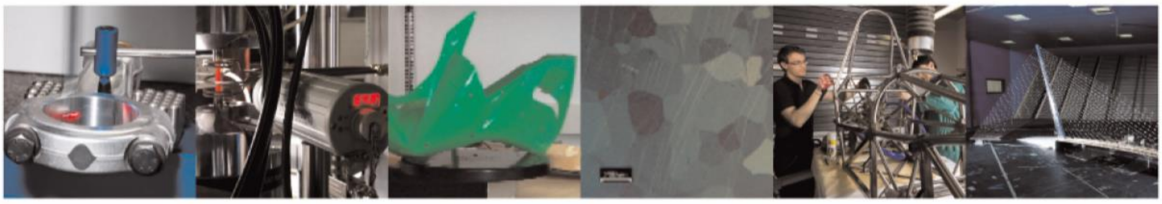
POLITECNICO
MILANO 1863

DIPARTIMENTO DI MECCANICA



A robust cardinality-constrained model to address the machine loading problem

Lugaresi, G.; Lanzarone, E.; Frigerio, N.; Matta, A.

This is a post-peer-review, pre-copyedit version of an article published in ROBOTICS AND COMPUTER-INTEGRATED MANUFACTURING. The final authenticated version is available online at: http://dx.doi.org/10.1016/j.rcim.2019.101883]

DIPARTIMENTO DI MECCANICA ■ POLITECNICO DI MILANO
via G. La Masa, 1 ■ 20156 Milano ■ EMAIL (PEC): pecmecc@cert.polimi.it
http://www.mecc.polimi.it
Rev. 0

A Robust Cardinality-Constrained Model to Address the Machine Loading Problem

Giovanni Lugaresi[1], Ettore Lanzarone[2*], Nicla Frigerio[1], Andrea Matta[1]
[1] *Politecnico di Milano, Dept. of Mechanical Engineering, Milan, Italy.*
[2] *CNR-IMATI, Milan, Italy.*
[*] *Corresponding author – ettore@mi.imati.cnr.it*

Abstract

Several deterministic models have been proposed in the literature to solve the Machine Loading Problem (MLP), which considers a set of product types to be produced on a set of machines using a set of tool types, and determines the quantity of each product type to be produced at each time period and the corresponding machine tool loading configuration. However, processing times are subject to random increases, which could impair the quality of a deterministic solution. Thus, we propose a robust MLP counterpart, searching for an approach that properly describes the uncertainty set of model parameters and, at the same time, ensures practical application. We exploit the Cardinality-Constrained approach, which considers a simple uncertainty set where all uncertain parameters belong to an interval, and allows tuning the robustness level by bounding the number of parameters that assume the worst value. The resulting plans provide accurate estimations on the minimum production level that a system achieves even in the worst conditions. The applicability of the robust MLP and the impact of robustness level have been tested on several problem variants, considering single- vs multi-machine and single- vs multi-period MLPs. We also consider the execution of the plans in a set of scenarios to evaluate the practical implications of MLP robustness. Results show the advantages of the robust formulation, in terms of improved feasibility of the plans, identification of the most critical tools and products, and evaluation of the maximum achievable performance in relation to the level of protection. Moreover, low computational times guarantee the applicability of the proposed robust MLP counterpart.

Keywords: Machine Loading Problem; Robust Optimization; Cardinality-Constrained Ap-

proach; Production Planning.

---

1. Introduction

Despite factories being considered the quintessence of clock-like accuracy, manufacturing systems must face several uncertain disruptive events, such as failures and variations of processing times. The level of uncertainty can be reduced by a proper management of the system, including maintenance interventions to prevent failures or standardization of the processes to reduce processing time uncertainty. However, even in modern manufacturing systems, uncertainty cannot be neglected and might have a relevant impact on system throughput, affecting profits and costs. Shapiro [1] already introduced stochastic models for accounting uncertainty in production planning in 1993. Nevertheless, at that time he also discouraged the introduction of stochastic models, pointing out that production managers had just begun to use deterministic models. After two decades, uncertainty is still often neglected by production planners, as reminded in [2]: "*Most systems for production planning do not recognize or account for uncertainty. Yet these systems are implemented in uncertain contexts*". The reason why models handling uncertainty have not been broadly applied yet probably stems from their complexity. A proper balance between the complexity of the model and a fair description of data uncertainty is a key point for a successful application of such models in practice. Too simple models can be ineffective and provide poor results, while overcomplicated models can remain confined to research papers.

We focus on the Machine Loading Problem (MLP), which is a relevant problem usually encountered in manufacturing. It considers a set of product types to be produced over a set of machines by using a set of available tools. The goal is to define a production plan that maximizes the profit or another function of interest over a given timeframe. Many variants of the problem are available in the literature, as well as several solution approaches. However, almost all the approaches proposed in the literature for the MLP do not take system uncertainty into account. However, from an industrial viewpoint processing times are subject to uncertain increases with respect to their nominal value due to several factors, such as tool wear, breakages of tools, and so on. Neglecting such uncertainty can result in production delays and unmet delivery dates. Properly addressing uncertain processing times would allow companies to implement more robust production plans and ensure that the minimum planned production levels be reached when plans are implemented.

In this paper, we address the MLP under uncertain processing times, and we develop the robust counterpart of a deterministic mathematical programming model exploiting the Cardinality-Constrained (CC) approach [3]. In particular, we refer to the deterministic MLP model of [4], but the same approach can also be used for other MLP models.

Our aims are to define a robust model that fairly describes the uncertainty set of model parameters while at the same time being not too complex (to ensure practical application), and above all to analyze the impact of the robustness in several MLP alternatives.

The fair description and the practical application together are the reason why we focus on robust optimization rather than on stochastic programming or distributionally robust optimization, and the reason why, among the robust approaches, we choose the CC approach. In fact, the CC approach considers a simple uncertainty set in which all uncertain parameters belong to an interval. Moreover, the level of robustness can be tuned by simply bounding the number of model parameters that assume the worst value in the interval (i.e., the solution corresponds to the worst combination of parameter values); such constrained number of parameters, denoted as *cardinality*, also lends the name to the approach. Finally, the CC formulation includes the uncertainty with a reasonable computational effort, i.e., the robust model is solved once and the counterpart of a linear model remains linear. Thus, the CC approach provides a trade-off between complexity and robustness, which can be easily tuned to take into account the specific degree of risk the decision maker accepts.

To validate our robust model and evaluate the impact of robustness with regard to the deterministic formulation, we apply the model to three problem alternatives that cover several realistic application scenarios. Moreover, for the most general one we also execute the solutions in a set of simulated scenarios, to analyze the practical impact and the trade-offs of including the robustness in practice. Results show the advantages of the robust formulation, and the low computational times guarantee its applicability.

To the best of our knowledge, almost all previous approaches do not include uncertainty in the MLP and, at the same time, the CC approach has so far been marginally exploited in the manufacturing context (see Section 2.2 for references and details). Thus, the contribution of our work stands in the integration of a widely adopted MLP deterministic model with a robust approach to include uncertain processing times. From a theoretical viewpoint, we choose the most suitable robust approach (i.e., the CC) and discuss the potentialities of such integration in a general context, which may also include other manufacturing problems. From an application viewpoint, we show the benefits and the trade-offs of such integration in the specific MLP case, supported by several numerical experiments. Thus, our work has practical implications in the potential applicability of the MLP to current manufacturing problems, since the robust formulation is better suited to practice and answers the needs of actual industrial scenarios, which are affected by uncertainty.

This work extends a previous conference paper [5] in three directions: 1) we deepen the discussion about the application of the CC to the MLP and to manufacturing in general; 2) we apply our robust MLP to practical problems characterized by different features and dimensions; 3) we execute the deterministic and robust solutions in a set of scenarios to evaluate the practical impact of including robustness in the plan.

The paper is organized as follows. Section 2 analyzes the literature on the MLP and the CC approach in manufacturing. Section 3 presents the deterministic MLP formulation [4] and the proposed robust counterpart. The experimental plan is described in Section 4, together with a discussion about the numerical results. Finally, Section 5 draws the conclusions of the work.

2. Literature review

In this section, we revise the existing literature related to this work. The MLP literature is analyzed in Section 2.1, while the CC approach and its applications in manufacturing are discussed in Section 2.2.

2.1. The Machine Loading Problem

Given a set of product types to produce, a set of machines and a set of tools, the MLP consists in deciding the quantities to produce on each machine and the tools to be assigned to the machines, in compliance with technological and capacity constraints. The MLP idea was firstly introduced by [6] and [7]. Stecke and Solberg [6] conducted an experimental investigation on the operating strategies for a real computer-controlled Flexible Manufacturing System (FMS) consisting of nine machines, an inspection station, and a centralized queueing area. They developed loading policies and real-time flow control strategies, and tested them through simulation. Results showed a strong dependence of the system performance on the loading and control strategies. Stecke [7] defined five production planning problems to be solved for efficient use of a FMS. Moreover, she proposed a mixed integer programming model for the grouping problem, and another one for the loading problem; finally, she first developed a nonlinear formulation and introduced several linearization methods.

Following these works, the deterministic MLP has been studied in-depth and the number of publications on the subject is quite extensive. Detailed surveys are available in [8, 9] and, more recently, in [10, 11]. Among the others, Shanker and Tzen [12] compared a heuristic approach with the exact formulation. Nagarjuna et al. [13] proposed a heuristic based on multi-stage programming for the loading problem in random type FMS, to select a subset of jobs and allocate them among the available machines. Chan and Swarnkar [14] proposed a fuzzy goal programming model for the machine tool selection and operation allocation problem of FMS, and applied an approach based on the ant colony optimization to solve it. Rossi et al. [15] proposes an ant colony optimization system for solving FMS scheduling with routing flexibility, sequence-dependent setup and transportation time, parallel machines and operation lag times. Das et al. [16] analyzed the problem in a comprehensive way, including machine loading, product part type grouping, and operations sequencing. Zeballos [17] proposed a constraint programming methodology to deal with FSM scheduling, which handles several industrial features, such as limitations on the number of tools, lifetime of tools, and tool magazine capacity. Abazari et al. [18] compared two solution strategies based on a genetic algorithm.

However, in practice, processing times are subject to a large extent to uncertain increases due to a variety of reasons (e.g., failures, unexpected tool breaks, unplanned maintenance interventions), and perturbations on such input data might impair the deterministic MLP solution, worsening the objective function value or even making the solution infeasible. The only relevant contributions addressing uncertainty in the MLP are listed below. Vidyarthi and Tiwari [19] formulated a fuzzy-based solution methodology to address the MLP uncertainty in the tool-machines and piece-tool assignments, to minimize system imbalance and maximize the throughput. Aldaihani and Savsar [20] considered a flexible manufacturing cell consisting of two machines, a pallet handling system, and a loading/unloading robot; they developed a stochastic model to determine the performance of the cell under variable operational conditions, including random machining times, random loading and unloading times, and random pallet transfer times. Mandal et al. [21] included machine breakdowns in the dynamic MLP with a view to maximizing the throughput and minimizing system unbalance and makespan; the results recorded under breakdowns validated the robustness of their model. Lugaresi et al. [5] firstly addressed uncertainty of processing times in the MLP with the goal of computing the price of robust solutions against a fixed number of disruptions.

2.2. Robust optimization and the Cardinality-Constrained approach

Different approaches have been proposed in the literature to deal with uncertain parameters in optimization problems, which belong to three groups: stochastic programming, distributionally robust optimization, and robust optimization [22]:

- *Stochastic programming* [23, 24].
  Uncertain parameters are modeled as random variables whose probability density functions are assumed to be known. The advantage is to provide a comprehensive and detailed description of the uncertain parameters; thus, stochastic programming models usually produce solutions that are neither over- nor under-conservative and protect against likely realizations, since they are tailored on the exact probability density. However, they require a deep knowledge of the real problem to derive the probability densities, and the resulting optimization problems can be difficult to solve, as they usually involve a wide number of scenarios to be evaluated. Finally, the risk of producing

bad solutions when the probability densities are not reliable is not negligible.

- *Distributionally robust optimization* (and *ambiguous chance-constrained approach*) [25, 26].

  These approaches assume that the probability densities are not known but lie in a known family of densities. The solution provided protects against the worst-case realization given by the admissible probability densities. The resulting problems are usually difficult to solve, but there are computationally tractable approximations [26].

- *Robust optimization* [27, 28, 29].

  These approaches assume that the uncertain parameters belong to a given (convex) *uncertainty set*, without any knowledge of the probability distribution over this set. The provided solution is guaranteed to be feasible for all realizations of the parameters within the set. The resulting problems are usually computationally more tractable than under stochastic programming; thus, robust optimization approaches represent a good compromise between the alternatives. Robust optimization is associated with the risk of producing over-conservative solutions, as they protect against all possible realizations in the uncertainty set. However, the uncertainty set can be easily adjusted to exclude unlikely realizations, thus tuning the level of robustness.

We focus on robust optimization approaches, as we aim to address the trade-off between the complexity of the model and a fair description of the uncertainty set for the model parameters. There are several robust optimization approaches in the literature, based on the shape and the assumptions that define the uncertainty set. In all the approaches, two contrasting aspects must be properly balanced, i.e., the level of robustness versus the efficiency and cost of the solution. The first is related to the *size* of the uncertainty set and refers to the feasibility of the solution in all scenarios given by the possible realizations of the parameters; the latter refers to the deterioration of the solution when applied to actual realizations of the uncertain parameters. On the one hand, a very conservative solution pushed by unlikely scenarios (when too large uncertainty sets are adopted) may turn out to be highly expensive for more likely scenarios. On the other hand, too weak solutions (when too strict uncertainty sets are adopted) easily become unfeasible even for small variations of the parameters. It is therefore essential to tune the level of robustness by means of proper uncertainty sets.

One of the earliest contributions was proposed by Soyster [27], who considered a linear optimization model to construct a solution that is feasible for all the realizations of the uncertain parameters. He assumed that each model parameter lies in an interval, and the robust solution protects against the case in which all parameters assume their worst value. However, it is usually unlikely that all parameters assume the worst value together; thus, the produced solutions are usually over-conservative and associated with bad values of the objective function. More recently, Ben-Tal and Nemirovski [28, 30] and Ben-Tal et al. [31] considered an uncertainty set in which the parameters of each constraint lie in an ellipsoidal uncertainty set. Thus, the parameters appearing in the same constraint are restricted from taking their worst values simultaneously. However, this approach leads to difficult conic quadratic problems, which are nonlinear though still convex.

Bertsimas and Sim [3] considered a different uncertainty set in which all uncertain parameters belong to an interval and the level of robustness is tuned by superiorly bounding, for each constraint, the cardinality of the subset of parameters that may assume the worst value in the interval instead of the nominal value. This approach keeps the simple description of the uncertainty set proposed by Soyster [27] while, at the same time, avoiding its over-conservatism. We believe that the CC approach is suitable for addressing the uncertainty of processing times typical of the MLP. In fact, production managers can easily estimate the number of disruptive events expected over a given timeframe, and they can use this knowledge to make reasonable assessments on the cardinality values to adopt, thus tuning the level of robustness of the solution.

Despite the CC approach has been widely applied by several researchers from heterogeneous backgrounds [32], there are several fields in which it has not been exploited yet, or where it has been employed only recently. Even in manufacturing problems, such as production planning, applications of the CC are not common. The valuable contributions, directly or indirectly related to manufacturing, that can be found in the literature are presented below.

Bertsimas and Thiele [33] addressed the optimal control of a supply chain in discreet time under stochastic demands that are not identically distributed over time. Hazir et al. [34] addressed the discrete time/cost trade-off problem and formulated three robust models, which were solved via exact and heuristic algorithms. Moon and Yao [35] developed a robust mean absolute deviation model for portfolio optimization that controls the impact of the estimation errors. Alem and Morabito [36] derived robust combined lot-sizing and cutting-stock models for furniture companies with uncertain production costs and product demands. Solyali et al. [37] proposed two mixed integer programming formulations for a robust inventory routing problem that faces dynamic uncertain demands over a finite discrete timeframe. Hazir and Dolgui [38] addressed the line balancing problem under operation times with uncertain intervals, and they developed a decomposition-based algorithm to solve large instances. Lu et al. [39] worked on the single machine scheduling problem with uncertain and correlated processing times to obtain robust job sequences with the minimum worst-case total flow time. Moreira et al. [40] considered assembly lines with uncertain and worker-dependent task execution times, with the goal of finding an assignment of tasks and workers to a minimal number of stations, such that the resulting productivity level abides by a desired robust measure.

These works, together with their main features, are summarized in Table 1. We may observe that the uncertainty is usually included in processing/operation times and demands, which are the most relevant factors affecting the feasibility and the quality of the production plans in practice. We can furthermore observe that the CC has not been applied to the MLP to date.

3. Model formulation

In this section, we integrate a deterministic MLP model available in the literature with the CC approach. As a deterministic model, we consider the one proposed by Sodhi et al. [4], which includes all general features of the MLP and addresses the core issues identified in the literature [9] in a compact formulation. Moreover, it has been used as reference in several applications, and it allows an easy integration of uncertain processing times (Section 3.2).

Reference	**Problem**	**Uncertainty set**
[33]	Supply chain management	Product demand
[34]	Project scheduling	Cost
[35]	Portfolio optimization	Return rate of assets
[36]	Production planning	Production costs, demand
[37]	Inventory routing	Product demand
[38]	Assembly line balancing	Operation times
[39]	Single-machine job scheduling	Processing times
[40]	Assembly line balancing	Workers' task execution times

Table 1: Literature works that apply the CC approach to manufacturing.

We present the deterministic MLP, denoted as MLP-D, in Section 3.1, and the developed robust counterpart, denoted as MLP-R, in Section 3.2. Sets, parameters and decision variables are summarized in Table 2.

3.1. Deterministic model (MLP-D)

We address the short-term production planning and the allocation of tools to machines. We consider a set $\mathbb{I}$ of product types to be produced on a set $\mathbb{M}$ of machines, using a set $\mathbb{J}$ of tool types. We divide the production planning timeframe $\mathbb{T}$ into a finite number of time periods $t \in \mathbb{T}$. Within each period t, each machine $m \in \mathbb{M}$ processes the product types assigned to it using the selected tools with a given time availability A_{mt}. The processing time of one unit of product type $i \in \mathbb{I}$ over the tool of type $j \in \mathbb{J}$ is denoted by O_{ij}.

The tool storage capacity of each machine is limited, and K_m is the total number of slots available in the tool magazine of machine m. The number of slots occupied by a tool of type j (independent of the machine) is denoted by k_j, while the number of available copies of j by α_j, with $\alpha_j \leq |\mathbb{M}|$. However, only one tool copy can be loaded on machine m at the same time. The latter assumption is not a limitation, because tools are sequentially used on each machine and tool wear is not included in the model. In general, α_j is strictly lower than $|\mathbb{M}|$ to account for a common situation in production environments where some operations require expensive tools and only a limited number of copies are kept on site. The marginal

MLP-D	Sets	
	$\mathbb{I}$	set of product types
	$\mathbb{J}$	set of tool types
	$\mathbb{M}$	set of machines
	$\mathbb{T}$	set of time periods (timeframe)
	Parameters	
	A_{mt}	time availability of machine $m \in \mathbb{M}$ within period $t \in \mathbb{T}$
	C_i	penalty shortage cost per unit of product type $i \in \mathbb{I}$
	D_i	demand of product type $i \in \mathbb{I}$ to be satisfied over the timeframe $\mathbb{T}$
	h_{it}	holding cost of a unit of product type $i \in \mathbb{I}$ from period $t \in \mathbb{T}$ until the end of the time horizon
	k_j	number of slots occupied by a tool of type $j \in \mathbb{J}$
	K_m	total number of slots available in the tool magazine of machine $m \in \mathbb{M}$
	O_{ij}	processing time of one unit of product type $i \in \mathbb{I}$ over tool type $j \in \mathbb{J}$
	w_i	marginal profit per unit of product type $i \in \mathbb{I}$
	α_j	number of available copies of tool of type $j \in \mathbb{J}$
	Decision variables	
	L_{jmt}	Boolean variable equal to 1 if tool type $j \in \mathbb{J}$ is loaded on machine $m \in \mathbb{M}$ in period $t \in \mathbb{T}$; 0 otherwise
	p_{jmt}	aggregate machining time spent by tool type $j \in \mathbb{J}$ on machine $m \in \mathbb{M}$ in period $t \in \mathbb{T}$
	S_i	shortage of product type $i \in \mathbb{I}$ with regard to D_i
	x_{it}	quantity of product type $i \in \mathbb{I}$ produced in period $t \in \mathbb{T}$
Additional for MLP-R	Parameters	
	$\bar{O}_{ij}$	nominal processing time for one unit of product type $i \in \mathbb{I}$ over tool type $j \in \mathbb{J}$
	$\widehat{O}_{ij}$	maximum increase of the processing time for one unit of product type $i \in \mathbb{I}$ over tool type $j \in \mathbb{J}$
	Γ_{jt}	cardinality parameter for tool $j \in \mathbb{J}$ in period $t \in \mathbb{T}$
	Decision variables	
	r_{ij}	auxiliary dual variable
	q_{ijt}	auxiliary dual variable

Table 2: Sets, parameters and decision variables of the MLP-D and the MLP-R.

profit per unit of product type i is denoted by w_i, while the demand of product type i to be satisfied over the timeframe by D_i. C_i is the penalty cost per unit of product type i for not satisfying the demand (e.g., the sub-furniture cost if outsourcing is allowed). Finally, h_{it} is the holding cost of a unit of product type $i \in \mathbb{I}$ from period $t \in \mathbb{T}$ (when it is produced) until the end of the timeframe.

The model determines the production plan in terms of the quantity of each product type i to be produced in each period t, which is denoted by x_{it}. Moreover, it also determines the machine tool loading, which is expressed by the Boolean variables L_{jmt} equal to 1 if a tool of type j is loaded on machine m in period t, and 0 otherwise. Finally, variables p_{jmt} represent the aggregate machining time spent by tool j on machine m in period t, and S_i denotes the production shortage of product type i with regard to its demand D_i. The goals are to maximize the total profit related to the production and to minimize shortage and holding costs, while the workload balancing among the machines is not pursued. The obtained plan is static, decided once at the beginning of the entire timeframe.

The assumptions at the basis of the model are listed below.

- Planning and scheduling problems are at two different levels, in agreement with several literature works [41], and the model considers the MLP at the planning level only, as in [4], addressing the short-term production planning and the allocation of tools to machines. Thus, scheduling and sequencing problems are not considered, and specific scheduling issues (as preemption within a period and resource starvation) are not modeled, since they are addressed at the next scheduling level. Planning is static, as it determines the production plan once for the entire timeframe $\mathbb{T}$.

- Production quantities x_{it} are continuous, according to [4], enabling the partial production of a product of type i over a time period t. In general, they can be limited to the integer domain to represent single-piece production cases; anyway, the considered MLP would provide similar insights for integer variables as well.

- The set $\mathbb{I}$ of product types and the set $\mathbb{J}$ of tool types have been previously composed, and no tool transportation system is present; thus, it is not possible to change the tool set without stopping the system. On the contrary, tools loaded on a certain machine can be changed between periods t.

- Tools are sequentially used, i.e., the time to process a unit of product type i is the sum of O_{ij} values over j.
- Machining non-operative times are included in the processing times, and the times required to set up the machines are already deducted from machine availability.

The deterministic model MLP-D is formulated as follows:

$$\max \quad \sum_i \sum_t (w_i - h_{it}) x_{it} \; - \; \sum_i C_i S_i \tag{1}$$

subject to:

$$\begin{aligned}
\sum_j L_{jmt} k_j \leq K_m & \qquad \forall m, t, & (2)\\
\sum_t x_{it} = D_i - S_i & \qquad \forall i, & (3)\\
\sum_i O_{ij} x_{it} \leq \sum_m p_{jmt} & \qquad \forall j, t, & (4)\\
p_{jmt} \leq L_{jmt} A_{mt} & \qquad \forall j, m, t, & (5)\\
\sum_j p_{jmt} \leq A_{mt} & \qquad \forall m, t, & (6)\\
\sum_m L_{jmt} \leq \alpha_j & \qquad \forall j, t, & (7)\\
L_{jmt} \in \{0, 1\} & \qquad \forall j, m, t, &\\
p_{jmt} \geq 0 & \qquad \forall j, m, t, &\\
x_{it} \geq 0 & \qquad \forall i, t, &\\
S_i \geq 0 & \qquad \forall i. &
\end{aligned}$$

The objective function (1) maximizes the total profit related to the production and minimizes holding and shortage costs. Constraints (2) limit the number of tools that can be loaded onto each machine, as machines have limited tool storage capacity (up to the number K_m of available slots in the machine tool magazine; see Sodhi et al. [4] for further details).

Constraints (3) compute the shortage S_i of each product i, as the difference between the production of part type i over the entire timeframe ($\sum_t x_{it}$) and the demand D_i. Constraints (4)-(6) guarantee that the production is made within the time availability of the machines. Specifically, constraints (4) limit the total production time on tool j in time period t, and constraints (5) guarantee that, for each tool j, machine m, and period t, the production can be done only if the necessary tool has been loaded; constraints (6) guarantee that, at each time period t, the production time at machine m does not exceed the availability A_{mt}. Constraints (7) limit the number of tool copies α_j that are available for type j in each period t. Finally, the remaining constraints define the domain of the variables.

3.2. Robust model formulation (MLP-R)

We include the robustness to protect from disruptive events that perturb the processing times from their nominal value up to a worst-case value. According to the CC approach [3], we model the processing times as random variables $\tilde{O}_{ij}$ characterized by a nominal value $\bar{O}_{ij}$ and a random increase up to a maximum value $\widehat{O}_{ij}$; thus, they may take values in the interval $\left[\bar{O}_{ij}\,,\,\bar{O}_{ij}+\widehat{O}_{ij}\right]$. By applying the CC approach, we assume that, for each tool j and time period t, up to Γ_{jt} processing times deviate from the nominal value $\bar{O}_{ij}$ up to the worst value $\bar{O}_{ij}+\widehat{O}_{ij}$. Both maximum increases $\hat{O}_{ij}$ and cardinalities Γ_{jt} are easy to derive in practical applications, from the information commonly available in industry. For instance, the variation ranges can be derived from historical data, or the maximum increase can be obtained using the expected Mean Time To Repair ($MTTR_j$) of each tool j, by assuming $\widehat{O}_{ij} \cong MTTR_j$. Furthermore, we can estimate the cardinality parameters from the number of failures observed for each tool j, which is related to the expected Mean Time Between Failures ($MTBF_j$) and the time availability of machine m. Bertsimas and Sim [3] assumed that the random parameters are distributed according to a symmetric density around the nominal value $\bar{O}_{ij}$, while we only consider positive increases from $\bar{O}_{ij}$[1].

[1]The CC approach itself does not require the assumption of symmetric distribution, while it relies on stochastic parameters that are simply characterized by a nominal value and a maximum increase, as we deal with in our model. Bertsimas and Sim [3] included the assumption of symmetric distributions over $\left[\bar{O}_{ij}-\widehat{O}_{ij}\,,\,\bar{O}_{ij}+\widehat{O}_{ij}\right]$ in order to determine probability bounds of constraint violation, which is not of

Considering the MLP-D, the increase of total processing time affects constraints (4). We denote by E_{jt} the set of the processing times over tool j at period t and define the cardinality matrix $\mathbf{\Gamma} = \{\Gamma_{jt}\}$, where each Γ_{jt} represents the maximum number of product types $i \in \mathbb{I}$ subject to an increment of the processing time over tool j at period t, up to the maximum value $\bar{O}_{ij} + \widehat{O}_{ij}$. Set $U_{jt} \subseteq E_{jt}$ represents the subset of product types whose processing time increase to the worst value $\bar{O}_{ij} + \widehat{O}_{ij}$, while processing times in $E_{jt} \setminus U_{jt}$ assume the nominal value $\bar{O}_{ij}$. Subset of E_{jt} is constrained to have cardinality lower than or equal to Γ_{jt}, i.e., $|U_{jt}| \leq \Gamma_{jt}$; at the optimum, set U_{jt} has cardinality Γ_{jt} and contains the worst combination of times that mostly affects the total usage time of tool j in period t.

The robust counterpart of constraints (4) is:

$$\sum_i \bar{O}_{ij} x_{it} + \max_{U_{jt}} \left\{ \widehat{O}_{ij} x_{it} \right\} \leq \sum_m p_{jmt} \qquad \forall j, t. \tag{8}$$

The selection of the processing times to include in set U_{jt} is an inner maximization problem, which is represented by a specific set of auxiliary decision variables z_{ijt}; we set $z_{ijt} = 1$ if product type i is included in U_{jt}, and $z_{ijt} = 0$ if not included. Moreover, continuous values between 0 and 1 allow a partial increase of the processing time between $\bar{O}_{ij}$ and $\bar{O}_{ij} + \widehat{O}_{ij}$ [3].

The inner maximization problem, denoted as *Primal*, can be written as:

$$\max \sum_i \widehat{O}_{ij} \tilde{x}_{it} z_{ijt} \tag{9}$$

subject to:

$$\sum_i z_{ijt} \leq \Gamma_{jt} \qquad \forall j, t, \tag{10}$$

$$0 \leq z_{ijt} \leq 1 \qquad \forall i, j, t, \tag{11}$$

where $\tilde{x}_{it}$ refers to a given solution of the MLP problem.

The dual of problem *Primal*, denoted as *Dual*, is:

$$\min \ \Gamma_{jt} r_{jt} + \sum_i q_{ijt} \tag{12}$$

interest for our work.

subject to:

$$r_{jt} + q_{ijt} \geq \widehat{O}_{ij}\tilde{x}_{it} \qquad \forall i, j, t, \tag{13}$$
$$r_{ij} \geq 0 \qquad \forall i, j,$$
$$q_{ijt} \geq 0 \qquad \forall i, j, t,$$

where r_{jt} and q_{ijt} are the dual variables referring to constraints (10) and (11), respectively.

Since the problem *Primal* is bounded and feasible, for the Strong Duality Theorem the optimal solution z^*_{ijt} of problem *Primal* and the optimal solution $\left(r^*_{jt}, q^*_{ijt}\right)$ of problem *Dual* are equivalent in terms of objective functions (9) and (12). The property holds for any $\tilde{x}_{it}$; therefore, also for the optimal x^*_{it}:

$$\Gamma_{jt} r^*_{jt} + \sum_i q^*_{ijt} = \widehat{O}_{ij} x^*_{it} z^*_{ijt}.$$

In this way, the maximization term $\max_{U_{jt}} \left\{\widehat{O}_{ij} x_{it}\right\}$ in (8) is the solution of problem *Dual*, and we may replace the maximization term with the *Dual* objective function. Moreover, this substitution requires us to add the constraints of problem *Dual*. Thus, the robust problem MLP-R is:

$$\max \quad \sum_i \sum_t (w_i - h_{it}) x_{it} \; - \sum_i C_i S_i \tag{14}$$

subject to:

$$\sum_j L_{jmt} k_j \leq K_m \qquad \forall m, t, \tag{15}$$

$$\sum_t x_{it} = D_i - S_i \qquad \forall i, \tag{16}$$

$$\sum_i \bar{O}_{ij} x_{it} + \Gamma_{jt} r_{jt} + \sum_i q_{ijt} \leq \sum_m p_{jmt} \qquad \forall j, t, \tag{17}$$

$$r_{jt} + q_{ijt} \geq \widehat{O}_{ij} x_{it} \qquad \forall i, j, t, \tag{18}$$

$$p_{jmt} \leq L_{jmt} A_{mt} \qquad \forall j, m, t, \tag{19}$$

$$\sum_j p_{jmt} \leq A_{mt} \qquad \forall m, t, \tag{20}$$

$$\sum_m L_{jmt} \leq \alpha_j \qquad \forall j, t, \tag{21}$$

$$L_{jmt} \in \{0, 1\} \qquad \forall j, m, t,$$

$$p_{jmt} \geq 0 \qquad \forall j, m, t,$$

$$x_{it} \geq 0 \qquad \forall i, t,$$

$$S_i \geq 0 \qquad \forall i,$$

$$r_{ij} \geq 0 \qquad \forall i, j,$$

$$q_{ijt} \geq 0 \qquad \forall i, j, t.$$

As mentioned, (17) is given by (8) in which the maximization term is replaced by the objective function of problem *Dual*, and (18) are the additional constraints required by this term. The other constraints and the objective function of the MLP-R are the same as in the MLP-D. The MLP-R solves the same problem of the MLP-D, but in a scenario where the worst realization of the times occurs. Such worst realization is automatically determined by the MLP-R, thanks to the optimal solution of problem *Dual* that is embedded in (17).

4. Experimental analyses

Three variants of the MLP, which cover the range of possible problem alternatives, have been considered to evaluate the behavior of the proposed MLP-R with regard to the MLP-D:

- **Single-Machine Single-Period (SM-SP) problem** with $|\mathbb{M}| = 1$ and $|\mathbb{T}| = 1$;

- **Multi-Machine Single-Period (MM-SP) problem** with $|\mathbb{M}| > 1$ and $|\mathbb{T}| = 1$;
- **Multi-Machine Multi-Period (MM-MP) problem** with $|\mathbb{M}| > 1$ and $|\mathbb{T}| > 1$.

The MM-MP is the most general one, while the SM-SP and the MM-SP are derived by constraining $|\mathbb{M}| = |\mathbb{T}| = 1$ and $|\mathbb{T}| = 1$, respectively. The goal is to first outline relevant insights in simpler problems (SM-SP and MM-SP) that are anyway of general validity for the MLP. Afterwards, the goal is to analyze the behavior of the approach in the most general case (i.e., the MM-MP problem).

We separately present the three problems, specifically discussing the related results. Several instances are created, with a range of problem parameters that realistically represent applications in manufacturing of non-standardized products, characterized by high variability both in terms of demand and product mix (ranges of parameter values are based on the realistic cases reported in [16]). As for the general MM-MP, we also consider the execution of the solutions in a set of scenarios. Finally, we provide details about the computational times in Section 4.4.

Briefly, the main findings obtained from the tested cases are: 1) the price of robustness increases with the cardinality, and the solutions converge to the most conservative one; 2) the robust production plan is not trivial to determine (e.g., with empirical rules), as the ranking of product types dynamically changes according to the assigned quantities; 3) machines are equally saturated on account of no cost difference among them in both MLP-D and MLP-R; 4) failures are equally balanced among machines, and the most critical tools are not assigned to the same machine; 5) machine availability is saturated starting from the last periods of the timeframe, to reduce the holding costs; 6) the CC approach easily enables the identification of critical product types.

In the presentation of the results, we denote the optimal quantity of product type i in period t as x^*_{it} and we define:

- the Nominal Production Time per unit of product i as $NPT_i = \sum_{j\in\mathbb{J}} \bar{O}_{ij}$;
- the Total Nominal Processing Time allocated to produce parts as $TNPT = \sum_{i\in\mathbb{I}} \left(NPT_i \cdot \sum_{t\in\mathbb{T}} x^*_{it}\right)$;

- the Total Failure Time (total time increase for failures) as
  $TFT = \sum_{i\in\mathbb{I},j\in\mathbb{J},t\in\mathbb{T}} \left(\max_{U_{jt}} \left\{\widehat{O}_{ij} \cdot x^*_{it}\right\}\right)$;
- the Total Processing Time as $TPT = TNPT + TFT$.

4.1. Single-Machine Single-Period (SM-SP) problem

We consider the SM-SP to compare the MLP-R solution with regard to the MLP-D under different levels of robustness, focusing on the impact of Γ_{j1} and δ only, where δ is the relative increase of the processing times defined as:

$$\delta = \frac{\widehat{O}_{ij} - \bar{O}_{ij}}{\bar{O}_{ij}}.$$

4.1.1. Problem description

Let us consider a set of $|\mathbb{I}| = 12$ product types to be produced over a single time period with $A_{tot} = \sum_{mt} A_{mt} = 2700$ *minutes*. A set of $|\mathbb{J}| = 12$ tool types is required. The demand D_i and the nominal times NPT_i are reported in Table 3a for each product type i. With the given A_{tot}, it is not possible to produce all product types; thus, even the MLP-D solution is not trivial. The production of each product type i requires between 2 and 4 tools, and the nominal processing times $\bar{O}_{ij}$ required for each tool j are reported in Table 3b. Without loss of generality, we consider equal marginal profits ($w_i = 30\ \forall i$) and null shortage costs ($C_i = 0\ \forall i$). As the problem is single-period, we do not consider the holding costs. We relax constraints (2) and (15) by assuming that the machine tool magazine has enough tool storage capacity. Otherwise, a strict limit on tool capacity K_m might limit the set of feasible solutions and, as a consequence, machine availability might not be saturated [4].

We consider three different values for δ, i.e., $\delta = \{0.1, 0.5, 1\}$. Then, for each δ, we solve the MLP-R under different cardinality vectors $\mathbf{\Gamma} = \{\Gamma_{j1}\}$. The maximum number of disruptions for a tool is equal to the number of product types requiring the tool, and from Table 3b we observe that its highest value over the tools is $\Gamma_{max} = 5$. Thus, $\Gamma_{j1} \leq \Gamma_{max}\ \forall j$, and in the most conservative case with $\Gamma_{j1} = \Gamma_{max}\ \forall j$ we include up to $\Gamma_{tot} = |\mathbb{J}|\,\Gamma_{max} = 60$ events. However, only 37 of them correspond to an actual increase of processing times, because the increase is null when a tool is not required (no value in Table 3b). In the

Product type i	1	2	3	4	5	6	7	8	9	10	11	12
D_i [$units$]	160	4	8	8	40	4	4	20	20	8	8	4
NPT_i [$min/unit$]	14.08	11.86	11.88	7.71	7.97	14.86	11.32	7.81	8.32	6.55	20.37	15.48

(a)

$\bar{O}_{ij}$		Tool j											
		1	2	3	4	5	6	7	8	9	10	11	12
Product type i	1	4.61	2	2.74	4.73	–	–	–	–	–	–	–	–
	2	2.46	2.51	–	4.19	2.7	–	–	–	–	–	–	–
	3	4.4	–	–	2.86	4.62	–	–	–	–	–	–	–
	4	–	–	–	–	–	3.12	1.95	2.64	–	–	–	–
	5	–	–	–	–	–	–	–	–	4.29	3.68	–	–
	6	–	–	–	–	–	–	–	–	3.15	5.39	2.18	4.14
	7	–	–	–	–	–	–	–	–	–	2.7	2.44	6.18
	8	–	–	1.92	2.59	3.3	–	–	–	–	–	–	–
	9	–	–	3.04	5.28	–	–	–	–	–	–	–	–
	10	4.45	2.1	–	–	–	–	–	–	–	–	–	–
	11	–	–	–	–	5.42	4	–	–	–	–	5.48	5.47
	12	–	–	–	–	–	–	–	4.22	3.62	3.92	3.72	–

(b)

Table 3: SM-SP problem – Product demand D_i [units] and NPT_i [min/unit] of each product type (a); nominal processing times $\bar{O}_{ij}$ [min/unit], where "–" denotes that the tool is not required for the product (b).

experiments, we consider both cases with equal cardinalities ($\Gamma_{j1} = \Gamma\ \forall j$) and cases with cardinalities that vary from tool to tool.

4.1.2. Results

The deterministic solution ($\Gamma = 0$ in Table 4) achieves an objective function of 7103. As expected, with continuous production quantities, the machine availability is saturated. Moreover, with equal w_i, h_{it} and C_i over i, the selection of product types is based only on a ranking of the nominal production times NPT_i, i.e., the product types i with shorter NPT_i are firstly selected to be produced. In our case, the order of selection is 10, 4, 8, 5, 9, 7, 2, 3, 1, 6, 12 and then 11; as the availability is not enough to produce all product types, the demand of types $i = \{6, 12, 11\}$ is not satisfied and that of $i = 1$ is only partially satisfied. This result is in line with the machine loading literature.

We now consider the solutions from the robust approach. The objective functions are first shown in Figure 1 in relation to Γ_{tot} and δ. The gap between the value of the objective function in the deterministic solution and that in any robust solution represents the *price of robustness*, i.e., the cost to protect from unfortunate events, which is paid in terms of a reduced planned production. As expected, the price of robustness monotonically increases over both Γ_{tot} and δ, because of a larger uncertainty set in which the number of disruptive events and the impact of each event increase. As the Γ_{tot} increases, the solution converges for each δ to that of the worst-case, in which all processing times increase. Furthermore, it can be noticed that, on average, the marginal decrease of the performance is higher for low cardinality values, since the most disruptive events are firstly selected by the robust model, according to the CC approach. Moreover, let us remark that, as the increased times in (9) are selected in order from the worst to the least relevant, the latest increases up to $\Gamma_{j1} = 5$ are not always related to an actual use of the tool once all processing times are selected (empty entries in columns of Table 3b).

Table 4 collects the results obtained for equal cardinalities $\Gamma_{j1} = \Gamma\ \forall j$. They show that, while product types $i = \{2, 3, 4, 5, 7, 8, 10\}$ are always top ranked and their demand completely satisfied, the ranking based on NPT_i does not hold in the robust cases; for example, it happens that product types $i = \{6, 11, 12\}$ are produced for $\Gamma > 0$ although

i	$\Gamma=0$	$\delta=0.1$					$\delta=0.5$					$\delta=1$				
		$\Gamma=1$	$\Gamma=2$	$\Gamma=3$	$\Gamma=4$	$\Gamma=5$	$\Gamma=1$	$\Gamma=2$	$\Gamma=3$	$\Gamma=4$	$\Gamma=5$	$\Gamma=1$	$\Gamma=2$	$\Gamma=3$	$\Gamma=4$	$\Gamma=5$
1	124.8	103.8	108.6	107.6	107.4	107.3	63.3	58.5	62	61.2	60.8	37.2	27.4	30.6	29.4	28.8
2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
3	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
4	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
5	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
6	0	4	0	0	0	0	4	4	0	0	0	4	4	0	0	0
7	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
8	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
9	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
10	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
11	0	0	0	0	0	0	2.7	0	0	0	0	4.5	1.8	0	0	0
12	0	2.6	0	0	0	0	4	3.5	0	0	0	4	4	0	0	0

Table 4: SM-SP problem – Optimal solutions x^*_{i1} in relation to Γ and δ, with $\Gamma_{j1}=\Gamma\ \forall j$; the case $\Gamma=0$ refers to the MLP-D.

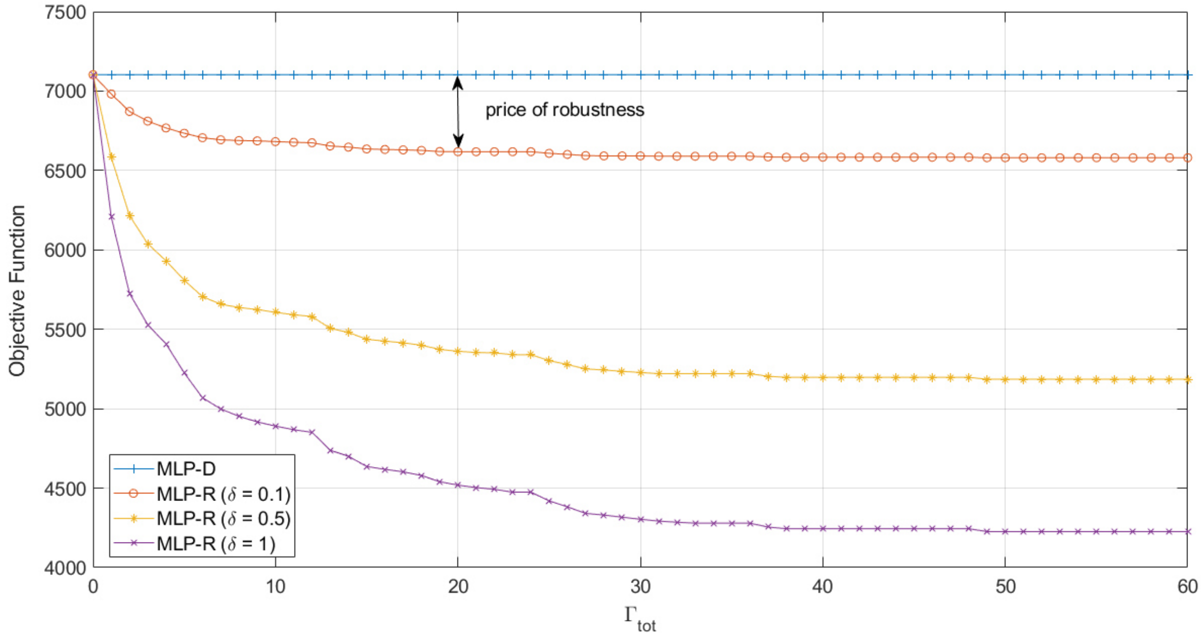


Figure 1: SM-SP problem – Objective function while varying Γ_{tot} for different δ values. While Table 4 only includes equal cardinalities $\Gamma_{j1} = \Gamma\ \forall j$, the abscissa values are given by cardinalities Γ_{j1} that depend on j, with higher Γ_{j1} values associated with higher $\bar{O}_{ij}$. The worst case corresponds to $\Gamma_{j1} = 5\,\forall j$ with $\Gamma_{tot} = 60$.

the demand of type $i = 1$ is not completely satisfied. Indeed, the selection depends on the cardinality and finding rules to guide the selection is not trivial. Let us denote by PT_i^R the *robust* processing time to produce a unit of type i, which includes the disruptive events and depends on the subset of processing times $O_{ij} : j \in E_{j1}$. Thus, the ranking of product types is based on times PT_i^R that dynamically change with Γ (and δ).

To describe the phenomenon, let us consider an example with $\Gamma = 1$, $\delta = 1$ and a solution $x = \{0, 0, 0, 8, 40, 0, 0, 20, 0, 4, 0, 0\}$ where each tool is used by only one product type (see Table 4b); therefore, all product types are affected by disrupting events (one event for each j and thus for each i). The next product type in the deterministic ranking is $i = 9$, with $NPT_9 = 8.32$ $min/part$. However, since types $i = 9$ and $i = 8$ share some tools, as the quantity of type $i = 9$ increases it becomes the critical product type for the shared tools instead of type $i = 8$. Thus, the ranking changes because the robust processing time for a unit of type $i = 9$ is $PT_9^R = 13.6$ $min/unit$ and other product types might be selected although the demand of type $i = 9$ is not satisfied yet.

Figure 2 shows the TPT (cases with $\Gamma_{j1} = \Gamma$ $\forall j$ and $\delta = 0.5$) divided among tools; we may observe that the $TNPT$ decreases with Γ, because less parts are loaded, while the TFT increases to account for more disruptive events. Although $TPT = A_{tot}$, more time is accounted for possible failures; thus, the plan is more robust because the machine is less saturated.

From a practical viewpoint, the CC approach allows us to evaluate the most critical products by analyzing how the disruptive events are distributed over the product types. The higher the number of events, the more critical the product type is. Thus, the tools required to produce critical product types should be subject to particular maintenance to moderate the risk of failure occurrence. Let us consider the cases with $\delta = 1$ in Table 4. Figure 3 represents the number of events allocated to the production of each product type i. All product types are produced for $\Gamma = 1$ and, according to the CC approach, the failures are allocated on products types $i = \{1, 3, 4, 5, 7, 11\}$. Hence, a particular attention must be paid to the production of these products because failures associated with them determine the highest detriment of the objective function. Product types $i = \{2, 6, 8, 9, 10, 12\}$ are less critical: failures occurring during their production have a smaller impact.

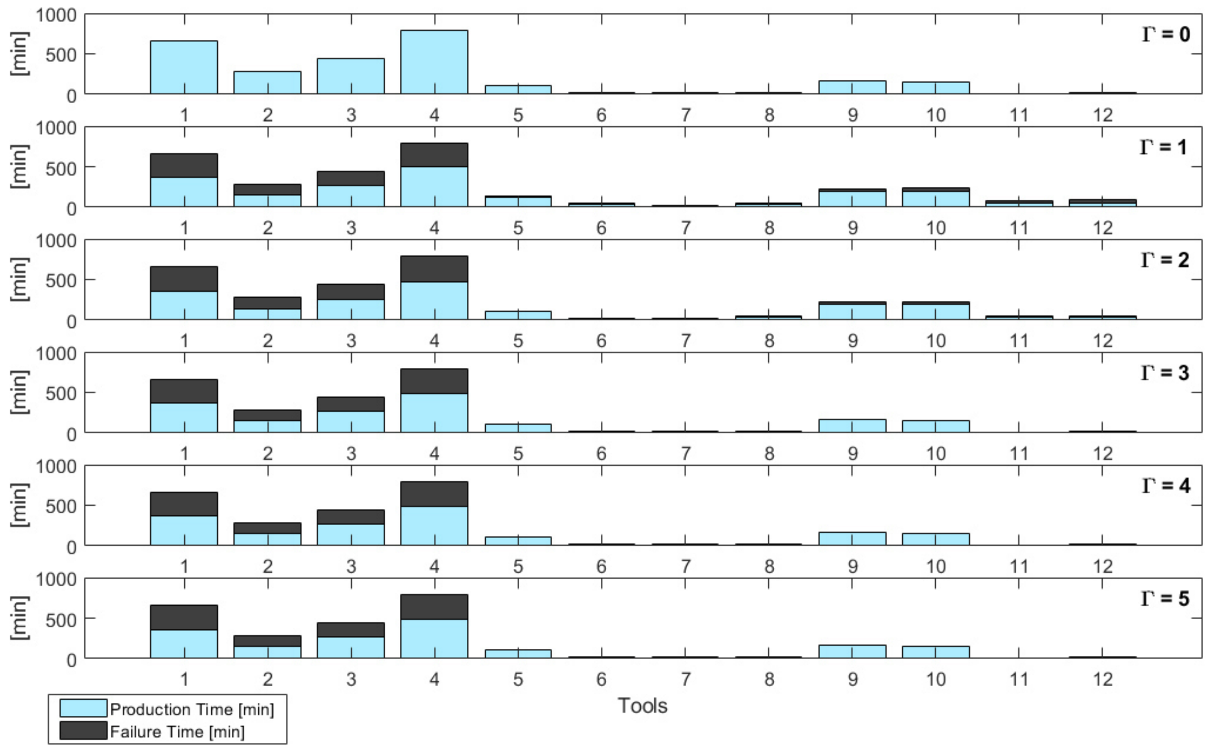


Figure 2: SM-SP problem – $TNPT$ (light blue) and TFT (black) divided among tools at different cardinality values $\Gamma_{j1} = \Gamma \, \forall j$ (case $\delta = 0.5$).

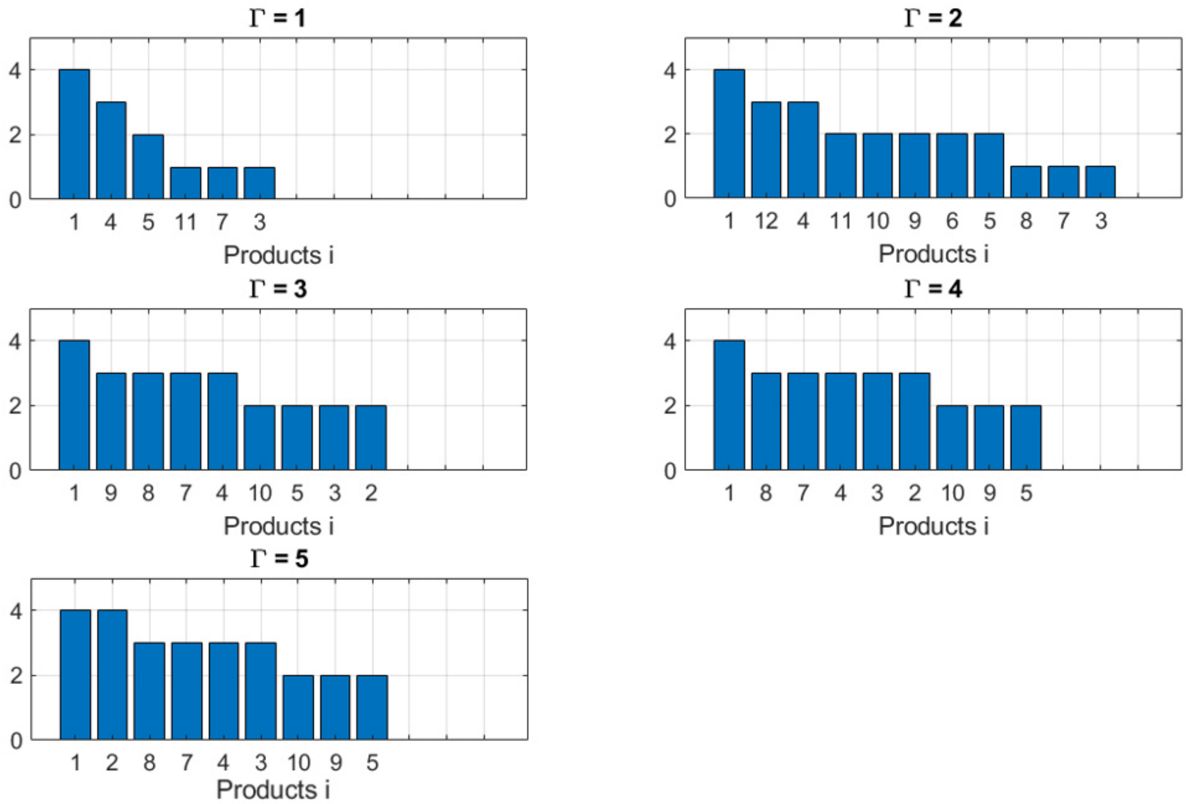


Figure 3: SM-SP problem – Number of failures affecting product types for different cardinalities $\Gamma_{j1} = \Gamma \, \forall j$ (case $\delta = 1$).

Furthermore, it is worth mentioning that the effect of other problem parameters can be trivially derived. For example, the satisfied demand and the objective function increase with the availability, the objective function decreases while costs increase, and increased values w_i modify the ranking of product types such that the product types i with higher w_i are selected beforehand.

4.2. Multi-Machine Single-Period (MM-SP) problem

Although the MM-MP is the most general problem, single period applications represent common situations for practitioners. As for the SM-SP, we analyze the effect of the robustness level by solving each problem instance for different cardinalities. The analysis is mostly focused on the effect that multi-machine features might have on the robust solution, by investigating the assignment of tools to machines. This analysis will also hold for the MM-MP problem.

4.2.1. Problem description

We consider a set of $|\mathbb{I}| = 4$ product types to be produced, and 30 instances are created by combining two factors: the number of machines (i.e., $|\mathbb{M}| = \{4 : 8\}$) and the number of tools (i.e., $|\mathbb{J}| = \{15 : 20\}$). Product demands D_i are independently sampled $\forall i$ from a uniform distribution over the interval $[1, 100]$, the nominal processing times $\bar{O}_{ij}$ are independently sampled from a uniform distribution over the interval $[50, 250]$, and the number of operations per part randomly varies between 6 and 8. In each instance, the total availability allows us to satisfy the demands under the deterministic setting (i.e., $\sum_{m\in\mathbb{M},t\in\mathbb{T}} A_{mt} = \sum_{i\in\mathbb{I}} D_i \cdot NPT_i$). As in the SM-SP experiments, we assume equal w_i (100 per unit), equal C_i (10 per unit) and null h_{i1} $\forall i$. We also relax constraints (2) and (15), and we assume that only one copy is available for each tool type (i.e., $\alpha_j = 1$ $\forall j$).

For each instance, we solve the MLP-R under different cardinality vectors $\mathbf{\Gamma} = \{\Gamma_{j1}\}$; in particular, we consider equal elements $\Gamma_{j1} = \Gamma > 0$, $\forall j$. Since the effect of δ is purely proportional (see results in Section 4.1), without loss of generality we consider only $\delta = 0.5$ in the analyses.

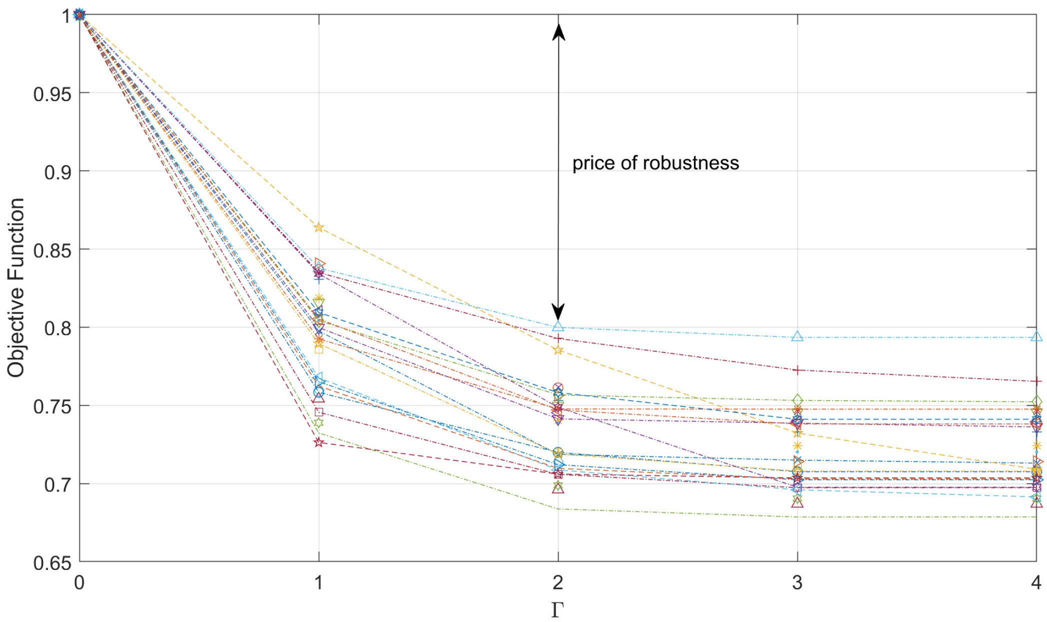


Figure 4: MM-SP problem – Normalized objective functions for increasing cardinality $\Gamma_{j1} = \Gamma\,\forall j$.

4.2.2. Results

The values of the objective functions are normalized to provide comparable results, by dividing each one by that of the corresponding deterministic solution with $\Gamma = 0$. Figure 4 collects the normalized objective functions and shows the price of robustness, which increases with the cardinality in all solved instances. The worst-case corresponds to $\Gamma = 4$, since $|\mathbb{I}| = 4$. Results are aligned with those of the SM-SP (Section 4.1.2).

We now provide the detailed results for one MM-SP instance (i.e., that with $|\mathbb{J}| = 15$ and $|\mathbb{M}| = 4$) in Table 5; insights are aligned with those of other MM-SP instances.

The TPT is equally apportioned among the machines and the machine availability is saturated ($TPT = \sum_{m\in\mathbb{M},\, t\in\mathbb{T}} A_{mt}$). Since only one copy is available for each tool, each product type might need to be processed by several machines. Although a scheduling problem arises, as mentioned earlier, we do not consider this problem at the planning level. Actually, machines might starve with consequences on the feasibility of the plan. Anyway, robustness also caters from these situations.

When uncertainty is considered, the impact of failures is balanced among the machines

	$\Gamma = 0$	$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 3$	$\Gamma = 4$
x^*_{11}	71.9	71.9	71.6	72.0	72.0
x^*_{21}	86.0	42.4	28.1	19.2	19.2
x^*_{31}	29.0	28.4	22.8	26.0	26.0
x^*_{41}	74.0	74.0	74.0	74.0	74.0
Objective function	26086	21670	19646	19122	19122

Table 5: MM-SP problem – Optimal solutions x^*_{i1} for the instance with $|\mathbb{J}| = 15$ and $|\mathbb{M}| = 4$ in relation to Γ, with $\Gamma_{j1} = \Gamma \ \forall j$; the case $\Gamma = 0$ refers to the MLP-D.

m	$\Gamma = 0$	$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 3$	$\Gamma = 4$
1	0	13070	23186	25197	25197
2	0	14707	23556	25197	25311
3	0	18727	22040	25311	25197
4	0	17591	24293	25209	25209

Table 6: MM-SP problem – TFT for each machine $m \in \mathbb{M}$ at different Γ values (with $\Gamma_{j1} = \Gamma \, \forall j$) for the instance with $|\mathbb{J}| = 15$ and $|\mathbb{M}| = 4$.

(Table 6) and the tool assignment to machines varies with the cardinality level (Figure 5). Moreover, tools having the highest impact on the TFT are not assigned to the same machine.

4.3. Multi-Machine Multi-Period (MM-MP) problem

Multi-period analysis focuses on the temporal distribution of product quantities over time for the robust solution. In the multi-period MLP, products are produced in a time period depending on the time availability in the period and the holding costs, which incentives the production delay to later periods. As in the MM-SP problem analysis (Section 4.2), several instances are created with a range of problem parameters that realistically represent applications in manufacturing.

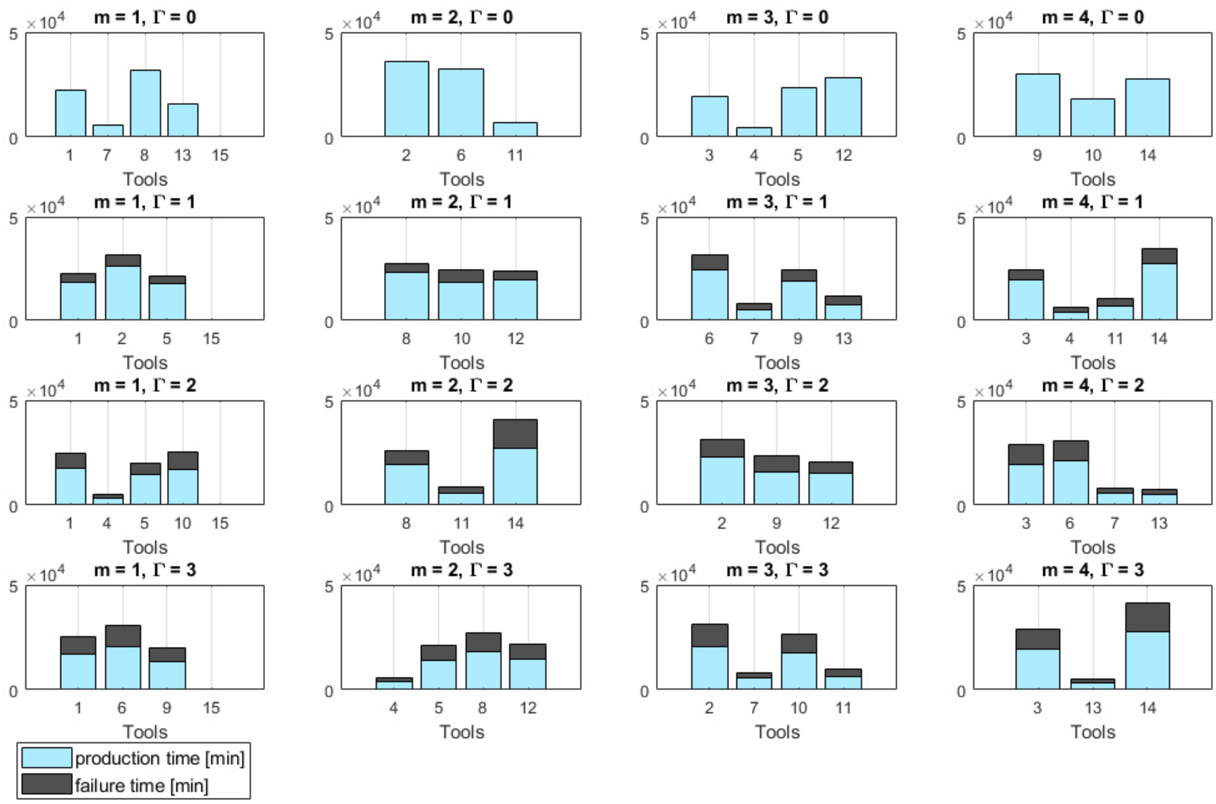


Figure 5: MM-SP problem – Assignment of tools to machines with nominal processing times (light blue) and failure times (black) for each tool j. Results are shown at different Γ values (with $\Gamma_{j1} = \Gamma\,\forall j$) and refer to the instance with $|\mathbb{J}| = 15$ and $|\mathbb{M}| = 4$.

4.3.1. Problem description

We consider a timeframe of two weeks (i.e., 7400 minutes), which is divided into several time periods. We create 30 instances using a space filling design (Latin Hypercube Design) by varying the problem dimension, i.e., the number of product types from $|\mathbb{I}| = 6$ to 18, the number of machines from $|\mathbb{M}| = 2$ to 8, the number of tools from $|\mathbb{J}| = 5$ to 25, and the number of periods from $|\mathbb{T}| = 2$ to 5.

Product demands D_i are independently sampled from a uniform distribution over the interval $[4, 50]$, nominal processing times $\bar{O}_{ij}$ are independently sampled from a uniform distribution over the interval $[4, 25]$, and the number of operations per product type randomly vary between 1 and 10. Holding costs h_{it} linearly decrease with the period t, i.e., $h_{it} = H_i\left(|\mathbb{T}| - t + 1\right)$ with values H_i independently sampled for each i from a uniform distribution over the interval $[0.001, 0.02]$. As in the other problems, we assume equal marginal profits ($w_i = 1\ \forall i$) and null shortage costs ($C_i = 0\ \forall i$). Furthermore, we relax constraints (2) and (15), and assume that multiple copies of tools are available (α_j randomly chosen in the interval $[1, 4]\ \forall j$). Finally, we assume that the total availability is equally distributed among the machines (same $A_{mt}\ \forall m$) and that such availability allows to meet the demands under the deterministic setting only in some instances, while in the others the demand of some products cannot be met completely even in the deterministic setting.

For each instance, we solve the MLP-R by assuming $\delta = 0.5$ and under different cardinality matrices $\mathbf{\Gamma} = \{\Gamma_{jt}\}$ with equal elements, i.e., $\Gamma_{jt} = \Gamma\ \forall j, t$.

4.3.2. Results

As in the MM-SP, the values of the objective functions are normalized by dividing each of them by that of the corresponding deterministic solution with $\Gamma = 0$. Figure 6 collects these values, which decrease with the cardinality of all solved instances, as the total produced quantity $\sum_{i\in\mathbb{I},t\in\mathbb{T}} x^*_{it}$ decreases.

Detailed results for one of the MM-MP instances (i.e., with $|\mathbb{J}| = 18$, $|\mathbb{I}| = 6$, $|\mathbb{M}| = 4$, $|\mathbb{T}| = 3$ and H_i as in Table 7) are provided in Table 8, which shows how the increased robustness forces to split the production over time periods in order to saturate the availability A_{mt} in late periods firstly. Moreover, the production of product types i with higher H_i values

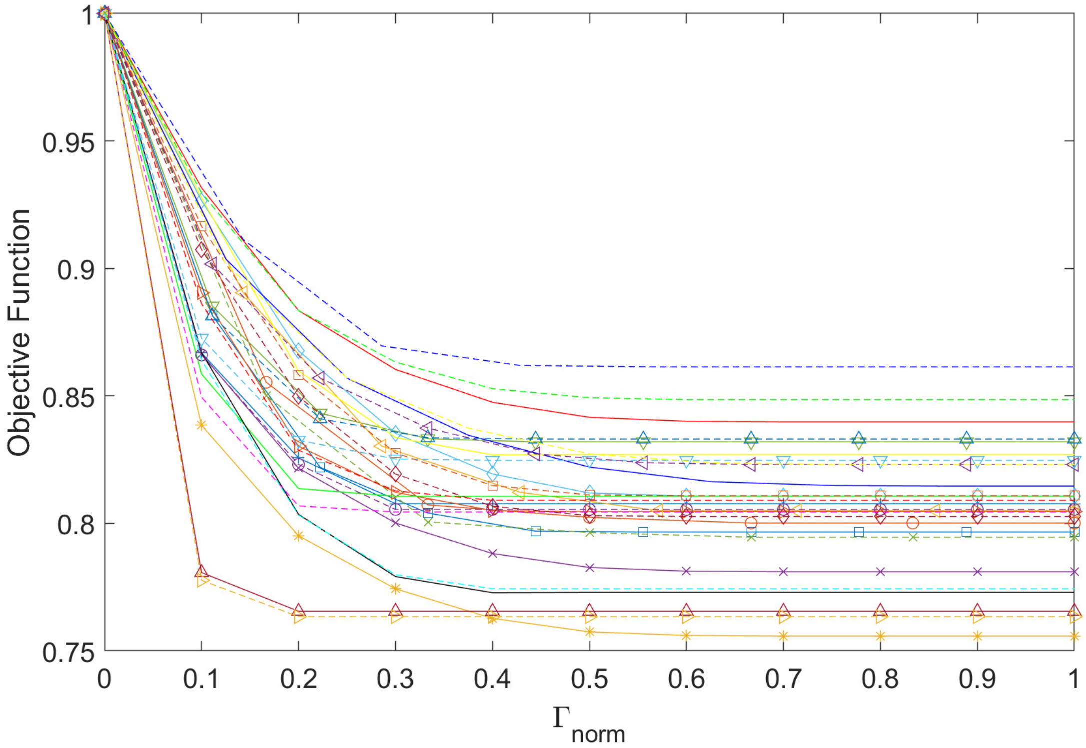


Figure 6: MM-MP problem – Normalized objective functions for increasing normalized cardinality Γ/Γ_{max}.

i	1	2	3	4	5	6
H_i	0.0162	0.0151	0.0164	0.0082	0.0127	0.0119

Table 7: MM-MP problem – Values H_i for the instance MM-MP with $|\mathbb{J}| = 18$, $|\mathbb{I}| = 6$, $|\mathbb{M}| = 4$, $|\mathbb{T}| = 3$.

is postponed to later periods to reduce the impact of the holding costs on the objective function.

The most critical product types for the deterministic case are $i = \{1, 3\}$ and their production is entirely assigned to period $t = 3$. However, the ranking of products in the robust solution is more complex (see discussion in Section 4.1) and it might happen that quantities x^*_{it} do not have a monotonic behavior in t and Γ, due to the dynamic ranking of products. As the cardinality varies, the production is differently assigned to periods. Let us consider product type $i = 3$: for the deterministic solution $\{x^*_{31}, x^*_{32}, x^*_{33}\}|_{\Gamma=0} = \{0, 0, 9\}$ units, while for the robust solutions $\{x^*_{31}, x^*_{32}, x^*_{33}\}|_{\Gamma=1} = \{0, 5.59, 3.41\}$ units and $\{x^*_{31}, x^*_{32}, x^*_{33}\}|_{\Gamma\geq 2} = \{0, 0, 9\}$ units. Although this product type has the highest holding cost H_i, its production is partially anticipated to period $t = 2$ in the solution with $\Gamma = 1$, whilst product type $i = 1$ is always kept in period $t = 3$.

Similar insights can be derived from the other MM-MP instances, and these results are also aligned with those of SM-SP problems (Section 4.1.2) and MM-SP problems (Section 4.2.2).

4.3.3. Execution of the solutions

In this section, we evaluate the performance of the robust plans obtained with the MLP-R against realistic variations of the parameters. Notice that the total monetary cost of a robust plan remains unchanged since the parameters of the objective function (1) are not subject to uncertainty. However, when real processing times are considered, the slacks of the constraints with time availability are significantly reduced, when they do not actually become negative. Therefore, we have chosen the saturation of the time availability A_{tot} as key performance index of the robust plans.

Let us define the *robust plan* $x^*_{it}(\Gamma, \delta)$ $\forall i, t$ as the solution of the MLP-R model with input parameters Γ, δ, and the Actual Time (AT) needed to produce a robust plan as the time necessary to produce all products as stated in the plan $x^*_{it}(\Gamma, \delta)$ when facing realistic realizations of the processing times $\tilde{\mathbf{O}}_{ij}$:

$$AT(\Gamma, \delta) = \sum_{i\in\mathbb{I},\, j\in\mathbb{J},\, t\in\mathbb{T}} x^*_{it}\,(\Gamma, \delta)\, \tilde{\mathbf{O}}_{ij}.$$

	Γ = 0			Γ = 1			Γ = 2			Γ = 3			Γ = 4			Γ = 5		
i \ t	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
1			42.0			42.0			42.0			42.0			42.0			42.0
2		7.0	39.0	18.4	14.2	13.4	8.2	37.1	0.7		34.9	11.1		25.6	20.4		25.6	20.4
3			9.0		5.6	3.4			9.0			9.0			9.0			9.0
4	39.1	6.9		16.1	9.5	11.7	20.3		8.9	24.3		4.1	26.1	1.9		26.1	1.9	
5		15.2																
6		8.0			8.0				8.0	2.3	2.0	3.7		8.0			8.0	

Table 8: M-MP problem – MLP-D ($\Gamma_{jt} = \Gamma = 0$, $\forall j, t$) and MLP-R ($\Gamma_{jt} = \Gamma > 0$, $\forall j, t$) solutions x^*_{it} in relation to Γ. Results of the MM-MP instance with $|\mathbb{J}| = 18$, $|\mathbb{I}| = 6$, $|\mathbb{M}| = 4$, $|\mathbb{T}| = 3$.

Hence, we define:

$$\eta(\Gamma,\delta) = \frac{AT(\Gamma,\delta)}{A_{tot}} - 1 \tag{22}$$

as performance index of the robust plan $x^*_{it}(\Gamma,\delta)$ for a given realization of processing times $\tilde{\mathbf{O}}_{ij}$. The value $\eta = 0$ is a feasibility threshold. A positive η value means that the production time required is larger than the total available time, and that the solution of the MLP-R is unfeasible with the encountered realization. On the other hand, a negative η value means that the available time is larger than the required one.

We execute the solutions in a setup where the processing times may increase up to twice their nominal value, and where smaller increases are more likely than higher ones. Indeed, we have independently generated 5 matrices of processing times $\tilde{O}_{ij}$ by sampling them from a triangular density function with parameters $a = b = \bar{O}_{ij}$ and $c = 2\,\bar{O}_{ij}$ (where a denotes the minimum value, b the mode, and c the maximum value). Notice that other densities might be used as well, with no loss of generality for our conclusions.

We have selected the MM-MP instance with $|\mathbb{J}| = 18$, $|\mathbb{I}| = 6$, $|\mathbb{M}| = 4$, and $|\mathbb{T}| = 3$, whose robust plans are in Table 8. We have computed $\eta(\Gamma,\delta)$ for the MLP-R solutions with $\Gamma = \{0,1,2,3,4,5,6\}$ and $\delta = 0.5$ under the 5 generated matrices of processing times $\tilde{O}_{ij}$. Figure 7 shows the behavior of $\eta(\Gamma,\delta)$ over Γ. The deterministic solution is always unfeasible when simulated, as the required time is larger than the total availability $\sum_{mt} A_{mt}$. By increasing the robustness of the plan with higher Γ values, solutions become feasible as the temporal slack becomes larger. Indeed, the plan $x^*_{it}(1,0.5)$ is feasible for 2 out of 3 realizations of $\tilde{O}_{ij}$ and the plans $x^*_{it}(\Gamma,0.5)$ are feasible in all realizations for $\Gamma \geq 2$. Table 9 summarizes the mean η value ($\bar{\eta}$), its standard deviation (σ_η) and its standard error ($SE(\eta)$) over the experiments, which show the low variability of the index with respect to its mean value.

These results confirm the effectiveness of the CC approach. Indeed, even small values of Γ guarantee the feasibility of the production plans in the tested case. Moreover, results show that the MLP-R performs well even if the impact of the variations is underestimated by using $\delta = 0.5$ (though processing times may increase up to twice their nominal value in the realizations). From the application viewpoint, the $\eta(\Gamma,\delta)$ values obtained in a specific

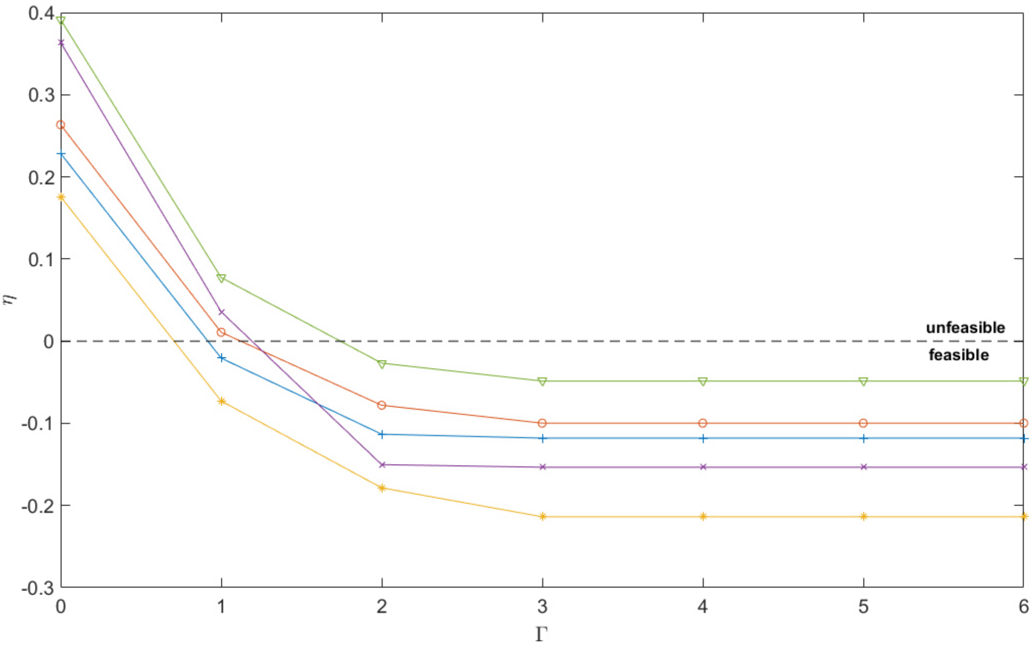


Figure 7: MM-MP problem – Simulation results obtained with the robust plans $x^*_{it}(\Gamma, \delta)$ with $\delta = 0.5$ and increasing Γ values. Results refer to the 5 replications for the MM-MP instance with $|\mathbb{J}| = 18$, $|\mathbb{I}| = 6$, $|\mathbb{M}| = 4$, and $|\mathbb{T}| = 3$.

case allows to evaluate the advantage or disadvantage for a production manager in adopting the robust solution. The deterministic solution is worthy of being implemented when $\eta \leq 0$, while the robust solution is beneficial when $\eta > 0$. When $\eta \cong 0$ other measures can be also considered (e.g., time availability) to decide between the deterministic and the robust solution.

4.4. Computational times

In this section, we summarize the computational times required to run the MLP-R for all instances presented in this paper. Experiments have been run with CPLEX 12.6 on a computer equipped with processor Intel Core i7 @ 2.5 GHz and 16 GB of installed RAM.

The observed computational times are all below 35 seconds, thus ensuring the practical applicability even for decision support tools that must provide a solution quickly. Moreover, the 40.9% of the times are below 6 seconds and the 90.6% below 7 seconds. Figure 8 shows all computational times in function of the number of decision variables; we may observe

Γ	δ	$\bar{\eta}(\Gamma,\delta)$	σ_η	$SE(\eta)$
0	0.5	0.340	0.090	0.040
1	0.5	-0.022	0.080	0.036
2	0.5	-0.102	0.075	0.034
3	0.5	-0.102	0.075	0.034
4	0.5	-0.102	0.075	0.034
5	0.5	-0.102	0.075	0.034
6	0.5	-0.102	0.075	0.034

Table 9: MM-MP problem – Performance η obtained with the robust plans $x^*_{it}(\Gamma,\delta)$. Results refer to 5 simulations of instance MM-MP with $|\mathbb{J}| = 18$, $|\mathbb{I}| = 6$, $|\mathbb{M}| = 4$, and $|\mathbb{T}| = 3$.

that they are not affected by the problem size even for the largest instance considered that includes 6646 decision variables (with $|\mathbb{J}| = 25$, $|\mathbb{I}| = 11$, $|\mathbb{M}| = 7$ and $|\mathbb{T}| = 10$).

5. Conclusions and future developments

In this work, we develop the robust counterpart of an existing MLP model. The goals are to allow companies to implement more robust production plans and guarantee minimum production levels that the system can achieve even in the worst conditions when plans are implemented. The robust model has been defined exploiting the CC approach to address the trade-off between the complexity of the model and a fair description of the uncertainty set for the model parameters, and to easily tune the specific degree of risk that the decision maker accepts. Thus, the resulting model provides a robust solution against a given number of unfortunate events that the production planner is expecting or against which he/she wishes to cover.

The behavior of the model and the impact of robustness level have been analyzed by means of several numerical experiments, considering single- vs multi-machine and single- vs multi-period problems. Outcomes show the capability of the approach to cope with uncertainty, and allow us to analyze the trade-off between robustness (the protection against uncertainty) and its price. In particular, the numerical analyses have confirmed that *i*) the

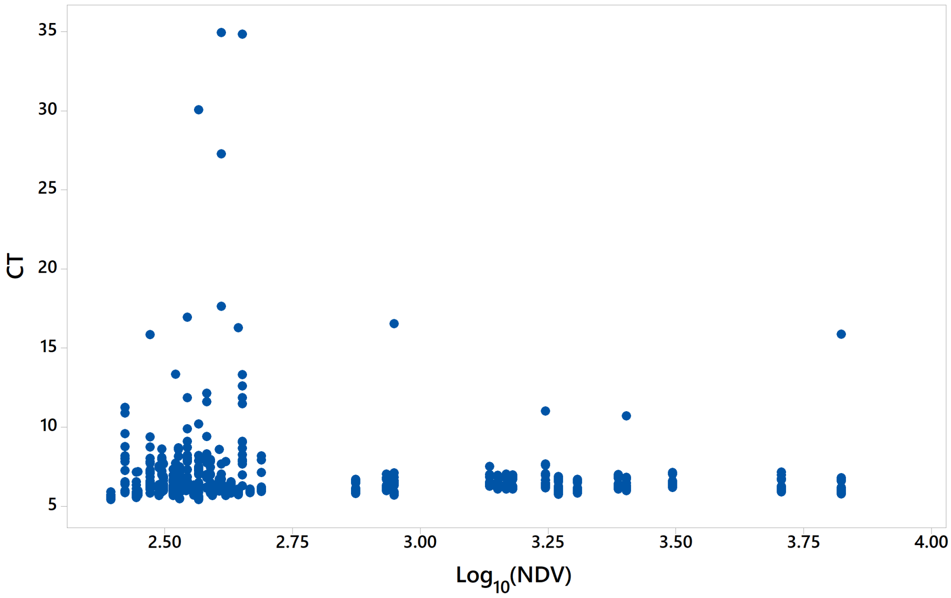


Figure 8: Scatterplot of the computational times in function of the decimal logarithm of the number of decision variables $Log_{10}(NDV)$.

robustness of the MLP solution can be easily tuned with the CC approach, *ii*) the most critical tools and products can be investigated from the robust solution, as the CC approach provides the worst case scenario, and *iii*) the impact of the amount of unfortunate events on the performance can be easily assessed.

The results achieved can be useful in terms of impact analysis at the first stages of a production planning process. Indeed, the robust model allows us to evaluate the maximum performance that can be obtained in the absence of disruptive events and the best performance once a level of protection is set (in terms of cardinality and entity of the disruptions).

It is worth mentioning that the reference MLP of Sodhi et al. [4] does not consider the detailed production scheduling within a period t, which is usually addressed at the next decision level. Thus, we neglect specific scheduling issues as preemption within a period and resource starvation. Our future work will address the integration of the proposed approach with the scheduling level. Even though some issues can be easily considered (e.g., the robustness of solution can absorb the starvation time that can be seen as part of the processing time uncertainty), a complete integration will require us to modify the MLP-D at the basis of our current work.

In the future, we will also evaluate the behavior of the robust model both in a rolling horizon perspective and in a dynamic context. In the former case, the plan will be defined at a fixed frequency and, if less events occur, the remaining time will be used to reschedule the remaining products and the newly arrived orders before the definition of the new plan. In the latter case, the solution at the MLP will define how to react at time t given the events that occurred in the previous time instants, as the decision variables will be replicated over a set of possible realizations. Finally, future development will be devoted to include a comparison with other existing deterministic MLP models, by applying the CC approach to them.

References

[1] J. F. Shapiro, Mathematical programming models and methods for production planning and scheduling, Handbooks in operations research and management science 4 (1993) 371–443.

[2] S. C. Graves, Uncertainty and production planning, in: Planning production and inventories in the extended enterprise, Springer, 2011, pp. 83–101.

[3] D. Bertsimas, M. Sim, The price of robustness, Operations research 52 (1) (2004) 35–53.

[4] M. Sodhi, R. Asking, S. Sen, Multiperiod tool and production assignment in flexible manufacturing systems, The International Journal of Production Research 32 (6) (1994) 1281–1294.

[5] G. Lugaresi, E. Lanzarone, N. Frigerio, A. Matta, A cardinality-constrained approach for robust machine loading problems, Procedia Manufacturing 11 (2017) 1718–1725.

[6] K. E. Stecke, J. J. Solberg, Loading and control policies for a flexible manufacturing system, The International Journal of Production Research 19 (5) (1981) 481–490.

[7] K. E. Stecke, Formulation and solution of nonlinear integer production planning problems for flexible manufacturing systems, Management Science 29 (3) (1983) 273–288.

[8] F. Guerrero, Machine loading and part type selection in flexible manufacturing systems, International Journal of Production Research 37 (6) (1999) 1303–1317.

[9] A. Grieco, Q. Semeraro, T. Tolio, A review of different approaches to the FMS loading problem, International Journal of Flexible Manufacturing Systems 13 (4) (2001) 361–384.

[10] H. Babu Nanvala, G. K. Awari, Approaches for solving machine loading problem in FMS: A review, International Journal of Engineering and Technology 3 (2) (2011) 64–73.

[11] R. Singh, R. Singh, B. K. Khan, A critical review of machine loading problem in flexible manufacturing system, World Journal of Engineering and Technology 3 (2015) 271–290.

[12] K. Shanker, Y.-J. J. Tzen, A loading and dispatching problem in a random flexible manufacturing system, International Journal of Production Research 23 (3) (1985) 579–595.

[13] N. Nagarjuna, O. Mahesh, K. Rajagopal, A heuristic based on multi-stage programming approach for machine-loading problem in a flexible manufacturing system, Robotics and Computer-Integrated Manufacturing 22 (4) (2006) 342–352.

[14] F. T. Chan, R. Swarnkar, Ant colony optimization approach to a fuzzy goal programming model for a machine tool selection and operation allocation problem in an fms, Robotics and Computer-Integrated Manufacturing 22 (4) (2006) 353–362.

[15] A. Rossi, G. Dini, Flexible job-shop scheduling with routing flexibility and separable setup times using ant colony optimisation method, Robotics and Computer-Integrated Manufacturing 23 (5) (2007) 503–516.

[16] K. Das, M. Baki, X. Li, Optimization of operation and changeover time for production planning and scheduling in a flexible manufacturing system, Computers & Industrial Engineering 56 (1) (2009) 283–293.

[17] L. Zeballos, A constraint programming approach to tool allocation and production scheduling in flexible manufacturing systems, Robotics and Computer-Integrated Manufacturing 26 (6) (2010) 725–743.

[18] A. M. Abazari, M. Solimanpur, H. Sattari, Optimum loading of machines in a flexible manufacturing system using a mixed-integer linear mathematical programming model and genetic algorithm, Computers & Industrial Engineering 62 (2) (2012) 469–478.

[19] N. Vidyarthi, M. Tiwari, Machine loading problem of fms: a fuzzy-based heuristic approach, International Journal of Production Research 39 (5) (2001) 953–979.

[20] M. M. Aldaihani, M. Savsar, A stochastic model for the analysis of a two-machine flexible manufacturing cell, Computers & Industrial Engineering 49 (4) (2005) 600–610.

[21] S. K. Mandal, M. K. Pandey, M. Tiwari, Incorporating dynamism in traditional machine loading problem: an AI-based optimisation approach, International Journal of Production Research 48 (12) (2010) 3535–3559.

[22] B. Addis, G. Carello, A. Grosso, E. Lanzarone, S. Mattia, E. Tànfani, Handling uncertainty in health care management using the cardinality-constrained approach: Advantages and remarks, Operations Research for Health Care 4 (2015) 1–4.

[23] A. Shapiro, D. Dentcheva, A. Ruszczyński, Lectures on stochastic programming: modeling and theory, SIAM, 2009.

[24] J. R. Birge, F. Louveaux, Introduction to stochastic programming, Springer Science & Business Media, 2011.

[25] A. Ben-Tal, D. Bertsimas, D. B. Brown, A soft robust model for optimization under ambiguity, Operations research 58 (4-part-2) (2010) 1220–1234.

[26] J. Goh, M. Sim, Distributionally robust optimization and its tractable approximations, Operations

research 58 (4-part-1) (2010) 902–917.

[27] A. L. Soyster, Convex programming with set-inclusive constraints and applications to inexact linear programming, Operations research 21 (5) (1973) 1154–1157.

[28] A. Ben-Tal, A. Nemirovski, Robust convex optimization, Mathematics of operations research 23 (4) (1998) 769–805.

[29] V. Gabrel, C. Murat, A. Thiele, Recent advances in robust optimization: An overview, European journal of operational research 235 (3) (2014) 471–483.

[30] A. Ben-Tal, A. Nemirovski, Robust solutions of linear programming problems contaminated with uncertain data, Mathematical programming 88 (3) (2000) 411–424.

[31] A. Ben-Tal, A. Goryashko, E. Guslitzer, A. Nemirovski, Adjustable robust solutions of uncertain linear programs, Mathematical Programming 99 (2) (2004) 351–376.

[32] D. Bertsimas, D. B. Brown, C. Caramanis, Theory and applications of robust optimization, SIAM review 53 (3) (2011) 464–501.

[33] D. Bertsimas, A. Thiele, A robust optimization approach to supply chain management, in: International Conference on Integer Programming and Combinatorial Optimization, Springer, 2004, pp. 86–100.

[34] Ö. Hazır, E. Erel, Y. Günalay, Robust optimization models for the discrete time/cost trade-off problem, International Journal of Production Economics 130 (1) (2011) 87–95.

[35] Y. Moon, T. Yao, A robust mean absolute deviation model for portfolio optimization, Computers & Operations Research 38 (9) (2011) 1251–1258.

[36] D. J. Alem, R. Morabito, Production planning in furniture settings via robust optimization, Computers & Operations Research 39 (2) (2012) 139–150.

[37] O. Solyalı, J.-F. Cordeau, G. Laporte, Robust inventory routing under demand uncertainty, Transportation Science 46 (3) (2012) 327–340.

[38] Ö. Hazır, A. Dolgui, Assembly line balancing under uncertainty: Robust optimization models and exact solution method, Computers & Industrial Engineering 65 (2) (2013) 261–267.

[39] C.-C. Lu, K.-C. Ying, S.-W. Lin, Robust single machine scheduling for minimizing total flow time in the presence of uncertain processing times, Computers & Industrial Engineering 74 (2014) 102–110.

[40] M. C. O. Moreira, J.-F. Cordeau, A. M. Costa, G. Laporte, Robust assembly line balancing with heterogeneous workers, Computers & Industrial Engineering 88 (2015) 254–263.

[41] K. E. Stecke, Design, planning, scheduling, and control problems of flexible manufacturing systems, Annals of Operations research 3 (1) (1985) 1–12.