Railway Collaborative Ecodrive via Dissension Based Switching Nonlinear Model Predictive Control

Article *in* European Journal of Control · November 2019

DOI: 10.1016/j.ejcon.2019.04.005

Railway Collaborative Ecodrive via Dissension Based Switching Nonlinear Model Predictive Control

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Abstract

This paper deals with the design of a switched Nonlinear Model Predictive Controller (NMPC) for collaborative ecodrive control of railway vehicles. Relying on a discrete, switched and nonlinear model of the train, the NMPC optimizes the handle position while fulfilling constraints on velocity and journey time. Specifically, the optimizer provides a set of operating modes, which the human driver is able to implement to modulate traction or braking forces and such that the corresponding driving style is constrained by predefined driving sequences. At network level, a Dissension based Adaptive Law (DAL) is then proposed to adjust the parameters of the NMPC cost so as to efficiently share the available regenerated braking energy among the trains connected to the same substation, while negotiating between constraint satisfaction and control aggressiveness. The effectiveness of the proposed strategy is finally demonstrated on a realistic simulation case study.

Keywords: Train control, predictive control, nonlinear control systems, switching algorithms.

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This is the final version of the accepted paper submitted to European Journal of Control.

1. Introduction

Nowadays, while transportation is in general one of the major contributors of emissions and pollution, railway remains by far the most efficient means of transportation from the point of view of energy consumption and therefore a strategic sector. In fact, analyzing the railway energy consumption, more than 70 percent usually corresponds to traction requirements while the rest to non-traction [1]. This has lead to most of the new technologies being developed aiming at the reduction of energy consumption. In electrical traction systems, one way to achieve this goal is to recover braking energy, that otherwise would be lost as heat into the environment. Typically, the recovered energy is referred as to *regenerative energy* while the phenomenon of energy recovery through braking is known as *regenerative braking*. During regenerative braking, the traction motor acts as a generator and restores part of the kinetic energy into electrical energy [2]. The principle usually involves a source pantograph, the traction motor/generator and the trains connected to the pantograph.

Historically speaking, one of the earliest examples in railways where regenerative braking was introduced is the Baku-Tbilisi-Batumi railway (Transcaucasus railway or Georgian railway) in the 1930s. Usually, the generated energy can either be fed back to the pantograph to be used instantaneously by other connected trains or saved on an energy storage device (e.g., batteries, supercapacitors, flywheels or fuel cells [3]) for later use. This stored energy could be used in the next acceleration phase by trains. While storing regenerative energy for later use can be a good option, it requires huge changes in the present railway infrastructure and can be economically quite demanding. Another direction to investigate is the instantaneous use of this regenerated energy, for instance incorporating it into an Energy Efficient Train Control (EETC) or ecodrive problem. EETC specifically refers to the development of control based methods with focus on techniques aimed at reducing traction energy consumption and emissions, which are effected by the behavior of the train driver, without necessarily upgrading the vehicle technology and usually involving single train control. In the literature, regenerative energy was first introduced directly into the EETC problem in [4]. This was followed by further developments as reported in [5, 6, 7, 8, 9, 10]. This use of regeneration energy can be extended to energy sharing by collaboration among trains connected to the same substation and is a relatively new concept from the point of view of EETC or ecodrive problem. In this framework, sharing of regenerative braking energy among trains can be quite useful to reduce the energy consumption of the whole network. Specifically, consuming the regenerative braking energy instead of demanding energy from the substation not only reduces the load on the substation and overall network energy consumption but also guarantees benefits in terms of losses during the energy transfer from the substation to the trains. This idea has given rise to the paradigm of *collaborative ecodrive*. Existing works mostly focus on optimizing the train timetables, so as to synchronize the acceleration and deceleration of trains as much as possible to be able to use the available regenerated energy by the accelerating trains in the network [2, 11, 12, 13].

In this paper, we focus on two main topics of interest: the single train ecodrive control and the collaborative ecodrive. Although for electrical trains the input force is usually continuous in nature, however, in the manual assistance scenario, where the algorithm is specifically designed to advice the driver, the train dynamics can be modeled as switching systems due to the switching nature of the input handle. In fact, the input handle which decides the amount of traction or braking force is enforced to belong to a set of discrete values or operating modes (acceleration, cruising, coasting and braking) which are implementable in practice by the driver and are constrained by predefined driving sequences. Since the train dynamics can be captured by a switching model [14, 15, 16] a switched Nonlinear Model Predictive Controller (NMPC) is designed at local level (see [17, 18, 19, 20, 21]). In fact, predictive control is a suitable approach, thanks to its capability to deal with state and input constraints and economic objectives [22, 23]. In particular, NMPC has already been applied to the energy efficient operation of trains in [24] but it has not been explored much in the context of the EETC problem. The second topic of interest explored in this paper is the collaborative ecodrive described above. For this purpose, at network level a Dissension based Adaptive Law (DAL), which resembles a consensus algorithm with Markov stochastic processes, is introduced (see Figure 1). The DAL assigns the weights of the parameters constituting the cost function and related to normalized traction force, resistance force and jour-

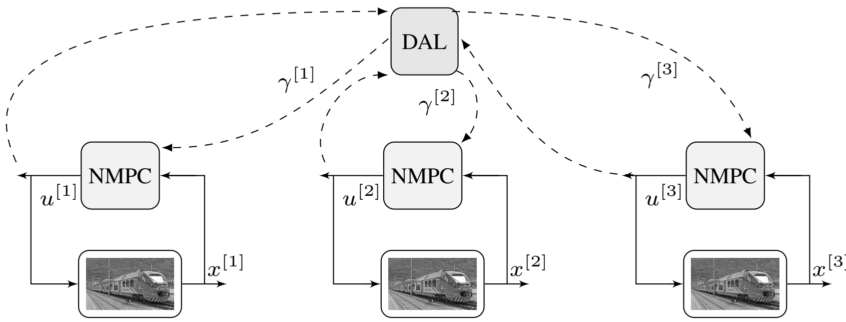

Figure 1: The proposed collaborative ecodrive NMPC based architecture

ney time. A different idea to coordinate agents in railway systems has already been investigated in [25, 26]. More specifically, in [25], a hierarchical predictive control is proposed for coordinating the electric traction substation energy flows at a higher level, and on-route trains energy consumption at a lower level. In [26], instead, a distributed cooperative model predictive control is designed for energy-efficient trajectory planning in case of multiple high-speed trains. The cooperative optimization algorithm is aimed at saving energy, explicitly taking into account comfort and cooperation among the trains in the cost function. Differently, the proposed NMPC with DAL makes the trains exploit the regenerative energy shared in the network in order to accelerate if they need, thus avoiding to lose it as heat in the rheostats. Future work will be devoted to the inclusion of the grid model to explicitly take into account its energy flow, and to the introduction of energy storage devices in the substations or on board the trains. Finally, a realistic case study shows the effectiveness of the proposed NMPC, even in comparison with a standalone ecodrive solution.

The paper is organized as follows. In Section 2, some preliminaries on switched systems and switching NMPC are introduced. In Section 3, the overall train network model, the considered single train model and the considered ecodrive control problem are discussed. Section 4 and Section 5 present the proposed NMPC based collaborative ecodrive in detail, where the single train problem is recast into the switching NMPC framework and the DAL, respectively. In Section 6 simulation results on a realistic scenario are illustrated and analyzed. Some conclusions are gathered in Section 7.

2. Preliminaries

In this section some preliminary issues are introduced. Specifically, the notation used in the paper and basics of switching model predictive control are reported.

2.1. Notation

The notation used in the paper is mostly standard. Let $\mathbb{N}$ denote the set of natural numbers, $\mathbb{Z}$ be the set of all integers, while $\mathbb{R}$ be the set of real numbers. Let x be a vector and x_i its entry and x' its transpose. Given a signal w, then $w_{p|w}$ is its prediction trajectory with initial condition w, so that at the current sampling time instant k, $w_{k|w} = w$. Moreover, let $\mathbf{w}_{[k,k+p]}$ be the signal w defined from k to $k+p$. Finally, given two vectors a and b, the Kronecker product is denoted as $a \otimes b$, while $\mathbb{1}$ is a vector with all ones.

2.2. Switching Finite-Horizon Optimal Control Problem

Consider the discrete time switched nonlinear system

$$x_{k+1} = f_{s_k}(k, x_k), \quad \forall\, k \in \mathbb{N} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state at time k, $s_k \in \mathbb{N}$ is the switching rule and f_i, $i \in \mathcal{S}$, is a Lipschitz continuous vector function, with $\mathcal{S} = \{1, \cdots, M\}$. The active model at the time instant k is determined by the integer $s_k \in \mathcal{S}$. The Finite-Horizon Open-Loop Optimal Control Problem (FHOCP) consists in minimizing at each sampling instant k with respect to the sequence $\mathbf{s}_{[k,k+N_\mathrm{p}-1|k]}$ a predefined prediction cost, i.e.,

$$\begin{aligned}
\min_{\mathbf{s}\in\mathcal{W}} J_{\mathbf{s}}(x) = & \sum_{p=k}^{k+N_\mathrm{p}-1} \gamma_1 \ell_{s_k}(x_{p|x}) + \gamma_2 V_\mathrm{f}(x_{p+N_\mathrm{p}|x}) \\
& \text{s.t.} \\
\mathbf{s}_{[k,k+N_\mathrm{p}-1|k]} &= [s_{k|k}, \ldots, s_{k+N_\mathrm{p}-1|k}] \\
x_{p+1|x} &= f_{s_p}(p, x_{p|x}) \\
x_{k|x} &= x \\
x_{p|x} &\in \mathcal{X}_p, \quad \forall\, p \in [k, k+N_\mathrm{p}] \\
x_{k+N_\mathrm{p}|x} &\in \mathcal{X}_\mathrm{f}\ ,
\end{aligned} \tag{FHOCP}$$

where $x_{p|x}$ in turn depends on the predicted switching strategy $\mathbf{s}$ over the prediction horizon N_{p}, V_{f} is the terminal cost, $\mathcal{X}_p \subset \mathbb{R}^n$ is the state constraint set, the terminal constraint set is $\mathcal{X}_{\mathrm{f}} \subset \mathbb{R}^n$ and γ_1, γ_2 are weights, possibly time-varying, such that $\gamma_1 + \gamma_2 = 1$. At each sampling instant k, the optimal switching policy, denoted by $\mathbf{s}^{\mathrm{o}} = [s^{\mathrm{o}}_k, \ldots, s^{\mathrm{o}}_{k+N_{\mathrm{p}}-1}]$ will be inside the set of feasible sequences $\mathcal{F} \subseteq \mathcal{W}$, with $\mathcal{W}$ being the set of all possible sequences. Finally, only the first element of the resulting optimal control switching strategy is used at each step, while the remaining entries are discarded. In the following sections it will be illustrated how to make the formulation of the train dynamics under consideration fit the structure (1), and be therefore eligible to be solved via a NMPC. The solution of the problem (FHOCP) can be computed according to Algorithm 1.

Algorithm 1 Switching NMPC - Pseudo Algorithm

Require: k, x_k

for each $\mathbf{s}_i \in \mathcal{W}$ with index i **do**
  compute $J_{\mathbf{s}_i}$ through simulation of the system using the sequence $\mathbf{s}_i$, time k and initial state x_k
  if $\mathbf{s}$ is feasible **then**
    add i to the set of feasible indexes $\mathcal{I}_{\mathcal{F}}$
  end if
end for
compute the index i^{o} optimal sequence

$$i^{\mathrm{o}} = \arg\min_{i \in \mathcal{I}_{\mathcal{F}}} (J_i)$$

return the first element of the optimal sequence $\mathbf{s}^{\mathrm{o}} = \mathbf{s}_{i^{\mathrm{o}}}$

3. Motivating application: collaborative ecodrive of trains

The proposal of this work is motivated by a real-word application under study in collaboration with the rail transport company Alstom. More specifically, the scope of

Figure 2: One of the Italian trains considered as case study

the work is to design a control strategy for the efficient operation of trains in a collaborative fashion, possibly exploiting the braking energy regenerated by the vehicles connected to the same substation.

3.1. Substation network modeling

Consider a network of N trains belonging to the same substation and capable to communicate each other. The network is described through a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \ldots, N\}$ is the set of nodes and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ are the edges associated to communication links such that (r, s) belongs to $\mathcal{E}$ if and only if the train r transmits information to train s. Moreover, let $\mathcal{N}_{\text{rec}}^{[r]} = \{s \in \mathcal{N} : (r, s) \in \mathcal{E}\}$ be the set of receiving neighbors of train r, while $\mathcal{N}_{\text{tra}}^{[r]} = \{s \in \mathcal{N} : (s, r) \in \mathcal{E}\}$ be the set of transmitting neighbors of train r.

3.2. Train modeling

Consider now the rth electric train (see for example the one depicted in Figure 2) controlled by a digital control unit with sampling time T_{s}. Let $k \in \mathbb{Z}$ be the discrete time variable and $x^{[r]}(k) = \left[x_1^{[r]}(k)\, x_2^{[r]}(k)\right]'$ be the state of the train, where $x_1^{[r]}$ is its position and $x_2^{[r]}$ its velocity. Each train has to move from one station with position $x_1^{[r]} = 0$ to the next one with position $x_1^{[r]} = x_{\mathrm{f}}$, in a prescribed time t_{f}. Furthermore, assume that the track features (slopes, curvature) are known in advance. Thus, in nominal conditions (i.e., with rated values of the train parameters, like its mass and the specifications of the powertrain and braking systems), using the forward Euler discretization method, the dynamics of the train is captured by

$$\begin{aligned} x_1(k+1) = &\quad x_1(k) + T_\mathrm{s} x_2(k) \\ x_2(k+1) = &\quad x_2(k) + T_\mathrm{s} \left(\tfrac{F_\mathrm{T}(x(k),u(k)) - F_\mathrm{B}(x(k),u(k)) - F_\mathrm{R}(x(k))}{M_\mathrm{tot}} \right) \end{aligned} \tag{2}$$

where the apex $[r]$ is omitted for the sake of simplicity and M_tot is the total mass of the train, F_T is the traction force, F_B is the braking force, F_R is the resistance force and $u(k)$ is the input handle of the train. As for the traction and braking forces, they are functions of the handle and velocity and their models can be captured by

$$\begin{aligned} F_\mathrm{T} = &\quad F_{\mathrm{T}_\mathrm{max}}(x_2) u_k \\ F_\mathrm{B} = &\quad F_{\mathrm{B}_\mathrm{max}}(x_2) u_k \ , \end{aligned} \tag{3}$$

with $F_{\mathrm{T}_\mathrm{max}}$ and $F_{\mathrm{B}_\mathrm{max}}$ being the maximum allowable traction and braking forces, respectively. Moreover, the resistance force F_R is given as a combination of frictional effects due to velocity, described by the famous Davis equation, and the frictional effects due to gravity and slopes of the track, i.e.,

$$F_\mathrm{R}(x) = R_v(x_2) + R_g(x_1) \tag{4}$$

$$R_v(x_2) = A + Bx_2 + Cx_2^2 \tag{5}$$

$$R_g(x_1) = M_\mathrm{s} \left(g \tan(\alpha(x_1)) + \frac{D}{r_\mathrm{c}(x_1)} \right) \tag{6}$$

where R_v is the frictional force due to velocity, A, B, C are the Davis equation parameters, R_g is the frictional force due to slope of the track and gravity g, M_s is the static mass of the train, r_c is the radius of the curve of the track, α is the slope and D is a train dependent parameter.

Besides the prescribed arrival time t_f and position x_f, additional state constraints pertaining to the maximum allowed velocity must be fulfilled. Let $\overline{x}_2(x_1)$ be the maximum velocity depending on the position x_1, as imposed for the sake of safety by the authority according to the track features. Letting $k_\mathrm{f} := \left\lfloor \frac{t_\mathrm{f}}{T_\mathrm{s}} \right\rfloor$ be the terminal step, with

$\lfloor \cdot \rfloor$ being the flooring operation to the closest integer, the state constraints are

$$\begin{aligned} x(0) = &\quad [0\,0]' \\ x(k_\mathrm{f}) = &\quad [x_\mathrm{f}\,0]' \\ x_2(k) \geq &\quad 0, \quad k = 0, \ldots, k_\mathrm{f} \\ x_2(k) \leq &\quad \overline{x}_2(x_1(k)), \quad k = 0, \ldots, k_\mathrm{f}\ . \end{aligned} \tag{7}$$

3.3. The ecodrive control problem

We are now in a position to formulate the control objective which is to maximize the energy efficiency of each train of the network while satisfying the constraints previously introduced in a collaborative way. This aim is translated into the minimization of a cost, representing the discretized integral of the square value of the traction power. Hence, the ecodrive problem is generally defined as

$$\begin{aligned} &\min_{u_k} J = \sum_{k=0}^{k_\mathrm{f}} \left(\frac{F_\mathrm{T}(x_k, u_k) x_2(k)}{\eta(x_k, u_k)} \right)^2 \\ &\text{s.t.} \quad x_{k+1} = \bar{f}(k, x_k, u_k) \\ &\qquad x(0) = [0\,0]' \\ &\qquad x(k_\mathrm{f}) = [x_\mathrm{f}\,0]' \\ &\qquad x_2(k) \geq 0, \quad k = 0, \ldots, k_\mathrm{f} \\ &\qquad x_2(k) \leq \overline{x}_2(x_1(k)), \quad k = 0, \ldots, k_\mathrm{f}\ . \end{aligned} \tag{EcoCP}$$

with $\bar{f}(x_k, u_k)$ being the system dynamics (2) and $\eta(x_k, u_k)$ being the energy efficiency. Note that there are different possibilities to take into account the constraint on journey time, for instance relaxing it with an additional term in the cost function pursuing a receding horizon approach or implementing a shrinking horizon strategy as introduced e.g., in [27] and [28], respectively.

4. Recasting the ecodrive problem into the NMPC framework

In this section the previous collaborative ecodrive problem (EcoCP) is recast according to the NMPC formulation previously presented.

4.1. The switching train model

Since we are working with electric trains, the input handle $u_k \in [1, -1]$, which represents the allowed traction (positive) and braking (negative) force that the train can use at a particular instant, is continuous in nature. However, in this case, since we have in mind the manual assistance scenario, that is an algorithm to assist the driver, only discrete values which are easily implementable by the latter, defining $m = 4$ different operational modes of the train, are considered and described hereafter: in acceleration (AC) the handle can assume three values, i.e., $u_k \in \{0.5, 0.75, 1\}$; in coasting (CO) the engine is switched off, i.e., $u_k = 0$; in cruising (CR) the train maintains a constant velocity, i.e., the corresponding handle $u_k = u_{\mathrm{CR}}$ is chosen such that $F_{\mathrm{R}} = F_{\mathrm{T}}$ for positive slopes and $F_{\mathrm{R}} = F_{\mathrm{B}}$ for negative slopes; in braking (BR), due to safety reasons, whenever this mode is activated, it is preferred to use maximum braking force, i.e., the handle is chosen such that $u_k = -1$.

Hence, letting $u_{s_k}(k, x_k) \in \{-1, 0, 0.5, 0.75, 1, u_{\mathrm{CR}}\}$ be the switching input, this makes our system, which is actually continuous, switching in nature, i.e., perfectly fitting structure (1),

$$f_{s_k}(k, x_k) = \bar{f}(k, x_k, u_{s_k}(k, x_k)) \tag{8}$$

where $s_k \in \{1, \ldots, 6\}$ is the externally updated switching signal. For example, $s_k = 1$ implies that $u_1(k, x_k) = -1$ is chosen to be applied to the system at the sampling instant k. Also, due to the nature of traction, braking and resistance forces described above, the considered train model is nonlinear.

Now, we are in a position to formulate the (FHOCP) as previously stated, with $\mathcal{X}_p$ as the constraints in (EcoCP) and $\mathcal{X}_{\mathrm{f}}$ unused. In our case, since energy efficiency as well as journey time constraints need to be ensured while adopting a driving style which is compliant with the requirements of the ecodrive, the cost function for the rth train of the network is chosen as a combination of line energy, the energy losses due to resistance and horizon space error, which represents the equivalent horizon time term. More specifically, with reference to FHOCP, for each train r belonging to the substation, the terms of the cost function are defined as

$$\ell_{s_p}^{[r]}\left(x_{p|x^{[r]}}^{[r]}\right)=\left(\frac{\frac{F_{\mathrm{T}}\left(x_p^{[r]},u_{s_p}^{[r]}\right)}{\eta_p}-F_{\mathrm{R}}\left(x_p^{[r]}\right)}{F_{\mathrm{T}_{\max}}}\right)^2 \tag{9}$$

$$V_{\mathrm{f}}^{[r]}\left(x_{p+N_{\mathrm{p}}|x^{[r]}}^{[r]}\right)=\left(\frac{S_{\mathrm{h}}-x_1^{[r]}}{S_{\mathrm{h}}}\right)^2 , \tag{10}$$

where $\gamma_1 = \gamma_k^{[r]}$ and $\gamma_2 = 1-\gamma_k^{[r]}$, with $0 \leq \gamma_k^{[r]} \leq 1$ being a scalar weight. The horizon S_{h} is the distance that the train needs to cover in a specific prediction horizon in order to ensure the prescribed arrival time, constrained to speed limits and track features. A heuristic procedure based on the knowledge of the final distance, journey time and resistance force is adopted to compute S_{h}. Furthermore, the reason for including losses due to the resistance energy in the cost is to provide a solution which takes into account the acceleration of the system, in order to favor cruising operation mode for reducing energy consumption. The cruising mode u_{cr} is suitably computed by an inner control loop in order to have $F_{\mathrm{R}} = F_{\mathrm{T}}$ for positive slopes and $F_{\mathrm{R}} = F_{\mathrm{B}}$ for negative slopes.

Remark 1. *Note that, another important aspect in the ecodrive problem is that of comfort in order to ensure the adequate speed profiles from the passengers' point of view (see e.g., the solutions proposed in [26, 29]). In the proposed NMPC algorithm, since the term in* (9) *represents the acceleration of the train, in this way we also take into account the comfort issue explicitly in the cost function. Indeed, acceleration (or its derivative, i.e., the jerk) is an index of comfort. Moreover, the latter is implicitly ensured also by the control sequences, selected according to the ecodrive style.*

4.2. Simplification of the proposed NMPC

In this section, a simplification of the proposed solution is introduced in order to improve the NMPC performance. Specifically, we introduce a modification to reduce the computational complexity of the algorithm. This was in accordance to the requirement of our industrial partner Alstom. Moreover, an adaptation strategy is introduced

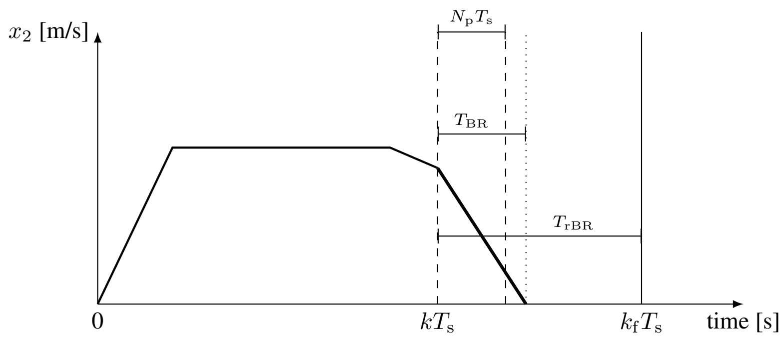

Figure 3: Graphical representations of the prediction times T_rBR and T_BR needed for braking time precomputation

in order to deal with braking dynamics which is very difficult to capture with a receding horizon approach.

4.2.1. *Reduction of computational complexity*

The cardinality of $\mathcal{W}$, which is strictly related to the computational complexity of solving the NMPC problem, is equal to $\mathrm{card}(\mathcal{W}) = M^{N_\mathrm{p}}$. Due to the exponential growth, this number can become very large for longer prediction horizon. *Yet, by virtue of the predictive braking approach discussed in the following, the prediction ahead does not need to capture the braking dynamics, so that N_p may be set as short as possible, while considering the track features.* Given the prediction horizon N_p, we set $\mathrm{card}(\mathcal{W}) = 15$ as reported in Table A.2. Note that, the choice of the sequences has been provided by our industrial partner Alstom, following the requirements imposed for current field driver assistance systems, which also take into account energy saving according to the ecodrive style, e.g., by avoiding to brake immediately after acceleration.

4.2.2. *Braking time precomputation*

A predictive braking approach which adapts to the samples so that the train stops at the arrival station is also included. The adaptive precomputation is adopted not just for capturing braking dynamics but also to capture the track characteristics such as the slopes and the curvature which are a function of the space as well as the changing velocity limits. At any step k, the braking time $T_{\mathrm{BR}}(k)$ needed for the train to stop based on the current velocity $x_2(k)$ can be computed through the following equation with input $u_{s_k} = -1$, i.e.,

$$0 = x_2(k) + T_{\mathrm{BR}} \left(\frac{-F_{\mathrm{B}}(x(k), u(k)) - F_{\mathrm{R}}(x(k))}{M_{\mathrm{tot}}} \right) . \tag{11}$$

Let T_{rBR} be the remaining time to be covered by the train to reach the arrival station (see Figure 3 as an illustrative example), then one can compute

$$\Delta T_{\mathrm{BR}} = T_{\mathrm{rBR}} - T_{\mathrm{BR}} . \tag{12}$$

If ΔT_{BR} is greater than zero, then Algorithm 1 is used, otherwise braking mode is forced posing $s_k = 1$, that is $u_1 = -1$.

5. Dissension based regenerative braking

The aim of the proposed approach is also to make the trains of the network cooperate in order to exploit the regenerative braking energy when at least one of the trains is in braking operation mode. In this case the other trains are allowed to accelerate if they need, while fulfilling all the constraints. The main idea underlying this work is to induce this behavior through the tuning of the weights $\gamma^{[r]}$ of the NMPC cost function. More specifically, in order to tune these parameters, we introduce a dissension strategy so as to induce the trains which are not braking to accelerate. The proposed control scheme is reported in Figure 1.

5.1. Dissension based adaptive law

Consider that each train $r \in \mathcal{N}$ of the substation can assume two possible states: state 1 indicates the braking (BR) operation mode; state 2 indicates all the other possible modes (AC, CR, CO). Specifically, hereafter we assume that the state of the network is detected by a supervisor or each train r has full knowledge of the actual state of the other ones. Moreover, the communication graph is symmetric and complete, i.e., $\mathcal{N}^{[r]}_{\text{tra}} = \mathcal{N}^{[r]}_{\text{rec}} = \mathcal{N}$ and $\mathcal{N}^{[r]} = \mathcal{N} \setminus r$, respectively.

Having in mind a dissension behavior of the trains in the network, since the real train dynamics is continuous, we define the following Dissension based Adaptive Law (DAL) in order to determine the weights $\gamma^{[r]}$, i.e.,

$$\dot{\pi}^{[r]}(t) = \left(Q^{[r]\prime} + A^{[r]\prime}\right)\pi^{[r]}(t) \tag{DAL}$$

where $\pi^{[r]}(t) = \left[\gamma^{[r]}(t)\,(1-\gamma^{[r]}(t))\right]'$, $Q^{[r]}$ is a Metzler matrix satisfying $\mathbb{1}'Q^{[r]} = 0$, while the matrix $A^{[r]}$ represents the interaction with neighbors with entries

$$a_{ij}^{[r]}(t) = \frac{\lambda_i}{N-1}\sum_{\kappa\in\mathcal{N}\setminus r}\mathcal{I}^{[\kappa]}(t) \tag{13}$$

$$\mathcal{I}^{[\kappa]}(t) = \begin{cases} 1 & i=1, \quad \text{(i.e., BR)} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

with $\lambda_i > 0$ being the influence strength intensity, while $\mathcal{I}^{[\kappa]}$ being an indicator function equal to one when the κth train is braking, zero otherwise. Furthermore, by defining

$$a_{ii}^{[r]}(t) = -\sum_{j=1,\, j\neq i}^{S} a_{ij}^{[r]}(t) \tag{15}$$

with $S = 2$ equal to the number of states, also the matrix $A^{[r]}$ is Metzler. This model describes the desired dissension behavior, namely the transition rate to state 2 (AC, CR, CO) undergo an increase which is proportional to the number of neighbors that are in state 1 (i.e., they are braking). Note that matrices $Q^{[r]}$ may be different each other according to the train characteristics imposed by the authority. Furthermore, we assume that $\lambda_2 = 0$, that is trains cannot be induced to brake if the other ones are accelerating. Moreover, simultaneous state jumps are not allowed. Finally, the discrete value of $\gamma^{[r]}(t)$ is given by

$$\gamma_k^{[r]} = \gamma^{[r]}(t), \quad \forall t \in [kT_{\mathrm{s}},\, (k+1)T_{\mathrm{s}}] \ . \tag{16}$$

Remark 2. *Note that, in principle the case $\lambda_2 > 0$ could be possible. In fact, if for some reason, the authority decided to force some delayed trains to accelerate, it could exploit also the regenerative energy of trains which are on time and accelerating, making them brake. Hence, with $\lambda_2 > 0$ the transition rate to state 1 (BR) undergo an* *increase which is proportional to the number of neighbors that are in state 2 (i.e., AC, CR, CO) becomes possible.*

5.2. Interpretation of trains as stochastic agents

The proposed adaptive strategy (DAL) resembles a consensus algorithm characterized by time-homogeneous Markov stochastic processes (see [30]). More specifically, a train r could be considered as a Markov stochastic process where the matrix $Q^{[r]} \in \mathbb{R}^{2\times 2}$ describes the behavior of the train when is not connected to the regenerative line, such that $\bar{\pi}^{[r]\prime} Q^{[r]} = 0$, with $\bar{\pi}^{[r]} = [\bar{\gamma}^{[r]} \;\; (1-\bar{\gamma}^{[r]})]'$ being the unique unit-sum Frobenius left eigenvector of $Q^{[r]}$ associated with the Perron-Frobenius null eigenvalue (see Chapter 2 of [31]). Hence, the entries $q_{ij}^{[r]}$ represent the transition rates between S^N states $\sigma^{[r]}(t)$, with $\sigma^{[r]}(t) \in \mathcal{M}$, while the weights $\gamma^{[r]}$ are the probabilities of being in state 1 (i.e., in braking mode) at time t, i.e.,

$$\gamma^{[r]} = \mathbb{P}\left\{\sigma^{[r]}(t) = 1\right\} \, . \tag{17}$$

Analogously, the probability distribution would obey the differential equation (DAL) with entries of the Metzler matrix $A^{[r]}$ such that

$$a_{ij}^{[r]}(t) = \frac{\lambda_i}{N-1} \sum_{k\in\mathcal{N}\setminus r} \mathcal{I}_{\sigma^{[\kappa]}(t)} \tag{18}$$

$$\mathcal{I}_{\sigma^{[\kappa]}(t)} = \begin{cases} 1 & \sigma^{[\kappa]}(t) = 1 \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

with $\lambda_1 > 0$ and $\lambda_2 = 0$. In this case, the model describes a dissension behavior, namely the transition rate to state j undergo an increase which is proportional to the number of neighbors that share opinion i.

Now, let $\Sigma(t) \in \mathcal{M}^N$, $\mathcal{M} = \{1, 2\}$, be the state of the network and $\pi(t) = \otimes_{r=1}^{N} \pi^{[r]}(t)$ denote the probability distribution of $\Sigma(t)$. The entries of $\pi(t)$ represent the condition at time t of a given configuration of the train states in the substation. The dynamics of $\pi(t)$ in absence of interaction due to the network obeys the differential equation

$$\dot{\pi}(t) = Q_0' \pi(t) \tag{20}$$

with

$$Q_0 = \sum_{r=1}^{N} I_{M^{r-1}} \otimes Q^{[r]} \otimes I_{M^{N-r}} \ . \tag{21}$$

Given the interaction model (DAL), the whole dynamics is given by

$$\dot{\pi}(t) = (Q_0' + A_0')\pi(t) \ , \tag{22}$$

where A_0 is the matrix which takes into account the substation topology and the interaction among vehicles.

Example 1. *As an example, consider a substation with three trains ($N = 3$) and let us compute the matrix A_0, given by*

$$\left[\begin{array}{cccc|cccc}
-3\lambda_1 & \lambda_1 & \lambda_1 & 0 & \lambda_1 & 0 & 0 & 0 \\
0 & -\lambda_1 & 0 & \frac{\lambda_1}{2} & 0 & \frac{\lambda_1}{2} & 0 & 0 \\
0 & 0 & -\lambda_1 & \frac{\lambda_1}{2} & 0 & 0 & \frac{\lambda_1}{2} & 0 \\
0 & 0 & 0 & -\lambda_1 & 0 & 0 & 0 & \lambda_1 \\
\hline
0 & 0 & 0 & 0 & -\lambda_1 & \frac{\lambda_1}{2} & \frac{\lambda_1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & -\lambda_1 & \lambda_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\lambda_1 & \lambda_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] .$$

Note that, for instance the first row represents the transition starting from $\Sigma(t) = [1\,1\,1]'$, that is when all the trains are in braking and only one train at time is allowed to accelerate if the other ones are braking. Conversely, the last row is zero because it corresponds to transitions starting from $\Sigma(t) = [2\,2\,2]'$, that is all the trains are in acceleration, cruising or coasting mode. Analogously, the bottom left 4×4 block is also zero.

6. Assessment of the collaborative ecodrive NMPC

In this section, simulation results on a realistic scenario are presented in order to assess the proposed collaborative NMPC.

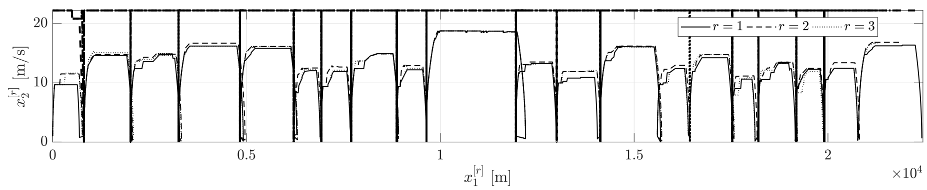

Figure 4: Evolution of the velocity profiles $x_2^{[r]}$ with respect to space $x_1^{[r]}$, $r = 1,\ 2,\ 3$ over all the track and velocity limits

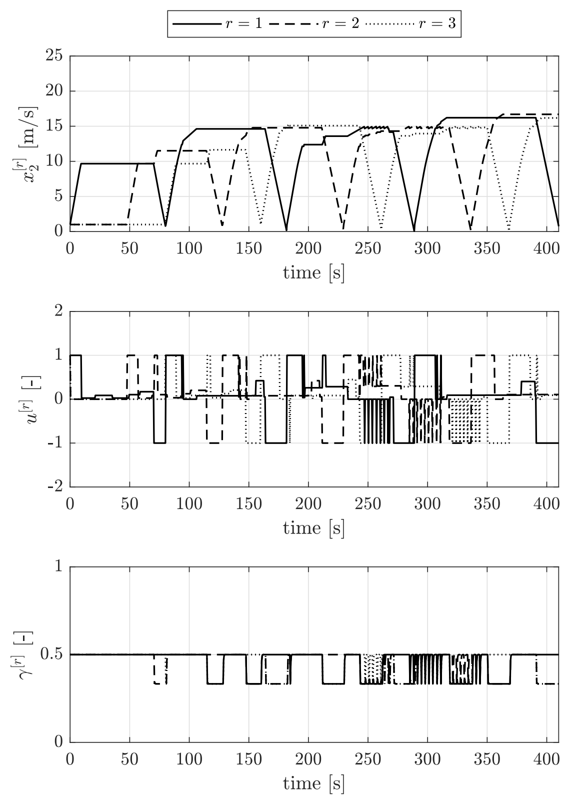

Figure 5: Time evolution of the states, inputs and cost weights for each train over the first four stops. From the top: velocity profiles $x_2^{[r]}$, handle $u^{[r]}$ and cost weights $\gamma^{[r]}$, $r = 1,\ 2,\ 3$

6.1. Description of the scenario

Simulations have been carried out by considering three trains ($N = 3$), with the total track having 20 stops. The route has a total length of $22.4\,\mathrm{km}$. The parameters of the track, which are curve radius, slope and velocity limit, have been provided by the company Alstom on the basis of real data and these are shown in Figure A.7 in the appendix. All the trains have identical parameters, provided by the company Alstom and reported in Table B.3 and Figure A.6, where the maximum traction and braking forces are illustrated. They refer to Italian trains like that depicted in Figure 2. Furthermore, in the considered scenario the consecutive trains run shifted in time by $48\,\mathrm{s}$ and $80\,\mathrm{s}$, respectively with respect to the first one. The sampling time is $T_\mathrm{s} = 0.4\,\mathrm{s}$ and the initial weights for each train are chosen as $\gamma_0^{[r]} = \bar{\gamma}^{[r]}$ with $\bar{\gamma}^{[r]} = 0.5$ being the first entry of the Frobenius left eigenvector associated to $Q^{[r]}$ ($r = 1, 2, 3$), with $q_{ij} = 1$.

6.2. Performance of the proposed collaborative NMPC

The performance of the proposed NMPC are now discussed. The prediction horizon has been set $N_\mathrm{p} = 3$ and the DAL is activated in order to make the system behave in a collaborative way. Figure 4 shows the evolution of the train speed over all the track. It can be observed that the velocity limits are always fulfilled and sometime trains behave in different way depending on the weights $\gamma^{[r]}$ used in the cost function. More specifically, the effect of the DAL can be better appreciated in Figure 5, where for the sake of clarity only the first four stops ($5\,\mathrm{km}$) are illustrated. Figure 5, in fact, reports the time evolution of the velocity $x_2^{[r]}$ for all the trains, the corresponding inputs $u^{[r]}$ and weights $\gamma^{[r]}$, $r = 1,\ 2,\ 3$. As expected, after $48\,\mathrm{s}$ the second train (dashed line) starts to run and analogously the third train (dotted line) after $80\,\mathrm{s}$. While initially all the profiles are identical, the effect of DAL is visible at $70.8\,\mathrm{s}$ when the second train is induced to accelerate by virtue of the braking energy provided by the first train (solid line). The same situation occurs at the time instant $115.2\,\mathrm{s}$ when the second train starts to brake and the third one accelerates. At the time $214\,\mathrm{s}$ the first train instead accelerates thanks to the braking energy provided by the second one. In correspondence of these events the values of $\gamma^{[r]}$ of the braking trains decrease from 0.5 to 0.3. Note also that $\gamma^{[r]}$ is reset equal to 0.5 after each stop.

Table 1: Performances when using the proposed NMPC with (w/) and without (w/o) DAL

r	DAL	J [-]	E_{T} [kW h]	E_{B} [kW h]
1	w/	50572	545.46	401.54
	w/o	50610	511.29	372.22
2	w/	37455	581.82	415.13
	w/o	44886	503.58	349.26
3	w/	43126	557.93	397.42
	w/o	47971	500.09	349.26

Numerical results for the performances in terms of value of the cost function (J), traction energy (E_{T}) and braking energy (E_{B}), both expressed in $\mathrm{kW\,h}$, are reported in Table 1. Notice that the computed value of traction energy E_{T} includes the energy from the main grid and the one regenerated by the other trains in the network. As a consequence of the DAL mechanism, the traction energy of all the trains is higher with respect to the case without DAL, thus implying shorter arrival time. Therefore, it can be observed that the values of the cost function, which include both the energy term and the one related to arrival time, are reduced for all the trains when the proposed NMPC with DAL is adopted. As a further consequence of the regenerated energy shared among the trains, higher braking energy is required. Then, by virtue of this strategy a better grid utilization is achieved.

Finally, the NMPC strategy standalone has been successuflly tested on the Alstom simulator CITHEL, featuring coupled mechanical, electric and thermal calculations of the full train with its electro-mechanical back-end (including detailed models of the drives and electric motors), able to provide accurate time and energy consumption predictions. NMPC was compared with the “all-out” solution (i.e., the one giving the shortest arrival time compatible with the system parameters and speed constraints) and with Alstom current ecodrive strategy. The results have shown that the proposal is comparable with the current Alstom strategy in terms of energy, thus being in principle eligible to field implementation [32]. Moreover, the merit of the proposal is to provide a valid real-time solution for the ecodriving task as alternative to the computationally onerous current ecodrive approach.

7. Conclusion

This work considered the problem of energy efficient train operation with ecodrive in a collaborative way. To address this problem, an optimal control solution to predict the velocity profile of a train by using switching NMPC algorithm in a collaborative fashion was proposed. For the purpose, a DAL was introduced to manage all the trains governed by the same substation and tune the NMPC law in order to use the braking energy, while taking into account constraints on velocity and journey times. Results show that the proposed NMPC is able to minimize the traction energy, which depends on the input handle and on the characteristics of the track while fulfilling all the constraints. Moreover, the DAL allows trains to effectively use the regenerated energy which is available in the substation when regenerative braking occurs.

Appendix A. NMPC settings

The sequences of modes belonging to the set $\mathcal{W}$ and selected to improve the performance of the proposed NMPC, as described in Section 4.2, are hereafter reported in Table A.2.

Appendix B. Scenario settings

The parameters of the single train, maximum traction and braking forces profile and track features (velocity limits, slope and curve radius) are reported in Table B.3, Figure A.6 and Figure A.7, respectively.

Acknowledgment

The authors gratefully acknowledge the contribution of the rail transport company Alstom for providing models and track data employed in the simulation case study.

Table A.2: Selected sequences belonging to the set $\mathcal{W}$

	$\mathbf{s}_{[k,k+N_\mathrm{p}-1\|k]}$			
	$p\|k$			
#	k	$k+1$	$\dots$	$k+N_\mathrm{p}-1$
1	-1	-1	-1	-1
2	0	0	0	0
3	u_CR	u_CR	u_CR	u_CR
4	1	1	1	1
5	0.5	0.5	0.5	0.5
6	0.75	0.75	0.75	0.75
7	u_CR	u_CR	0	0
8	u_CR	u_CR	-1	-1
9	u_CR	u_CR	0.5	0.5
10	u_CR	0.5	0.5	0.5
11	u_CR	-1	0	0
12	0	-1	-1	-1
13	-1	-1	0	0
14	-1	0	0	0
15	-1	0.5	0.5	0.5

Table B.3: Parameters of the train

M_tot	$267\,464\,\mathrm{kg}$
M_s	$255\,200\,\mathrm{kg}$
A	$3597.6\,\mathrm{N}$
B	$119.5\,\mathrm{N\,s\,m^{-2}}$
C	$6.97\,\mathrm{N\,s\,m^{-2}}$

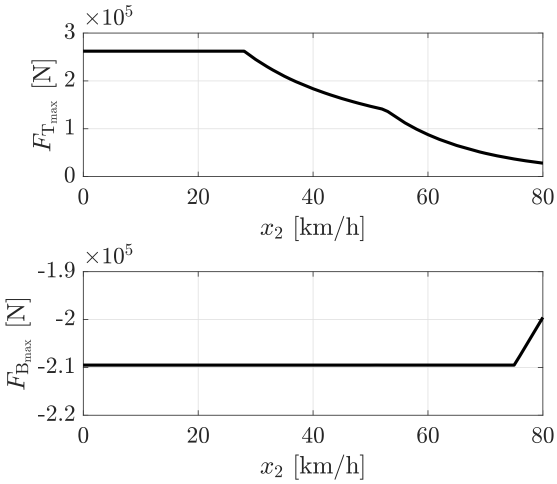

Figure A.6: Maximum traction and braking forces as functions of velocity

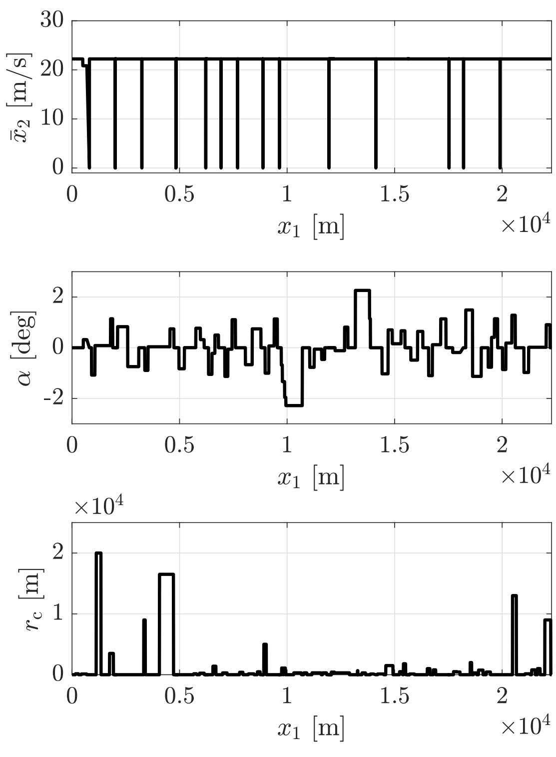

Figure A.7: Model of the track given by speed limit, slope and curve radius

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