Finite-Time Event-Triggered $\mathcal{H}_\infty$ Control for T-S Fuzzy Markov Jump Systems

Hao Shen, Feng Li, Hamid Reza Karimi, *Senior Member, IEEE,* and Hak-Keung Lam, *Senior Member, IEEE*

Abstract—This paper investigates the finite-time event-triggered $\mathcal{H}_\infty$ control problem for Takagi-Sugeno Markov jump fuzzy systems. Because of the sampling behaviors and the effect of the network environment, the premise variables considered in this paper are subject to asynchronous constraints. The aim of this work is to synthesize a controller via an event-triggered communication scheme such that not only the resulting closed-loop system is finite-time bounded and satisfies a prescribed $\mathcal{H}_\infty$ performance level, but also the communication burden is reduced. Firstly, a sufficient condition is established for the finite-time bounded $\mathcal{H}_\infty$ performance analysis of the closed-loop fuzzy system with fully considering the asynchronous premises. Then, based on the derived condition, the desired controller design method is presented. Finally, two illustrative examples are given to explain the practicability and availability of the designed controller.

Index Terms—Markov jump fuzzy systems, finite-time control, event-triggered $\mathcal{H}_\infty$ control, asynchronous premises.

I. INTRODUCTION

IN the last several decades, Markov jump systems (MJSs) have developed into an actively investigated topic in the control and system community. This is due mainly to the fact that many practical systems experiencing random changes in their parameters or structures are very appropriately described by MJSs. A Markov jump system is composed of many interconnected subsystems, and the information exchange among these subsystems is governed by the Markov chain or Markov process. Hence, MJSs could be regarded as a particular class of hybrid stochastic systems. The typical applicable examples for such systems can be found in a broad range of areas, such as networked control systems (NCSs), economic systems and power systems [1], [4], [26]. In this context, therefore, a large number of researchers have endeavored to investigate MJSs and quantities of related results have been acquired, see, e.g., [6], [11], [13], [16], [22], [37], [38]. However, it is recognized that many obtained research results are just available for the linear MJSs owing to the inherent complex characteristics of nonlinear MJSs. Thanks to the presented Takagi-Sugeno (T-S) fuzzy model [5], [8], [9], [19], [24], [31], [41], such complex

nonlinear MJSs could be successfully represented in an easy-to-investigate form. As a consequence, it is not surprising that the past years have witnessed a surge of research interest in the study of nonlinear MJSs by using the fuzzy-model-based approach. For details on research advances about this issue, we refer readers to [3], [20], [28] and the references therein.

It is worth noting that all the aforementioned results related to the primary stability analysis and control issues for MJSs, especially for fuzzy Markov jump systems (FMJSs), are concentrated on Lyapunov asymptotic stability (LAS). The LAS, as is known, is mainly described as the infinite time interval asymptotic behavior of the state trajectories. Actually, it would not be suitable in some scenarios, where the control objective is to make sure that during a fixed time-interval, the states of system keep in assigned threshold values. For example, the orbital control of a space vehicle from an initial position to the object region was discussed in [2]. Therefore, the finite-time stability emerged from the far-reaching work of [7] has received extensive attention during the past decades, such as finite-time stability and stabilization [2], finite-time filtering and state estimation [40], finite-time control [12], to name a few. Despite the significance of the finite-time stability has been acknowledged, there appears little progress toward on this issue for FMJSs, which is one of the motivations behind this work.

On the other hand, as an irresistible trend, in modern control systems, the data among the sensors, controllers, and actuators are frequently transmitted through a shared communication network. The insertion of the network can offer many advantages including reduced equipment weight, lower deployment costs and easy maintenance. Such benefits have given a tremendous amount of motivations to extensive applications of NCSs [10], [21], [29], [32], [39]. It should be remarkable that the communication bandwidth of the shared communication network, as a scarce resource in NCSs, is usually limited. In the widely used time-triggered control, all the sampled data (SD) are required to be transmitted and the actuator state is adjusted at each sampling instant (SI). However, when the states of the controlled plant have arrived or are close to the equilibrium point and no exogenous disturbance input exists, there is little effect of the sensor measurement signal on system performances over those specific time interval. In this case, some *unnecessary* SD may be generated. Undoubtedly, transmitting those *unnecessary* SD reduces the efficiency of communication and leads to *unnecessary* energy consumptions. To cope with this problem, the event-triggered communication scheme (ETCS), as an alternative communication scheme, has been introduced to overcome the shortcomings

of the time-triggered control strategy. Such a communication scheme can efficiently utilize the communication bandwidth since that the transmissions of the SD between the controller and the plant occur only when a predefined event triggering condition is satisfied rather than periodically as the case of traditional setups. Up to now, a growing number of works have been devoted to this topic [23]. To mention a few, to reduce communication burdens, Wen et al. used an event-triggered control method to study the load frequency control for multi-area power systems in [30]; a discrete event-triggered scheme was proposed to investigate the $\mathcal{H}_\infty$ controller design for linear NCSs in [33] and then expanded to the case of networked T-S fuzzy systems (NTSFSs) in [18]. It should be mentioned that how to use such a method to deal with the $\mathcal{H}_\infty$ controller design for FMJSs in the sense of finite-time stability is still challenging.

Moreover, many works reported in the area of NTSFSs are on the basis of the assumption that the controller/filter completely shares the same premise variables with plant [25]. However, such an assumption cannot be satisfied in NTSFSs because of the sampling behaviors and the effect of network environment. Owing to this reason, the asynchronous premises have been taken into consideration when concerning with the modeling, analysis and control of NTSFSs. For instance, the stability analysis was carried out in [15], the event-triggered $\mathcal{H}_\infty$ filtering issue was considered in [36], and the event-triggered control problem was resolved in [18]. To the best of our knowledge, however, there are no solutions available to cope with the finite-time event-triggered $\mathcal{H}_\infty$ control problems for FMJSs when such an asynchronous constraint is fully considered, and accordingly seeking satisfactory solutions to this problem is the another primary motivation of the present study.

Summarizing the above discussions, in this paper we are interested in addressing the finite-time event-triggered $\mathcal{H}_\infty$ control problem for T-S FMJSs. The main contributions of this work lie in the following three aspects: 1) The ETCS, which covers the traditional time-triggered communication scheme (TTCS) as a special case by choosing some appropriate scalars (i.e., setting $\sigma_l \equiv 0$ in (5)), is employed to address the finite-time $\mathcal{H}_\infty$ control problem for T-S FMJSs as a first attempt. 2) Different from some previous works in NTSFSs where the controller/filter completely shares the same premise variables with the plant, the asynchronous premises are here taken into account here. Due to the effect of the network environment, considering the asynchronous premises in NTSFSs is more appropriate with the actual situation than considering the synchronous ones. 3) Some auxiliary function-based integral inequalities are established, that provide much tighter bounds than Jensen inequality and the Wirtinger-based inequality. With the help of those inequalities, some double integral terms could be reserved and less conservative results can be obtained, consequently. The rest of this paper is organized as follows. Section II formulates the problem. The design of the finite-time event-triggered $\mathcal{H}_\infty$ controller for the considered FMJSs is carried out in Section III. The effectiveness and superiority of our developed scheme are demonstrated via two examples in Section IV. Section V concludes this paper, finally.

Notations. In this paper, the notations employed are fairly standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; for symmetric matrices $\mathbf{M}$, the notation $\mathbf{M} \geq 0$ (respectively, $\mathbf{M} > 0$) means that the matrix $\mathbf{M}$ is positive semidefinite (respectively, positive definite); $\lambda_{\min}(\mathbf{P})$ (respectively, $\lambda_{\max}(\mathbf{P})$) represents the smallest (respectively, largest) eigenvalue of symmetric matrix $\mathbf{P}$; $\mathcal{E}\{\cdot\}$ denotes the expectation operator; $\mathrm{Sym}\{\mathbf{M}\}$ represents $\mathbf{M}+\mathbf{M}^T$ and $\otimes$ denotes Kronecker product; 0 and I stand for, respectively, the zero matrix and identity matrix with appropriate dimensions. The transpose of the matrix $\mathbf{M}$ is represented by the notation $\mathbf{M}^T$. In complex matrix expressions or symmetric block matrices, we use an asterisk $(*)$ to describe a term which is induced by symmetry.

II. PROBLEM FORMULATION

Fixing a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, consider the following nonlinear Markov jump system represented by a T-S fuzzy model:

Plant Rule i: IF $\vartheta_1(t)$ is μ_{i1}, and $\vartheta_2(t)$ is μ_{i2}, and $\cdots$, and $\vartheta_g(t)$ is μ_{ig}, THEN

$$\dot{x}(t) = A_i(\delta(t))x(t) + B_i(\delta(t))u(t) + C_i(\delta(t))\omega(t), \tag{1}$$

$$z(t) = D_i(\delta(t))x(t) + F_i(\delta(t))u(t), \tag{2}$$

where the scalar $i = 1, 2, \cdots, r$, r is the number of IF-THEN rules of the system; $\mu_{ij}(i = 1, 2, \cdots, r; j=1, 2, \cdots, g)$ present the fuzzy sets and $\vartheta_j(t)(j=1, 2, \cdots, g)$ are the premise variables. $x(t) \in \mathbb{R}^n$ and $z(t) \in \mathbb{R}^p$ are the system state vector and system output vector, respectively. $\omega(t) \in \mathbb{R}^q$ is the exogenous disturbance input that belongs to $\mathcal{L}_2[0, \infty)$ and subjects to $\int_0^{t_p} \omega^T(t)\omega(t)\,dt \leqslant \hat{\omega}$. The parameter $\delta(t)$, which takes discrete values in a finite state space $\mathbb{M} = \{1, 2, \cdots, \bar{m}\}$, is a right continuous homogeneous Markov process and the jumping of $\delta(t)$ satisfies the transition probability matrix (TPM) $\Pi \triangleq \{\pi_{lm}\}$ determined by

$$\begin{aligned} &\Pr\{\delta(t + \Delta t) = m \,|\, \delta(t) = l\} \\ &= \begin{cases} \pi_{lm}\Delta t + o(\Delta t), & l \neq m, \\ 1 + \pi_{ll}\Delta t + o(\Delta t), & l = m. \end{cases} \end{aligned}$$

The dynamics of the fuzzy system model (1)-(2) can be derived as follows:

$$\dot{x}(t) = \sum_{i=1}^r h_i(\vartheta(t))[A_{il}x(t) + B_{il}u(t) + C_{il}\omega(t)], \tag{3}$$

$$z(t) = \sum_{i=1}^r h_i(\vartheta(t))[D_{il}x(t) + F_{il}u(t)], \tag{4}$$

where $h_i(\vartheta(t)) = \frac{\Pi_{j=1}^g \mu_{ij}(\vartheta_j(t))}{\sum_{i=1}^r \Pi_{j=1}^g \mu_{ij}(\vartheta_j(t))}$ is the fuzzy basis function, in which $\mu_{ij}(\vartheta_j(t))$ is defined as the grade of membership of $\vartheta_j(t)$ in μ_{ij}. For each $\delta(t) = l \in \mathbb{M}$, A_{il}, B_{il}, C_{il}, D_{il} and F_{il} are denoted as $A_{il} \triangleq A_i(\delta(t))$, $B_{il} \triangleq B_i(\delta(t))$, $C_{il} \triangleq C_i(\delta(t))$, $D_{il} \triangleq D_i(\delta(t))$, $F_{il} \triangleq F_i(\delta(t))$, respectively, to simplify the notation. In this work, it is denoted that $\Pi_{j=1}^g \mu_{ij}(\vartheta_j(t)) \geqslant 0$, $\sum_{i=1}^r \Pi_{j=1}^g \mu_{ij}(\vartheta_j(t)) > 0$. Thus, we

have $h_i(\vartheta(t)) \geqslant 0 \ (i = 1, 2, \cdots, r)$ and $\sum_{i=1}^{r} h_i(\vartheta(t)) = 1$ for all t.

In this paper, inspired by the work in [33], the event-triggered mechanism is that all the transmitted SD are time-stamped and only when the following condition is unsatisfied, the SD are transmitted to the controller:

$$e_k^T(i_k h) \Omega_l e_k(i_k h) < \sigma_l x^T(i_k h) \Omega_l x(i_k h), \qquad (5)$$

where $e_k(i_k h)$ is the error between the state $x(i_k h)$ at the current SI and the state $x(t_k h)$ at the latest transmitted SI, i.e., $e_k(i_k h) = x(i_k h) - x(t_k h)$. For $l \in \mathbb{M}$, σ_l are given positive scalars and Ω_l are symmetric positive matrices to be determined later.

Remark 1: According to (5), one can see that the next release time $t_{k+1} h$ is determined by

$$t_{k+1} h = t_k h + \min_{m \geqslant 1} \left\{ mh \middle| \bar{e}_k^T \Omega_l \bar{e}_k \geqslant \sigma_l x^T(i_k h) \Omega_l x(i_k h) \right\},$$

where $\bar{e}_k^T = x(t_k h + mh) - x(t_k h)$. Therefore, the transmit interval is $t_{k+1} h - t_k h$ and the SD obtained by sensors during the time interval $(t_k h, t_{k+1} h)$ is not transmitted to the controller completely, which means the ETCS can reduce the bandwidth utilization.

Remark 2: It is easy to see that if we set $\sigma_l \equiv 0$, the condition (5) cannot be satisfied all the time. In other words, all the SD are transmitted to the controller. In this case, the ETCS degenerates to the case of TTCS.

Before further theoretical development, in this paper, some necessary assumptions are given below.

Assumption 1: [18] In the communication network, the sensors are assumed to be time-triggered with the trigger period h. Meanwhile, the controller and actuators are event-triggered.

Assumption 2: The total network-induced delay considered in this paper is $\tilde{\tau}_{t_k}$, which is bounded and satisfies $0 < \tilde{\tau}_{t_k} \leqslant \tilde{\tau}_M$, where $\tilde{\tau}_M$ is a known constant. The holding time of a logic ZOH at the actuator is $t \in \Omega_k = [i_k h + \tilde{\tau}_{t_k}, i_{k+1} h + \tilde{\tau}_{t_{k+1}}]$, where $i_k h + \tilde{\tau}_{t_k}$ is the instant when the control data packet reaches the ZOH.

Under the above assumptions, when $t \in \bar{\Omega}_k = [t_k^n h + \tilde{\tau}_{t_k+n}, t_k^n h + h + \tilde{\tau}_{t_k+n+1})$, the error between the future transmitted SI $i_{k+1} h$ and the current transmitted SI $i_k h$ can be deduced as

$$e_k(t_k^n h) = x(t_k^n h) - x(t_k h),$$

where $t_k^n h = i_k h + nh$ is the SI between the above two states.

Define $\tau(t) = t - t_k^n h$, then the expression of the transmitted state $x(t_k h)$ can be described:

$$x(t_k h) = x(t - \tau(t)) - e_k(t_k^n h). \qquad (6)$$

In light of the above definition about $\tau(t)$, it yields that $\dot{\tau}(t) = 1$ for $t \in \bar{\Omega}_k$ and

$$0 < \tilde{\tau}_k \leqslant \tau(t) \leqslant h + \max\{\tilde{\tau}_k, \tilde{\tau}_{k+1}\} = h + \tilde{\tau}_M \triangleq \tau_M.$$

In this study, the fuzzy-model based state feedback controllers are designed as

Control Rule i: IF $\vartheta_1(t_k h)$ is μ_{i1}, and $\vartheta_2(t_k h)$ is μ_{i2}, and $\cdots$, and $\vartheta_g(t_k h)$ is μ_{ig}, THEN

$$u(t) = K_i x(t_k h), t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}),$$

where K_i are the desired controller gains to be determined for each $i = 1, 2, \cdots, r$. Then, the overall fuzzy controller can be represented by

$$u(t) = \sum_{i=1}^{r} h_i(\vartheta(t_k h)) K_i x(t_k h). \qquad (7)$$

Remark 3: Most of the existing works in the area of NTSFSs contain the implicit assumption that the premise variables of fuzzy controller are consistent with the fuzzy model. Then the fuzzy controller is chosen as $u(t) = \sum_{i=1}^{r} h_i(\vartheta(t)) K_i x(t_k h)$. However, due to the sampling behaviors and the influence of the network environment, the membership functions of the controller and the plant are no longer equivalent in NBTSFSs. For this consideration, in this work, the designed controller is chosen as the representation of (7) rather than $u(t) = \sum_{i=1}^{r} h_i(\vartheta(t)) K_i x(t_k h)$.

Substituting (6) into (7), we can obtain

$$u(t) = \sum_{i=1}^{r} h_j(\vartheta(t_k h)) K_i [x(t - \tau(t)) - e_k(t_k^n h)]. \qquad (8)$$

Inspired by [18], the membership functions are also subject to the following constraints

$$\begin{cases} h_j(\vartheta(t_k h)) = \rho_j h_j(\vartheta(t)), \\ |h_j(\vartheta(t_k h)) - h_j(\vartheta(t))| \leqslant \Delta_j, \end{cases} \qquad (9)$$

where $\rho_j > 0$, $\Delta_j \geqslant 0 \ (j = 1, 2, \ldots, r)$.

Using the same way in [18], on the basis of the asynchronous constraints, it is clear that

$$\varkappa_1^j \triangleq 1 - \frac{\Delta_j}{h_j(\vartheta(t))} \leqslant \rho_j$$

$$\leqslant 1 + \frac{\Delta_j}{h_j(\vartheta(t))} \triangleq \varkappa_2^j, \quad \varkappa_2^j \geqslant 1,$$

which exhibits that

$$\frac{\varkappa_1^i}{\varkappa_2^j} = \frac{\min\{\rho_i\}}{\max\{\rho_j\}} \leqslant \min\left\{\frac{\rho_i}{\rho_j}\right\} \leqslant \frac{\rho_i}{\rho_j}$$

$$\leqslant \max\left\{\frac{\rho_i}{\rho_j}\right\} \leqslant \frac{\max\{\rho_i\}}{\min\{\rho_j\}} = \frac{\varkappa_2^i}{\varkappa_1^j}.$$

Denoting $\varkappa_2 = \max_{j=1,2,\ldots r}\left\{\varkappa_2^j\right\}$, $\varkappa_1 = \min_{j=1,2,\ldots r}\left\{\varkappa_1^j\right\}$ and $\phi = \frac{\varkappa_2}{\varkappa_1}$, we can get that

$$\frac{1}{\phi} \leqslant \frac{\rho_i}{\rho_j} \leqslant \phi, \quad \phi \geqslant 1.$$

Remark 4: For asynchronous constraints (9), it is not difficult to find that when $\phi = 1$, $\max_{j=1,2,\ldots r}\left\{\varkappa_2^j\right\} = \varkappa_2 = \varkappa_1 = \min_{j=1,2,\ldots r}\left\{\varkappa_1^j\right\}$, which means $\rho_j \equiv 1$. In other words, the membership functions of a fuzzy controller are always the same with the fuzzy model (1)-(2), i.e. $h_j(\vartheta(t_k h)) = h_j(\vartheta(t))$, which has been widely considered in the existing

literature. In this case, the controller is simplified into the general point-to-point connected controller. Thus, the synchronous premise can be deemed as a peculiar case of asynchronous ones, which means that the considered asynchronous premise for NTSFSs is more general in this paper.

Combining (1)-(2), (8) and (9), the closed-loop fuzzy system (CLFS) $\left(\tilde{\Sigma}\right)$ can be formulated as

$$\dot{x}(t) = \mathcal{A}_l x(t) + \mathcal{B}_l x(t - \tau(t)) - \mathcal{B}_l e(t_k^n h) + \mathcal{C}_l \omega(t) \quad (10)$$
$$z(t) = \mathcal{D}_l x(t) + \mathcal{F}_l x(t - \tau(t)) - \mathcal{F}_l e(t_k^n h), \quad (11)$$
$$x(t) = \varphi(t), t \in [-\tau_M, 0], \quad (12)$$

where

$$\mathcal{A}_l \triangleq \sum_{i=1}^r h_i A_{il}, \quad \mathcal{B}_l \triangleq \sum_{i=1}^r \sum_{j=1}^r \rho_j h_i h_j B_{il} K_j,$$

$$\mathcal{C}_l \triangleq \sum_{i=1}^r h_i C_{il}, \quad \mathcal{D}_l \triangleq \sum_{i=1}^r h_i D_{il},$$

$$\mathcal{F}_l \triangleq \sum_{i=1}^r \sum_{j=1}^r \rho_j h_i h_j F_{il} K_j.$$

To describe the objective of this paper, the following lemmas and definitions are introduced at first.

Lemma 1: [17] For any vectors ζ_1, ζ_2, matrices $\Upsilon > 0$, F, and real scalars $k_1 > 0$, $k_2 > 0$ satisfying $k_1 + k_2 = 1$, the following inequality holds:

$$-\frac{1}{k_1}\zeta_1^T \Upsilon \zeta_1 - \frac{1}{k_2}\zeta_2^T \Upsilon \zeta_2 \leqslant - \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}^T \begin{bmatrix} \Upsilon & F \\ * & \Upsilon \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix},$$

subject to

$$\begin{bmatrix} \Upsilon & F \\ * & \Upsilon \end{bmatrix} > 0.$$

Lemma 2: [35] The following condition:

$$\begin{cases} \Delta + \vartheta \Delta_1 < 0 \\ \Delta + \vartheta \Delta_2 < 0 \end{cases}$$

are equivalent to the inequality as follows

$$\Delta + \epsilon \Delta_1 + (\vartheta - \epsilon)\Delta_2 < 0,$$

where constant matrices Δ, Δ_1 and Δ_2 are with appropriate dimensions, $\epsilon \in [0, \vartheta]$ and $\vartheta > 0$.

Definition 1: [12] Given two fixed scalars $a_1 > 0$, $a_2 > 0$ and a positive matrix $R > 0$, system $\left(\tilde{\Sigma}\right)$ with $\omega(t) \equiv 0$ is of finite-time stable (FTS) with regard to (a_1, a_2, t_p, R), if the following condition holds

$$\sup_{-\tau_M \leqslant s \leqslant 0} \mathcal{E}\left\{x^T(s) R x(s), \dot{x}^T(s) R \dot{x}(s)\right\} \leqslant a_1$$
$$\Longrightarrow \mathcal{E}\left\{x^T(t) R x(t)\right\} \leqslant a_2, \quad \forall t \in [0, t_p].$$

Definition 2: [12] Given three fixed scalars $a_1 > 0$, $a_2 > 0$, $\hat{\omega} > 0$ and a positive matrix $R > 0$, system $\left(\tilde{\Sigma}\right)$ is of finite-time bounded (FTB) with regard to $(a_1, a_2, t_p, R, \hat{\omega})$, if the following condition holds

$$\begin{cases} \sup_{-\tau_M \leqslant s \leqslant 0} \mathcal{E}\left\{x^T(s) R x(s), \dot{x}^T(s) R \dot{x}(s)\right\} \leqslant a_1 \\ \int_0^{t_p} \omega^T(t)\omega(t) dt \leqslant \hat{\omega} \end{cases}$$
$$\Longrightarrow \mathcal{E}\left\{x^T(t) R x(t)\right\} \leqslant a_2, \quad \forall t \in [0, t_p].$$

Definition 3: [12] Given three fixed scalars $a_1 > 0$, $a_2 > 0$, $\hat{\omega} > 0$ and a positive matrix $R > 0$, system $\left(\tilde{\Sigma}\right)$ is of FTB with regard to $(a_1, a_2, t_p, R, \hat{\omega})$ and satisfies a prescribed $\mathcal{H}_\infty$ performance level γ, if (a) system $\left(\tilde{\Sigma}\right)$ is of FTB with regard to $(a_1, a_2, t_p, R, \hat{\omega})$ in Definition 2; (b) under zero initial state and for any non-zero $\omega(t) \in \mathcal{L}_2[0, \infty)$, the following condition is ensured:

$$\mathcal{E}\left\{\int_0^{t_p} z^T(t) z(t) dt\right\} < \gamma^2 \mathcal{E}\left\{\int_0^{t_p} \omega^T(t)\omega(t) dt\right\}.$$

With those definitions, the issue to be addressed in this work could be formulated as follows: design a controller of form (7) for the given system (1)-(2) such that the CLFS (10)-(12) is FTB with regard to $(a_1, a_2, t_p, R, \hat{\omega})$ and satisfies the given $\mathcal{H}_\infty$ performance level γ in Definition 3.

III. MAIN RESULTS

For the addressed problem in this paper, this section will develop a controller design method. Firstly, a FTB $\mathcal{H}_\infty$ performance analysis criterion for the CLFS (10)-(12) is established. For presentation convenience, throughout this paper we denote the following functions for $\varsigma = 1, 2$, $\kappa = 1, 2, \ldots, 12$ and any matrix $\mathcal{X}$

$$\varrho_\kappa^T \triangleq \begin{bmatrix} \underbrace{0 \quad \cdots \quad 0}_{\kappa-1} & I & \underbrace{0 \quad \cdots \quad 0}_{12-\kappa} \end{bmatrix},$$

$$\Gamma \triangleq \begin{bmatrix} \Gamma_1 & \Gamma_2 \end{bmatrix}, \Xi(\mathcal{X}) \triangleq \text{diag}\{\mathcal{X}, 3\mathcal{X}, 5\mathcal{X}\},$$

$$\Gamma_\varsigma \triangleq \begin{bmatrix} (\varrho_{2\varsigma-1} - \varrho_{\varsigma+2}) & (\varrho_{2\varsigma-1} + \varrho_{\varsigma+2} - 2\varrho_{\varsigma+4}) \\ (\varrho_{2\varsigma-1} - \varrho_{\varsigma+2} + 6\varrho_{\varsigma+4} - 12\varrho_{\varsigma+6}) \end{bmatrix},$$

$$f_\varsigma(\mathcal{X}) \triangleq 2(\varrho_{2\varsigma-1} - \varrho_{\varsigma+4})\mathcal{X}(\varrho_{2\varsigma-1} - \varrho_{\varsigma+4})^T$$
$$-4(\varrho_{2\varsigma-1} + 2\varrho_{\varsigma+4} - 6\varrho_{\varsigma+6})\mathcal{X}(\varrho_{2\varsigma-1} + 2\varrho_{\varsigma+4} - 6\varrho_{\varsigma+6})^T,$$

$$g_\varsigma(\mathcal{X}) \triangleq 2(\varrho_{\varsigma+2} - \varrho_{\varsigma+4})\mathcal{X}(\varrho_{\varsigma+2} - \varrho_{\varsigma+4})^T$$
$$-4(\varrho_{\varsigma+2} - 4\varrho_{\varsigma+4} + 6\varrho_{\varsigma+6})\mathcal{X}(\varrho_{\varsigma+2} - 4\varrho_{\varsigma+4} + 6\varrho_{\varsigma+6})^T,$$

$$\mathcal{I} \triangleq \text{diag}\{I, I\},$$

and denote the following functions, which will be used in Theorem 1 in the sequel,

$$\Theta_{1ijl} \triangleq \varrho_1 \left(\sum_{m \in \mathbb{M}} \pi_{lm} P_{1m}\right) \varrho_1^T - \lambda \left[\varrho_1 P_{1l}\varrho_1^T + \varrho_{11} P_3 \varrho_{11}^T\right]$$
$$+ Sym\left\{\varrho_1 P_{1l}\varrho_2^T + \varrho_1 P_2(\varrho_1 - \varrho_4)^T + \lambda\varrho_1 P_2\varrho_{11}^T\right\}$$
$$- \frac{2\lambda}{\tau_M^2}\varrho_{12} T_1 \varrho_{12}^T + \varrho_1(Q_{1l} + \tau_M T_1)\varrho_1^T - \varrho_4 Q_{1l}\varrho_4^T$$
$$+ \frac{\tau_M^2}{2}\varrho_2(2Q_{2l} + Q_{3l} + Q_{4l} + T_2 + \tau_M T_3)\varrho_2^T$$
$$- \sum_{\varsigma=1,2}\left[f_\varsigma(\Phi_{2l}) + g_\varsigma(Q_{3l}) + \mathcal{U}_{\varsigma ijl}\right] + \sigma_l \varrho_3 \Omega_l \varrho_3^T$$
$$- \Gamma \begin{bmatrix} \Xi(Q_{2l}) & \mathcal{G}_l \\ * & \Xi(Q_{2l}) \end{bmatrix} \Gamma^T - \varrho_9 \Omega_l \varrho_9^T - \tilde{\gamma}^2 \varrho_{10}\varrho_{10}^T,$$

$$\Theta_{2ijl} \triangleq D_{il}\varrho_1^T + F_{il}K_j(\varrho_3 - \varrho_9)^T,$$

$$\Psi_{\varsigma l} \triangleq -3(\varrho_{\varsigma+4} - 2\varrho_{\varsigma+6})\Phi_{1l}(\varrho_{\varsigma+4} - 2\varrho_{\varsigma+6})^T$$

$$-\varrho_{\varsigma+4}\Phi_{1l}\varrho_{\varsigma+4}^T + Sym\left\{\varrho_2 P_2\varrho_{\varsigma+4}^T + (\varrho_1 - \varrho_4)P_3\varrho_{\varsigma+4}^T\right\},$$ where

$$\mathcal{S} \triangleq (1+\tau_M)\max_{l\in\mathbb{M}}\{\lambda_{\max}(\mathcal{P}_l)\} + \tau_M\max_{l\in\mathbb{M}}\{\lambda_{\max}(\mathcal{Q}_{1l})\}$$

$$+\frac{1}{2}\tau_M^3\max_{l\in\mathbb{M}}\{\lambda_{\max}(\mathcal{Q}_{2l})\} + \frac{1}{2}\tau_M^2\lambda_{\max}(\mathcal{Z}_1)$$

$$+\frac{1}{3}\tau_M^3\max_{l\in\mathbb{M}}\{\lambda_{\max}(\mathcal{Q}_{3l})\} + \frac{1}{6}\tau_M^3\max_{l\in\mathbb{M}}\{\lambda_{\max}(\mathcal{Q}_{4l})\}$$

$$+\frac{1}{6}\tau_M^3\lambda_{\max}(\mathcal{Z}_2) + \frac{1}{6}\tau_M^4\lambda_{\max}(\mathcal{Z}_3),$$

with

$$\mathcal{U}_{\varsigma ijl} \triangleq Sym\left\{\varrho_\varsigma Y_\varsigma^T A_{il}\varrho_1^T + \varrho_\varsigma Y_\varsigma^T B_{il}K_j(\varrho_3 - \varrho_9)^T\right\}$$

$$+Sym\left\{\varrho_\varsigma Y_\varsigma^T C_{il}\varrho_{10}^T - \varrho_\varsigma Y_\varsigma^T \varrho_2^T\right\},$$

$$\mathcal{P}_l \triangleq \mathcal{R}^{-\frac{1}{2}}\hat{P}_l\mathcal{R}^{-\frac{1}{2}},$$

$$\mathcal{Z}_\zeta \triangleq R^{-\frac{1}{2}}T_\zeta R^{-\frac{1}{2}}\ (\zeta = 1,2,3),$$

$$\mathcal{Q}_{\upsilon l} \triangleq R^{-\frac{1}{2}}Q_{\upsilon l}R^{-\frac{1}{2}}\ (\upsilon = 1,2,3,4),$$

$$\Phi_{1l} \triangleq \lambda Q_{1l} + T_1 - \sum_{m\in\mathbb{M}}\pi_{lm}Q_{1m}, \mathcal{R} \triangleq \text{diag}\{R,R\},$$

$$\Phi_{2l} \triangleq \lambda\tau_M Q_{2l} + Q_{4l} + T_2 - \tau_M\sum_{m\in\mathbb{M}}\pi_{lm}Q_{2m},$$

$$\Phi_{3l} \triangleq \lambda Q_{4l} + (\lambda\tau_M + 1)T_3 + \lambda T_2 - \sum_{m\in\mathbb{M}}\pi_{lm}Q_{4m},$$

$$\Phi_{4l} \triangleq \lambda Q_{3l} + T_3 - \sum_{m\in\mathbb{M}}\pi_{lm}Q_{3m}.$$

Theorem 1: Given scalars $\lambda > 0$, $\tilde{\gamma} > 0$, $a_1 > 0$, $a_2 > 0$, $\hat{\omega} > 0$, $t_p > 0$, $\tau_M > 0$, $\phi \geqslant 1$, $\sigma_l \geqslant 0$ and matrix $R > 0$. Then system (10)-(12) is of FTB with regard to $(a_1, a_2, t_p, R, \hat{\omega})$ with a prescribed $\mathcal{H}_\infty$ performance level $\gamma = \tilde{\gamma}\sqrt{\exp(\lambda t_p)}$, if there exist matrices Ω_l, $\mathcal{G}_l$, symmetric matrices $\hat{P}_l = \begin{bmatrix} P_{1l} & P_2 \\ * & P_3 \end{bmatrix} > 0$, $Q_{1l} > 0$, $Q_{2l} > 0$, $Q_{3l} > 0$, Q_{4l}, $T_1 > 0$, T_2, T_3, satisfying $Q_{4l} + T_2 + \tau_M T_3 > 0$, such that the following conditions hold for each $l \in \mathbb{M}$, $i,j = 1,2,\cdots,r$, $i < j$, $\varpi \in \left\{\phi, \frac{1}{\phi}\right\}$, $\varsigma = 1,2$,

$$\begin{bmatrix} \Theta_{1iil} + \tau_M\Psi_{\varsigma l} & \Theta_{2iil}^T \\ * & -I \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} \Theta_{1ijl} + \varpi\Theta_{1jil} + \tau_M\Psi_{\varsigma l} & \Theta_{2ijl}^T & \sqrt{\phi}\Theta_{2jil}^T \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} \Xi(Q_{2l} + \Phi_{2l}) & \mathcal{G}_l \\ * & \Xi(Q_{2l} + Q_{3l}) \end{bmatrix} > 0, \tag{15}$$

$$\Phi_{\upsilon l} > 0, \quad \upsilon = 1,2,3,4, \tag{16}$$

$$\frac{\exp(\lambda t_p)\mathcal{S}a_1 + \exp(\lambda t_p)\tilde{\gamma}^2\hat{\omega}}{\min_{l\in\mathbb{M}}\{\lambda_{\min}(\mathcal{P}_l)\}} \leqslant a_2. \tag{17}$$

Proof: Choose a stochastic Lyapunov functional for system $\left(\tilde{\Sigma}\right)$ as follows:

$$V(x(t),\delta(t),t) \triangleq \sum_{f=1}^4 V_f(x(t),\delta(t),t), \tag{18}$$

$$V_1(x(t),\delta(t),t) \triangleq \eta^T(t)\hat{P}_l\eta(t),$$

$$\eta(t) \triangleq \begin{bmatrix} x^T(t) & \int_{t-\tau_M}^t x^T(\alpha)\,d\alpha \end{bmatrix}^T,$$

$$V_2(x(t),\delta(t),t) \triangleq \int_{t-\tau_M}^t x^T(\alpha)Q_{1l}x(\alpha)\,d\alpha$$

$$+\int_{-\tau_M}^0\int_{t+\beta}^t x^T(\alpha)T_1 x(\alpha)\,d\alpha d\beta,$$

$$V_3(x(t),\delta(t),t) \triangleq \tau_M\int_{-\tau_M}^0\int_{t+\beta}^t \dot{x}^T(\alpha)Q_{2l}\dot{x}(\alpha)\,d\alpha d\beta$$

$$+\int_{-\tau_M}^0\int_{-\tau_M}^\gamma\int_{t+\beta}^t \dot{x}^T(\alpha)Q_{3l}\dot{x}(\alpha)\,d\alpha d\beta d\gamma,$$

$$V_4(x(t),\delta(t),t) \triangleq \int_{-\tau_M}^0\int_\gamma^0\int_{t+\beta}^t \dot{x}^T(\alpha)$$

$$\times(Q_{4l} + T_2 + \tau_M T_3)\dot{x}(\alpha)\,d\alpha d\beta d\gamma.$$

As operated in [34], we define the weak infinitesimal operator $\mathcal{L}$. Thus, we can get

$$\mathcal{E}\{\mathcal{L}\{\exp(-\lambda t)V(x(t),\delta(t),t)\}\}$$

$$= \mathcal{E}\left\{\xi^T(t)\left\{\exp(-\lambda t)\left[\varrho_1\left(\sum_{m\in\mathbb{M}}\pi_{lm}P_{1m}\right)\varrho_1^T\right.\right.\right.$$

$$+Sym\left\{\varrho_1 P_{1l}\varrho_2^T + \varrho_1 P_2(\varrho_1 - \varrho_4)^T\right\}$$

$$+Sym\left\{\tau(t)\varrho_5\left[P_2^T\varrho_2^T + P_3(\varrho_1 - \varrho_4)^T\right]\right\}$$

$$+Sym\left\{(\tau_M - \tau(t))\varrho_6\left[P_2^T\varrho_2^T + P_3(\varrho_1 - \varrho_4)^T\right]\right\}$$

$$-\lambda\left[\varrho_1 P_{1l}\varrho_1^T + Sym(\varrho_1 P_2\varrho_{11}^T) + \varrho_{11}P_3\varrho_{11}^T\right]$$

$$+\varrho_1(Q_{1l} + \tau_M T_1)\varrho_1^T - \varrho_4(t-\tau_M)Q_{1l}\varrho_4^T$$

$$\left.\left.+ \frac{\tau_M^2}{2}\varrho_2(2Q_{2l} + T_2 + Q_{4l} + Q_{3l} + \tau_M T_3)\varrho_2^T\right]\right\}\xi(t)$$

$$+\exp(-\lambda t)\left[-\int_{t-\tau_M}^{t-\tau(t)}x^T(\alpha)\Phi_{1l}x(\alpha)\,d\alpha\right.$$

$$-\int_{t-\tau(t)}^t x^T(\alpha)\Phi_{1l}x(\alpha)\,d\alpha$$

$$-\lambda\int_{-\tau_M}^0\int_{t+\beta}^t x^T(\alpha)T_1 x(\alpha)\,d\alpha d\beta$$

$$-\tau_M\int_{t-\tau_M}^t \dot{x}^T(\alpha)Q_{2l}\dot{x}(\alpha)\,d\alpha$$

$$-\int_{-\tau(t)}^0\int_{t+\beta}^t \dot{x}^T(\alpha)\Phi_{2l}\dot{x}(\alpha)\,d\alpha d\beta$$

$$-\int_{-\tau_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)} \dot{x}^T(\alpha)\Phi_{2l}\dot{x}(\alpha)\,d\alpha d\beta$$

$$-(\tau_M - \tau(t))\int_{t-\tau(t)}^t \dot{x}^T(\alpha)\Phi_{2l}\dot{x}(\alpha)\,d\alpha$$

$$-\int_{-\tau(t)}^0\int_{t-\tau(t)}^{t+\beta} \dot{x}^T(\alpha)Q_{3l}\dot{x}(\alpha)\,d\alpha d\beta$$

$$-\int_{-\tau_M}^{-\tau(t)}\int_{t-\tau_M}^{t+\beta} \dot{x}^T(\alpha)Q_{3l}\dot{x}(\alpha)\,d\alpha d\beta$$

$$-\tau\left(t\right)\int_{t-\tau_M}^{t-\tau(t)}\dot{x}^T\left(\alpha\right)Q_{3l}\dot{x}\left(\alpha\right)d\alpha$$

$$-\int_{-\tau_M}^{0}\int_{\gamma}^{0}\int_{t+\beta}^{t}\dot{x}^T\left(\alpha\right)\Phi_{3l}\dot{x}\left(\alpha\right)d\alpha d\beta d\gamma$$

$$\left.\left.-\int_{-\tau_M}^{0}\int_{-\tau_M}^{\gamma}\int_{t+\beta}^{t}\dot{x}^T\left(\alpha\right)\Phi_{4l}\dot{x}\left(\alpha\right)d\alpha d\beta d\gamma\right]\right\}, \quad (19)$$

where

$$\xi\left(t\right)\triangleq\left[\begin{array}{cccccc}\xi_1\left(t\right) & \xi_2\left(t\right) & \xi_3\left(t\right) & \xi_4\left(t\right) & \xi_5\left(t\right) & \xi_6\left(t\right)\end{array}\right]^T,$$

$$\xi_1\left(t\right)\triangleq\left[\begin{array}{cccc}x^T\left(t\right) & \dot{x}^T\left(t\right) & x^T\left(t-\tau\left(t\right)\right) & x^T\left(t-\tau_M\right)\end{array}\right],$$

$$\xi_2\left(t\right)\triangleq\left[\begin{array}{cc}c_1\int_{t-\tau(t)}^{t}x^T\left(\alpha\right)d\alpha & c_2\int_{t-\tau_M}^{t-\tau(t)}x^T\left(\alpha\right)d\alpha\end{array}\right],$$

$$c_1\triangleq\frac{1}{\tau\left(t\right)},c_2\triangleq\frac{1}{\tau_M-\tau\left(t\right)},$$

$$\xi_3\left(t\right)\triangleq c_1^2\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}x^T\left(\alpha\right)d\alpha d\beta,$$

$$\xi_4\left(t\right)\triangleq c_2^2\int_{-\tau_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)}x^T\left(\alpha\right)d\alpha d\beta,$$

$$\xi_5\left(t\right)\triangleq\left[\begin{array}{cc}e_k^T\left(t_k^n h\right) & \omega^T\left(t\right)\end{array}\right],$$

$$\xi_6\left(t\right)\triangleq\left[\begin{array}{cc}\int_{t-\tau_M}^{t}x^T\left(\alpha\right)d\alpha & \int_{-\tau_M}^{0}\int_{t+\beta}^{t}x^T\left(\alpha\right)d\alpha d\beta\end{array}\right].$$

From (16), we can see that $\Phi_{1l}>0$, $\Phi_{2l}>0$, $\Phi_{3l}>0$, $\Phi_{4l}>0$. Then applying the inequalities in Lemma 5.1 in [17], it leads to

$$-\int_{t-\tau_M}^{t-\tau(t)}x^T\left(\alpha\right)\Phi_{1l}x\left(\alpha\right)d\alpha\leqslant-\left(\tau_M-\tau\left(t\right)\right)\xi^T\left(t\right)$$
$$\times\left[\varrho_6\Phi_{1l}\varrho_6^T+3\left(\varrho_6-2\varrho_8\right)\Phi_{1l}\left(\varrho_6-2\varrho_8\right)^T\right]\xi\left(t\right), (20)$$

$$-\int_{t-\tau(t)}^{t}x^T\left(\alpha\right)\Phi_{1l}x\left(\alpha\right)d\alpha\leqslant-\tau\left(t\right)\xi^T\left(t\right)$$
$$\times\left[\varrho_5\Phi_{1l}\varrho_5^T+3\left(\varrho_5-2\varrho_7\right)\Phi_{1l}\left(\varrho_5-2\varrho_7\right)^T\right]\xi\left(t\right), (21)$$

$$-\lambda\int_{-\tau_M}^{0}\int_{t+\beta}^{t}x^T\left(\alpha\right)T_1x\left(\alpha\right)d\alpha d\beta$$
$$\leqslant-\frac{2\lambda}{\tau_M^2}\xi^T\left(t\right)\varrho_{12}T_1\varrho_{12}^T\xi\left(t\right), \quad (22)$$

$$-\tau_M\int_{t-\tau_M}^{t-\tau(t)}\dot{x}^T\left(\alpha\right)Q_{2l}\dot{x}\left(\alpha\right)d\alpha$$
$$\leqslant-\frac{\tau_M}{\tau_M-\tau\left(t\right)}\xi^T\left(t\right)\Gamma_2\Xi\left(Q_{2l}\right)\Gamma_2^T\xi\left(t\right), \quad (23)$$

$$-\tau_M\int_{t-\tau(t)}^{t}\dot{x}\left(\alpha\right)Q_{2l}\dot{x}\left(\alpha\right)d\alpha$$
$$\leqslant-\frac{\tau_M}{\tau\left(t\right)}\xi^T\left(t\right)\Gamma_1\Xi\left(Q_{2l}\right)\Gamma_1^T\xi\left(t\right), \quad (24)$$

$$-\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{x}^T\left(\alpha\right)\Phi_{2l}\dot{x}\left(\alpha\right)d\alpha d\beta$$
$$\leqslant-\xi^T\left(t\right)f_1\left(\Phi_{2l}\right)\xi\left(t\right), \quad (25)$$

$$-\int_{-\tau_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)}\dot{x}^T\left(\alpha\right)\Phi_{2l}\dot{x}\left(\alpha\right)d\alpha d\beta$$
$$\leqslant-\xi^T\left(t\right)f_2\left(\Phi_{2l}\right)\xi\left(t\right), \quad (26)$$

$$-\left(\tau_M-\tau\left(t\right)\right)\int_{t-\tau(t)}^{t}\dot{x}^T\left(\alpha\right)\Phi_{2l}\dot{x}\left(\alpha\right)d\alpha$$

$$\leqslant-\left(\frac{\tau_M}{\tau\left(t\right)}-1\right)\xi^T\left(t\right)\Gamma_1\Xi\left(\Phi_{2l}\right)\Gamma_1^T\xi\left(t\right), \quad (27)$$

$$-\int_{-\tau(t)}^{0}\int_{t-\tau(t)}^{t+\beta}\dot{x}^T\left(\alpha\right)Q_{3l}\dot{x}\left(\alpha\right)d\alpha d\beta$$
$$\leqslant-\xi^T\left(t\right)g_1\left(Q_{3l}\right)\xi\left(t\right), \quad (28)$$

$$-\int_{-\tau_M}^{-\tau(t)}\int_{t-\tau_M}^{t+\beta}\dot{x}^T\left(\alpha\right)Q_{3l}\dot{x}\left(\alpha\right)d\alpha d\beta$$
$$\leqslant-\xi^T\left(t\right)g_2\left(Q_{3l}\right)\xi\left(t\right), \quad (29)$$

$$-\tau\left(t\right)\int_{t-\tau_M}^{t-\tau(t)}\dot{x}^T\left(\alpha\right)Q_{3l}\dot{x}\left(\alpha\right)d\alpha$$

$$\leqslant-\left(\frac{\tau_M}{\tau_M-\tau\left(t\right)}-1\right)\xi^T\left(t\right)\Gamma_2\Xi\left(Q_{3l}\right)\Gamma_2^T\xi\left(t\right). \quad (30)$$

Additionally, from (10), for matrices Y_1 and Y_2 with any compatible dimensions the following equality holds:

$$\begin{aligned}0=&Sym\left\{\exp\left(-\lambda t\right)\left[x^T\left(t\right)Y_1^T+\dot{x}^T\left(t\right)Y_2^T\right]\right.\\&\times\left[\mathcal{A}_lx\left(t\right)+\mathcal{B}_lx\left(t-\tau\left(t\right)\right)\right.\\&\left.\left.-\mathcal{B}_le\left(t_kh\right)+\mathcal{C}_l\omega\left(t\right)-\dot{x}\left(t\right)\right]\right\}.\end{aligned} \quad (31)$$

In view of (19)-(31), it follows that

$$\mathcal{E}\left\{\mathcal{L}\left\{\exp\left(-\lambda t\right)V\left(x\left(t\right),\delta\left(t\right),t\right)\right\}\right\}$$
$$\leqslant\mathcal{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\rho_jh_ih_j\xi^T\left(t\right)\exp\left(-\lambda t\right)\left[\Theta_{1ijl}+\Theta_{2ijl}^T\Theta_{2ijl}\right.\right.$$
$$+\tau\left(t\right)\Psi_{1l}+\left(\tau_M-\tau\left(t\right)\right)\Psi_{2l}]\xi\left(t\right)$$
$$\left.-\exp\left(-\lambda t\right)\left[z^T\left(t\right)z\left(t\right)-\tilde{\gamma}^2\omega^T\left(t\right)\omega\left(t\right)\right]\right\}$$
$$=\mathcal{E}\left\{\sum_{i=1}^{r}\sum_{j>i}^{r}\rho_jh_ih_j\xi^T\left(t\right)\left[\exp\left(-\lambda t\right)\left(\Theta_{1ijl}+\frac{\rho_i}{\rho_j}\Theta_{1jil}\right.\right.\right.$$
$$+\Theta_{2ijl}^T\Theta_{2ijl}+\frac{\rho_i}{\rho_j}\Theta_{2jil}^T\Theta_{2jil}$$
$$+\tau\left(t\right)\Psi_{1l}+\left(\tau_M-\tau\left(t\right)\right)\Psi_{2l}]\xi\left(t\right)$$
$$+\sum_{i=1}^{r}\sum_{j=i}^{r}\rho_ih_i^2\xi^T\left(t\right)\left[\exp\left(-\lambda t\right)\left(\Theta_{1iil}+\Theta_{2iil}^T\Theta_{2iil}\right.\right.$$
$$+\tau\left(t\right)\Psi_{1l}+\left(\tau_M-\tau\left(t\right)\right)\Psi_{2l}]\xi\left(t\right)$$
$$\left.-\exp\left(-\lambda t\right)\left[z^T\left(t\right)z\left(t\right)-\tilde{\gamma}^2\omega^T\left(t\right)\omega\left(t\right)\right]\right\}. \quad (32)$$

Using Schur complement and Lemma 2, (13)-(14) imply that

$$0>\Theta_{1iil}+\Theta_{2iil}^T\Theta_{2iil}+\tau\left(t\right)\Psi_{1l}+\left(\tau_M-\tau\left(t\right)\right)\Psi_{2l}, \quad (33)$$

$$0>\Theta_{1ijl}+\frac{1}{\phi}\Theta_{1jil}+\Theta_{2ijl}^T\Theta_{2ijl}+\phi\Theta_{2jil}^T\Theta_{2jil}$$
$$+\tau\left(t\right)\Psi_{1l}+\left(\tau_M-\tau\left(t\right)\right)\Psi_{2l}, \quad (34)$$

$$0>\Theta_{1ijl}+\phi\Theta_{1jil}+\Theta_{2ijl}^T\Theta_{2ijl}+\phi\Theta_{2jil}^T\Theta_{2jil}$$
$$+\tau\left(t\right)\Psi_{1l}+\left(\tau_M-\tau\left(t\right)\right)\Psi_{2l}. \quad (35)$$

When $\phi=1$, as pointed out in Remark 2, the membership functions used in the controller are the same with the fuzzy plant and $\frac{\rho_i}{\rho_j}\equiv1$. In this case, it can be obtained from (34) and (35) that for $i<j$, the following inequality holds:

$$0>\Theta_{1ijl}+\Theta_{1jil}+\Theta_{2ijl}^T\Theta_{2ijl}+\Theta_{2jil}^T\Theta_{2jil}$$
$$+\tau\left(t\right)\Psi_{1l}+\left(\tau_M-\tau\left(t\right)\right)\Psi_{2l}. \quad (36)$$

When $\phi > 1$, define $\varrho_1 = \frac{\phi - \left(\frac{\rho_i}{\rho_j}\right)}{\phi - \frac{1}{\phi}} \geqslant 0$, $\varrho_2 = \frac{\left(\frac{\rho_i}{\rho_j}\right) - \frac{1}{\phi}}{\phi - \frac{1}{\phi}} \geqslant 0$. Since $\phi \geqslant \frac{\rho_i}{\rho_j}$, it could be obtained from (34) and (35) that for $i < j$, the following inequality holds:

$$
\begin{aligned}
0 \;>\; & \varrho_1 \left(\Theta_{1ijl} + \frac{1}{\phi}\Theta_{1jil} + \Theta_{2ijl}^T \Theta_{2ijl} \right. \\
& \left. + \frac{\rho_i}{\rho_j}\Theta_{2jil}^T \Theta_{2jil} + \tau(t)\Psi_{1l} + (\tau_M - \tau(t))\Psi_{2l} \right) \\
& + \varrho_2 \left(\Theta_{1ijl} + \phi\Theta_{1jil} + \Theta_{2ijl}^T \Theta_{2ijl} \right. \\
& \left. + \frac{\rho_i}{\rho_j}\Theta_{2jil}^T \Theta_{2jil} + \tau(t)\Psi_{1l} + (\tau_M - \tau(t))\Psi_{2l} \right),
\end{aligned}
$$

that is,

$$
\begin{aligned}
0 \;>\; & \Theta_{1ijl} + \frac{\rho_i}{\rho_j}\Theta_{1jil} + \Theta_{2ijl}^T\Theta_{2ijl} + \frac{\rho_i}{\rho_j}\Theta_{2jil}^T\Theta_{2jil} \\
& + \tau(t)\Psi_{1l} + (\tau_M - \tau(t))\Psi_{2l}. \tag{37}
\end{aligned}
$$

It follows from (32), (33), (36) and (37) that for $\phi \geqslant 1$, the inequality is established as below:

$$
\begin{aligned}
& \mathcal{E}\{\mathcal{L}\{\exp(-\lambda t) V(x(t), \delta(t), t)\}\} \\
& \leqslant -\exp(-\lambda t)\left[z^T(t)z(t) - \tilde{\gamma}^2\omega^T(t)\omega(t)\right]. \tag{38}
\end{aligned}
$$

Integrating both sides of (38) from 0 to t and taking the expectation, it may be calculated that for each $l \in \mathbb{M}$

$$
\begin{aligned}
& \mathcal{E}\{\exp(-\lambda t) V(x(t), \delta(t), t)\} \\
& \leqslant \mathcal{E}\left\{\int_0^t \exp(-\lambda s)\left[-z^T(s)z(s) + \tilde{\gamma}^2\omega^T(s)\omega(s)\right]ds\right\} \\
& + \mathcal{E}\{V(x(0), \delta(0), 0)\}. \tag{39}
\end{aligned}
$$

Since $\lambda > 0$ and $\exp(-\lambda s) < 0$ for $0 < s < t \leqslant t_p$, it follows from (39) that

$$
\begin{aligned}
0 \;<\; & \mathcal{E}\{V(x(t), \delta(t), t)\} \\
\;<\; & \mathcal{E}\left\{-\exp(\lambda t)\int_0^t z^T(s)z(s)ds\right\} + \exp(\lambda t) \\
& \times \mathcal{E}\left[V(x(0), \delta(0), 0) + \tilde{\gamma}^2\int_0^t \omega^T(s)\omega(s)ds\right] \tag{40}
\end{aligned}
$$

$$
\leqslant \exp(\lambda t_p)\left(a_1 \mathcal{S} + \tilde{\gamma}^2\hat{\omega}\right). \tag{41}
$$

What's more, it is hard to observe from (18) that

$$
\mathcal{E}\{V(x(t), \delta(t), t)\} \geqslant \min_{l \in \mathbb{M}}\{\lambda_{\min}(\mathcal{P}_l)\}\mathcal{E}\{x^T(t)Rx(t)\},
$$

which is combined to (41) and (17) implies that

$$
\mathcal{E}\{x^T(t)Rx(t)\} \leqslant \frac{a_1\exp(\lambda t_p)\mathcal{S} + \exp(\lambda t_p)\tilde{\gamma}^2\hat{\omega}}{\min_{l \in \mathbb{M}}\{\lambda_{\min}(\mathcal{P}_l)\}\mathcal{E}\{x^T(t)Rx(t)\}} \leqslant a_2.
$$

According to Definition 2, the CLFS is of FTB with regard to $(a_1, a_2, t_p, R, \hat{\omega})$, i.e. the condition (a) in Definition 3 is guaranteed.

Moreover, under zero initial condition, it can deduce from (40) that

$$
\begin{aligned}
0 \;<\; & \mathcal{E}\left\{-\exp(\lambda t)\int_0^t z^T(s)z(s)ds \right. \\
& \left. + \exp(\lambda t)\tilde{\gamma}^2\int_0^t \omega^T(s)\omega(s)ds\right\}
\end{aligned}
$$

$$
\begin{aligned}
\;<\; & \mathcal{E}\left\{-\int_0^t z^T(s)z(s)ds \right. \\
& \left. + \exp(\lambda t_p)\tilde{\gamma}^2\int_0^t \omega^T(s)\omega(s)ds\right\},
\end{aligned}
$$

which means that

$$
\mathcal{E}\left\{\int_0^{t_p} z^T(t)z(t)dt\right\} < \gamma^2\mathcal{E}\left\{\int_0^{t_p} \omega^T(t)\omega(t)dt\right\},
$$

where $\gamma = \tilde{\gamma}\sqrt{\exp(\lambda t_p)}$. Here, one can see that the condition (b) in Definition 3 is also ensured. Therefore, in light of Definition 3, the CLFS $\left(\tilde{\Sigma}\right)$ is of FTB with regard to $(a_1, a_2, t_p, R, \hat{\omega})$ and accords with a prescribed $\mathcal{H}_\infty$ performance level γ. This completes the proof.

Remark 5: Theorem 1 provides a bounded real lemma for the resulting CLFS (10)-(12). To reduce the conservatism of the obtained condition, some integral inequalities introduced in Lemma 5.1 in [17], which present tighter upper bounds than those gained by extended Wirtinger inequality or Jensen inequality in [27], are employed in this paper. For the same reason, some double integral terms, e.g., $\frac{1}{\tau^2(t)}\int_{-\tau(t)}^0 \int_{t+\beta}^t x^T(\alpha)d\alpha d\beta$, $\frac{1}{(\tau_M - \tau(t))^2}\int_{-\tau_M}^{-\tau(t)}\int_{t+\beta}^{t-\tau(t)} x^T(\alpha)d\alpha d\beta$ are introduced in the state-augmented vector $\xi(t)$.

Remark 6: Like most of the literature on the analysis of Markov jump delayed systems, some mode-dependent matrices (that is, $\hat{P}_l$, Q_{1l}, Q_{2l}, Q_{3l}, Q_{4l}) are employed in the proposed Lyapunov functional. In the derivation of the weak infinitesimal operator $\mathcal{L}$, one has to ensure that $\Phi_{vl} > 0$, $v = 1, 2, 3, 4$. In the existing results, such a condition (i.e., (16)) was not fully considered in presenting the main conditions (like (13) and (14) in Theorem 1). Since it is easy to see that $-\Phi_{vl} < 0$, $v = 1, 2, 3, 4$, the inequality conditions (13) and (14) can become more feasible by adding some terms including Φ_{vl}, $v = 1, 2, 3, 4$, such as $-\varrho_{\varsigma+4}\Phi_{1l}\varrho_{\varsigma+4}^T$, $-3(\varrho_{\varsigma+4} - 2\varrho_{\varsigma+6})\Phi_{1l}(\varrho_{\varsigma+4} - 2\varrho_{\varsigma+6})^T$, $-f_\varsigma(\Phi_{2l})$.

On the basis of the FTB $\mathcal{H}_\infty$ performance analysis criterion conducted above, we are now in a position to address the design issue of an $\mathcal{H}_\infty$ controller by using an ETCS. The following theorem therefore puts forward the sufficient condition which ensures that the solutions to such an issue are existed. Before presenting the Theorem 2, we first denote the following functions, which will be used in Theorem 2 in the sequel,

$$
\begin{aligned}
\bar{\Theta}_{1ijl} \triangleq\; & \varrho_1\left(\sum_{m \in \mathbb{M}}\pi_{lm}\bar{P}_{1m}\right)\varrho_1^T + Sym\{\varrho_1\bar{P}_{1l}\varrho_2^T\} \\
& + Sym\{\varrho_1\bar{P}_2(\varrho_1 - \varrho_4)^T\} - \lambda\varrho_1\bar{P}_{1l}\varrho_1^T \\
& - \lambda Sym(\varrho_1\bar{P}_2\varrho_{11}^T) - \lambda\varrho_{11}\bar{P}_3\varrho_{11}^T \\
& - \frac{2\lambda}{\tau_M^2}\varrho_{12}\bar{T}_1\varrho_{12}^T + \varrho_1\left(\bar{Q}_{1l} + \tau_M\bar{T}_1\right)\varrho_1^T - \varrho_4\bar{Q}_{1l}\varrho_4^T \\
& + \frac{\tau_M^2}{2}\varrho_2\left(2\bar{Q}_{2l} + \bar{Q}_{3l} + \bar{Q}_{4l} + \bar{T}_2 + \tau_M\bar{T}_3\right)\varrho_2^T \\
& - \sum_{\varsigma=1,2}\left[f_\varsigma(\bar{\Phi}_{2l}) + g_\varsigma(\bar{Q}_{3l}) + \bar{\mathcal{U}}_{\varsigma ijl}\right] + \sigma_l\varrho_3\bar{\Omega}_l\varrho_3^T
\end{aligned}
$$

$$-\Gamma \begin{bmatrix} \Xi\left(\bar{Q}_{2l}\right) & \bar{\mathcal{G}}_l \\ * & \Xi\left(\bar{Q}_{2l}\right) \end{bmatrix} \Gamma^T - \varrho_9 \bar{\Omega}_l \varrho_9^T - \tilde{\gamma}^2 \varrho_{10} \varrho_{10}^T,$$

$$\bar{\Theta}_{2ijl} \triangleq D_{il}\mathcal{V}\varrho_1^T + F_{il}\tilde{K}_j\left(\varrho_3 - \varrho_9\right)^T,$$

$$\bar{\Psi}_{\varsigma l} \triangleq -3\left(\varrho_{\varsigma+4} - 2\varrho_{\varsigma+6}\right)\bar{\Phi}_{1l}\left(\varrho_{\varsigma+4} - 2\varrho_{\varsigma+6}\right)^T$$
$$-\varrho_{\varsigma+4}\bar{\Phi}_{1l}\varrho_{\varsigma+4}^T + Sym\left\{\varrho_2\bar{P}_2\varrho_{\varsigma+4}^T + \left(\varrho_1 - \varrho_4\right)\bar{P}_3\varrho_{\varsigma+4}^T\right\},$$

$$\hat{\mathcal{S}} \triangleq \left(1 + \tau_M\right)\lambda_2 + \tau_M\lambda_3 + \frac{1}{2}\tau_M^3\lambda_4 + \frac{1}{3}\tau_M^3\lambda_5$$
$$+\frac{1}{6}\tau_M^3\lambda_6 + \frac{1}{2}\tau_M^2\lambda_7 + \frac{1}{6}\tau_M^3\lambda_8 + \frac{1}{6}\tau_M^3\lambda_9,$$

$$\bar{\mathcal{U}}_{\varsigma ijl} \triangleq Sym\left\{\theta_\varsigma\varrho_\varsigma A_{il}\mathcal{V}\varrho_1^T + \theta_\varsigma\varrho_\varsigma B_{il}\tilde{K}_j\left(\varrho_3 - \varrho_9\right)^T\right\}$$
$$+Sym\left\{\theta_\varsigma\varrho_\varsigma C_{il}\mathcal{V}\varrho_{10}^T - \theta_\varsigma\varrho_\varsigma\mathcal{V}\varrho_2^T\right\},$$

$$\mathcal{R} \triangleq \mathrm{diag}\left\{R, R\right\}, \mathcal{I} \triangleq \mathrm{diag}\left\{I, I\right\},$$

$$\bar{\Phi}_{1l} \triangleq \lambda\bar{Q}_{1l} + \bar{T}_1 - \sum_{m\in\mathbb{M}}\pi_{lm}\bar{Q}_{1m},$$

$$\bar{\Phi}_{2l} \triangleq \lambda\tau_M\bar{Q}_{2l} + \bar{Q}_{4l} + \bar{T}_2 - \tau_M\sum_{m\in\mathbb{M}}\pi_{lm}\bar{Q}_{2m},$$

$$\bar{\Phi}_{3l} \triangleq \lambda\bar{S}_l + \left(\lambda\tau_M + 1\right)\bar{T}_3 + \lambda\bar{T}_2 - \sum_{m\in\mathbb{M}}\pi_{lm}\bar{Q}_{4m},$$

$$\bar{\Phi}_{4l} \triangleq \lambda\bar{Q}_{3l} + \bar{T}_3 - \sum_{m\in\mathbb{M}}\pi_{lm}\bar{Q}_{3m}.$$

Theorem 2: Given scalars $\lambda > 0$, $\tilde{\gamma} > 0$, $a_1 > 0$, $a_2 > 0$, $\hat{\omega} > 0$, $t_p > 0$, $\tau_M > 0$, $\theta_1, \theta_2, \phi \geqslant 1$, $\sigma_l \geqslant 0$, matrices $\mathcal{J}_1 = \begin{bmatrix} \mathcal{J}_{11} & \mathcal{J}_{12} \\ \mathcal{J}_{13} & \mathcal{J}_{14} \end{bmatrix}$, $\mathcal{J}_{\varphi_1}\left(\varphi_1 = 2, 3, \ldots, 8\right)$ and $R > 0$. Then system (10)-(12) is of FTB with regard to $\left(a_1, a_2, t_p, R, \hat{\omega}\right)$ and accords with a fixed $\mathcal{H}_\infty$ performance level $\gamma = \tilde{\gamma}\sqrt{\exp\left(\lambda t_p\right)}$. The gain matrices of the controller (8) are $K_j = \tilde{K}_j\mathcal{V}^{-1}$ with the event-triggerd parameters $\Omega_l = \mathcal{V}^{-T}\bar{\Omega}_l\mathcal{V}^{-1}$, if there exist scalars $\lambda_{\varphi_2}\left(\varphi_2 = 1, 2, \ldots, 9\right)$, matrices $\tilde{K}_j$, $\bar{\Omega}_l$, $\mathcal{V}$, $\bar{\mathcal{G}}_l$, symmetric matrices $\bar{P}_l = \begin{bmatrix} \bar{P}_{1l} & \bar{P}_2 \\ * & \bar{P}_3 \end{bmatrix} > 0$, $\bar{Q}_{1l} > 0$, $\bar{Q}_{2l} > 0$, $\bar{Q}_{3l} > 0$, $\bar{Q}_{4l}, \bar{T}_1 > 0$, $\bar{T}_2, \bar{T}_3$ satisfying $\bar{Q}_{4l} + \bar{T}_2 + \tau_M\bar{T}_3 > 0$, such that the following LMIs hold for $\varsigma = 1, 2$, $\upsilon = 1, 2, 3, 4$, $\zeta = 1, 2, 3$, each $l \in \mathbb{M}$, $i, j = 1, 2, \cdots, r$, $i < j$, $\varpi \in \left\{\phi, \frac{1}{\phi}\right\}$

$$\begin{bmatrix} \bar{\Theta}_{1iil} + \tau_M\bar{\Psi}_{\varsigma l} & \bar{\Theta}_{2iil}^T \\ * & -I \end{bmatrix} < 0, \quad (42)$$

$$\begin{bmatrix} \bar{\Theta}_{1ijl} + \varpi\bar{\Theta}_{1jil} + \tau_M\bar{\Psi}_{\varsigma l} & \bar{\Theta}_{2ijl}^T & \sqrt{\phi}\bar{\Theta}_{2jil}^T \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (43)$$

$$\begin{bmatrix} \Xi\left(\bar{Q}_{2l} + \bar{\Phi}_{2l}\right) & \bar{\mathcal{G}}_l \\ * & \Xi\left(\bar{Q}_{2l} + \bar{Q}_{3l}\right) \end{bmatrix} > 0, \quad (44)$$

$$\bar{\Phi}_{\upsilon l} > 0, \quad \upsilon = 1, 2, 3, 4, \quad (45)$$

$$\exp\left(\lambda t_p\right)\left(\hat{\mathcal{S}}a_1 + \tilde{\gamma}^2\hat{\omega}\right) < a_2\lambda_1, \quad (46)$$

$$\begin{bmatrix} -\bar{P}_l & \mathcal{V}^T \otimes \mathcal{I} \\ * & -\frac{R^{-1}}{\lambda_1} \otimes \mathcal{I} \end{bmatrix} < 0, \quad (47)$$

$$\begin{bmatrix} -\lambda_2\mathcal{R} & \mathcal{I} \\ * & \mathcal{J}_1\bar{P}_l\mathcal{J}_1^T - Sym\left\{\mathcal{J}_1^T\left(\mathcal{V} \otimes \mathcal{I}\right)\right\} \end{bmatrix} < 0, \quad (48)$$

$$\begin{bmatrix} -\lambda_{\upsilon+2}R & I \\ * & \mathcal{J}_{\upsilon+1}\bar{Q}_{\upsilon l}\mathcal{J}_{\upsilon+1}^T - Sym\left\{\mathcal{J}_{\upsilon+1}\mathcal{V}^T\right\} \end{bmatrix} < 0, \quad (49)$$

$$\begin{bmatrix} -\lambda_{\zeta+6}R & I \\ * & \mathcal{J}_{\zeta+5}\bar{T}_\zeta\mathcal{J}_{\zeta+5}^T - Sym\left\{\mathcal{J}_{\zeta+5}\mathcal{V}^T\right\} \end{bmatrix} < 0. \quad (50)$$

Proof: Define $Y_1 \triangleq \theta_1\mathcal{V}^{-1}$, $Y_2 \triangleq \theta_2\mathcal{V}^{-1}$, $\tilde{K}_j \triangleq K_j\mathcal{V}$, $\bar{V} \triangleq \mathrm{diag}\left\{\mathcal{V}, \mathcal{V}, \mathcal{V}\right\}$, $\bar{P}_l \triangleq \left(\mathcal{V} \otimes \mathcal{I}\right)^T\hat{P}_l\left(\mathcal{V} \otimes \mathcal{I}\right)$, $\bar{\mathcal{G}}_l \triangleq \bar{V}^T\mathcal{G}_l\bar{V}$, $\bar{\Omega}_l \triangleq \mathcal{V}^T\Omega_l\mathcal{V}$, $\bar{Q}_{\upsilon l} \triangleq \mathcal{V}^TQ_{\upsilon l}\mathcal{V}$, $\upsilon = 1, 2, 3, 4$, $\bar{T}_\zeta \triangleq \mathcal{V}^TT_\zeta\mathcal{V}$, $\zeta = 1, 2, 3$, and $\hat{\mathcal{V}} \triangleq \mathrm{diag}\left\{\mathcal{V}, \mathcal{V}, \mathcal{V}, \mathcal{V}, \mathcal{V}, \mathcal{V}, \mathcal{V}, \mathcal{V}, I\right\}$. Then, pre- and post-multiplying (42) by $\mathrm{diag}\left\{\hat{\mathcal{V}}^{-T}, I\right\}$ and its transpose respectively, it is easy to obtain (13). Using the same way, pre- and post-multiplying (43) by $\mathrm{diag}\left\{\hat{\mathcal{V}}^{-T}, I, I\right\}$ and its transpose respectively, (44) by $\mathrm{diag}\left\{\mathcal{V}, \mathcal{V}, \mathcal{V}, \mathcal{V}, \mathcal{V}, \mathcal{V}\right\}^{-T}$ and its transpose, (45) by $\mathrm{diag}\left\{\mathcal{V}, \mathcal{V}, \mathcal{V}, \mathcal{V}\right\}^{-T}$ and its transpose, respectively, we can get (14)-(16).

Using Schur complement, it could be observed from (47) that

$$-\bar{P}_l + \lambda_1\left(\mathcal{V} \otimes \mathcal{I}\right)^T\mathcal{R}\left(\mathcal{V} \otimes \mathcal{I}\right) \leqslant 0. \quad (51)$$

Pre- and post-multiplying (51) by $\left(\mathcal{V} \otimes \mathcal{I}\right)^{-1}$ and its transpose, we can get that $-\hat{P}_l + \lambda_1\mathcal{R} \leqslant 0$, which implies that

$$\min_{l\in\mathbb{M}}\left\{\lambda_{\min}\left(\mathcal{P}_l\right)\right\} \geqslant \lambda_1. \quad (52)$$

Additionally,

$$\mathcal{J}_1\bar{P}_l\mathcal{J}_1^T - Sym\left\{\mathcal{J}_1^T\left(\mathcal{V} \otimes \mathcal{I}\right)\right\}$$
$$\geqslant -\left(\mathcal{V} \otimes \mathcal{I}\right)\bar{P}_l^{-1}\left(\mathcal{V} \otimes \mathcal{I}\right)^T$$
$$= -\left\{\left(\mathcal{V} \otimes \mathcal{I}\right)^{-T}\bar{P}_l\left(\mathcal{V} \otimes \mathcal{I}\right)^{-1}\right\}^{-1}. \quad (53)$$

Combining (48) with (53) and using Schur complement, it follows that $-\lambda_2\mathcal{R} + \left(\mathcal{V} \otimes \mathcal{I}\right)^{-T}\bar{P}_l\left(\mathcal{V} \otimes \mathcal{I}\right)^{-1} \leqslant 0$, that is, $-\lambda_2\mathcal{I} + \mathcal{P}_l \leqslant 0$, which means that

$$\max_{l\in\mathbb{M}}\left\{\lambda_{\max}\left(\mathcal{P}_l\right)\right\} \leqslant \lambda_2. \quad (54)$$

Following a similar line, one can obtain from (49)-(50) that

$$\max_{l\in\mathbb{M}}\left\{\lambda_{\max}\left(\mathcal{Q}_{\upsilon l}\right)\right\} \leqslant \lambda_{\upsilon+2}, \upsilon = 1, 2, 3, 4, \quad (55)$$

$$\lambda_{\max}\left(\mathcal{Z}_\zeta\right) \leqslant \lambda_{\zeta+6}, \zeta = 1, 2, 3. \quad (56)$$

Combining (44), (52) and (54)-(56), one can get (17). This completes the proof.

Remark 7: By means of Theorem 2, the designed method to the desired fuzzy-model-based controller of the form in (7) is presented for the underlying system (1)-(2). In order to convert the design problem into a convex optimization problem readily solved by the standard software, some scalars are introduced in the derivation, such as $\lambda_{\varphi_2}\left(\varphi_2 = 1, 2, \ldots, 9\right)$. A possible way to reduce the conservativeness is to replace these scalars with some matrices, which could be considered as an extension of the paper in the future.

IV. NUMERICAL EXAMPLES

Two examples are given in this section. In the first example, we investigate the relationship of σ_l, ϕ, τ_M, γ and obtain some conclusions. In the second example, a nonlinear mass-spring-damper mechanical system (MSDMS) is provided to explain the effectiveness and practicability of our theoretical results.

Example 1: Consider the FMJSs (1)-(2) with $r = 2$, $\bar{m} = 2$ and the following parameters:

$$A_{11} = \begin{bmatrix} -2.4 & 1.2 \\ -0.9 & -2.25 \end{bmatrix}, A_{12} = \begin{bmatrix} -2.1 & 0.45 \\ 1.95 & 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -1.35 & 0.75 \\ -0.48 & -1.2 \end{bmatrix}, A_{22} = \begin{bmatrix} -1.575 & 1.2 \\ -0.225 & -1.95 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} 0.24 & 0.5 \end{bmatrix}^T, B_{12} = \begin{bmatrix} 0.15 & 0.3 \end{bmatrix}^T,$$

$$B_{21} = \begin{bmatrix} 0.63 & 1.4 \end{bmatrix}^T, B_{22} = \begin{bmatrix} 0.45 & 0.13 \end{bmatrix}^T,$$

$$C_{11} = \begin{bmatrix} 0.5 & 0.4 \\ 0.2 & 0.3 \end{bmatrix}, C_{12} = \begin{bmatrix} 0.1 & 0.6 \\ 0.8 & 0.7 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} 0.15 & 0.32 \\ 0.54 & 0.21 \end{bmatrix}, C_{22} = \begin{bmatrix} 0.16 & 0.23 \\ 0.3 & 0.45 \end{bmatrix},$$

$$D_{11} = D_{12} = \begin{bmatrix} 0 & -0.1 \end{bmatrix}, D_{21} = D_{22} = \begin{bmatrix} 0.1 & 0 \end{bmatrix},$$

$$F_{11} = F_{12} = 0.1, \quad F_{21} = F_{22} = 0.3,$$

$$\pi_{11} = -0.1, \quad \pi_{22} = -0.8.$$

The fuzzy basis functions are given as

$$h_1(x_1(t)) = 1 - \frac{x_1^2(t)}{0.8165^2}, \quad h_2(x_1(t)) = \frac{x_1^2(t)}{0.8165^2},$$

where $x_1(t) \in [-0.8165, 0.8165]$.

Algorithm I: Maximizing τ_M

Step I: Given some values for scalars λ, $\tilde{\gamma}$, a_1, a_2, $\hat{\omega}$, t_p, θ_1, θ_2, ϕ, σ_l, matrices $\mathcal{J}_{\varphi_1}$ $(\varphi_1 = 1, 2, \ldots, 8)$ and R.
Step II: Given an accuracy coefficient $\Delta\tau$ and a search upper bound τ_+ for τ_M. Then let $\tau_- = 0$.
Step III: Let $\tau = \frac{\tau_- + \tau_+}{2}$ and solve the LMIs in Theorem 2.
Step IV: If LMIs in Theorem 2 are feasible and $\tau - \tau_- \leqslant \Delta\tau$, then $\tau_{M\max} = \tau$, **End**; else go to **Step V**.
Step V: If LMIs in Theorem 2 are feasible and $\tau - \tau_- > \Delta\tau$, then let $\tau_- = \tau$ and go back to **Step III**; else go to **Step VI**.
Step VI: If LMIs in Theorem 2 are unfeasible and $\tau - \tau_- > \Delta\tau$, then let $\tau_+ = \tau$ and go back to **Step III**; else let $\tau = \tau_-$ and solve the LMIs in Theorem 2.
Step VII: If LMIs in Theorem 2 are feasible, then $\tau_{M\max} = \tau$, **End**; else restart **Step I** with other values.

Let $\lambda = 0.1$, $\theta_1 = \theta_2 = 1$, $a_1 = 1$, $a_2 = 1.5$, $t_p = 10$, $\hat{\omega} = 0.64$, $\mathcal{J}_{11} = \mathcal{J}_{14} = \text{diag}\{0.1, 0.1\}$, $\mathcal{J}_{12} = \mathcal{J}_{13} = 0$, $\mathcal{J}_{\varphi_1} = \text{diag}\{1, 1\}$ $(\varphi_1 = 2, 3, \ldots, 8)$ and the exogenous disturbance input $\omega(t) = \begin{bmatrix} \frac{\sqrt{0.032}}{1+t^2} & \frac{\sqrt{0.032}}{1+t^2} \end{bmatrix}^T$. Then we can see that $\int_0^{t_p} \omega^T(t)\omega(t)\,dt \leqslant \hat{\omega} = 0.64$. Firstly, applying Theorem 2 with $\tilde{\gamma} = 0.5$, the relationship between τ_M, σ_l $(l = 1, 2)$ and ϕ is analyzed with the aid of Algorithm I. The

$\tau_{M\max}$	$\sigma_l = 0$	$\sigma_l = 0.10$	$\sigma_l = 0.15$	$\sigma_l = 0.25$
$\phi = 1.0$	1.2416	0.6992	0.6143	0.4471
$\phi = 1.5$	0.7609	0.1718	0.0703	0.0123
$\phi = 2.0$	0.7298	0.1246	0.0421	0.0058

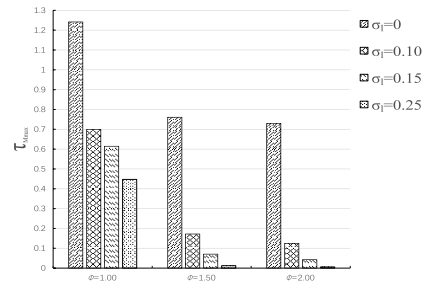| $\gamma_{\min}$	$\sigma_l = 0$	$\sigma_l = 0.10$	$\sigma_l = 0.15$	$\sigma_l = 0.25$
$\phi = 1.0$	0.0239	0.1810	0.2640	0.5492
$\phi = 1.5$	0.1340	0.9014	1.3300	2.5034
$\phi = 2.0$	0.1570	1.0041	1.6495	2.5974

Fig. 1. $\tau_{M\max}$ for different σ_l $(l = 1, 2)$ and ϕ in Example 1.

corresponding computation results are shown in Table I and Fig. 1, respectively. As clearly shown in Table I and Fig. 1, the maximum value of τ_M is decreasing with σ_l or ϕ increasing.

Moreover, using the similar method, the minimum optimal $\mathcal{H}_\infty$ performance γ can be computed by Theorem 2 with $\tau_M = 0.3$ for different σ_l $(l = 1, 2)$ and ϕ, that are presented in Table II and Fig. 2, respectively. From Table II and Fig. 2, it is obvious that the minimum value of γ is increasing with σ_l or ϕ increasing.

Remark 8: It should be pointed out that from Table I and Table II, we can find that the parameter ϕ affects the performance of the system. It indirectly shows that the consideration of asynchronous premise variables is necessary and reasonable.

Remark 9: From Table I and Table II, one can also observe that the maximum value of τ_M and the minimum value of γ are obtained at $\sigma_l = 0$. It is worth noting that when $\sigma_l = 0$, as pointed out in Remark 2, the communication scheme is time-triggered, i.e., all the SD are transmitted to the controller. These facts imply that when the network bandwidth is adequate, the system can acquire better performance by using TTCS rather than ETCS. However, it is worth pointing out that in NCSs, the communication bandwidth is a scarce resource and in this case, the TTCS may be unavailable. Fortunately, the ETCS can be employed to effectively reduce the usage of bandwidth, which will be demonstrated in the later.

Further, fixing $\tau_M = 0.3$, $\sigma_1 = \sigma_2 = 0.3$, $\tilde{\gamma} = 0.7$, $\phi = 1$, the sampling time $h = 0.12$ and the other parameters are
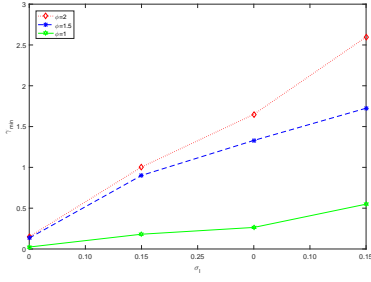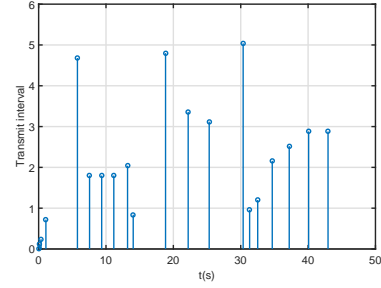
Fig. 2. Minimum value of γ for different σ_l $(l = 1, 2)$ and ϕ in Example 1.

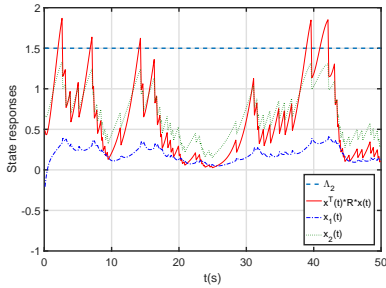Fig. 4. Transmit interval in Example 1.

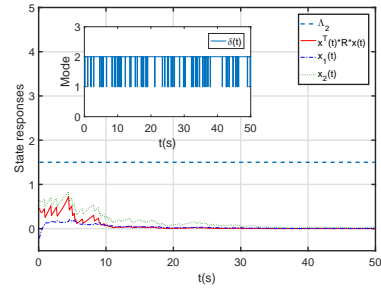Fig. 3. State responses of the open-loop system in Example 1.

Fig. 5. State responses of the CLFS and mode evolution in Example 1.

the same as the former. The corresponding trigger parameters and gain matrices of the controller (8) can be computed by Theorem 2, which are listed as below:

$$\Omega_1 = \begin{bmatrix} 11.5949 & -14.0135 \\ -14.0135 & 30.1296 \end{bmatrix}, K_1 = \begin{bmatrix} 0.2022 \\ -0.4383 \end{bmatrix}^T,$$

$$\Omega_2 = \begin{bmatrix} 68.9458 & -29.6763 \\ -29.6763 & 36.4576 \end{bmatrix}, K_2 = \begin{bmatrix} 0.1740 \\ -0.4112 \end{bmatrix}^T.$$

Firstly, the state responses of the open-loop system in Fig. 3 indicate that the open-loop system is not FTB. Then, under the obtained trigger parameters and gain matrices of the controller (7), the transmit interval is plotted in Fig. 4. The state responses of the CLFS, $x^T(t) Rx(t)$ and possible mode evolution are plotted in Fig. 5, where $x(0) = \begin{bmatrix} -0.4 & 0.8 \end{bmatrix}^T$. From Fig. 4, we can compute that only about 4.8077% of all SD are transmitted to the controller within simulation duration 50s, which testifies that the ETCS can effectively reduce the usage of bandwidth. From Fig. 5, we can see that the CLFS is of FTB, which means that the developed design approach in this work is effective.

Example 2: Consider the following nonlinear MSDMS modified from [14], which is illustrated in Fig. 6.

$$M\ddot{\theta}(t) + D\left(\dot{\theta}(t)\right)\dot{\theta}(t) + k\theta(t) = \psi_1 u(t) + \psi_2 \omega_1(t),$$

where the physical meaning and nominal values of the parameters are given in Table III.

Moreover, in this paper, we assume that k is unknown but have four different modes, i.e. 100%, 102%, 104% and 108% of its nominal values. The switching between four different
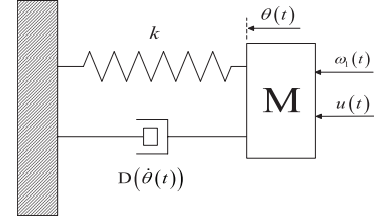

Fig. 6. MSDMS in Example 2.

TABLE III
PHYSICAL MEANING AND NOMINAL VALUES OF THE MSDMS'S PARAMETERS

Physical meaning	Parameter	Nominal value
Relative position of the mass	$\theta(t)$	——
External force	$u(t)$	——
Exogenous disturbance input	$\omega_1(t)$	——
Mass of this system	M	$1kg$
Stiffness of the spring	k	$0.1N/mm$
Input coefficient	ψ_1	1
Exogenous disturbance input coefficient	ψ_2	1
Damping coefficient of the nonlinear damper	$D\left(\dot{\theta}(t)\right)$	$0.5 + 0.75\dot{\theta}^2(t)$

modes is assumed to follow a Markov process with the TPM

Π given as:

$$\Pi = \begin{bmatrix} -0.3 & 0.15 & 0.05 & 0.1 \\ 0.1 & -0.2 & 0.05 & 0.05 \\ 0.03 & 0.04 & -0.1 & 0.03 \\ 0.3 & 0.1 & 0.4 & -0.8 \end{bmatrix}.$$

Choosing the augmented state as $x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T = \begin{bmatrix} \dot\theta(t) & \theta(t) \end{bmatrix}^T$, a state-space representation of the MSDMS can be yielded as follows:

$$\dot x_1(t) = \left(-0.75 x_1^2(t) - 0.5\right) x_1(t) + u(t)$$
$$\qquad\qquad -0.1\left(1 + \mu(\delta(t))\right) x_2(t) + \omega_1(t), \quad (57)$$
$$\dot x_2(t) = x_1(t), \qquad\qquad\qquad\qquad\qquad (58)$$

where

$$\mu(\delta(t) = 1) = 0.0, \quad \mu(\delta(t) = 2) = 0.02,$$
$$\mu(\delta(t) = 3) = 0.04, \quad \mu(\delta(t) = 4) = 0.08.$$

Choosing $\omega(t)$ as $\omega(t) = \begin{bmatrix} \omega_1(t) & \omega_1(t) \end{bmatrix}^T$ and by adopting the same fuzzy sets used in [14], the system (57)-(58) can be reconstructed as follows:

Plant rule 1: IF $x_1(t)$ is about Γ_1, THEN

$$\dot x(t) = A_1(\delta(t)) x(t) + B_1(\delta(t)) u(t) + C_1(\delta(t))\omega(t),$$
$$z(t) = D_1(\delta(t)) x(t) + F_1(\delta(t)) u(t).$$

Plant rule 2: IF $x_1(t)$ is about Γ_2, THEN

$$\dot x(t) = A_2(\delta(t)) x(t) + B_2(\delta(t)) u(t) + C_2(\delta(t))\omega(t),$$
$$z(t) = D_2(\delta(t)) x(t) + F_2(\delta(t)) u(t),$$

where Γ_1, Γ_2 are defined in [14] and

$$A_1(\delta(t)) = \begin{bmatrix} -0.5 & -0.1(1 + \mu(\delta(t))) \\ 1.0 & 0.0 \end{bmatrix},$$
$$A_2(\delta(t)) = \begin{bmatrix} -1.0 & -0.1(1 + \mu(\delta(t))) \\ 1.0 & 0.0 \end{bmatrix},$$
$$B_1(\delta(t)) = B_2(\delta(t)) = \begin{bmatrix} 1.0 & 0.0 \end{bmatrix}^T,$$
$$C_1(\delta(t)) = C_2(\delta(t)) = \mathrm{diag}\{1.0, 0.0\},$$
$$D_1(\delta(t)) = D_2(\delta(t)) = \begin{bmatrix} 1.0 & 2.0 \end{bmatrix},$$
$$F_{11} = 0.1, \quad F_{12} = 0.2, \quad F_{13} = -0.1,$$
$$F_{14} = 0.3, \quad F_{21} = 0.2, \quad F_{22} = 0.3,$$
$$F_{23} = -0.7, \quad F_{24} = 0.7, \quad \delta(t) = 1, 2, 3, 4.$$

In addition, let the exogenous disturbance input be $\omega_1(t) = \frac{\sqrt{0.032}}{1+t^2}$. Applying Theorem 2 with the following parameters $\tilde\gamma = 0.8$, $\sigma_l = 0.3\,(l = 1, 2, 3, 4)$, $\lambda = 0.1$, $\phi = 1$, $\theta_1 = \theta_2 = 1$, $\tau_M = 0.3$, $a_1 = 1$, $a_2 = 3$, $t_p = 10$, $\hat\omega = 0.64$, $\mathcal{J}_{11} = \mathcal{J}_{14} = \mathrm{diag}\{0.1, 0.1\}$, $\mathcal{J}_{12} = \mathcal{J}_{13} = 0$, $\mathcal{J}_{\varphi_1} = \mathrm{diag}\{1, 1\}\,(\varphi_1 = 2, 3, \ldots, 8)$, the corresponding trigger parameters and gain matrices of the controller (8) are obtained:

$$\Omega_1 = \begin{bmatrix} 36.3029 & 21.3056 \\ 21.3056 & 14.7233 \end{bmatrix}, \Omega_2 = \begin{bmatrix} 36.0814 & 21.2761 \\ 21.2761 & 14.7033 \end{bmatrix},$$
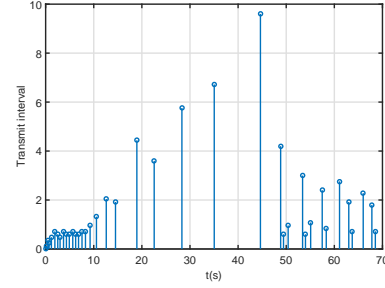

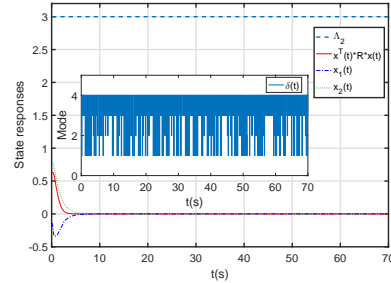Fig. 7. Transmit interval in Example 2.

Fig. 8. State responses of the CLFS and mode evolution in Example 2.

$$\Omega_3 = \begin{bmatrix} 31.8305 & 19.4141 \\ 19.4141 & 13.1587 \end{bmatrix}, \Omega_4 = \begin{bmatrix} 43.9538 & 25.2538 \\ 25.2538 & 18.5945 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -1.5243 & -0.9808 \end{bmatrix}, K_2 = \begin{bmatrix} -1.5330 & -1.0134 \end{bmatrix}.$$

Under the obtained trigger parameters and gain matrices of the controller (7), the transmit interval is plotted in Fig. 7. The state responses of the CLFS, $x^T(t) R x(t)$ and possible mode evolution are plotted in Fig. 8 where $x(0) = \begin{bmatrix} -0.1 & 0.8 \end{bmatrix}^T$. From Fig. 7, it can compute that only about 6.6895% of all SD are transmitted to the controller within simulation duration 70s, which means that the communication burden is largely reduced by using the proposed ETCS. From Fig. 8, one can see that the CLFS is of FTB, which indicates the availability of the developed design approach.

V. CONCLUSIONS

The issue of event-triggered finite-time $\mathcal{H}_\infty$ control for T-S FMJSs with asynchronous premises is investigated in this paper. The ETCS is employed to reduce the communication burdens. By using the asynchronous premise reconstruct method, the asynchronous constraints on membership functions are well utilized. Moreover, by utilizing finite-time analysis theory and Lyapunov–Krasovskii approach, some sufficient conditions for finite-time boundedness with a prescribed $\mathcal{H}_\infty$ performance analysis of the considered system have been derived and the corresponding $\mathcal{H}_\infty$ controller design have been carried out. Finally, both a numerical example and a nonlinear MSDMS are provided to show the utilizability and practicability of our theoretical results. It is noteworthy that the

results presented in this work can be broadly extended to more complex FMJSs, such as FMJSs with parametric uncertainties, fuzzy semi-MJSs and delayed FMJSs, which deserves further exploration.

REFERENCES

[1] F. Abdollahi and K. Khorasani, "A decentralized Markovian jump control routing strategy for mobile multi-agent networked systems," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 2, pp. 269–283, Mar. 2011.

[2] F. Amato, M. Ariola, and P. Dorato, "Finite-time control of linear systems subject to parametric uncertainties and disturbances," *Automatica*, vol. 37, no. 9, pp. 1459–1463, 2001.

[3] N. Arrifano and V. Oliveira, "Robust fuzzy control approach for a class of Markovian jump nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 6, pp. 738–754, Dec. 2006.

[4] W. Blair Jr and D. Sworder, "Feedback control of a class of linear discrete systems with jump parameters and quadratic cost criteria," *Int. J. Control*, vol. 21, no. 5, pp. 833–841, 1975.

[5] M. Chadli and TM. Guerra, "LMI solution for robust static output feedback control of discrete Takagi-Sugeno fuzzy models," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 6, pp. 1160–1165, Dec. 2012.

[6] O. L. d. V. Costa, M. D. Fragoso, and M. G. Todorov, "A detector-based approach for the control of Markov jump linear systems with partial information," *IEEE Trans. Automat. Control*, vol. 60, no. 5, pp. 1219–1234, May 2015.

[7] P. Dorato, "Short-time stability in linear time-varying systems," *Proc. of IRE Int. Convention Record, part 4*, pages 83–87, 1961.

[8] J. Dong, G.H. Yang, and H. Zhang, "Stability analysis of T–S fuzzy control systems by using set theory," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 4, pp. 827–841, Aug. 2015.

[9] G. Feng, "A survey on analysis and design of model-based fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 5, pp. 676–697, Oct. 2006.

[10] H. Gao, T. Chen, and J. Lam, "A new delay system approach to network-based control," *Automatica*, vol. 44, no. 1, pp. 39–52, 2008.

[11] J. C. Geromel and G. W. Gabriel, "Optimal H_2 state feedback sampled-data control design of Markov jump linear systems," *Automatica*, vol. 54, no. 4 pp. 182–188, 2015.

[12] S. He and F. Liu, "Finite-time fuzzy H_∞ control of nonlinear jump systems with time delays via dynamic observer-based state feedback," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 4, pp. 676–697, Aug. 2012.

[13] H.R. Karimi, "Robust delay-dependent control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 58, no. 8, pp. 1910-1923, Aug. 2011.

[14] H. J. Lee, J. B. Park, and Y. H. Joo, "Robust control for uncertain Takagi–Sugeno fuzzy systems with time-varying input delay," *ASME J. Dyn. Syst., Meas., Control*, vol. 127, no. 2, pp. 302–306, 2005.

[15] H.K. Lam and F. H. F. Leung, "Stability analysis of fuzzy control systems subject to uncertain grades of membership," *IEEE Trans. Syst. Man Cybernet. Part B*, vol. 35, no. 6, pp. 1322-1325, Dec. 2005.

[16] H. Li, P. Shi, D. Yao, and L. Wu, "Observer-based adaptive sliding mode control for nonlinear Markovian jump systems," *Automatica*, vol. 64, no. 1, pp. 133–142, 2016.

[17] P. Park, W. I. Lee, and S. Y. Lee, "Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems," *J. Franklin Inst.*, vol. 352, no. 4, pp. 1378–1396, 2015.

[18] C. Peng, D. Yue, and Q.-L. Han, *Communication and Control for Networked Complex Systems*, Berlin, Heidelberg, Springer-Verlag, 2015.

[19] J. Qiu, S. Ding, H. Gao, and S. Yin, "Fuzzy-model-based reliable static output feedback H_∞ control of nonlinear hyperbolic PDE systems," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 2, pp. 388–400, Apr. 2014.

[20] R. Sakthivel, S. Selvi, K. Mathiyalagan and P. Shi, "Reliable mixed H_∞ and passivity-based control for fuzzy Markovian switching systems with probabilistic time delays and actuator failures," *IEEE Trans. Cybernet.*, vol. 45, no. 12, pp. 2720–2731, Dec. 2015.

[21] Y. Shi and B. Yu, "Output feedback stabilization of networked control systems with random delays modeled by Markov chains," *IEEE Trans. Automat. Control*, vol. 54, no. 7, pp. 1668–1674, Jul. 2009.

[22] H. Shen, Y. Zhu, L. Zhang, and J. Park, "Extended dissipative state estimation for Markov jump neural networks with unreliable links," *IEEE Trans. Neural Netw. Learn. Syst.*, DOI:10.1109/TNNLS.2015.2511196, 2015.

[23] P. Shi, H. Wang, and C.-C. Lim, "Network-based event-triggered control for singular systems with quantizations," *IEEE Trans. Industrial Elec.*, vol. 63, no. 2, pp. 1230–1238, Feb. 2016.

[24] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst. Man Cybernet.*, vol. SMC-15, no. 1, pp. 116–132, Jan.-Feb. 1985.

[25] S. Tong, B. Huo, and Y. Li, "Observer-based adaptive decentralized fuzzy fault-tolerant control of nonlinear large-scale systems with actuator failures," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 1–15, Feb. 2014.

[26] V. Ugrinovskii and H. R. Pota, "Decentralized control of power systems via robust control of uncertain Markov jump parameter systems," *Int. J. Control*, vol. 78, no. 9, pp. 662–677, 2005.

[27] H. Wang, P. Shi, and C.-C. Lim, "Decentralized event-triggering passive control of networked T-S fuzzy systems," *IEEE Trans. Fuzzy Syst.*, DOI:10.1109/TFUZZ.2015.2453394, 2015.

[28] J.-W. Wang, H.-N. Wu, L. Guo, and Y.-S. Luo, "Robust H_∞ fuzzy control for uncertain nonlinear Markovian jump systems with time-varying delay," *Fuzzy Sets and Systems*, vol. 212, pp. 41–61, 2013.

[29] Z. Wang, H. Dong, B. Shen, and H. Gao, "Finite-horizon H_∞ filtering with missing measurements and quantization effects," *IEEE Trans. Automat. Control*, vol. 58, no. 7, pp. 1707–1718, Jul. 2013.

[30] S. Wen, X. Yu, Z. Zeng, and J. Wang, "Event-triggering load frequency control for multi-area power systems with communication delays," *IEEE Trans. Industrial Elec.*, vol. 63, no. 2, pp. 1308–1317, Feb. 2016.

[31] L. Wu, X. Yang, and H. Lam, "Dissipativity analysis and synthesis for discrete-time T–S fuzzy stochastic systemswith time-varying delay," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 2, pp. 380–394, Apr. 2014.

[32] Z. Wu, P. Shi, H. Su, and J. Chu, "Sampled-data fuzzy control of chaotic systems based on a T–S fuzzy model," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 1, pp. 153–163, Feb. 2014.

[33] D. Yue, E. Tian, and Q.-L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Trans. Automat. Control*, vol. 58, no. 2, pp. 475–481, Feb. 2013.

[34] B. Zhang and W.-X. Zheng and S. Xu, "Filtering of Markovian jump delay systems based on a new performance index," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 60, no. 5, pp. 1250-1263, May 2013.

[35] H. Zhang, Z. Liu, G.-B. Huang, and Z. Wang, "Novel weighting-delay-based stability criteria for recurrent neural networks with time-varying delay," *IEEE Trans. Neural Networks*, vol. 21, no. 1, pp. 91–106, Jan. 2010.

[36] J. Zhang and C. Peng, "Event-triggered H_∞ filtering for networked Takagi–Sugeno fuzzy systems with asynchronous constraints," *IET Signal Processing*, vol. 9, no. 5, pp. 403–411, Jun. 2015.

[37] L. Zhang, Y. Zhu, Z. Ning, and X. Yin, "Resilient estimation for networked systems with variable communication capability," *IEEE Trans. Automat. Control*, DOI: 10.1109/TAC.2016.2550202, 2016.

[38] L. Zhang, Y. Leng, and P. Colaneri, "Stability and stabilization of discrete-Time semi-Markov jump linear systems via semi-Markov kernel approach," *IEEE Trans. Automat. Control*, vol. 61, no. 2, pp. 503–508, Feb. 2016.

[39] X. Zhang, Q. Han, and B. Zhang, "An overview and deep investigation on sampled-data-based event-triggered control and filtering for networked systems," *IEEE Trans. Industrial Info.*, DOI:10.1109/TII.2016.2607150, 2016.

[40] Z. Zhang, Z. Zhang, H. Zhang, P. Shi, and H. R. Karimi, "Finite-time filtering for T–S fuzzy discrete-time systems with time-varying delay and norm-bounded uncertainties," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 6, pp. 2427–2434, Dec. 2015.

[41] L. Zhao, H. Gao, H.R. Karimi, "Robust stability and stabilization of uncertain T–S fuzzy systems with time-varying delay: an input–output approach," *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 5, pp. 883-897, Oct. 2013.