A Mechanistic model to predict pressure drop and holdup pertinent to horizontal gas-liquid-liquid intermittent flow

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Nomenclature

Symbols	Denotes	Unit
A	pipe cross-sectional area	m^2
A_f	area of film section occupied by liquid	m^2
A_g	area of film section occupied by gas	m^2
C	regression coefficient (eq. 31)	[-]
C_e	coefficient depending on pipe inclination (eq. 32)	[-]
D	pipe diameter	m
e_{ri}	average relative error	-
f_g, f_l, f_i	gas-wall, liquid-wall, and interfacial friction factors	-
f_{LM}	liquid mixture friction factor	-
f_s	mixture-wall friction factor	-
Fr	Froude number (eq. 30)	-
g	gravitational acceleration	$m \cdot s^{-2}$
H_g	mean gas hold up	-
H_{lf}, H_{ls}	liquid holdup in film section and slug body	-
h_L	liquid height in film section	m
J_g, J_o, J_w	gas, oil, and water superficial velocity	$m \cdot s^{-1}$

J_L, J_t	total liquid and mixture superficial velocity	$m \cdot s^{-1}$
$J_{g,c}$	critical superficial velocity	$m \cdot s^{-1}$
J_{go}	superficial gas velocity corresponding to shortest slug length	$m \cdot s^{-1}$
L_b, L_s	bubble and slug body length	m
L_u	slug unit length	m
N_{Fr}	Froude number (eq. 21)	-
N_μ	Viscous number	-
Re_g, Re_l	gas and liquid Reynolds number in film section	-
Re_s	mixture Reynolds number in slug body	-
Re_{sl}	superficial liquid Reynolds number	-
S_f, S_g, S_i	liquid, gas, and interface perimeters	m
U_b	velocity of dispersed bubble in slug body	$m \cdot s^{-1}$
U_d	drift velocity	$m \cdot s^{-1}$
U_f	liquid velocity in film section	$m \cdot s^{-1}$
U_g	gas velocity in film section	$m \cdot s^{-1}$
U_l	liquid velocity in slug body	$m \cdot s^{-1}$
U_t	translational bubble velocity	$m \cdot s^{-1}$
$\Delta P/L$	three-phase pressure drop	Pa/m
α	coefficient depending on liquid height	-
ε	standard deviation of the relative errors	-
ε_{Lg}	ratio between gas to liquid superficial velocity	-
ε_{Lo}	ratio between oil to liquid superficial velocity	-
ε_{Lw}	ratio between water to liquid superficial velocity	-

ζ	distribution parameter	-
Θ	inclination angle	°
μ_{cont}	viscosity of continuous phase	Pa·s
μ_g	gas viscosity	Pa·s
μ_L	liquid viscosity	Pa·s
μ_o	oil viscosity	Pa·s
μ_w	water viscosity	Pa·s
μ_s	average mixture viscosity in slug body	Pa·s
ρ_g	gas density	$kg{\cdot}m^{-3}$
ρ_l	liquid density	$kg{\cdot}m^{-3}$
ρ_o	oil density	$kg{\cdot}m^{-3}$
ρ_s	average mixture density in slug body	$kg{\cdot}m^{-3}$
ρ_w	water density	$kg{\cdot}m^{-3}$
σ_{ow}	interfacial tension between oil and water	$N{\cdot}m^{-1}$
τ_f, τ_g, τ_i	liquid-wall, gas-wall, interface shear stresses	Pa
τ_s	mixture-wall shear stress in slug body	Pa
Φ	independent parameter	-
χ	Lockhart-Martinelli parameter	-

1. Introduction

Multiphase flow of high viscous oil-water and oil-water-gas within pipelines are a matter of high importance for petroleum industry. In the last decades, there have been a large number of research studies on oil-water flows in horizontal ducts, see for instance, Charles et al. (1961), Arney et al. (1993), Grassi et al. (2008), Sotgia et al. (2008), Colombo et al. (2015, 2017), Loh and Premanadham (2016), Shi et al. (2017), Babakhani (2017), and Babakhani et al. (2017a, 2017b, 2018). In many practical application though, the presence of gas together with oil and water is unavoidable, it would be necessary to investigate and predict the pressure drop and phase holdup during multiphase production at different flow conditions.

There are several experimental investigations on characterization of liquid-liquid-gas flows, considering much lower oil viscosity. A large part of these studies were conducted on a large-scale test facility (WASP) at Imperial College, London. Among them, studies by Acikgoz et al. (1992), Hall (1992), Pan (1996), Odozi (2000), and Hewitt (2005) can be mentioned. In most of these researches, oil viscosity varied from 4 mPa·s to 153 mPa·s in horizontal tubes ranging from 19 mm to 78 mm. The focus of their studies has been on flow pattern observation, considering interaction between gas and oil/water mixture.

Low viscous oil-water-gas flow may differ from those of high viscous oil-water-gas flow due to the fact that viscous forces might play an important role in the latter case. To the best of author' knowledge, information mostly concerning to pressure drop, liquid holdup and flow pattern for high viscous oil-water-gas is still lacking in the open literature: the author was able to find only six contributions, which summarily described in the following.

Bannwart et al. (2004) used oil, water, air with volumetric fluxes (superficial velocities) varying in the intervals of J_o=0.01-2.5 m/s, J_w=0.04-0.5 m/s, and J_g=0.03-10 m/s, respectively. Experimental tests were conducted with two different oil viscosities in two facilities with horizontal and vertical pipe orientation: heavy crude oil (μ_o=3.4 Pa·s, ρ_o=970 kg.m^{-3} at 20 ℃) within 28.4 mm (Laboratory scale) i.d. pipe and very heavy crude oil (μ_o=36.95 Pa·s, ρ_o=972.1 kg.m^{-3} at 20 ℃) within 77 mm (Field scale) i.d. pipe. To evaluate the effect of gas phase, the results of pressure drop were presented based on a parameter defined as the ratio between three-phase pressure drop to oil-water pressure drop at the same oil and water volumetric fluxes.

The work by Bannwart et al. (2009) can also be cited, used similar pipe configurations and test fluids as operated by Bannwart et al. (2004). They measured pressure drop and observed flow patterns, leading to identification of Nine flow patterns in horizontal pipe: Bubble gas-Bubble oil (B_g, B_o), Bubble gas-Annular oil (B_g, A_o), Bubble gas-Intermittent oil (B_g, I_o), Bubble gas-Stratified oil (B_g, S_o), Intermittent gas-Bubble oil (I_g, B_o), Intermittent gas-Annular oil (I_g, A_o), Intermittent gas-Intermittent oil (I_g, I_o), Stratified gas-Bubble oil (S_g, B_o), Stratified gas-Stratified oil (S_g, S_o). In the whole ranges of experimental conditions, water always were in the contact with pipe wall, preventing oil from sticking to the pipe and promoting the occurrence of core-annular flow. Furthermore, concerning pressure drop, they concluded that gas superficial velocity has a significant influence on frictional pressure loss.

Poesio et al. (2009) studied the flow of gas, water, and oil with two viscosities of 0.9 and 1.2 Pa·s at room temperature. Different pipe diameters were tested, including 21 mm, 28 mm and 40 mm i.d. Flow pattern under investigation was slug flow. Oil, water and air superficial velocities are in the ranges of J_o=0.46-1.08 m·s^{-1}, J_w=0.04-0.67 m·s^{-1}, and J_g=0.06-4 m·s^{-1}. The main measurement included pressure drop detected 6 m downstream of injector. They developed a hybrid model which computes overall pressure drop based on Lockhart-Martinelli model and the results of comparison between pressure drop predictions and measurements showed a fairly good agreement.

The work by Wang et al. (2013) deals with oil-water-gas flowing in a horizontal pipe with 52.5 mm i.d. pipe. Oil viscosity varied from 0.15 Pa.s to 0.57 Pa.s at temperatures ranging from 37.8 to 15.6 ℃, respectively. Oil, water, superficial velocities were experimented up to 1 m·s^{-1}, respectively, while gas superficial velocity was in the range of 1 to 5 m·s^{-1}. The flow patterns were observed and images recorded using high speed video camera. They classified three-phase flow according to the interactions between gas and liquid, and oil-water mixture within slug body and film regions, leading to four different flow patterns: INT (O/W-S&SOW-F): gas and liquid are in slug flow (Oil is dispersed in water within slug body and both are stratified in the film region); INT (O/W-S & O/W-F): gas and liquid are in slug flow (Oil is dispersed in water in slug body and film regions); INT (W/O-S & W/O-F): gas and liquid flow regime is slug (Water is dispersed in oil in both slug body and film regions); STR (O/W-F): gas and liquid flow regime is stratified (Oil and gas are entrained and dispersed in water, a thin layer of oil is also present at the pipe wall). Moreover, pressure drop measurements were performed and reported as a function of the water cut (defined as the ratio between superficial water velocity to superficial liquid velocity), showing increased trend of frictional pressure loss as gas velocity is increased.

Shmueli et al. (2015) investigated viscous oil-water-gas (μ_o=0.102 Pa·s at 20 °C; ρ_o=847.9 kg.m^{-3}) flow within a horizontal pipe with 69 mm i.d. pipe and 50 m long. Flow patterns were detected to be stratified-annular flow over the tested operating conditions. Liquid height, phase hold up and pressure gradients were measured by means of a traversing two-energy gamma densitometer. They concluded that there is a curvy interface between gas and liquid, which is contrary to the visual observations.

Babakhani (2017) measured experimental pressure drop for three-phase flow within horizontal 40 mm i.d. pipe, considering μ_o=0.838 Pa·s. In addition, Translational bubble velocity and geometrical characteristics of slug units were determined by means cross-correlation of the signal from two optical probes and video analysis. Based on experimental data, a new correlation to compute the slug unit length as a function gas and liquid superficial velocity as well as pipe diameter was suggested. Acceptable agreement between predicted slug unit length and measurements was observed. A summary of experimental studies on viscous oil-water-gas flows is listed in Table 1.

The most common flow pattern for oil-water-gas flows is slug flow, where a series of liquid slugs is separated by relatively large gas pockets. Up to our knowledge, there are few theoretical studies to characterize flow behavior of oil-water-gas flows, which is mostly related to stratified flow regime (see Taitel et al., 1995, Khor et al., 1997, Hanich and Thompson, 2001). Taitel et al. (1995) presented a theoretical approach for three layer stratified flow of liquid-liquid-gas to compute liquid and gas holdup. Steady state momentum equations for each phase were written and solution iteratively obtained by assuming a guess value for liquid height, that is, h_L=h_w+h_o. Once solution for liquid height was obtained, other important physical parameters of flow such as pressure drop and phase velocity can be calculated. The weakness of model is that is only applicable to stratified three phase flow. Later, Khor et al. (1997) compered stratified experimental data of liquid hold up with above model, considering different correlations for gas-wall shear stress, oil-water and water-wall shear stresses. They found satisfactory agreement between measured liquid holdup and prediction by the model.

Perhaps, the most complete work presented so far corresponding to Intermittent flow of three phases is the unified model developed by Zhang and Sarica (2006), which was presented after their proposed model for two-phase flow (see Zhang et al., 2003a). They divided slug unit into two sections, including slug body and gas pocket regions, assuming that oil and water are stratified in the gas pocket section. The model was tested against experimental pressure drop data of Hall (1992), Laflin and Oglesby (1976) where low oil viscosities were used (μ_o=0.005-0.083 Pa·s). Preliminary validations have been obtained between pressure drop prediction and measurements. Performance of the mechanics model is also compared against experimental data of Wang et al. (2013). However, the model was not able to predict high viscous oil-water-gas flows, probably due to the fact that oil and water is considered to be stratified flow regime in gas pocket region, which is not the case in the whole range of experimental data.

In the original Unified model proposed by Zhang et al (2003a), the length of liquid in film section (L_b) was either obtained based on trial and error procedure or from experimental data. Furthermore, the value of liquid holdup in slug body region (H_{ls}) was calculated in an iterative process. Hence, two numerical procedures are needed to compute L_b and H_{ls}. In the current work, by knowing that $L_b=L_u-L_s$, modifications are suggested to the Unified model so that L_b can be calculated, taking into account a correlation to compute total slug unit length (L_u) developed by

Babakhani (2017) for viscous oil-water-gas flow. By doing so, the complexity level of original Unified model is reduced. Another initiative of this study is that oil and water are summed to behave homogeneously, concerning flow regime under investigation (Slug flow with fully mixed oil/water and Slug flow with oil core/annular water). Hence, oil-water-gas three phase flow can be simplified into gas-liquid flow. In the following, a detailed description of the proposed mechanistic model for slug flow with different oil/water interactions is reported in section 2. The results of pressure drop prediction from mechanistic model are compared with experimental data banks of Poesio et al. (2009) and Babakhani (2017), as shown in section 4. It has to be remarked that all the available data were taken in plants with a “smooth” introduction of the phases and the measurements reported averages of the major quantities (phase holdup and pressure drop), which are independent of time, i.e. in quasi-steady state conditions.

Table 1. Summary of previous studies on viscous oil-water-gas flow in horizontal pipe

Author	Pipe I.D. (mm)	Pipe length (m)	μ_o (mPa·s)	ρ_o (kg·m^{-3})	Velocity range (m·s^{-1})
Bannwart et al. (2004)	28.4;77	5.40 274	3400; 36950	970	J_o: 0.01-2.5 J_w: 0.04-0.5 J_g=0.03-10
Bannwart (2009)	28.4; 77	5.40 274	3400; 36950	970	J_o: 0.01-2.5 J_w: 0.04-0.5 J_g=0.03-10
Poesio et al. (2009)	21;28;40	9	900;1200	886	J_o: 0.46-1.08 J_w:0.04-0.67 J_g=0.06-4.0
Wang et al. (2013)	52.5	24	150-570	884.4	J_o:0.1-1 J_w:0.1-1 J_g=1-5
Shmueli et al. (2015)	69	50	102	847.9	J_o:0.05-0.8

					J_w:0.05-0.8 J_g=1.3-10.7
Babakhani (2017)	40	12	838	890	J_o:0.36-0.71 J_w:0.44-1.32 J_g=0.22-2.10

2. Mechanistic model

The idea of considering oil and water mixture as a homogeneous flow for dispersed flow regime was presented by Picchi et al. (2015) for oil-water flows, where a steady two-fluid model is used to predict pressure gradient and phase holdup. This concept is extended for three-phase slug flow, making use of Zhang et al. (2003a) mechanistic model. Essentially, prediction by mechanistic models is more accurate than general correlations regardless of the number of phases within pipeline because most important hydrodynamic parameters are considered.

2.1 Mass conservation equations

The mathematical model presented here is based on the slug unit cell propagating with the translational velocity (U_t) in horizontal pipe proposed by Zhang et al. (2003a). The schematic geometry of slug is depicted in Fig. 1. The slug unit cell is divided into two sections: a liquid slug body with a length of L_s and a film section with elongated bubble length of L_b where gas and liquid are stratified. The slug body contains gas entrainment in the form of dispersed bubbles, on the other hand, no liquid is entrained and dispersed into the gas pocket. Thus, the current hybrid model is a combination of a two-fluid model for the segregated flow part and a drift-flux model for the dispersed component. The model is a steady state model in which liquid and gas are treated as incompressible flows. This assumption is still valid even for long pipelines where the density is not constant, see Taitel and Barnea (1990). In order to make use of two-fluid model in the film section with characteristic length of L_b, a homogeneous distribution of liquid phases is assumed and the effective viscosity of liquid is calculated according the Einstein's equation (1906):

$$\mu_L = \mu_{cont}(1 + 2.5\,\varepsilon_{Lo}) \qquad (1)$$

Where μ_{cont} and ε_o are viscosity of continuous phase (water) and input volume oil fraction ($\varepsilon_{Lo} = \frac{J_o}{J_L} = 1 - \varepsilon_{Lw}$), respectively. The application of eq. 1 requires that spherically dispersed bubbles are distributed evenly in a radial direction. In the present study, the combined continuity and momentum equations for gas/two phase liquid is adopted. If a reference frame with the same velocity as U_t is considered, the mass balance for both liquid and gas phases can be written by considering the liquid and gas mass flow rates entering and exiting control volume:

$$H_{ls}(U_t - U_l) = H_{lf}\,(U_t - U_f) \qquad (2.a)$$

$$(1 - H_{ls})(U_t - U_b) = (1 - H_{lf})\,(U_t - U_g) \qquad (2.b)$$

Where U_l, U_b are liquid and bubble velocities in slug body and U_f, U_g are velocities of liquid and gas in gas pocket (film section). The dispersed bubble velocity in slug body can be estimated by model of Wallis (1969) which is a drift-flux based approximation as $U_b = 1.2 \cdot J_t$.

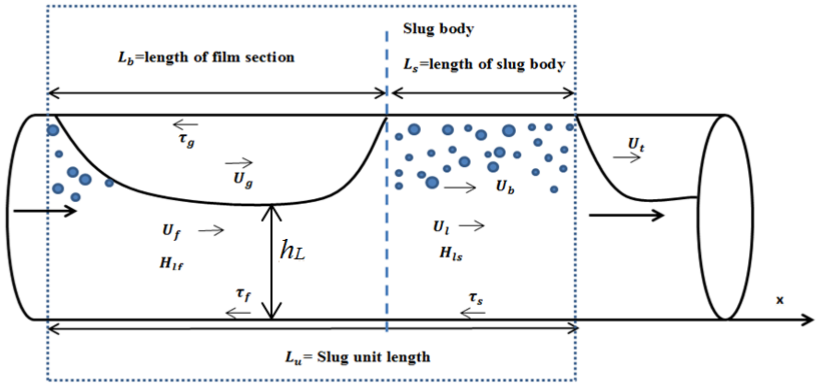

Fig 1. Schematic of slug flow structure

When a slug unit cell passes, the following equations can be written for gas and liquid:

$$L_u J_l = L_s H_{ls} J_t + L_b H_{lf} U_f \quad (3)$$

$$L_u J_g = L_s (1 - H_{ls}) J_t + L_b (1 - H_{lf}) U_g \quad (4)$$

$$L_u = L_s + L_b \quad (5)$$

The mean average gas holdup can be calculated based on following equation:

$$H_g = \frac{J_g}{U_g} \quad (6)$$

2.2 Momentum equations

For the sake of simplicity, the liquid height and its shape (h_L) along the liquid film is considered to be uniform. The shape of liquid film requires a special attention, because, at the bubble front the liquid holdup gradient differs from that at the bubble tail. Referring to Fig. 1, momentum equations can be derived according to the analysis of forces exerted at the inlet and out of control volume containing important information such as pressure loss. The entire film section as control volume is considered and momentum equations solved, see for instance, Zhang et al. (2003a). The momentum equation for liquid and gas pocket in horizontal pipe is given by:

$$\frac{\rho_l(U_t - U_f)(U_l - U_f)}{L_b} = \frac{\Delta p}{L_b} + \frac{\tau_f S_f}{A_f} - \frac{\tau_i S_i}{A_f} - \rho_l\, g\, \cos\theta\, \frac{\partial h_L}{\partial x} \qquad (7)$$

$$\frac{\rho_g(U_t - U_g)(U_b - U_g)}{L_b} = \frac{\Delta p}{L_b} + \frac{\tau_g S_g}{A_g} - \frac{\tau_i S_i}{A_g} - \rho_g\, g\, \cos\theta\, \frac{\partial h_L}{\partial x} \qquad (8)$$

The pressure drop terms and the last term in RHS of eq. 7 and 8 are eliminated from above equations. Thus, the combined momentum equation may be given by:

$$\frac{\rho_l(U_t - U_f)(U_l - U_f) - \rho_g(U_t - U_g)(U_b - U_g)}{L_b} - \frac{\tau_f S_f}{A_f} + \frac{\tau_g S_g}{A_g} + \tau_i S_i\left(\frac{1}{A_f} + \frac{1}{A_g}\right) = 0 \qquad (9)$$

The first term at LHS of equation (9) is the force due to momentum exchange between slug body and film section of unit. Zhao and Yeung (2015) reported that If there is low liquid film height (h_L in Fig 1), there is no considerable difference between gas pocket velocity (U_g) and liquid velocity in the film region (U_f) beneath it. Zhang and Sarica (2006) developed a unified model, taking into account the stratified gas-oil-water in both liquid slug body and film sections. They stated that L_b tends to be infinitely long in stratified flow of gas-oil-water. Thus, the momentum exchange term is neglected from equation (9), the original form of momentum equation, developed by Taitel and Barnea (1990) can be obtained. It is worth noting that liquid height calculated in this way is the one in its equilibrium level and can be iteratively computed according to equation 9.

Gas-wall (τ_g), liquid-wall (τ_f) and interfacial shear stresses between gas pocket and liquid in film region are defined as:

$$\tau_f = f_l \frac{\rho_l U_f^2}{2} \quad (10)$$

$$\tau_g = f_g \frac{\rho_g U_g^2}{2} \quad (11)$$

$$\tau_i = f_i \frac{\rho_g (U_g - U_f)|U_g - U_f|}{2} \quad (12)$$

To calculate shear stresses in film region, some geometrical parameters are required which are given in Appendix A. The friction factors in equations 10-12 can be directly linked to the phase Reynolds number for liquid film and gas pocket:

$$Re_l = \frac{4\, A_f U_f \rho_L}{S_f \mu_L}, \qquad Re_g = \frac{4\, A_g U_g \rho_g}{(S_g + S_i)\mu_g} \quad (13)$$

In definition of gas Reynolds number, the cord length at the interface, S_i is used as suggested by Taitel and Dukler (1976).

2.3 Pressure gradient prediction

The total pressure drop for slug unit length can be computed using three contributions as frictional, gravitational and acceleration pressure gradients:

$$-\frac{dP}{dx} = -\left(\frac{dP}{dx}\right)_F - \left(\frac{dP}{dx}\right)_G - \left(\frac{dP}{dx}\right)_A \quad (14)$$

We assumed that gas expansion would not occur from the entrance to downstream of pipeline (flow is incompressible) and acceleration contribution is negligible. Thus, the only contribution that remains is frictional term in horizontal pipe and computed as:

$$-\left(\frac{dP}{dx}\right)_F = \frac{\tau_s \pi D}{A}\frac{L_s}{L_u} + \frac{\tau_f S_f + \tau_g S_g}{A}\frac{L_b}{L_u} \quad (15)$$

The first term in equation above corresponds to frictional pressure drop in slug body and the second is frictional contribution to the pressure drop in the film zone. Rheological properties of mixture in slug zone are calculated based on weighted average of liquid and gas holdup, as proposed by Taitel and Barnea (1990) and Zhao and Yeung (2015)

$$\rho_s = H_{ls}\rho_l + (1 - H_{ls})\rho_g \quad (16)$$

$$\mu_s = H_{ls}\mu_l + (1 - H_{ls})\mu_g$$

The shear stress in slug body caused by interaction between homogeneous mixture (dispersed bubble entrained to slug body zone and liquid) and pipe wall in slug region, τ_s, is calculated considering total mixture superficial velocity for Reynolds number:

$$Re_s = \frac{\rho_s J_t D}{\mu_s} \quad (17)$$

$$\tau_s = f_s \frac{\rho_s J_t^2}{2} \quad (18)$$

To compute pressure drop and phase holdup in high viscous oil-water-gas flow based on mechanistic model presented in sections 2-1 to 2-3, some information are required, including slug body holdup (H_{ls}) and length (L_s), and closure relation for two-phase friction factor and translational velocity. All information is obtained from available models in the literature for gas-liquid flow due to the lack of suitable models for three-phase flow in horizontal pipes, discussed in sections 2.4 to 2.6. In addition to above parameters, an appropriate model for slug unit length is necessary to calculate actual velocity of phases and avoid iterative procedure in continuity equations. In the present study, a new formulation for computing the total unit length as a function of pipe diameter, and flow conditions are presented, explained in section 2.7.

2.4 Slug body holdup

The mechanistic model requires the information of slug body holdup. Some researchers have studied liquid body holdup for gas-liquid flow in the case of low viscosity oil, see for instance, Andreussi et al. (1993) and Nadler and Mewes (1995). The slug body region can be divided into two sub-regions, namely, developed body region and developing mixing region. When liquid moves from the layer beneath gas pocket to slug region, a sudden expansion occurs which, in turn, helps to form a jet and create a mixing region at the head of slug. As a result of mixing developing region and liquid loss, the generated liquid re-circulate from slug body and move toward the leading Taylor bubble tail. The rest of liquid is transported to the developed slug region (see Fig. 2 taken from Babakhani, 2017). This phenomenon has significant effect on the developing mixing length and its intensity as well as slug body liquid holdup.

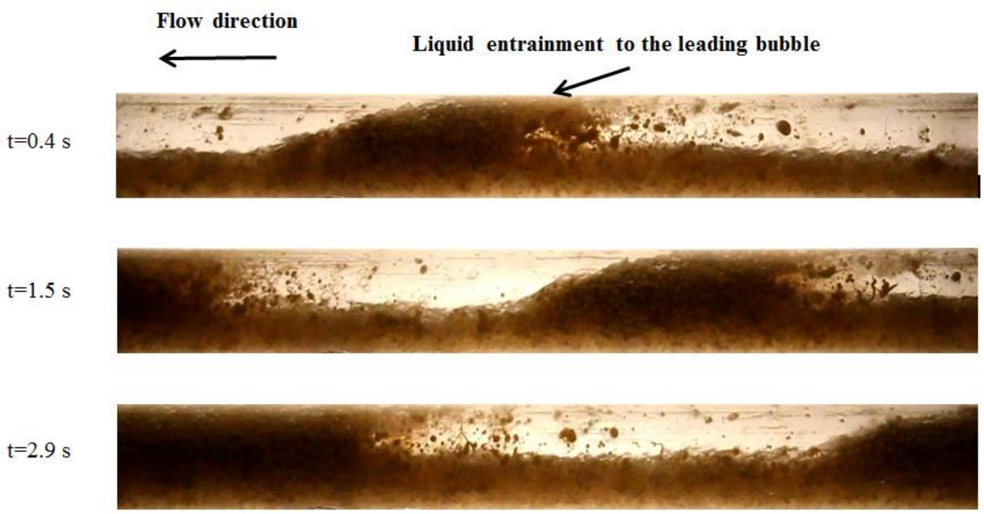

Fig 2. Frames of subsequent images to show the liquid entrainment mechanism, image from Babakhani, 2017.

Al-Safran et al. (2015) experimentally examined the influence of high liquid viscosity on slug liquid holdup in horizontal pipe. They concluded that viscous and inertia forces are responsible for bubble loss, fragmentation (changing the size of larger bubbles to dispersed bubbles) in slug body. According to their work, increase in liquid viscosity would result in increasing slug body liquid holdup. A new formulation for slug body liquid holdup was presented as:

$$H_{ls} = 0.85 - 0.075\,\varphi + 0.057\sqrt{\varphi^2 + 2.27} \quad (19)$$

$$\varphi = N_{Fr} N_{\mu}^{0.2} - 0.89 \quad (20)$$

$$N_{Fr} = \frac{J_t}{(gD)^{0.5}}\sqrt{\frac{\rho_L}{(\rho_L - \rho_g)}} \quad (21)$$

$$N_{\mu} = \frac{J_t\,\mu_L}{g\,D^2(\rho_L - \rho_g)} \quad (22)$$

2.5 Slug body length

In this section, comparisons have been made between experimental data of slug length (measured by optical sensor) from Babakhani (2017) and empirical correlations developed for viscous liquid-gas flow in the open literature, reported in Table 2. The results of comparisons are depicted in Fig. 3 (a-b). Among the models presented in Table 2, correlation by Barnea and Brauner (1985) has not been compared with experimental data because it was developed for low viscous liquid-gas flows, its application would lead to large overestimation of slug length. Furthermore, it is evident from Fig. 3 (a-b) that model by Al-safran et al. (2011) is insensitive to superficial gas velocity whilst slug length is immensely affected by gas velocity. It is worth remarking that weighted averages are defined for liquid density and viscosity. They concluded that average value of $L_s=10 \cdot D$ is a reasonable approximation for viscous oil-gas intermittent flow. In Fig. 3-b, an increase in slug length is observed at $J_g=0.95$ $m \cdot s^{-1}$, which is associated with increase in water cut (ε_{Lw}). When water (lower viscous phase) is added, turbulent kinetic energy overcomes viscous forces and slug front becomes more turbulent, more liquid is entrained into gas pocket from slug, which result in stretching slug into longer slugs. This phenomenon is in agreement with the prescribed behaviour in the works of Al-Safran et al. (2011) and Losi et al. (2016b). Losi et al. (2016b) measured slug length for high viscous oil-air within a horizontal pipe. From Fig. 3 (a-b), it can be seen that the approach by Losi et al. (2016b) is able to describe the behavior of experimental data, particularly at low superficial gas velocity where transition from slug to dispersed flow regime occurs. In the whole ranges of operating conditions, average relative error between experimental data and model by Losi et al.

(2016b) is found to be 20.8%, with larger deviation at higher superficial gas velocity. The lower average relative error of Losi et al. (2016b) model might be related to considering both pipe diameter and superficial gas velocity as compared to other empirical correlations in the open literature. Hence, this model is used as an input in the mechanistic model for prediction of slug length.

Table 2. Slug length models for viscous liquid-gas flows from literature

Author	model	Additional information
Barnea and Brauner (1985)	$\frac{l_s}{D} = 32$	
Al-Safran et al. (2011)	$\frac{l_s}{D} = 2.63\left(\frac{D^{3/2}\sqrt{\rho_l(\rho_l - \rho_g)g}}{\mu_l}\right)^{0.321}$	$\rho_l = \rho_w \varepsilon_{Lw} + \rho_o \varepsilon_{Lo}$ $\mu_l = \mu_w \varepsilon_{Lw} + \mu_o \varepsilon_{Lo}$
Losi et al. (2016b)	$\frac{l_s}{D} = K\left(J_g + \frac{J_{go}^2}{J_g}\right)$	$K = 5.3$ $J_{go} = 0.3$

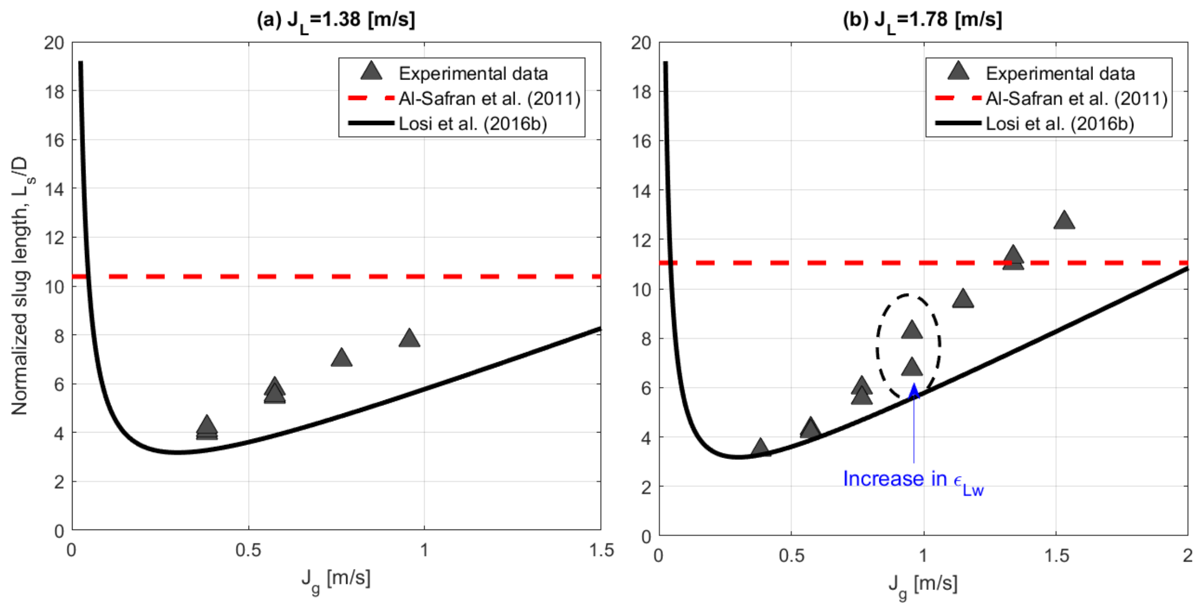

Fig. 3 Comparison between measured slug length (Babakhani, 2017) at (a) J_L=1.38 m/s, (b) J_L=1.78 m/s and the predictions of Al-Safran et al. (2011) and Losi et al. (2016b)

2.6 Closure relation

2.6.1 Two phase friction factor

Some empirical correlations to express two phase friction factors as a function of phase Reynolds number is required. For gas-wall friction factor, Blasius formulation is often used as described by Taitel and Dukler (1976). The validity of gas-liquid friction factors estimated by Blasius is assessed by Khor et al. (1997) for three phase stratified flow. These are:

$$f_g = \frac{16}{Re_g} \qquad Re_g \leq 2100 \tag{23}$$

$$f_g = \frac{0.046}{Re_g^{0.2}} \qquad Re_g > 2100 \tag{24}$$

Zhao et al. (2013b) developed a new expression for liquid-wall friction factor in the case of laminar liquid for gas-liquid flow over the large range of liquid viscosity

$$f_l = \frac{20.76}{Re_l} \qquad \text{for} \qquad Re_l \leq 2100 \tag{25}$$

Kowalski (1987) measured wall-to-liquid shear stresses and proposed a new correlation for turbulent liquid-wall friction factor as a function of liquid superficial Reynolds number and local liquid holdup for the large range of phase superficial velocity.

$$f_l = \frac{0.0262}{(H_{lf} Re_{sl})^{0.139}} \qquad \text{for} \qquad Re_l > 2100 \tag{26}$$

Regarding interfacial friction factor, no dependence of gas-wall shear stresses on interfacial characteristic of gas-liquid in film region was observed, see for instance Taitel and Dukler, 1976 and Kowalski (1987).

Andritsos and Hanratty (1987) studied the effect of large-amplitude wave on interfacial conditions of gas-liquid flows and concluded that interfacial shear stresses increases as a result of higher large-amplitude wave. They defined a critical superficial velocity at which large amplitude wave appears and proposed a new correlation as a function of non-dimensional liquid height and superficial gas velocity.

$$\frac{f_i}{f_g} = 1 \qquad \text{for} \qquad J_g \leq J_{g,c}$$

$$\frac{f_i}{f_g} = 1 + 15\left(\frac{h_L}{D}\right)^{0.5}\left(\frac{J_g}{J_{g,c}} - 1\right) \quad \text{for} \qquad J_g > J_{g,c} \tag{27}$$

$$J_{g,c} = 5\,(\frac{\rho_{go}}{\rho_g})^{0.5} \tag{28}$$

Where ρ_{go} is the gas density at atmospheric pressure.

2.6.2 Translational velocity of elongated bubble

Translational bubble velocity was first presented by Nicklin (1962) as a function of superficial mixture velocity (J_t) and drift velocity (U_d), based on drift-flux approach:

$$U_t = \zeta \cdot J_t + U_d \tag{29}$$

The other empirical correlations in the previous studies to calculate translational bubble velocity are the modification of Nicklin (1962) model. The distribution parameter, ζ was found to be 1.2 when flow is turbulent and 2 in the case of laminar flow. Benjamin (1968) suggested that drift velocity can be correlated to Froude number, diameter and gravitational acceleration in horizontal pipe.

$$Fr = \frac{U_d}{\sqrt{g\ D}} = 0.54 \tag{30}$$

However, this correlation does not take into account viscous effect. Losi and Poesio (2016) evaluated the influence of oil viscosity on drift velocity of a gas bubble in liquids for different axial positions in both horizontal and inclined pipes. They concluded that drift velocity for very viscous oil-gas flow (μ_o=0.804 Pa·s) is ranged between 0.0025-0.0065 $m \cdot s^{-1}$ for different axial positions in a horizontal pipe, which can be approximated equal to zero. Fig. 4 shows translational bubble velocity data from Babakhani (2017), measured by optical sensor, as a function of mixture superficial velocity. The square symbols denote the experimental data, while solid line shows the homogenous line. It is observed that experimental data is overestimated by Nicklin (1962) correlation, due to improper drift velocity expression introduced in this correlation which does not take into account the viscous and surface tension effects. However, the proposed model ($U_t=1.2 \cdot J_t$) for bubble translational velocity gives a satisfactory agreement, which is exactly equivalent to the model proposed by Wallis (1969). The experimental data of bubble velocity as a function of $\varepsilon_{Lg}=J_g/J_L$ is presented in Fig. 5 for different gas superficial velocities ranging from 0.38 $m \cdot s^{-1}$ to 2.10 $m \cdot s^{-1}$. It is interesting to see how data tends to be aligned in a regular trend, depending upon superficial gas velocity. However, a slight scattering tendency is observed at higher J_g which might be due to measurement uncertainty caused by the increasing flow disturbances. In Fig. 5 the data trends validated the assumption of equivalent two-phase flow where oil and water flow as a homogeneous mixture with the distribution parameter equal to 1.2, in line with the works of Zuber and Findlay (1965) as well as Wallis (1969). The agreement between proposed models and experimental data is excellent: e_{ri}=2.5 %, ε=2.8 %.

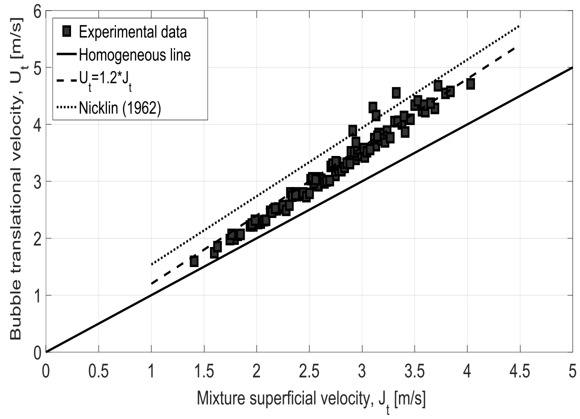

Fig. 4 Bubble translational velocity (U_t) versus mixture superficial velocity (J_t)

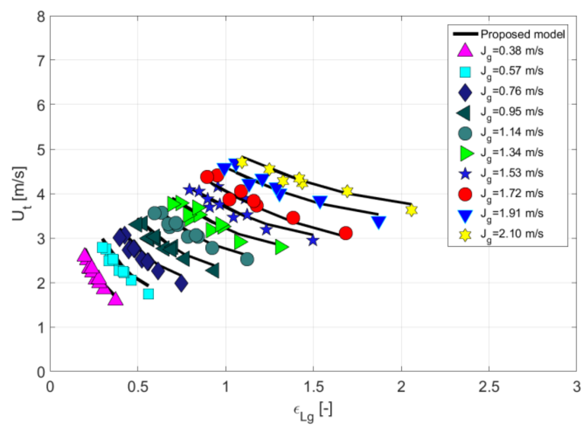

Fig 5. Translational bubble velocity (U_t) versus ε_{Lg} (ratio between superficial gas velocity and superficial liquid velocity) for J_g=0.38-2.10 $m{\cdot}s^{-1}$

2.7 Slug unit length

As there is no information regarding slug unit (cell) length, l_u for viscous oil-water-gas flow in the open literature, a model for l_u is formulated so that the slug unit length acts as an input parameter in the mechanistic model. The slug unit length is expressed taking into account the influences of pipe diameter and operating conditions, based on a power law functional form:

$$\frac{l_u}{D} = C \cdot (1 + \varepsilon_{LG})^n \qquad (31)$$

From regression analysis, the coefficients C and n were found to be 7.3 and 2.0, respectively. Babakhani (2017) and Babakhani et al. (2019) report a discussion about statistical significance of the collected data. In particular, the slug body length l_s shows a log-normal distribution, which shifts from right-skewed to normal like one as the gas superficial velocity is increased at constant oil and water superficial velocities. The mean slug body length increases from 3D to 27D, with a ratio between the standard deviation and the mean varying within about 0.20 and 0.33. It is evident that for each operating condition, l_s and l_u, as a consequence, are significantly variable. Nonetheless, owing to their statistical distribution it make sense to adopt the mean value as representative of the typical slug in a quasi-steady state model, at least in a range of operating conditions where the translational bubble velocity is well correlated to ε_{Lg}, as shown in Fig. 5. This implies that an equivalent liquid phase with suitable averaged properties is able to catch the more complex behavior of the two liquid phases (oil and water) and that it is reasonable to adopt empirical models developed for two-phase slug flows as closure relations. Fig. 6 shows comparison of measured slug cell length with eq. 31, considering four different superficial oil velocities (J_o=0.36-0.48-0.60-0.71 $m \cdot s^{-1}$), parameterized by two water superficial velocities (J_w=1.05-1.32 $m \cdot s^{-1}$). An increasing dependence of slug cell length on superficial gas velocity is observed. An analysis of 124 data points revealed that the agreement between experimental data and proposed model by Babakhani (2017) was reasonable: Mean Average Percentage Error (MAPE) resulted 10.3 %, and the standard deviation of the error was 9.5 %.

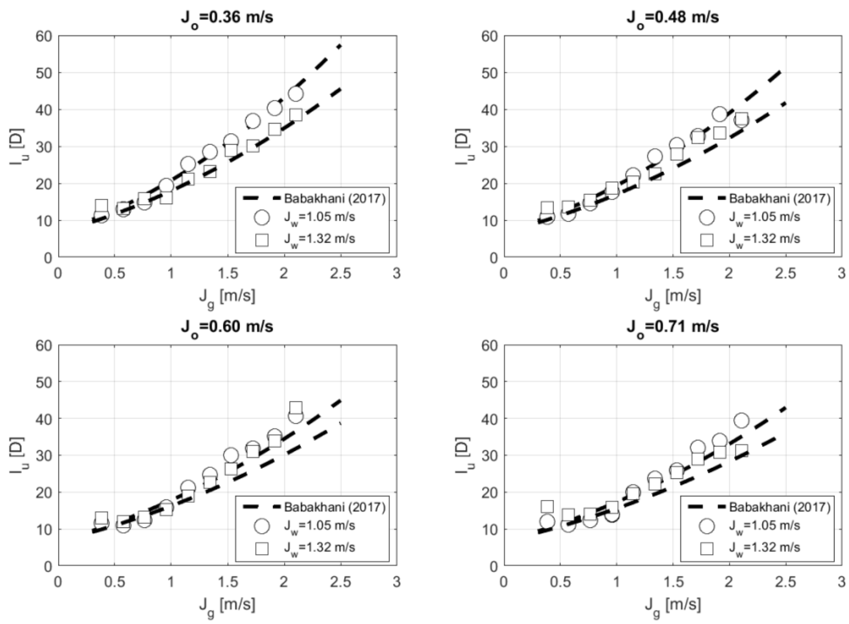

Fig. 6 Slug cell length versus superficial gas velocity for Jo=0.36-0.71 $m{\cdot}s^{-1}$

3. Solution procedure

In order to solve continuity and momentum equations of intermittent flow based on mechanistic model (sections 2-1 to 2-3), some closure relationships are required. This information are obtained from empirical correlations for two-phase flows because no references are available for some parameters such as slug length, slug body holdup, and friction factor for liquid-liquid-gas flows (a detailed description of models for two-phase flows are described in sections 2-4 to 2-7). It is worth noting that the following assumptions are made to use mechanistic model:

- Uniform liquid height, that is, $\frac{\partial h_L}{\partial x} = 0$
- Oil and water are assumed to be a homogeneous mixture
- Gas is entrained and dispersed into slug body
- Entrained gas into slug body move with the velocity equals to Translational bubble velocity
- Gas expansion will not occur from the pipe inlet
- Elongated bubbles move with a so-called translational bubble velocity (U_g=U_t)

Based on the balance between the total free-surface energy of dispersed gas bubbles and the total kinetic turbulent energy of liquid in slug body, Zhang and Sarica (2006) proposed a criterion when the assumption of an equivalent liquid for oil and water is valid:

$$J_L > \left(\frac{6.325 C_e \varphi_{Int} [\sigma_{ow}(\rho_w - \rho_o)]^{0.5}}{f_{LM} \rho_L}\right)^{0.5} \quad (32)$$

$$C_e = \frac{2.5 - |\sin(\theta)|}{2}$$

Where Θ, φ_{Int}, and σ_{ow} are pipe inclination, the volumetric fraction of dispersed phase, that is, oil in the present study (φ_{Int}=ε_{Lo}=J_o/J_o+J_w), and interfacial tension between oil and water, respectively. The friction factor for liquid mixture (f_{LM}) is obtained from Blasius formulation (eq. 23 and 24), taking into account Reynolds number for homogeneous mixture of oil and water. This correlation is widely used to compute friction factor for oil-water flow (see, e.g. Colombo et al., 2017). A flowchart for calculation of pressure drop and gas holdup based on mechanistic model for three-phase intermittent flow, considering fully mixed oil/water and core-annular is shown in Fig. 7. The solutions of continuity and momentum equations only require rheological properties of phases, pipe diameter, and flow conditions.

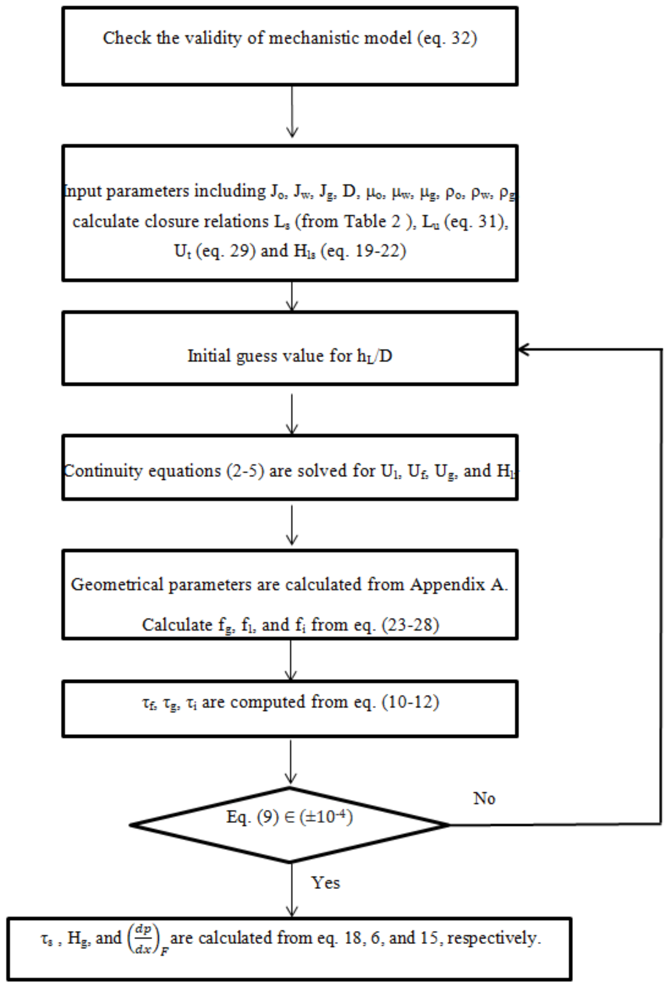

Fig. 7 Flowchart calculation of three-phase intermittent flow based on mechanistic model

4. Validation of mechanistic model

In the following sections, the results of three-phase pressure drop and gas holdup predicted by mechanistic model are presented.

4.1 Pressure drop prediction

As no independent data set of pressure drop was available for viscous oil-water-gas to validate the model, only two sets of experimental data are found to evaluate the model performance. One data set is the experimental data of Babakhani (2017). Four values of oil superficial velocity (J_o=0.36, 0.48, 0.60, 0.71 m/s) were considered, for each pair of J_o-J_w the values of superficial gas velocity ranging from 0.22-1.91 were investigated. Another source is data bank in the work of Poesio et al. (2009) who performed tests with μ_o=1.2 Pa·s within a 21 mm i.d. horizontal pipe. The details of data bank are reported in Table 3.

Table 3. Details of data sources used to evaluate model performance

Data source	Diameter [m]	Oil viscosity [Pa.s]	Gas velocity [m/s]	Liquid velocity [m/s]	Data points
Babakhani (2017)	0.040	0.83 @ room temp	0.22-1.91	1.02-2.05	131
Poesio et al. (2009)	0.021	1.2	0.29	0.13-3.4	30

As it is evident from Fig. 8, there is a fairly good agreement between predicted and measured pressure drop data of Poesio et al. (2009), considering the average relative error of -14.8% and standard deviation 14.7%. Almost all data predicted by the model are underestimated measurements. Considering acceleration pressure drop contribution in equation 14 might result in a better prediction of measured data by model and can be a topic of further investigation. About 87% of all data predicted by the model falls within ±30% of relative error. Larger deviation occurs at low oil superficial velocity. At low oil superficial velocity, oil and water tends to form core-annular flow and degree of stratification increases. Since the present model assumes an equivalent liquid for oil and water, which oil and water treated as a fully-mixed liquid, it is possible that the pressure drop predicted by the model shows slightly higher deviation.

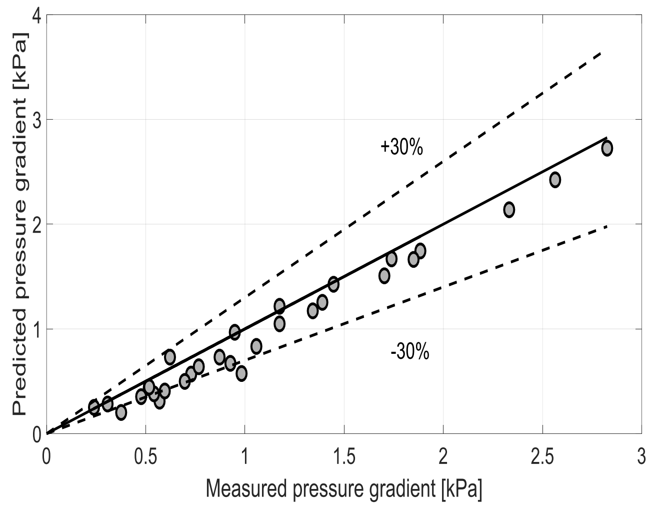

Fig 8. Pressure drop comparison between prediction and data of Poesio et al (2009) for D=21mm

The comparison of pressure gradient computed by model and measured data by Babakhani (2017) is depicted in Fig. 9 showing a good agreement with an average relative error of -15.4%, while standard deviation was found to be 10.2%. The wide range of operating conditions was considered for this comparison. To evaluate the performance of current mechanistic model, hybrid model developed by Poesio et al. (2009) is also compared in Fig. 9. They used Lockhart-Martinelli parameter (χ) modified by Chisholm (1973) to predict three-phase pressure drop. The hybrid model is based on solution of two-fluid model for liquid-liquid developed by Brauner

(1991), which is eventually substituted in Lockhart-Martinelli parameter to compute overall pressure drop. Table 4 lists the equations required for the hybrid model proposed by Poesio et al. (2009).

Table 4. Hybrid model proposed by Poesio et al. (2009)

Hybrid model	Additional information
$\Delta P_{o-w-g}=\phi_g^2 \cdot \Delta P_g$ $\phi_g^2 = 1 + C \cdot \chi + \chi^2$	$\chi=\sqrt{\frac{\Delta P_{liq}}{\Delta P_g}}$, C=15 (Lockhart-Martinelli parameter)

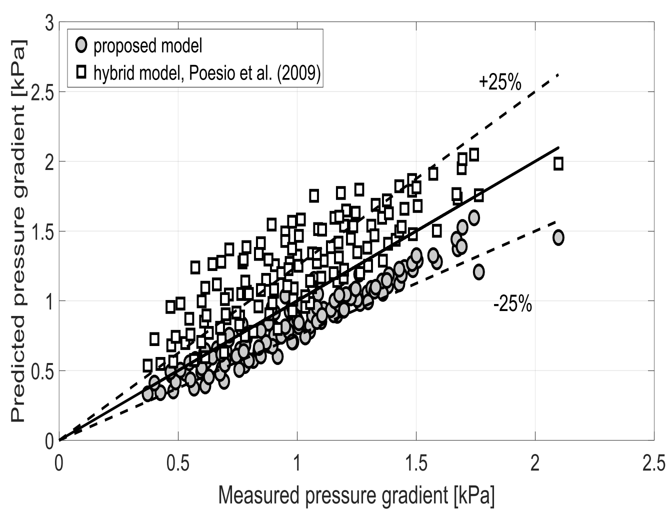

Fig 9. Predicted pressure drop versus measured data of Babakhani (2017) and comparison with hybrid model developed by Poesio et al. (2009)

The proposed mechanistic model is able to predict pressure drop better than hybrid model over entire range of operating conditions. Almost 84% of all data fall within 25% of relative error for proposed model while 54% of all data falls into 25% of relative error predicted by hybrid model. Table 5 shows statistical analysis of proposed model and comparison with hybrid model.

Table 5. Comparison of pressure drop between proposed mechanistic model and hybrid model

Models	e_{ri} (%)	Max e_{ri} (%)	Min e_{ri} (%)	Std deviation (%)
Poesio et al. (2009)	27.9	116	-13.1	26.4
Proposed model	-15.4	8.7	-39.1	10.2

The results of pressure drop prediction shows that in spite of complexity of three phase flow of high viscous oil-water-gas, the developed mechanistic model is able to predict pressure gradients with a reasonable average relative error. Hence, it can be used as an operative engineering tool to compute pressure drop.

4.2 Gas holdup prediction

As no time-space average technique for measuring gas hold up has been so far presented for viscous oil-water-gas flow in the open literature, all the comparisons of model performance was made, taking into account the definition of translational bubble velocity, that is, $H_g=J_g/U_g$ with $U_g=U_t$, reported in Table 6. Hence, the proposed model is based on a drift-flux concept with distribution parameter equals to 1.2 (eq. 6), which is derived from experimental analysis and it can be used as a reference value. A comparison has been made between prediction of gas holdup by eq. (6) and available models in literature for two-phase flow of gas-liquid to check the possibility to use in three-phase flow. The 68 void fraction correlations according to large data set have been reported by Melkamu and Ghajar (2007). Among all correlations presented in their work, three correlations in the families of slip ratio models (Lockhart and Martinelli, 1946, and Chen, 1986) and $K\varepsilon_H$ (Armand, 1946) are widely used. The latter was validated by Guilizzoni et al. (2018) who proposed an image-based technique by video camera, with a resolution of 1280×720 and frequency 50 fps, to measure average gas hold up. They concluded that Armand (1946) correlation has an outstanding performance, with mean average percentage error=3.1 % and ε=2.5 %.

Table 6. Performance of available correlations for mean gas holdup

Correlation	Avg absolute error (%)	Max absolute error (%)	St deviation (%)
Lockhart and Martinelli (1946)	24.0	32.1	8.9
Chen (1986)	16.1	23.6	6.6

Comparison between average gas hold up (eq. 6) and correlations of Lockhart and Martinelli (1946) and Chen (1986) for J_o=0.36-0.71 m/s are shown in Fig. 10. Both models underestimated the reference value, with larger deviation at higher superficial gas velocity. Fig. 10 and Table 6 revealed that although these models are on basis of slip ratio concept, they are incapable of predicting mean average hold up.

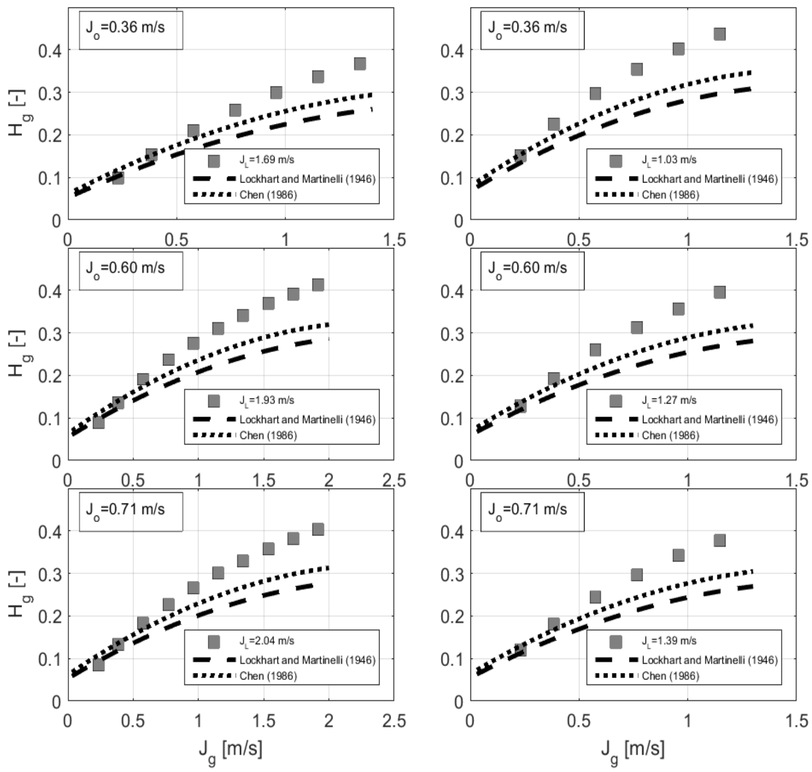

Fig 10. Comparison between average gas hold up at J_o=0.36-0.71 m/s and correlations of Lockhart and Martinelli (1946), and Chen (1986)

5.Conclusion

A mechanistic model based on the solution of continuity and momentum equations is proposed to compute phase velocity and pressure drop in slug body and film section (assuming uniform liquid height) for horizontal viscous oil-water-air flows at atmospheric pressure. The model requires the superficial velocity of phases and rheological properties as input parameters. Oil and water are treated as an equivalent fluid with suitably averaged properties. In particular, density was determined by assuming homogeneous distribution, whereas viscosity was calculated according to

Einstein's equation. Accordingly, empirical closure relations for gas-liquid flows (slug length and holdup) were used, due to the lack of information for viscous oil-water-gas flows. The major output of mechanistic model is the pressure drop across slug unit cell. Moreover, a correlation for calculation of mean gas holdup based on drift-flux concept is also proposed. The results of the predicted pressure drop were compared with the measurements reported in the literature survey, showing a promising approach for viscous oil-water-gas phase flows in horizontal pipes, under a variety of operating conditions where the statistical distribution of slug lengths indicates that the mean value is representative of a typical slug structure in a quasi-steady state model. Furthermore, it has to be taken into account that the current database available for validation is relatively scarce and all the experiments on three-phase flows were conducted in plants with a "smooth" introduction of the phases. Accordingly, it was not possible to investigate the effect of the inlet conditions on the flow development. In particular, the proposed approach seems to be valid until the translational bubble velocity is well correlated to the liquid-to-gas input volume fraction, which implies that the two liquids can be lumped into an equivalent liquid phase with suitable averaged properties.

Appendix A. Geometrical parameters

The geometrical parameters, $A_f, A_g, S_f, S_g, S_i, H_{lf}$ are presented by Aziz and Govier (1972), assuming that interface between gas-liquid is flat. They are all functions of liquid film height (h_L) given by:

$$\alpha = 2cos^{-1}[1 - \frac{2h_L}{D}] \quad \text{(A-1)}$$

$$A_f = \frac{A}{2\pi}(\alpha - \sin\alpha) \quad \text{(A-2)}$$

$$A_g = A - A_f \quad \text{(A-3)}$$

$$H_{lf} = \frac{A_f}{A} \quad \text{(A-4)}$$

$$s_f = \frac{D\,\alpha}{2} \quad \text{(A-5)}$$

$$s_g = \pi D - s_f \quad \text{(A-6)}$$

$$s_i = 2D\sqrt{[\frac{h_L}{D} - (\frac{h_L}{D})^2]} \quad \text{(A-7)}$$

Appendix B. Statistical parameters

The statistical parameters based on average relative error and standard deviation is used to evaluate performance of model with measured data as follows:

$$e_{ri} = \frac{(E_{i,cal} - E_{i,meas})}{E_{i,measured}} \times 100 \quad \text{(B-1)}$$

$$e = \frac{1}{N}\sum_{i=1}^{N} e_{ri} \quad \text{(B-2)}$$

Standard deviation related to average relative error is:

$$\varepsilon = \sqrt{\frac{\sum_1^N (e_{ri} - e)}{N-1}} \quad \text{(B-3)}$$

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