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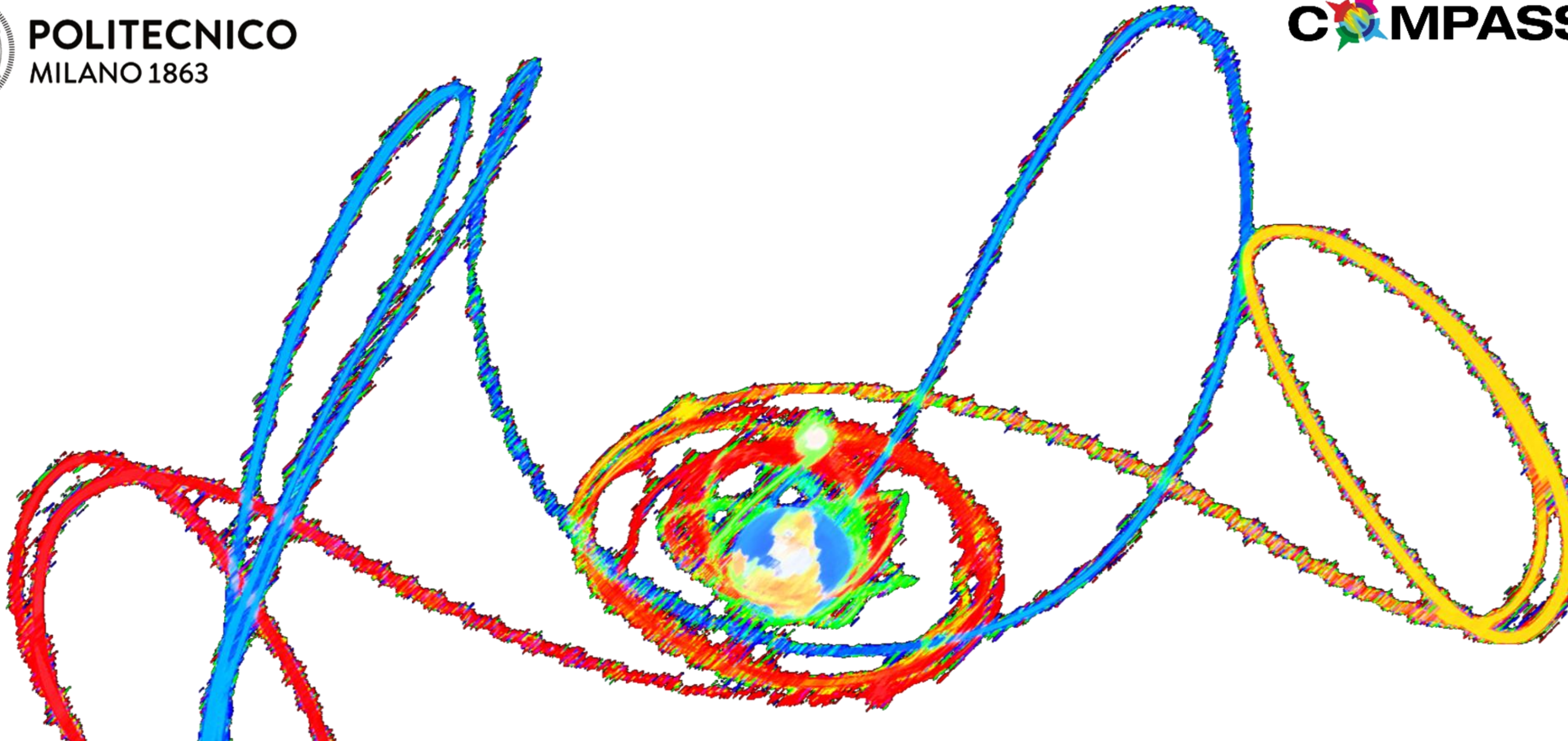

Drag- and SRP-induced effects in uncertainty evolution for close approaches

Juan Luis Gonzalo, Camilla Colombo and Pierluigi Di Lizia

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Introduction

The growing use of space-based assets is increasing technical and organizational requirements for Space Situational Awareness and collision avoidance activities

- Policy aspects (e.g. new guidelines from the IADC)
- Improvement/update of SSA capabilities (e.g. the USAF Space Fence system developed by Lockheed Martin)
- Increasing need of software tools (orbit determination, catalogue maintenance/update, close approach prediction, collision avoidance manoeuvre design, etc).

The COMPASS team is developing a new tool for close approach analysis and CAM design: MISS (Manoeuvre Intelligence for Space Safety) [1]

- Based on analytic and semi-analytic methods
- **In this talk, we will focus on the effect of drag and SRP in uncertainty evolution**

[1] J.L. Gonzalo, C. Colombo, and P. Di Lizia, "Analysis And Design Of Collision Avoidance Maneuvers For Passive De-Orbiting Missions," 2018 AAS/AIAA Astrodynamics Specialist Conference, Snowbird (UT), 19 -23 Aug 2018. No AAS 18-357

Earth picture: credit ESA

Outline

- Modelling of impulsive CAM
  - Analytic STM
- Effect of drag and SRP in uncertainty evolution
  - Numerical tests
- Semi-analytic STM including drag and SRP
  - Ongoing work
- Conclusion

Earth picture: credit ESA

Modelling of impulsive CAM

Displacement in the b-plane

Given a close approach (CA) between a manoeuvrable spacecraft and a debris

CAM given at lead time Δt before the CA, modelled through **Gauss planetary equations** [1]

$$\delta\boldsymbol{\alpha}(t_{\mathrm{CAM}}) = \mathbf{G}_v(t_{\mathrm{CAM}};\boldsymbol{\alpha})\,\delta\mathbf{v}(t_{\mathrm{CAM}})$$

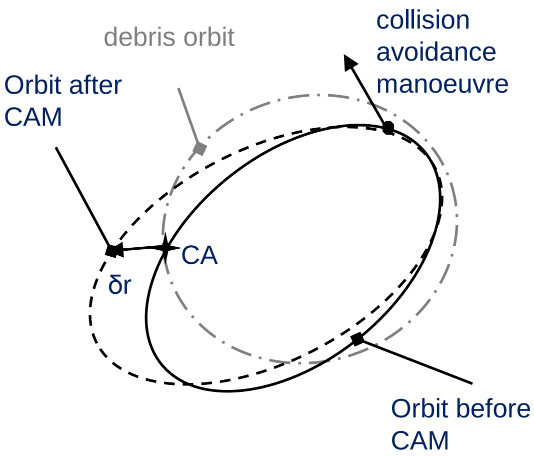
Analytic computation of miss distance at CA using **linearized relative motion** [2]

$$\delta\mathbf{r}(t_{\mathrm{CA}}) = \mathbf{A}_r(t_{\mathrm{CA}};\boldsymbol{\alpha},\Delta t)\,\delta\boldsymbol{\alpha}(t_{\mathrm{CAM}})$$

Displacement (total and projected on b-plane [3])

$$\delta\mathbf{r}(t_{\mathrm{CA}}) = \mathbf{A}_r\mathbf{G}_v\delta\boldsymbol{v}(t_{\mathrm{CAM}}) = \mathbf{T}\,\delta\mathbf{v}(t_{\mathrm{CAM}})$$

$$\delta\mathbf{b}(t_{\mathrm{CA}}) = \mathbf{M}(t_{\mathrm{CA}})\delta\mathbf{r}(t_{\mathrm{CAM}}) = \mathbf{Z}\,\delta\mathbf{v}(t_{\mathrm{CAM}})$$



[1] R. Battin, *An Introduction to the Mathematics and Methods of Astrodynamics*, 1999
[2] J. L. Junkins and H. Schaub, *Analytical mechanics of space systems*, 2009
[3] M. Petit, “Optimal Deflection Of Resonant Near-Earth Objects Using The b-Plane”, Master thesis, 2018

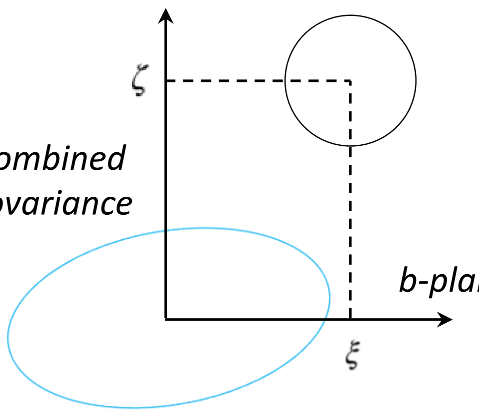
Modelling of impulsive CAM

Maximum deviation and minimum collision probability CAM

- **Maximum deviation CAM** reduces to an eigenvalue/eigenvector problem (Conway [1]):

$$\max \|\delta \mathbf{b}(t_{CA})\| \quad \Leftrightarrow \quad \delta \mathbf{v} \parallel \text{eigenvector associated to largest eigenvalue of } \mathbf{Z}^{T}\mathbf{Z}$$

- **Minimum collision probability CAM** can also be reduced to an eigenvalue/eigenvector problem (Bombardelli and Hernando-Ayuso [2])
- Collision probability computed using Chan's method
  - Combined circular envelope for s/c (no uncertainty)
  - **Combined covariance for debris** (point)



In parametric analysis for different lead times, should we consider fixed covariances or propagate them?

[1] B. A. Conway, "Near-optimal deflection of earth-approaching asteroids," *JGCD,* 24(5):1035-1037, 2001
[2] C. Bombardelli, and J. Hernando-Ayuso, "Optimal impulsive collision avoidance in low earth orbit", JGCD, 38(2):217-225, 2015

State Transition Matrix

Propagation of covariance matrix

Extending the model, the full **analytic State Transition Matrix** (STM) from $\delta\mathbf{s} = (\delta\mathbf{r}, \delta\mathbf{v})$ at t_{CAM} to $\delta\mathbf{s} = (\delta\mathbf{r}, \delta\mathbf{v})$ at t_{CA} is developed:

- Additional matrices $\mathbf{G}_{r}$ and $\mathbf{A}_{v}$
- Optimizing the miss distance only required a quarter of this matrix
- The **covariance matrix can be propagated**
- **Validated against Monte-Carlo simulations with nonlinear dynamics**
- **Drag and SRP not taken into account**

$$\delta\boldsymbol{\alpha}(t_{\mathrm{CAM}}) = \begin{bmatrix} \mathbf{G}_{\mathrm{r}}(t_{\mathrm{CAM}}, \boldsymbol{\alpha}) \\ \mathbf{G}_{\mathrm{v}}(t_{\mathrm{CAM}}, \boldsymbol{\alpha}) \end{bmatrix}^{\mathrm{T}} \delta\mathbf{s}(t_{\mathrm{CAM}})$$

$$\delta\mathbf{s}(t_{\mathrm{CA}}) = \begin{bmatrix} \mathbf{A}_{r}(t_{\mathrm{CA}}; \boldsymbol{\alpha}, \Delta t) \\ \mathbf{A}_{v}(t_{\mathrm{CA}}; \boldsymbol{\alpha}, \Delta t) \end{bmatrix} \delta\boldsymbol{\alpha}(t_{\mathrm{CAM}})$$

$$\delta\mathbf{s}(t_{\mathrm{CA}}) = \begin{bmatrix} \mathbf{A}_{\mathrm{r}}\mathbf{G}_{\mathrm{r}} & \mathbf{A}_{\mathrm{r}}\mathbf{G}_{\mathrm{v}} \\ \mathbf{A}_{\mathrm{v}}\mathbf{G}_{\mathrm{r}} & \mathbf{A}_{\mathrm{v}}\mathbf{G}_{\mathrm{v}} \end{bmatrix} \delta\mathbf{s}(t_{\mathrm{CAM}})$$

Effect of drag and SRP

Numerical evaluation

The **effect of drag and SRP in the combined covariance** is evaluated for a large set of test cases:

- Lead times Δt up to five orbital periods of the s/c
  - Initial covariance matrices of s/c and debris are known at t_{CAM}
- Spacecraft not affected by drag and SRP
- Debris affected by drag and SRP
  - Several area-to-mass ratios, and uncertainty levels for c_D and c_R

The combined covariance for each case is **obtained numerically**

- **Orbit propagation with PlanODyn** [1] (single-averaged, semi-analytical propagator)
- **10^5 Monte Carlo runs per case**

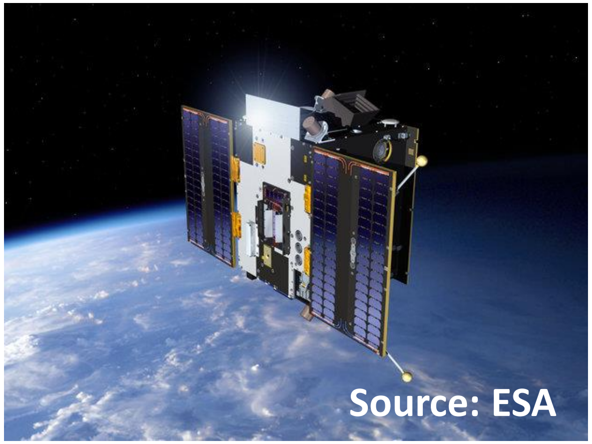
[1] C. Colombo, "Planetary Orbital Dynamics (PlanODyn) suite for long term propagation in perturbed environment," 6th ICATT, Darmstadt, Germany, 14-17 March 2016.

Effect of drag and SRP

Test case: Nominal close approach

A **nominal close approach** is defined as [1]:

- Spacecraft is PROBA-2 [2]
- Virtual debris constructed using statistical data from ESA's MASTER-2009
- Direct impact



	a [km]	e [-]	i [deg]	Ω [deg]	ω [deg]	θ_0 [deg]
PROBA-2	7093.637	0.0014624	98.2443	303.5949	109.4990	179.4986
Debris	7782.193	0.08716212	88.6896	142.7269	248.1679	1.2233

Debris has $c_D = 2.1$, $c_R = 1.8$, and several A/m from 1 m^2/kg up to 20 m^2/kg

[1] J.L. Gonzalo, C. Colombo, and P. Di Lizia, "Analysis And Design Of Collision Avoidance Maneuvers For Passive De-Orbiting Missions," 2018 AAS/AIAA Astrodynamics Specialist Conference, Snowbird (UT), 19 -23 Aug 2018. No AAS 18-357
[2] http://www.heavens-above.com/

Effect of drag and SRP

Test case: Initial covariance

Sample covariance matrix numerically constructed from TLEs for an Iridium 33 debris (NORAD ID 33874)

- OD based on state vectors from SGP4 and a high-fidelity propagator [1]

x [km]	y [km]	z [km]	vx [km/s]	vy [km/s]	vz [km/s]
1.1554603E-02	-2.3144336E-03	-1.1731962E-03	+4.5252955E-07	-5.6795909E-07	-1.0945466E-05
-2.3144336E-03	+1.9146944E-02	+1.4167202E-02	-1.2286501E-05	-2.5535536E-06	-3.3049394E-06
-1.1731962E-03	+1.4167202E-02	+3.0870284E-01	-2.8750137E-04	-8.6187779E-05	-1.2493173E-06
+4.5252955E-07	-1.2286501E-05	-2.8750137E-04	+2.8850680E-07	+7.9940433E-08	+1.1511416E-09
-5.6795909E-07	-2.5535536E-06	-8.6187779E-05	+7.9940433E-08	+4.5996583E-08	+1.4570093E-09
-1.0945466E-05	-3.3049394E-06	-1.2493173E-06	+1.1511416E-09	+1.4570093E-09	+1.2022009E-08

[1] A. Morselli, R. Armellin, P. Di Lizia, and F. Bernelli Zazzera, "A High Order Method for Orbital Conjunctions Analysis: Sensitivity to Initial Uncertainties", Advances in Space Research, 53(3):490-508, 2014. doi: 10.1016/j.asr.2013.11.038

Effect of drag and SRP

Test case: Initial covariance

Sample covariance matrix numerically constructed from TLEs for an Iridium 33 debris (NORAD ID 33874)

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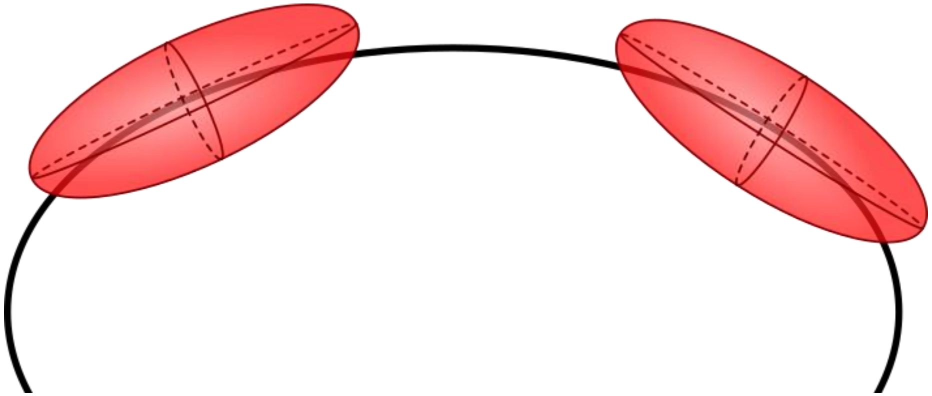
This sample covariance is linked to a particular orbit position (true anomaly)

- For other true anomalies, a translation process is applied keeping the eigenvalues but updating the orientation of the eigenvectors.

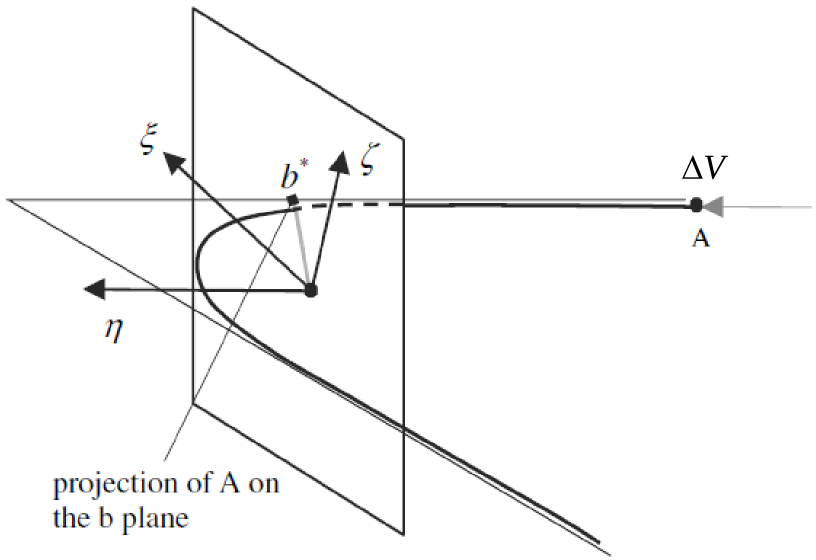


[1] A. Morselli, R. Armellin, P. Di Lizia, and F. Bernelli Zazzera, "A High Order Method for Orbital Conjunctions Analysis: Sensitivity to Initial Uncertainties", Advances in Space Research, 53(3):490-508, 2014. doi: 10.1016/j.asr.2013.11.038

Effect of drag and SRP

B-plane projection

B-plane: Plane **orthogonal** to the s/c **relative velocity** at conjunction



η-axis: parallel to the relative velocity

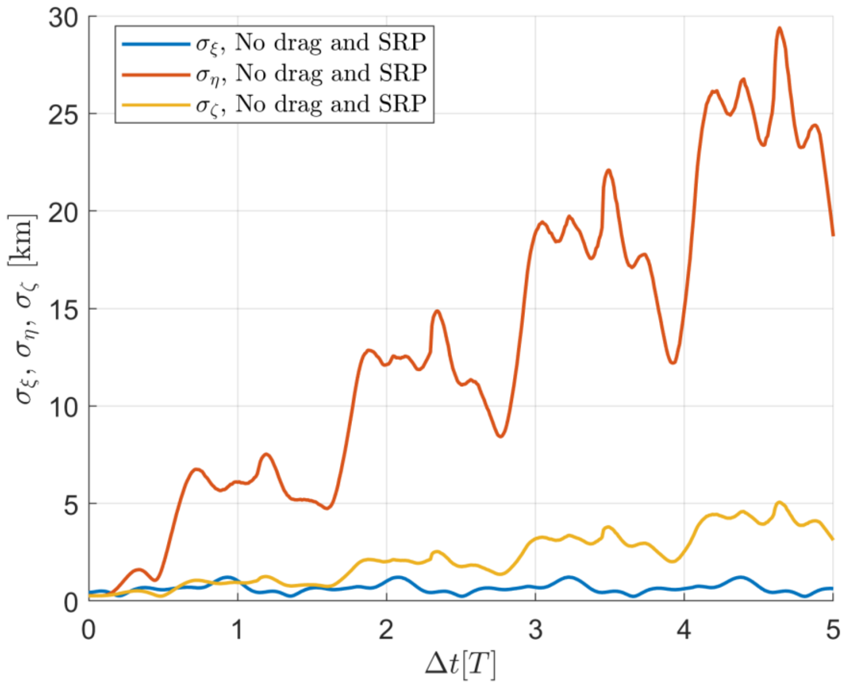
ζ-axis: parallel and opposite to the projection on the b-plane of the debris velocity

ξ-axis: to complete a positively oriented reference system

Effect of drag and SRP

B-plane projection

B-plane: Plane **orthogonal** to the s/c **relative velocity** at conjunction



The covariance experiments its largest variation along the η axis (normal to the b-plane).

Inside the b-plane, the covariance tends to grow along the time axis ζ.

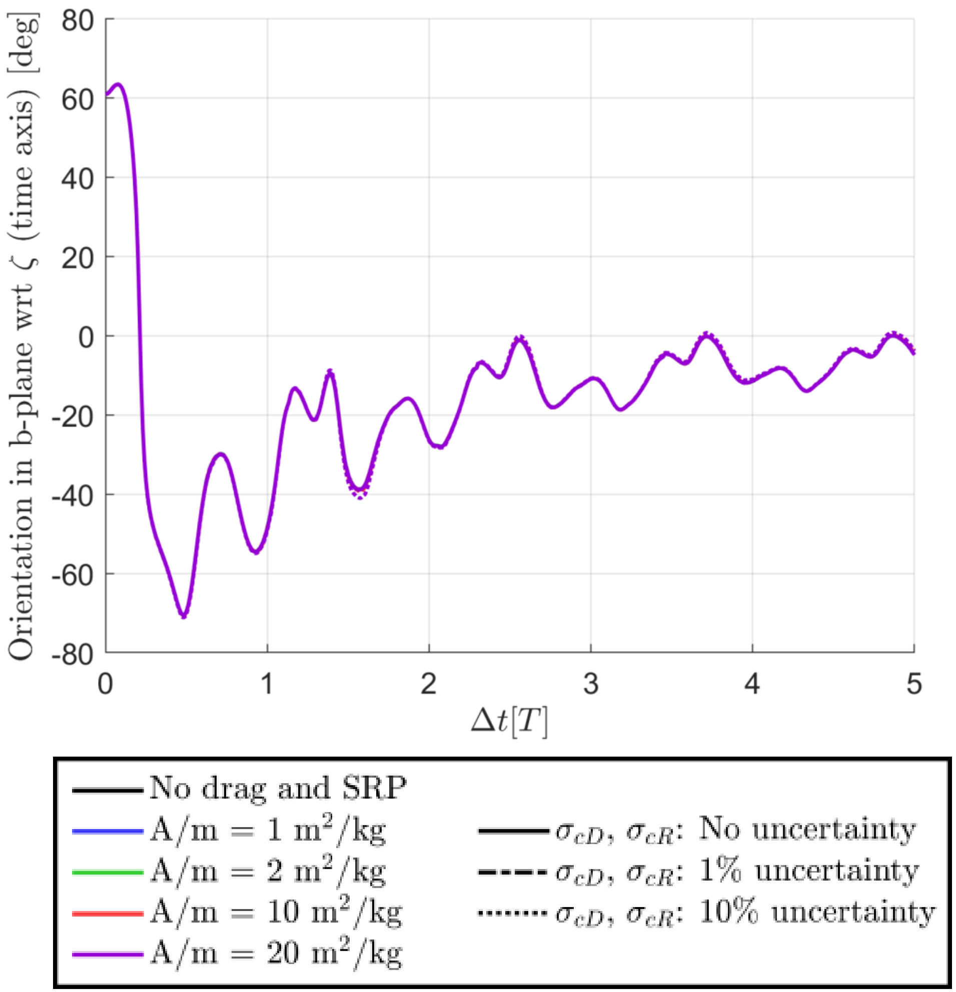
η-axis: parallel to the relative velocity

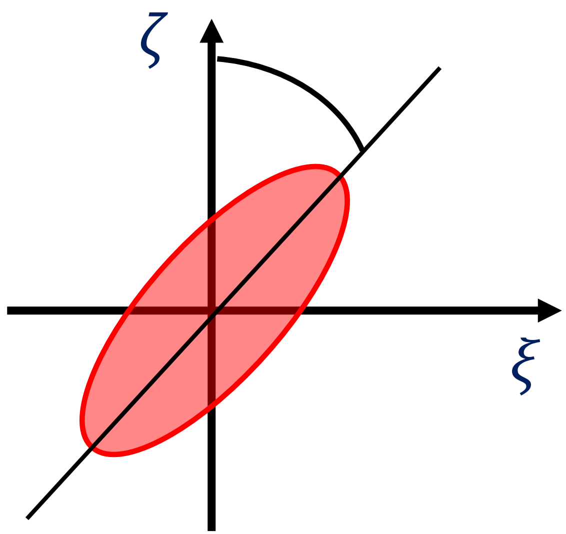
ζ-axis: parallel and opposite to the projection on the b-plane of the debris velocity

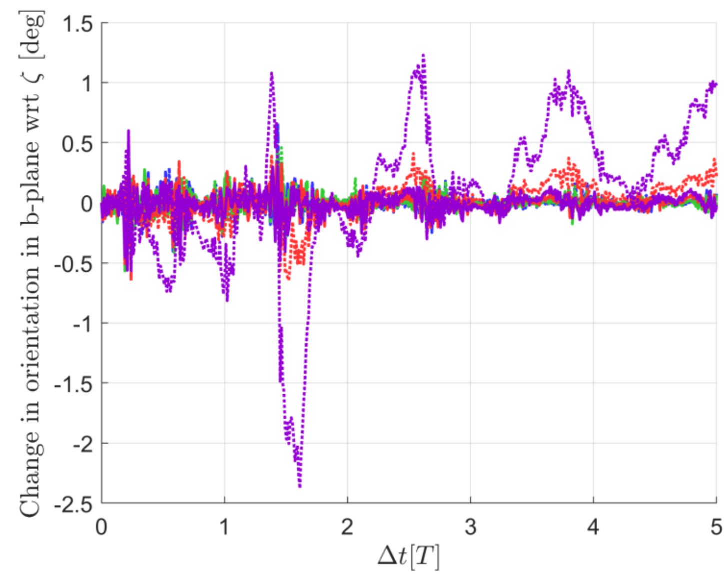
ξ-axis: to complete a positively oriented reference system

Effect of drag and SRP

Orientation in b-plane

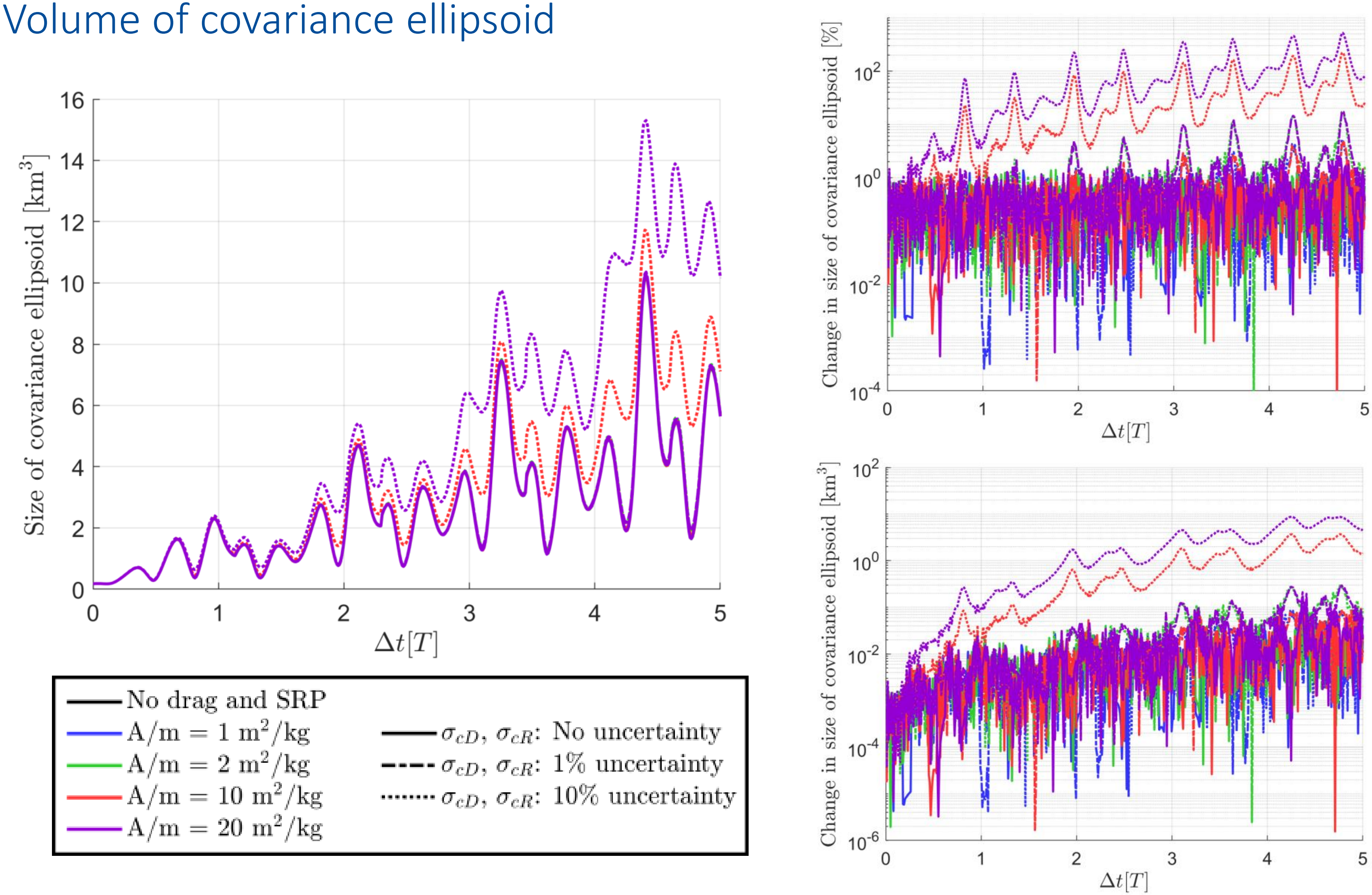






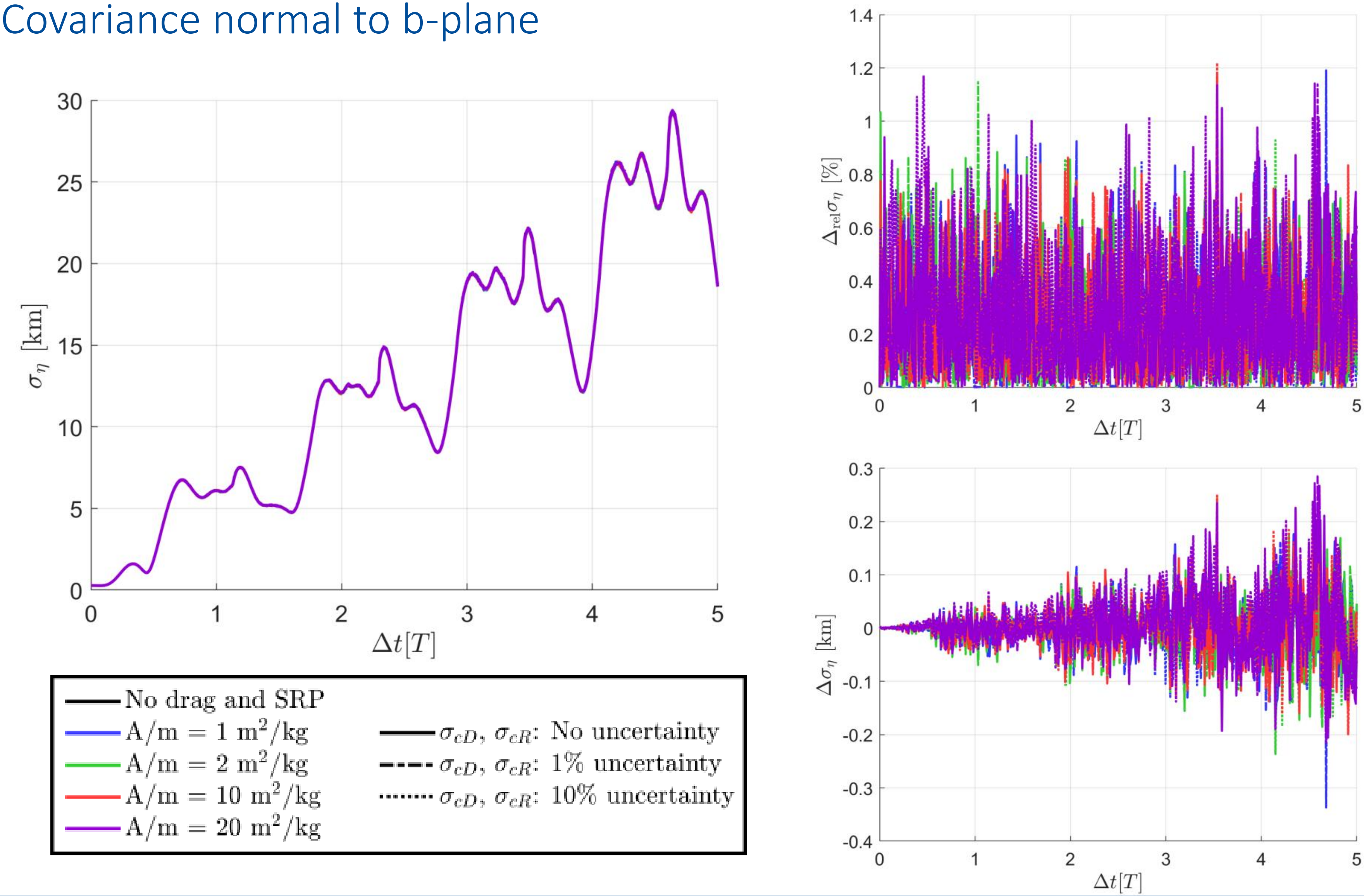
Effect of drag and SRP

Volume of covariance ellipsoid



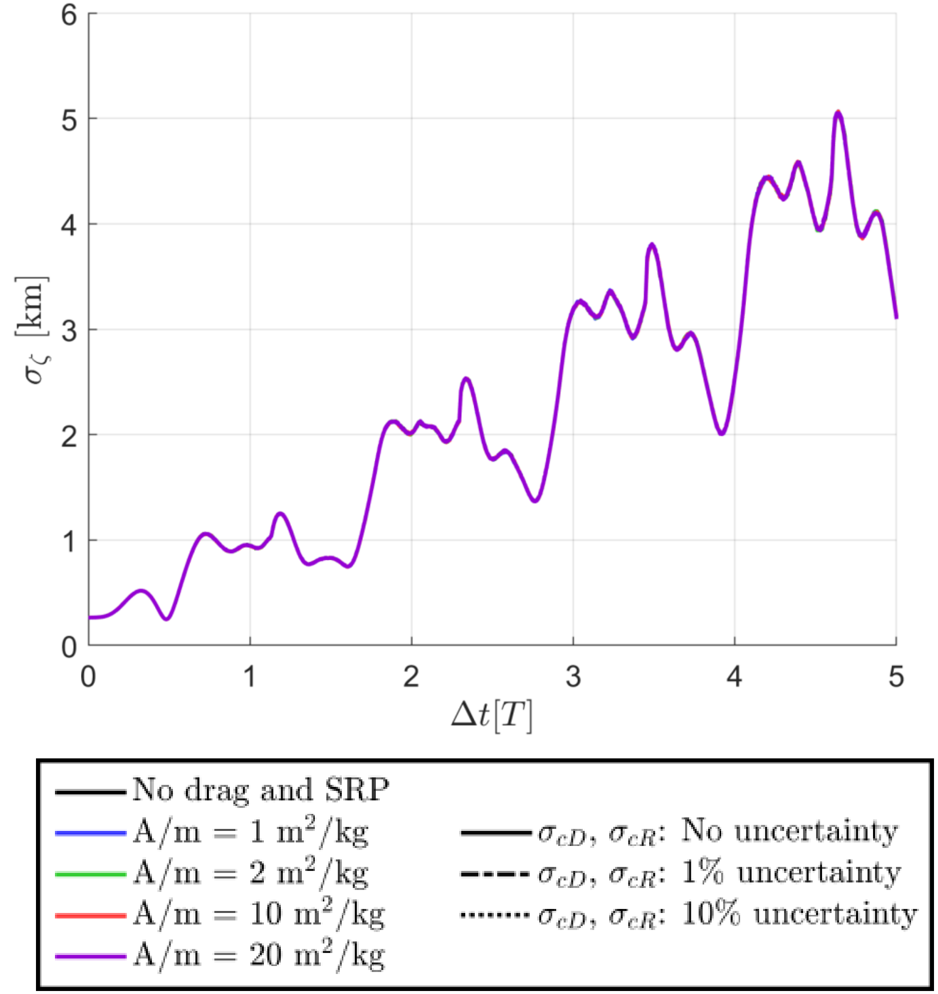
Effect of drag and SRP

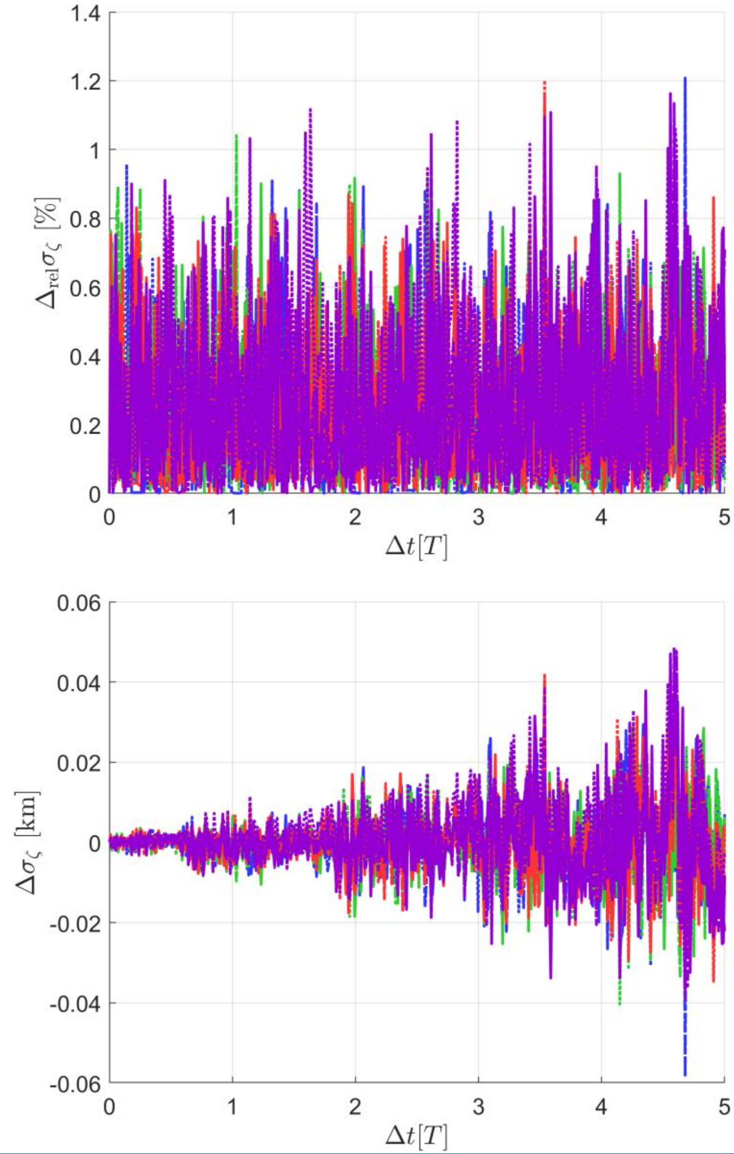
Covariance normal to b-plane



Effect of drag and SRP

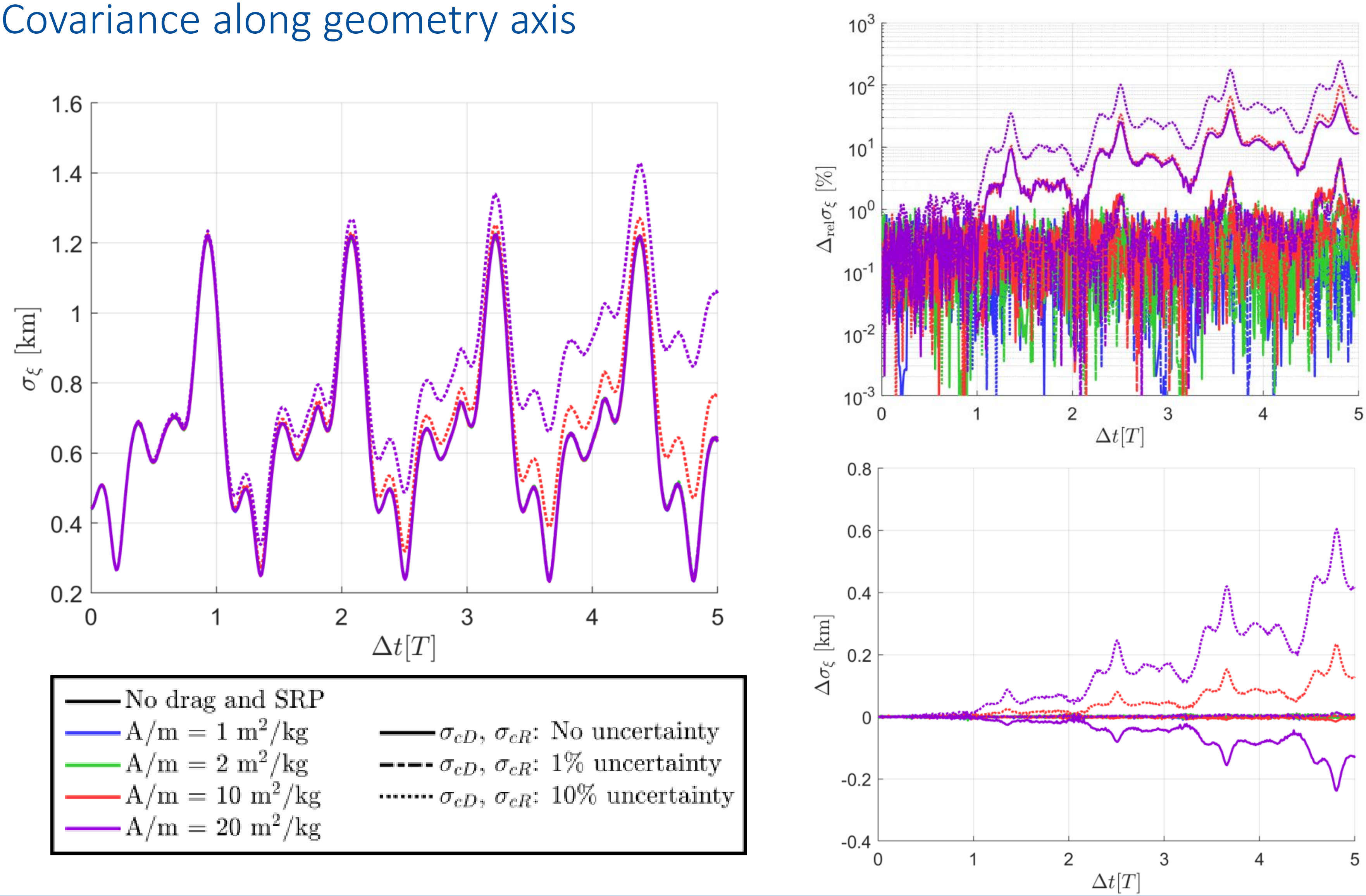
Covariance along time axis





Effect of drag and SRP

Covariance along geometry axis



Including drag and SRP in the STM

Ongoing work

PlanODyn can compute the STM numerically integrating the variational equations (semi-analytical, single-averaged model)

- Models for drag and SRP already available
- Does not take into account uncertainties in the drag and reflectivity coefficients

$$\boldsymbol{\delta\alpha}_f = \begin{bmatrix} x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \\ x & x & x & x & x & x \end{bmatrix} \boldsymbol{\delta\alpha}_0$$

Including drag and SRP in the STM

Ongoing work

PlanODyn can compute the STM numerically integrating the variational equations (semi-analytical, single-averaged model)

- Models for drag and SRP already available
- **Extend the state including c_D and c_R (implementation in process)**

$$\begin{bmatrix} \boldsymbol{\delta\alpha} \\ \delta c_D \\ \delta c_R \end{bmatrix}_f = \begin{bmatrix} x & x & x & x & x & x & 0 & 0 \\ x & x & x & x & x & x & 0 & 0 \\ x & x & x & x & x & x & 0 & 0 \\ x & x & x & x & x & x & 0 & 0 \\ x & x & x & x & x & x & 0 & 0 \\ x & x & x & x & x & x & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\delta\alpha} \\ \delta c_D \\ \delta c_R \end{bmatrix}_0$$

Conclusion

- **Drag and SRP have a limited effect on uncertainty evolution for close approaches**
  - Only relevant for (very) large A/m with significant uncertainties
  - Analytic STM (no drag & SRP) is cost-effective tool for sensitivity analyses

- Covariance evolution has been characterized using the b-plane
  - Drag & SRP affect mostly σ_ξ
  - Principal direction in the b-plane tends to align with ζ. Negligible influence of drag and SRP.

- **Future work**: Semi-analytic STM including uncertainties in drag and SRP
  - Applicable to design of CAMs by sails

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