines and multistage lines. Introducing the concept of robust performance analysis towards input parameter uncertainty, experiments have shown that decisions which are solely based on expected value to be less reliable in achieving target performance measures. The proposed method provides a framework for optimal uncertainty reduction to achieve a target confidence level on output performance. Future studies will focus on extending the application to a wider set of complex systems design issues and improving the computational efficiency of the proposed method.

9. Appendix

Proof 1: $E[e(p)] \geq e(E[p])$ for a single unreliable machine. The efficiency (throughput) of a single isolated machine is:

$$e = \frac{r}{p+r}$$

Assuming r is deterministic and p is known with uncertainty, this can be written in the 2^{nd} order Taylor approximation of the throughput as:

$$E[e(p)] = e(E[p]) + \frac{1}{2}e''(E[p])\sigma_p^2 + Remainder$$

Where:
$e''(p) = \frac{2r}{(p+r)^3} > Remainder \geq 0 \forall p, r$

Therefore $E[e(p)] \geq e(E[p])$ always holds.

Proof 2: $E[e(r)] \leq e(E[r])$ for a single unreliable machine. The efficiency (throughput) of a single isolated machine is again:

$e = \frac{r}{p+r}$

Assuming p is deterministic and r is known with uncertainty, this can be written in the 2^{nd} order Taylor approximation of the throughput as:

$E[e(r)] = e(E[r]) + 1/2e''(E[r])\sigma_r^2 + Remainder$

Where:
$e''(r) = -\frac{2p}{(p+r)^3} < Remainder \geq 0 \forall p, r$

Therefore $E[e(p)] \leq e(E[p])$ always holds.

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