

# Harmonic Distortion Compensation in Voltage Transformers for Improved Power Quality Measurements

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**Abstract**— Monitoring voltage harmonics represents one of the most important tasks in power quality assessment. In particular, the employed instrument transformer plays a key role in the achieved accuracy. Its harmonic measurement performance is typically evaluated by measuring its frequency response function. However, nonlinearities may have a non-negligible impact on measurement uncertainty: for example, this occurs as far as inductive voltage transformers are considered. This paper proposes a simple technique allowing the compensation of the most significant nonlinear effect, which is the harmonic distortion produced by the large fundamental primary voltage. The method is firstly derived and introduced by means of numerical simulations, and then implemented through a proper experimental setup. Results highlight remarkable accuracy improvements when realistic voltage waveforms are measured.

**Index Terms**—Error compensation, Harmonic distortion, Instrument transformers; Voltage transformers; Voltage measurement; Calibration; Nonlinear systems; Frequency response; Power system harmonics; Measurement uncertainty; Frequency-domain analysis

## I. INTRODUCTION

THE continuously increasing penetration of power converters, high voltage dc systems, nonlinear loads and generation from renewable energy sources (e.g. wind farms and photovoltaic plants) we experienced in the last years have dramatically boosted the importance of power quality assessment at all voltage levels. In this scenario, one of the crucial tasks is represented by voltage harmonics monitoring [1]-[3]. It is well-known that the metrological performance of the employed voltage transformer (VT) has a major impact on the achieved accuracy. The relevance of the topic is also highlighted by a recent IEC technical report [4] discussing the employment of instrument transformers for power quality measurements.

Thanks to their favorable mix in terms of performance, reliability and long-term stability, conventional inductive transformers are still widely employed as VTs in both

transmission and distribution grids. Therefore, their outputs are often used for harmonic monitoring, although it should be stressed that their metrological performance is guaranteed only at the rated frequency [5]. Many works discussing the suitability of inductive VTs to measure harmonic voltages can be found in the scientific literature [6]-[11]. Most of these papers model the transformer as a linear time invariant (LTI) system characterized by an uneven frequency response due to the windings leakage inductances, stray capacitances as well as the related resonances. Methods for extending their bandwidths by connecting external capacitors, by properly designing their windings [12] or by adding compensation filters [13], [14] have been proposed in the past.

However, inductive VTs also suffer from a nonlinear behavior due to the ferromagnetic core which may jeopardize their accuracy when employed for harmonic measurements [4], [15]. One of the simplest approaches that allows taking into account nonlinearities without introducing a nonlinear representation, is that based on the measurement of the Best Linear Approximation (BLA) [18], [19]. The BLA is defined as the frequency response function (FRF) which, when applied to the secondary voltage, guarantees the most accurate measurement (in the least squares sense) for a given class of periodic input voltage signals. With respect to a small-signal FRF, the BLA ensures better accuracy since it is able to include some nonlinear effects produced by the VT excited by this specific class of signals. These nonlinearities biasing the BLA estimate are often known as “systematic” [20]. On the other hand, it should be noticed that most part of the nonlinearities in a VT cannot be merged into a linear model. Therefore, the FRF of the VT appears to change according to the primary voltage waveform: this is symptom of undermodeling. Nonlinearities causing such effect are generally called “stochastic”. From these considerations, it is clear that a better reconstruction of the primary voltage spectrum is viable only by using a more complex model than a FRF. Only in this way a larger amount of nonlinearity can be included.

The effect of VT nonlinearities is magnified because of the typical spectral content of voltage waveforms in ac power systems, thus consisting of a largely prevailing fundamental component superimposed to harmonics that are much smaller in amplitude. As a result, the impact of nonlinearities is small

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at the fundamental component, but it becomes quite noticeable when measuring low-order harmonics. This behavior can be accurately represented by using proper nonlinear models that have been specifically developed for power system devices [16], [17] starting from the frequency-domain Volterra approach. The proposed method has been applied to the characterization of medium voltage inductive VTs [21], and results highlight the remarkable accuracy achieved in predicting the secondary voltage spectrum. Furthermore, analyzing the experimental data [21] it can be noticed that the strongest nonlinear effect in VTs is represented by the harmonic distortion (HD) produced by the large fundamental component. In [22], for the first time the authors proposed a simple and computationally effective technique allowing compensating the impact of this phenomenon. In this work, the procedure has been experimentally implemented in order to enhance the harmonic measurement capability of an inductive VT. Results are reported and deeply discussed, and confirm that the proposed method allows a strong accuracy improvement.

## II. HARMONIC DISTORTION COMPENSATION MODEL

Let us consider a VT operating in an ac power system characterized by the rated angular frequency  $\omega_0$ ; let us introduce  $v_1$  as the primary voltage waveform, supposed to be periodic with fundamental angular frequency  $\omega_0$  and containing harmonics up to the  $M$ -th order. In general, the voltage transformer can be considered as a (weakly) nonlinear time-invariant system. Excluding complex and chaotic behaviors (a typical example is ferroresonance in inductive VTs), the corresponding steady-state response is represented by a secondary voltage waveform  $v_2$  characterized by the same period. Under these assumptions, it is convenient to study the behavior of the VT in the frequency domain, thus introducing the primary and secondary voltage spectra  $V_1(jm\omega_0)$  and  $V_2(jm\omega_0)$ , respectively; for the sake of brevity, in the following  $jm\omega_0$  will be replaced with the corresponding harmonic index  $m$ . The generic  $m$ -th order harmonic  $V_2(m)$  (with  $m \geq 2$ ) appearing in the secondary voltage can be decomposed into the sum of three different contributions:

$$V_2(m) = V_{2,L}(m) + V_{2,NL}(m) + N_2(m) \quad (1)$$

The first term  $V_{2,L}(m)$  represents the linear contribution to the transformer output, and hence proportional to the primary voltage harmonic having the same frequency:

$$V_{2,L}(m) = H_L(m)V_1(m) \quad (2)$$

$H_L(m)$  is the FRF characterizing the underlying linear part of the VT. The second term,  $V_{2,NL}(m)$  is produced by the nonlinear behavior of the VT; in general, it is a function of all the primary voltage spectral components. Finally, the third contribution  $N_2(m)$  takes into account random measurement noise.

Let us suppose that measurement noise is negligible. If the

VT were a perfectly LTI system, a virtually exact reconstruction of the primary voltage spectrum could be performed from the secondary side by inverting the previously measured FRF  $H_L(m)$ , as proposed by several papers [6]-[8].

Voltage waveforms in ac power systems are *quasi sinusoidal*, namely made of a strong fundamental component superimposed to harmonics that are characterized by considerably smaller amplitudes. This peculiar spectral distribution makes that secondary voltage harmonics are considerably influenced by nonlinearity, while the fundamental component is much weakly affected in relative terms. For the same reason, the strongest nonlinear effect is represented by the harmonic distortion (HD) due the fundamental primary voltage [17], [21]. Under this assumption, other nonlinear phenomenon can be neglected; this means that it is possible to consider  $V_{2,NL}(m)$  as dependent on the fundamental primary voltage only. Since nonlinearity has small impact on the fundamental term,  $V_{2,NL}(m)$  can be written also as a function of the fundamental secondary voltage. Hence, substituting (2) into (1) it is possible to obtain an expression of the primary voltage components:

$$V_1(m) = K_L(m)V_2(m) + V_{1,HD}(m) \quad (3)$$

where  $V_{1,HD}(m) = -V_{2,NL}(m)/H_L(m)$  is a function of the fundamental secondary voltage only, while  $K_L(m)$  is the inverse of  $H_L(m)$ . It is clear that an explicit expression of  $V_{1,HD}$  would allow enhancing the accuracy of harmonic measurements: it permits to considerably reduce the definitional uncertainty which conventional frequency response compensation suffers from. In this respect, by adopting a frequency-domain polynomial approach to model the HD contributions [23], it can be written as:

$$V_{1,HD}(m) = \sum_{i=2}^I K(i_p, i_m) V_2(1)^{i_p} V_2^*(1)^{i_m} \quad (4)$$

subject to the constraints:

$$\begin{cases} i_p - i_m = m \\ i_p + i_m = i \end{cases} \quad (5)$$

\* denotes the complex conjugate operator,  $i_p$  and  $i_m$  are nonnegative integers while  $I \geq 2$  is the maximum degree of the employed polynomial model. Therefore, HD affecting the  $m$ -th order harmonic of the secondary voltage can be decomposed in up to  $I-1$  contributions. Each of them is characterized by its order  $i$ , representing the number of interactions of the fundamental component (or its negative frequency image) with itself. These terms appearing in the summation (4) can be rewritten as:

$$K(i_p, i_m) V_2(1)^{i_p} V_2^*(1)^{i_m} = K^i(m) |V_2(1)|^i e^{jm\varphi} \quad (6)$$

where:

$$\varphi = \angle V_2(1) \quad (7)$$

(5) together with the definitions of  $i_p$  and  $i_m$ , the bounds of the summation appearing in (4), results in the following system of equations:

$$\begin{cases} 2 \leq i \leq I \\ i = 2i_m + m \end{cases} \quad (8)$$

This implies that an even harmonic  $m$  is affected only by contributions characterized by even orders  $i$ ; the opposite happens for odd harmonics. Since  $m \geq 2$ , imposing the constraints resulting from (8), the HD contribution (4) can be rewritten as:

$$V_{1,HD}(m) = \sum_{i_m=0}^{\lfloor \frac{I-m}{2} \rfloor} K^i(m) |V_2(1)|^i e^{jm\varphi} \quad (9)$$

with  $\lfloor \cdot \rfloor$  denoting the floor function. Therefore, it is trivial to find that the number of coefficients defining the HD for the generic  $m$ -th order harmonic is given by:

$$c_{HD}(m) = \max \left\{ 0, \left\lfloor \frac{I-m}{2} \right\rfloor + 1 \right\} \quad (10)$$

It is also obvious to notice that  $I$ -th degree HD affects harmonics up to the order  $m = I$ . The overall number of terms ( $c_{HD}$  plus the linear contribution) as a function of the harmonic index  $m$  and the maximum order of the polynomial model  $I$  is reported in Fig. 1.

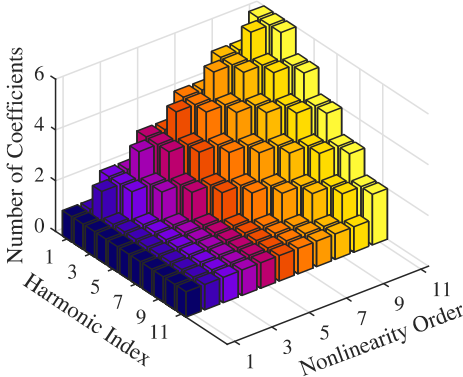


Fig. 1. Number of coefficients of the proposed compensation method as a function of the harmonic index  $m$  and the maximum order  $I$ .

Using (9) in (3), the expression of  $V_1(m)$  results:

$$V_1(m) = K_L(m)V_2(m) + \sum_{i_m=0}^{\lfloor \frac{I-m}{2} \rfloor} K^i(m) |V_2(1)|^i e^{jm\varphi} \quad (11)$$

Adopting vector notation, (11) can be rearranged as:

$$V_1(m) = \mathbf{W}^T(m) \mathbf{K}(m) \quad (12)$$

where:

$$\mathbf{W}(m) = \begin{bmatrix} V_2(m) \\ |V_2(1)|^m e^{jm\varphi} \\ \dots \\ |V_2(1)|^{2\lfloor \frac{I-m}{2} \rfloor + m} e^{jm\varphi} \end{bmatrix} \quad \mathbf{K}(m) = \begin{bmatrix} K_L(m) \\ K^m(m) \\ \dots \\ K^{2\lfloor \frac{I-m}{2} \rfloor + m}(m) \end{bmatrix} \quad (13)$$

Once having defined the compensation model, the vector of coefficients  $\mathbf{K}(m)$  has to be estimated for every considered harmonic order  $m$ . This mandates for at least  $L$  independent constraints, where  $L$  represents the maximum length of  $\mathbf{K}(m)$ , thus resulting:

$$L = \left\lfloor \frac{I-2}{2} \right\rfloor + 2 \quad (14)$$

Therefore, identification requires measuring the steady-state response to a set of  $P \geq L$  periodic, independent and quasi-sinusoidal primary voltage waveforms. The generic  $p$ -th observation allows writing an equation similar to (12) for each spectral component, thus relating the primary voltage harmonics  $V_{1,id}^{[p]}(m)$  with the corresponding column vector  $\mathbf{W}_{id}^{[p]}(m)$ . Considering all the injected signals, the following vector equation can be written:

$$\mathbf{V}_{1,id}(m) = \mathbf{W}_{id}(m) \mathbf{K}(m) \quad (15)$$

where:

$$\mathbf{W}_{id}(m) = \begin{bmatrix} \mathbf{W}_{id}^{[1]T}(m) \\ \vdots \\ \mathbf{W}_{id}^{[P]T}(m) \end{bmatrix} \quad \mathbf{V}_{1,id}(m) = \begin{bmatrix} V_{1,id}^{[1]}(m) \\ \vdots \\ V_{1,id}^{[P]}(m) \end{bmatrix} \quad (16)$$

Assuming that  $\mathbf{W}_{id}(m)$  is full-rank, it is possible to compute its Moore-Penrose pseudoinverse, and in turns obtaining the least-squares estimation of the coefficients:

$$\mathbf{K}_{id}(m) = \mathbf{W}_{id}^\dagger(m) \mathbf{V}_{1,id}(m) \quad (17)$$

After the identification procedure, the primary voltage harmonics can be easily reconstructed from a given secondary voltage spectrum by using (12).

### III. NUMERICAL SIMULATIONS

The proposed approach has been applied by means of numerical simulations for compensating nonlinearities in a low voltage, inductive VT having 50 Hz rated frequency and 20 VA rated burden. Said VT has been modeled with the usual Steinmetz equivalent circuit shown in Fig. 2. The values of the resistances, of the leakage inductances and of the turn ratio are reported in TABLE I. The nonlinear magnetizing inductance  $L_m$  has been represented with the single-valued flux linkage-current relation shown in Fig. 3; odd symmetry is assumed, hence the model produces purely odd nonlinearities.

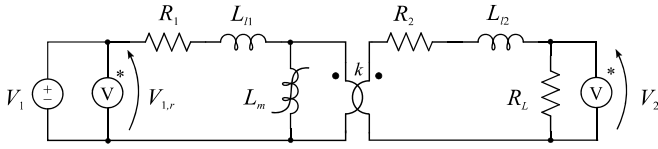
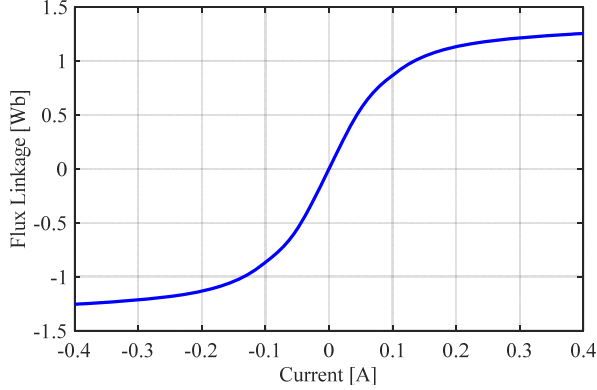


Fig. 2. Equivalent circuit of the VT.

TABLE I  
VOLTAGE TRANSFORMER PARAMETERS

$V_{1n}$ [V]	$k$	$R_1$ [ $\Omega$ ]	$R_2$ [ $\Omega$ ]	$L_{11}$ [mH]	$L_{12}$ [mH]	$R_L$ [ $\Omega$ ]
200	2	6	1.25	4.75	1.19	1600

Fig. 3. Flux linkage – current characteristic of the magnetizing inductance  $L_m$ .

The target of these simulations is comparing the accuracy achieved by employing three different approaches to reconstruct the primary voltage spectrum:

- Using the ratio obtained from a conventional calibration at the rated frequency
- Using the BLA of the VT to reconstruct the primary voltage
- Using the proposed HD compensation of the VT to reconstruct the primary voltage.

#### A. Design of the Excitation Signals

In order to compare the performances achieved by the aforementioned methods, a proper set of primary test voltages has to be considered. As in the previous section,  $v_1$  is assumed to be a multisine waveform having 50 Hz fundamental frequency and realistic harmonic content. A possible choice is using the limits for fundamental and harmonic voltages in public distribution grids prescribed by standard EN 50160 [24].

Limits up to the 25<sup>th</sup> harmonic order have been considered. [24] states that over a one week observation window, the ten minute average root mean square value of each voltage harmonic should remain below the corresponding limit for 95% of the time. Furthermore, [24] requires that in the same observation period, the ten minute average root mean square of the voltage should be within  $\pm 10\%$  of its rated value for

95% of the time. Hence, these limits can be considered as 95<sup>th</sup> percentile values of the probability density functions (PDFs) characterizing the amplitudes of both fundamental and harmonic voltages. On the other hand, the standard does not provide information about the shapes of these PDFs nor about the phases.

Fundamental amplitude has been supposed to be normally distributed with mean equal to its rated voltage and a standard deviation so that it falls within  $\pm 10\%$  of the rated value with 95% probability. Instead, harmonic phasors are supposed to follow zero-mean, circular complex normal distributions, thus having uncorrelated, normally distributed real and imaginary parts. Therefore, harmonic amplitudes follow Rayleigh distributions characterized by parameters that can be computed from the limits reported in [24], while the phases are assumed to be uniformly distributed in the interval  $[-\pi, \pi]$ .

#### B. Model Identification

A set of  $P = 100$  primary voltage waveforms has been generated by sampling the previous probability density functions. These excitation signals have been applied to the model of the voltage transformer, and the corresponding steady-state secondary voltages have been obtained.

Firstly, the conventional ratio has been computed by using the mean values of primary and secondary fundamental voltages over the whole set of identification signals. Then, the Best Linear Approximation has been evaluated by following the procedure presented in [18], [19]. Finally, the proposed HD compensation technique has been considered. The identification is carried out according to the procedure explained in Section II for degrees ranging from two to eleven.

#### C. Performance Comparison

In [22] comparison has been performed by using the same set of identification signals. However, it is interesting to evaluate the accuracies of the different approaches by sampling a new set of  $P = 500$  signals from the previously defined PDFs. For the generic  $p$ -th signal and  $m$ -th order harmonic, the Total Vector Error (TVE) has been computed. It represents the distance in the complex plane between the estimated primary voltage phasor,  $V_{1,e}^{[p]}(m)$ , and the actual one,  $V_1^{[p]}(m)$ , thus allowing to consider phase and ratio errors simultaneously. In measurement applications, it is significant to express it as percentage of the actual amplitude of  $V_1^{[p]}(m)$

$$\text{TVE}_m^{[p]}(m) = \frac{|V_{1,e}^{[p]}(m) - V_1^{[p]}(m)|}{|V_1^{[p]}(m)|} \quad (18)$$

As synthetic performance index,  $\text{TVE}_m^{95}(m)$  has been obtained as the 95<sup>th</sup> percentile value of  $\text{TVE}_m^{[p]}(m)$  for a given harmonic order and compensation method.

Fig. 4 reports the  $TVE_m^{95}$  values achieved by the different approaches. Since HD compensation works only when  $m \leq I$ , harmonic indexes ranging from two to eleven are reported. When conventional calibration is employed, the highest error value (4.5%) occurs at the third harmonic, which is typically the most affected by nonlinearity, as shown in other works [15], [18], [19]. Error is significantly lower at the fourth one (clearly not affected by odd harmonic distortion), but it becomes to rise for higher-order components mainly because of the filtering behavior of the VT. When the BLA is employed to reconstruct the primary voltage, the 95<sup>th</sup> percentile value of  $TVE_m$  at the third harmonic remains almost unchanged. On the contrary, considerable improvement is achieved at higher frequencies. For example, using conventional calibration,  $TVE_m^{95}$  exceeds 4.2% for the eleventh harmonic, but it can be reduced to 0.45% by using the BLA. According to what expected, HD compensation allows excellent results for low-order harmonic measurements. In particular, a reduction of the  $TVE_m^{95}$  of about an order of magnitude is obtained at the third (0.11 % vs 4.5%) and fifth harmonic (0.24% vs 2.48%) in spite of a slight increase in complexity (six coefficients for the third harmonic, five for the fifth). From another point of view, this remarkable accuracy improvement highlights the significant impact of HD on harmonic measurements.

Although the TVE represents an effective synthetic index of harmonic measurement performance, the accuracy of instrument transformers is conventionally expressed by ratio and phase errors, which can be employed both for fundamental [25] and harmonic components [26]. Therefore, for each measurement method,  $m$ -th order harmonic and  $p$ -th test signal, ratio error can be computed as:

$$e_{abs}^{[p]}(m) = \frac{|V_{1,e}^{[p]}(m)| - |V_1^{[p]}(m)|}{|V_1^{[p]}(m)|} \quad (19)$$

while phase error is defined as:

$$e_{\angle}^{[p]}(m) = \angle(V_{1,e}^{[p]}(m)) - \angle(V_1^{[p]}(m)) \quad (20)$$

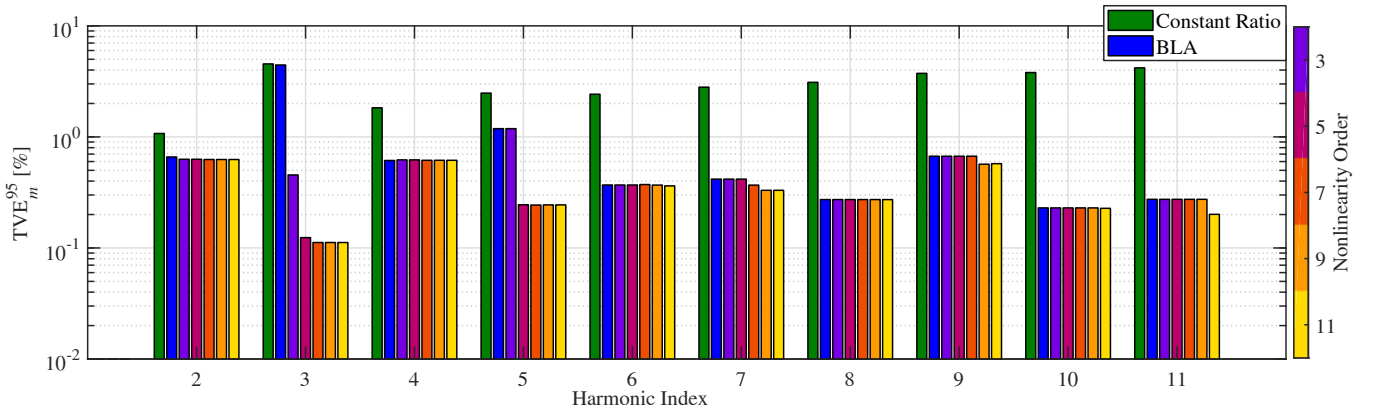


Fig. 4. Numerical simulations:  $TVE_m^{95}$  percentile values achieved by conventional calibration (first bar), BLA compensation (second bar) and HD compensation of different orders (last five bars).

For each method and harmonic order, the average values of ratio and phase errors have been computed over the  $P$  test signals, together with their 2.5<sup>th</sup> and 97.5<sup>th</sup> percentile values as dispersion indicators. Fig. 5 and Fig. 6 report average errors (continuous lines) as well as 2.5<sup>th</sup> and 97.5<sup>th</sup> percentiles values (dash-dot lines) when the primary voltage is reconstructed by using a constant ratio obtained by conventional calibration, BLA compensation and eleventh order HD compensation. When looking at the ratio errors (Fig. 5), it can be noticed that average values are small, namely they are virtually unbiased. However, there are very significant differences in terms of spread. When considering the conventional calibration and the BLA compensation, percentile bounds are very similar. The broadest dispersions occur for the third and the fifth order harmonics, resulting in 95<sup>th</sup> percentile error bands of 6.7% and 1.6%. Instead, by using the proposed eleventh order HD compensation, these values decrease to about 0.17% and 0.32%, respectively. Conversely, HD compensation is not able to increase the accuracy reached by using the BLA for even harmonics, since they are not affected by the purely odd nonlinearity produced by the modeled VT.

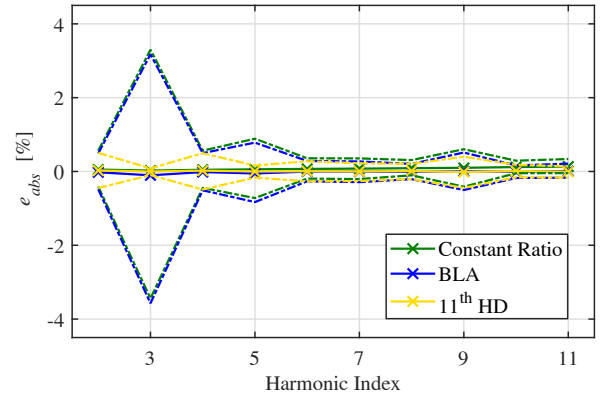


Fig. 5. Numerical simulations: ratio errors achieved by conventional calibration, BLA compensation and 11<sup>th</sup> order HD compensation.

Different considerations arise when looking at the phase errors, depicted in Fig. 6. In this case, using conventional calibration results in a biased phase error showing an almost linear increase with frequency. The reason is directly related with the filtering behavior of the VT, which can be

compensated by using the BLA approach. In fact, both BLA and HD compensations result in unbiased phase errors. However, thanks to the capability of HD models to take into account nonlinearity, the dispersions of the phase errors are reduced considerably, in particular for low-order, odd harmonics. As an example, when the third harmonic is considered, the 95<sup>th</sup> percentile interval is reduced from 5.9 crad (BLA compensation) to 0.17 crad (eleventh order HD compensation).

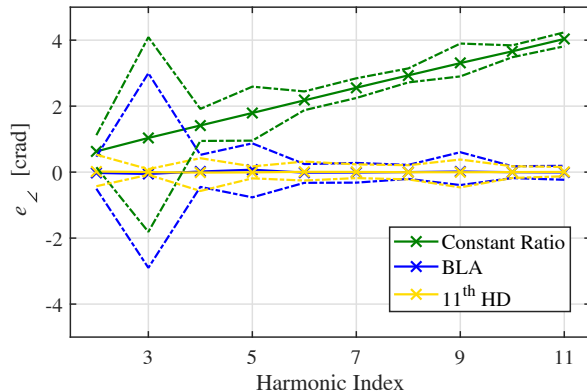


Fig. 6. Numerical simulations: phase errors achieved by conventional calibration, BLA compensation and 11<sup>th</sup> order HD compensation.

#### IV. EXPERIMENTAL SETUP

The proposed nonlinear compensation method has been experimentally applied to a conventional 50 Hz inductive VT, whose characteristics are reported in TABLE II. The experimental setup developed in [27] for the characterization of voltage transformers has been employed; its block diagram is depicted in Fig. 7.

$V_1$ [V]	$V_2$ [V]	BURDEN [VA]	CLASS
200	100	20	0.5

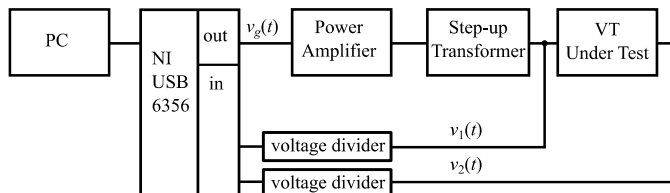


Fig. 7. Experimental setup.

A PC manages both generation and data acquisition by means of a National Instruments NI USB-6356 board characterized by 16-bit resolution and simultaneous sampling with maximum rate of 1.25 Msamples/s per channel. Its synchronized sampling and generation capability allows obtaining negligible spectral leakage. Input channels introduce a total harmonic distortion which is lower than -80dB, namely negligible with respect to that expected from the VT under

test. An analog output of the board is connected to an AETechron 7548 industrial power amplifier, whose specifications are listed in TABLE III.

TABLE III  
POWER AMPLIFIER SPECIFICATIONS

$V_{MAX}$ [V]	$I_{MAX}$ [A]	BANDWIDTH	THD	SNR	GAIN
200	43	DC–30 kHz, (+0.1, -0.5 dB)	below 0.1%	>120dB	20

Since the amplifier output voltage is not high enough to drive the VT under test above its rated voltage, a 100 V/400 V step up transformer has been employed. The secondary winding of the VT under test has been connected to its rated burden.

Both primary and secondary voltages have been acquired by means of calibrated resistive dividers which have been connected to the data acquisition board through Analog Devices AD215BY isolation amplifiers operating as voltage followers; their FRFs have been measured and compensated. After calibration, the two voltage measurement channels are characterized by gain uncertainty below  $10^{-4}$  and phase uncertainty lower than 0.2 mrad in the frequency range 50÷1250 Hz.

#### V. EXPERIMENTAL RESULTS

The target of the experimental activity is assessing the harmonic measurement performance that can be achieved by using the proposed HD compensation method considering different orders  $I$ , ranging from two to eleven. As for the simulations, results have been compared to those obtained by using conventional calibration or BLA compensation.

The input voltage  $v_1(t)$  that has to be applied to the VT under test may be significantly different from a scaled replica of  $v_g(t)$  because of the response of the power amplifier and, mostly, of the step up transformer. In order to compensate at least for linear effects (uneven frequency response in the considered range), the small-signal FRF between  $v_1(t)$  and  $v_g(t)$  has been estimated by injecting a random phase multisine signal [20]. Afterwards, the obtained FRF has been employed to prefilter the desired excitation signals. Of course, nonlinear artifacts cannot be cancelled with this approach. However, when both the power amplifier and the step-up transformer operate within their rated capabilities, these nonlinearities can be neglected since they just produce a minor change in the actual class of excitation signal. Therefore, their impact on the overall performance of the methods is negligible.

As for the numerical simulations,  $P = 100$  quasi-sinusoidal multisine signals with 50 Hz fundamental frequency have been sampled from the PDFs defined in Section III.A. These signals have been injected to the VT under test while measuring primary and secondary steady-state voltages. A 2 s observation interval been chosen, and each channel has been acquired with 200 kHz sampling rate. Discrete Fourier Transform (DFT) has been used to compute input and output spectra and frequency-domain averaging has been employed in order to reduce the impact of measurement noise. For each

generic  $m$ -th order harmonic,  $\mathbf{W}_{id}(m)$  has been obtained and the different models coefficients can be estimated as previously explained.

Once having identified the different models, they have been employed to compensate the response of the VT under test when supplied with a different set of  $P = 1000$  multisine signals belonging to the same class. Achieved harmonic measurement performances have been compared in terms of  $TVE_m$ , as well as by using conventional magnitude and phase errors.

The 95<sup>th</sup> percentile values of  $TVE_m$  are reported in Fig. 8, showing that experimental results confirms the trends already observed in numerical simulations. Focusing on the third harmonic, the error decreases from 4.8% to 0.2% by using the third order HD model, consisting of only two coefficients in this case. It can be noticed that experimental results denote even a better improvement with respect to simulations. On the other hand, no improvement is achieved by further increasing the degree of nonlinearity. The proposed model allows a slight error reduction for second and fourth harmonics; this means that the VT also suffers from very weak even nonlinearity.

The trend highlighted by  $TVE_m^{95}$  is very similar to what can be noticed when analyzing ratio and phase errors; results are shown in Fig. 9 and Fig. 10, respectively.

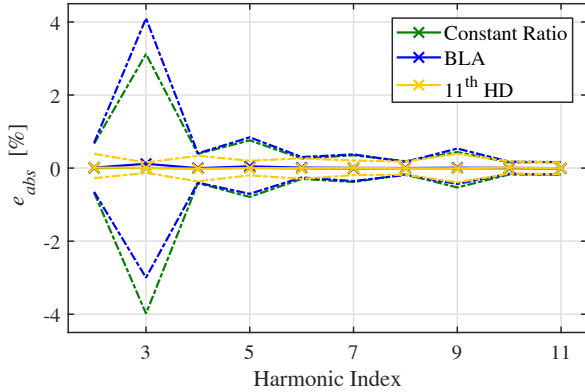


Fig. 9. Experimental results: ratio errors achieved by conventional calibration, BLA compensation and 11<sup>th</sup> order HD compensation.

The shapes of the ratio error versus harmonic order plots resemble what observed from simulation results. Constant ratio and BLA achieve unbiased ratio errors, but the dispersion

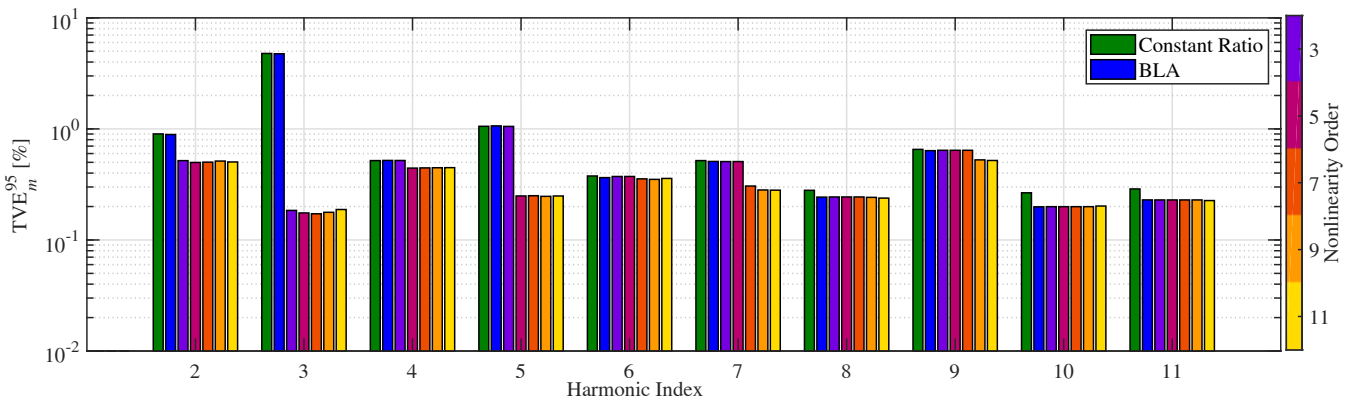


Fig. 8. Experimental results:  $TVE_m^{95}$  percentile values achieved by conventional calibration (first bar), BLA compensation (second bar) and HD compensations of different orders (last five bars).

is rather broad. As an example, the 95<sup>th</sup> percentile band is about 7.1% for the third harmonic. HD compensation permits to reduce this interval below 0.28%, hence more than 25 times. Conversely, HD models result in marginal improvements at even harmonics with respect to the BLA, since the VT suffers mostly from odd nonlinearity.

Conversely, phase errors show a different trend with respect to what previously observed from simulations results. In fact, while in simulations using a constant ratio results in a significantly biased phase error, this is not so evident from experimental results. This means that the modeled VT has a filtering behavior due to a rather large leakage inductance that, on the contrary, is just barely noticeable in the VT under test. As a consequence, the BLA compensation results in similar performance as that achieved by using a constant ratio. However, as in numerical simulations, harmonic distortion compensation allow a strong reduction of the 95<sup>th</sup> percentile band for low-order, odd harmonics. As examples, when third and fifth harmonics are considered, the corresponding 95<sup>th</sup> percentile bands are reduced from 6.8 crad to 0.28 crad, and from 1.8 crad to 0.35 crad, respectively.

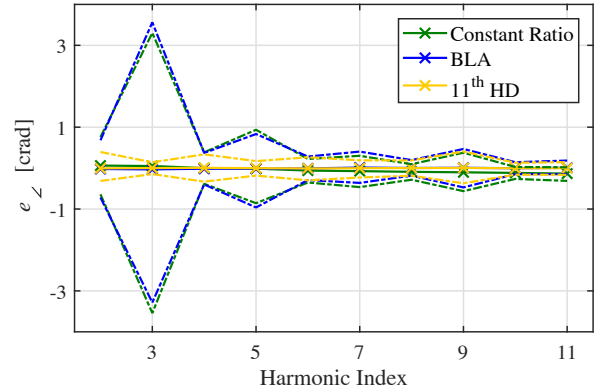


Fig. 10. Experimental results: phase errors achieved by conventional calibration, BLA compensation and 11<sup>th</sup> order HD compensation.

## VI. CONCLUSION

Conventional inductive VTs are often employed to measure voltage harmonics even if their metrological performances are guaranteed only at their rated frequency. Therefore, primary voltage spectrum may be measured with unsatisfactory

accuracy if a constant turn ratio is applied to the secondary voltage harmonics. The reason is related to the complex behavior of the VT; dynamics and nonlinear effects interact with each other and can be hardly separated. In order to enhance the measurement performance of inductive VTs devoted to power quality applications, the employment of linear compensating filters has been proposed for a long time. This work introduces an innovative method capable of drastically reducing also the strongest nonlinear effect, which is the harmonic distortion produced by the large fundamental component, a typical feature of electrical quantities in ac power systems. The technique has been firstly validated through numerical simulations by considering a large set of realistic primary voltage waveforms; the achieved accuracy is compared with that obtained with a conventional calibration or by using the best FRF compensation (BLA approach). Afterwards, an experimental activity performed on a low voltage inductive VT has been carried out. Results confirm the remarkable performance of the method: errors at the third and fifth harmonics are reduced by almost an order of magnitude in spite of the simplicity and the small set of coefficients to be estimated. It is worth noticing that the proposed method is general, so it can be applied for compensating the effects of harmonic distortion occurring in other types of voltage transformers, even including low-power instrument transformers. The reduced computational cost makes it suitable to be implemented in instruments allowing harmonic measurements as well as in merging units.

#### REFERENCES

- [1] *Electromagnetic Compatibility (EMC)—Part 4-7: Testing and Measurement Techniques—General Guide on Harmonics and Interharmonics Measurements and Instrumentation, for Power Supply Systems and Equipment Connected Thereto*, document BS EN 61000-4-7:2002+A1:2009, 2009.
- [2] M. Kaczmarek and R. Nowicz, "Application of instrument transformers in power quality assessment," in *Proc. Modern Electric Power Systems*, 2010, pp. 1-5.
- [3] G. Crotti et al., "Frequency Compliance of MV Voltage Sensors for Smart Grid Application," *IEEE Sensors J.*, vol. 17, no. 23, pp. 7621-7629, 1 Dec. 2017.
- [4] *Instrument transformers – the use of instrument transformers for power quality measurement*, document IEC TR 61869-103, 2012.
- [5] *Instrument transformers - Part 3: Additional requirements for inductive voltage transformers*, document IEC 61869-3:2011, 2011.
- [6] R. Stiegler, J. Meyer, J. Kilter and S. Konzelmann, "Assessment of voltage instrument transformers accuracy for harmonic measurements in transmission systems," in *Proc. 17th Int. Conf. on Harmonics and Quality of Power*, 2016, pp. 152-157.
- [7] M. I. Samesima, J. C. de Oliveira and E. M. Dias, "Frequency response analysis and modeling of measurement transformers under distorted current and voltage supply," in *IEEE Trans. Power Del.*, vol. 6, no. 4, pp. 1762-1768, Oct 1991.
- [8] M. Klatt, J. Meyer, M. Elst and P. Schegner, "Frequency Responses of MV voltage transformers in the range of 50 Hz to 10 kHz," in *Proc. Int. Conf. on Harmonics and Quality of Power*, 2010, pp. 1-6.
- [9] G. Olivier, R.-P. Bouchard, Y. Gervais, and D. Mukhedkar, "Frequency response of HV test transformers and the associated measurement problems," *IEEE Trans. Power App. Syst.*, vol. PAS-99, no. 1, pp. 141-146, Jan./Feb. 1980.
- [10] M. Kaczmarek, "Measurement error of non-sinusoidal electrical power and energy caused by instrument transformers," in *IET Generation, Transmission & Distribution*, vol. 10, no. 14, pp. 3492-3498, Nov. 2016.
- [11] J. Luszcz, R. Smolenski, "Voltage harmonic distortion measurement issue in smart-grid distribution system," in *Proc. Asia-Pacific Symp. on Electromagnetic Compatibility*, Singapore, 2012, pp. 841-844.
- [12] C. Buchhagen, L. Hofmann and H. Däumling, "Compensation of the first natural frequency of inductive medium voltage transformers," in *Proc. IEEE Int. Conf. on Power System Technology*, 2012, pp. 1-6.
- [13] L. Kadar, P. Hacksel and J. Wikston, "The effect of current and voltage transformers accuracy on harmonic measurements in electric arc furnaces," *IEEE Trans. on Ind. Appl.*, vol. 33, no. 3, pp. 780-783, May/June 1997.
- [14] B. Boulet, L. Kadar and J. Wikston, "Real-time compensation of instrument transformer dynamics using frequency-domain interpolation techniques," in *Proc. IEEE Instrumentation and Measurement Technology Conf.*, 1997, pp. 285-290.
- [15] G. Crotti, D. Gallo, D. Giordano, C. Landi, M. Luiso and M. Modarres, "Frequency Response of MV Voltage Transformer Under Actual Waveforms," *IEEE Trans. Instrum. Meas.*, vol. 66, pp. 1146-1154, June 2017.
- [16] M. Faifer, R. Ottoboni, M. Prioli and S. Toscani, "Simplified Modeling and Identification of Nonlinear Systems Under Quasi-Sinusoidal Conditions," *IEEE Trans. Instrum. Meas.*, vol. 65, pp. 1508-1515, June 2016.
- [17] M. Faifer, C. Laurano, R. Ottoboni, M. Prioli, S. Toscani and M. Zanoni, "Definition of Simplified Frequency-Domain Volterra Models With Quasi-Sinusoidal Input," in *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 65, pp. 1652-1663, May 2018.
- [18] M. Faifer, C. Laurano, R. Ottoboni, S. Toscani and M. Zanoni, "Voltage Transducers Testing Procedure Based on the Best Linear Approximation," in *Proc. IEEE Int. Workshop on Applied Measurements for Power Systems*, 2017, pp. 1-6.
- [19] M. Faifer, C. Laurano, R. Ottoboni, S. Toscani and M. Zanoni, "Characterization of Voltage Instrument Transformers Under Nonsinusoidal Conditions Based on the Best Linear Approximation," *IEEE Trans. Instrum. Meas.*, vol. 67, pp. 2392-2400, Oct. 2018.
- [20] R. Pintelon, J. Schoukens, "System Identification. A frequency domain approach", 2nd ed., Boston, MA, USA, Wiley, 2012.
- [21] M. Faifer et al., "Overcoming Frequency Response Measurements of Voltage Transformers: An Approach Based on Quasi-Sinusoidal Volterra Models," *IEEE Trans. Instrum. Meas.*, vol. PP, no. 99, pp 1-8.
- [22] M. Faifer, C. Laurano, R. Ottoboni, S. Toscani and M. Zanoni, "A Simple Method for Compensating the Harmonic Distortion Introduced by Voltage Transformers" in *Proc. IEEE Int. Workshop on Applied Measurements for Power Systems*, 2018, pp. 183-188.
- [23] V. J. Mathews and G. L. Sicuranza, *Polynomial Signal Processing* (Wiley series in Telecommunications and Signal Processing). New York, NY, USA: Wiley, 2000.
- [24] *Voltage characteristics of electricity supplied by public distribution networks*, Standard EN 50160, 2010.
- [25] *Instrument transformers - Part 1: General requirements*, document IEC 61869-1:2009, 2009.
- [26] *Instrument transformers – Part 6: Additional general requirements for low-power instrument transformers*, document IEC 61869-6:2016, 2016.
- [27] M. Faifer, R. Ottoboni, S. Toscani, C. Cherbaucich and P. Mazza, "Metrological Characterization of a Signal Generator for the Testing of Medium-Voltage Measurement Transducers," *IEEE Trans. Instrum. Meas.*, vol. 64, pp. 1837-1846, July 2015.

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