Optimal Deflection of Near-Earth Objects Using the B-Plane

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Presentation Outline

- Introduction
- B-Plane
  - Definition
  - Effect of resonances
- Deflection of a Near-Earth Object
  - Geometrical deviation formulation
  - Extension to deviation on the b-plane
- Results
  - Optimal deviation technique
  - Asteroid deflection preliminary mission design
- Conclusions

Introduction

Background - Near-Earth Objects (NEOs)

- Celestial bodies
  - Asteroids
  - Comets
- Close to or intersecting the Earth's orbit
  - Atiras: $a < 1$ AU, $r_a < 0.983$ AU
  - Atens: $a < 1$ AU, $r_a > 0.983$ AU
  - Apollos: $a > 1$ AU, $r_p < 1.017$ AU
  - Amors: $a > 1$ AU, 1.017 AU $< r_p < 1.3$ AU
- Over 16˙000 NEOs are present in the Solar System
- Relatively low catastrophic impact probability
  - Catastrophic ($d > 1$ km): 1 over millions of years
  - Severe ($d > 40$ m): 1 every 100 years or less

Image credits: NASA apod 14/11/2007 – Leonid Kulik Expedition

Introduction

Background - Deflection of Near-Earth Objects

- To avoid a possible impact
- Several techniques are possible
- Kinetic impactor
  - Deflect a NEO by hitting it with a spacecraft at high relative speed
  - Most mature technology
- Resonances
  - Possibility of the fly-by to insert the NEO on a return orbit to the Earth

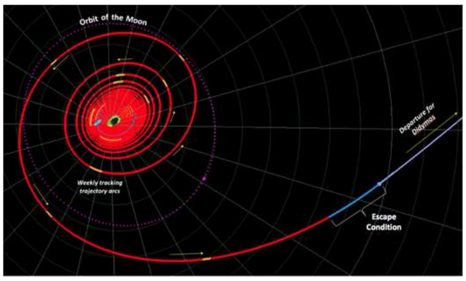

Image credits: NASA Planetary Defense - DART

Introduction

Aims of the Project

- Describe resonant returns by means of the b-plane
- Obtain a convenient analytical formulation correlating the deflection to the deviation on the b-plane
- Determine the optimal deflection direction to maximise the displacement on the b-plane
- Detail an optimal deflection strategy aimed at avoiding resonant returns of asteroids

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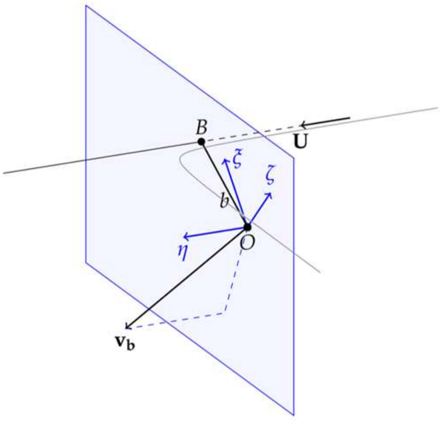
B-PLANE

B-Plane

Definition

- Reference frame centred in the Earth
- η-axis identified by the planetocentric velocity vector $\boldsymbol{U}$ of the NEO
- ζ-axis points in the opposite direction as the projection of the planet's velocity vector on the b-plane
- ξ-axis completes the right-handed reference frame
- Impact parameter $b = \sqrt{\xi^2 + \zeta^2}$



> Valsecchi et al., "Resonant returns to close approaches: Analytical theory", 2003
> Öpik, "Interplanetary Encounters", 1976

Image credits: F. Letizia, J. Van den Eynde and C. Colombo, "SNAPPshot ESA planetary protection compliance verification software, Final report," 2016

B-Plane

Definition

- ξ-axis represents the geometric distance between the two bodies' orbits at the encounter
  - Minimum Orbit Intersection Distance (MOID)
- ζ-axis represents a shift in the time of arrival of the object at the planet
- Very convenient description of an encounter

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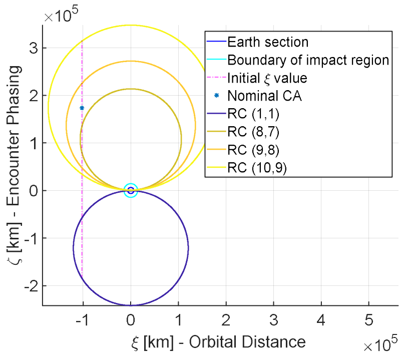
Image credits: F. Letizia, J. Van den Eynde and C. Colombo, "SNAPPshot ESA planetary protection compliance verification software, Final report," 2016

B-Plane

Resonances – Resonant Circles

- Resonant circles are regions of the b-plane corresponding to returns to Earth
  - $k\mathrm{T}_P = h\mathrm{T}' \longrightarrow a'$
  - A circle can be drawn on the b-plane for each couple of integers (h, k)
- Hypotheses
  - 2-Body Problem (2BP)
  - Circular Earth orbit
  - Coincident heliocentric positions of the Earth and the NEO

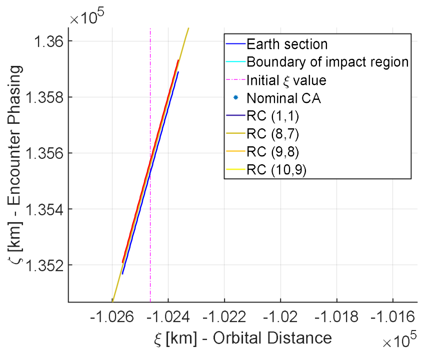


➢ Valsecchi et al., "Resonant returns to close approaches: Analytical theory", 2003

B-Plane

Resonances - Keyholes

- Keyholes are the regions of the b-plane leading to a subsequent encounter
  - Hit: pre-image of the Earth's cross-section
  - Return: pre-image of the Sphere of Influence (SOI)'s cross-section
- Close to the resonant circles



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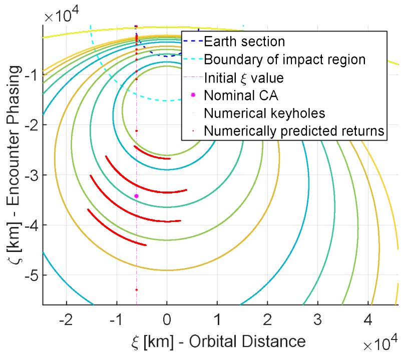
Resonances – Numerical Keyhole Determination

- Hypotheses are removed
  - Circular Earth orbit
  - Coincident heliocentric positions of the NEO and the Earth
- Numerical computation technique
  - Recording of the nominal encounter
  - Exploration of the post-fly-by conditions of a synthetic set of ζ values
  - If the resulting semi-major axis corresponds the period required to obtain a return after $h \cdot \mathrm{T}_{NEO}{}'$ and $k \cdot \mathrm{T}_{Earth}$, the point is part of the (h, k) keyhole
  - Extension to the ξ-axis



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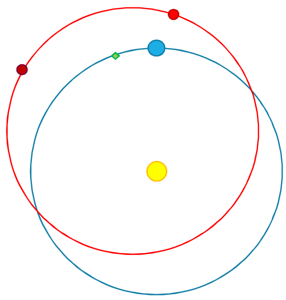
DEFLECTION OF A NEAR-EARTH OBJECT

Deflection

Introduction

- Deflection mission
  - Departure from Earth
  - Asteroid hit
  - Deflected NEO fly-by of the Earth
- Modeling
  - Deflection a certain amount of time before the close approach
    - Modeled through Gauss planetary equations
  - Study the effect at the close approach
    - Modeled through proximal motion equations

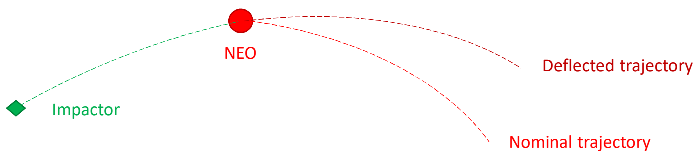


Earth orbit
NEO original orbit
Impactor
NEO modified

➢ Vasile and Colombo, "Optimal Impact Strategies for Asteroid Deflection", 2008

Deflection

Gauss Planetary Equations



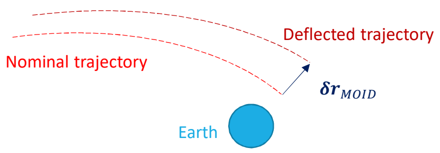
- The deflection can be modeled as a change in the orbital parameters in function of an instantaneous perturbation of the NEO's velocity vector
- The Gauss planetary equations can be written in matrix form

$$\boldsymbol{\delta\alpha}_d = \boldsymbol{G}_d \boldsymbol{\delta v}_d$$

- The variation in the asteroid's orbital parameters is obtained in function of the deflection velocity vector components

Deflection

Proximal Motion Equations



- The perturbed orbit of the NEO following the deflection can be considered as being proximal to the nominal one
- The proximal motion equations can be written in matrix form

$$\boldsymbol{\delta r}_{MOID} = \boldsymbol{A}_{MOID}\boldsymbol{\delta\alpha}_d$$

- The deviation at the encounter is expressed in function of the variation of the NEO's orbital parameters at the time of the deviation

Deflection

Compact Formulation

- Combining the proximal motion and Gauss planetary equations
- Analytical correlation between the deflection action and the geometric deviation

$$\begin{cases} \boldsymbol{\delta r}_{MOID} = \boldsymbol{A}_{MOID}\boldsymbol{\delta\alpha}_d \\ \boldsymbol{\delta\alpha}_d = \boldsymbol{G}_d\boldsymbol{\delta v}_d \end{cases} \Rightarrow \boldsymbol{\delta r}_{MOID} = \boldsymbol{A}_{MOID}\boldsymbol{G}_d\boldsymbol{\delta v}_d$$

$$\boldsymbol{\delta r}_{MOID} = \boldsymbol{T}\boldsymbol{\delta v}_d$$

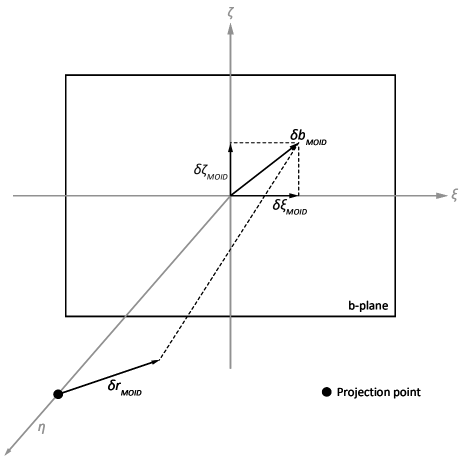
- Easy optimisation with the eigenvector method
  - Maximising $\|\boldsymbol{\delta r}_{MOID}\|$ is equivalent to maximising the quadratic form $\boldsymbol{T}^T\boldsymbol{T}$
  - Achieved by choosing $\boldsymbol{\delta v}_d$ parallel to the direction of the eigenvector of the matrix $\boldsymbol{T}^T\boldsymbol{T}$ conjugated to its maximum eigenvalue
    - The direction is constrained
    - The sign can be chosen to in order to increase the distance of the encounter

➢ M. Vasile and C. Colombo, "Optimal Impact Strategies for Asteroid Deflection", 2008

Deflection

Extension to the B-Plane

- The previously obtained analytical formulation can be extended to the deviation on the b-plane
  - Impact parameter δb
  - Variation along the ξ-axis $\delta\xi$
  - Variation along the ζ-axis $\delta\zeta$
- The analytical nature is retained
  - Matrix formulation
  - The same eigenvector-based maximisation can be applied



Deflection

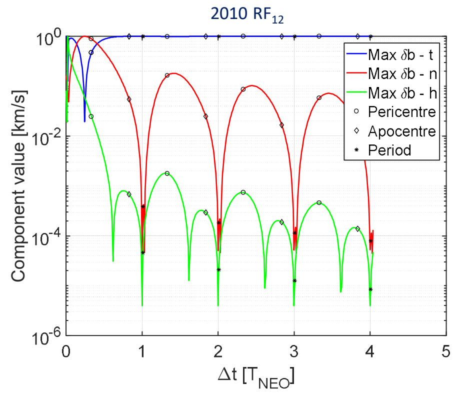
Optimal Deflection Direction - b

$$\boldsymbol{\delta b}_{MOID} = \boldsymbol{\delta r}_{MOID} - (\boldsymbol{\delta r}_{MOID} \cdot \boldsymbol{e}_\eta)\boldsymbol{e}_\eta = \boldsymbol{e}_\eta \times (\boldsymbol{\delta r}_{MOID} \times \boldsymbol{e}_\eta) = \boldsymbol{M}_{\delta b}\boldsymbol{\delta r}_{MOID}$$

$$\boldsymbol{M}_{\delta b} = \begin{bmatrix} {e_{\eta}}_2^{\,2} + {e_{\eta}}_3^{\,2} & -{e_{\eta}}_1 {e_{\eta}}_2 & -{e_{\eta}}_1 {e_{\eta}}_3 \\ -{e_{\eta}}_1 {e_{\eta}}_2 & {e_{\eta}}_1^{\,2} + {e_{\eta}}_3^{\,2} & -{e_{\eta}}_2 {e_{\eta}}_3 \\ -{e_{\eta}}_1 {e_{\eta}}_3 & -{e_{\eta}}_2 {e_{\eta}}_3 & {e_{\eta}}_1^{\,2} + {e_{\eta}}_2^{\,2} \end{bmatrix}$$

$$\boldsymbol{\delta b}_{MOID} = \boldsymbol{M}_{\delta b}\boldsymbol{T}\boldsymbol{\delta v}_d = \boldsymbol{T}_{\delta b}\boldsymbol{\delta v}_d$$

- The maximisation of $\boldsymbol{\delta b}_{MOID}$ is achieved by choosing $\boldsymbol{\delta v}_d$ parallel to the eigenvector conjugated to the maximum eigenvalue of $\boldsymbol{T}_{\delta b}{}^T\boldsymbol{T}_{\delta b}$



Deflection

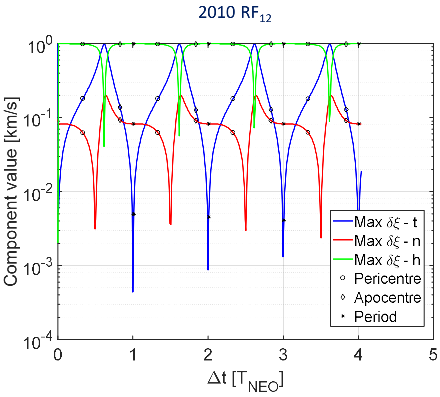
Optimal Deflection Direction - ξ

$$\delta\boldsymbol{\xi}_{MOID} = \delta\boldsymbol{b}_{MOID} - (\delta\boldsymbol{b}_{MOID} \cdot \boldsymbol{e}_{\zeta})\boldsymbol{e}_{\zeta}$$
$$= \boldsymbol{e}_{\zeta} \times (\delta\boldsymbol{b}_{MOID} \times \boldsymbol{e}_{\zeta}) = \boldsymbol{M}_{\delta\xi}\delta\boldsymbol{b}_{MOID}$$

$$\boldsymbol{M}_{\delta\xi} = \begin{bmatrix} {e_{\zeta_2}}^2 + {e_{\zeta_3}}^2 & -e_{\zeta_1}e_{\zeta_2} & -e_{\zeta_1}e_{\zeta_3} \\ -e_{\zeta_1}e_{\zeta_2} & {e_{\zeta_1}}^2 + {e_{\zeta_3}}^2 & -e_{\zeta_2}e_{\zeta_3} \\ -e_{\zeta_1}e_{\zeta_3} & -e_{\zeta_2}e_{\zeta_3} & {e_{\zeta_1}}^2 + {e_{\zeta_2}}^2 \end{bmatrix}$$

$$\delta\boldsymbol{\xi}_{MOID} = \boldsymbol{M}_{\delta\xi}\boldsymbol{T}_{\delta b}\delta\boldsymbol{v}_d = \boldsymbol{T}_{\delta\xi}\delta\boldsymbol{v}_d$$

- The maximisation of $\delta\boldsymbol{\xi}_{MOID}$ is achieved by choosing $\delta\boldsymbol{v}_d$ parallel to the eigenvector conjugated to the maximum eigenvalue of ${\boldsymbol{T}_{\delta\xi}}^T\boldsymbol{T}_{\delta\xi}$



Deflection

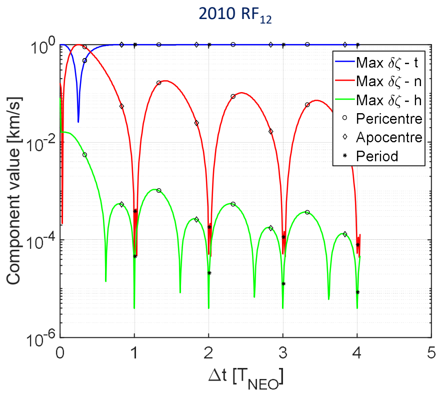
Optimal Deflection Direction - ζ

$$\delta\boldsymbol{\zeta}_{MOID} = \delta\boldsymbol{b}_{MOID} - (\delta\boldsymbol{b}_{MOID} \cdot \boldsymbol{e}_{\xi})\boldsymbol{e}_{\xi}$$
$$= \boldsymbol{e}_{\xi} \times (\delta\boldsymbol{b}_{MOID} \times \boldsymbol{e}_{\xi}) = \boldsymbol{M}_{\delta\zeta}\delta\boldsymbol{b}_{MOID}$$

$$\boldsymbol{M}_{\delta\zeta} = \begin{bmatrix} {e_{\xi_2}}^2 + {e_{\xi_3}}^2 & -e_{\xi_1}e_{\xi_2} & -e_{\xi_1}e_{\xi_3} \\ -e_{\xi_1}e_{\xi_2} & {e_{\xi_1}}^2 + {e_{\xi_3}}^2 & -e_{\xi_2}e_{\xi_3} \\ -e_{\xi_1}e_{\xi_3} & -e_{\xi_2}e_{\xi_3} & {e_{\xi_1}}^2 + {e_{\xi_2}}^2 \end{bmatrix}$$

$$\delta\boldsymbol{\zeta}_{MOID} = \boldsymbol{M}_{\delta\zeta}\boldsymbol{T}_{\delta b}\delta\boldsymbol{v}_d = \boldsymbol{T}_{\delta\zeta}\delta\boldsymbol{v}_d$$

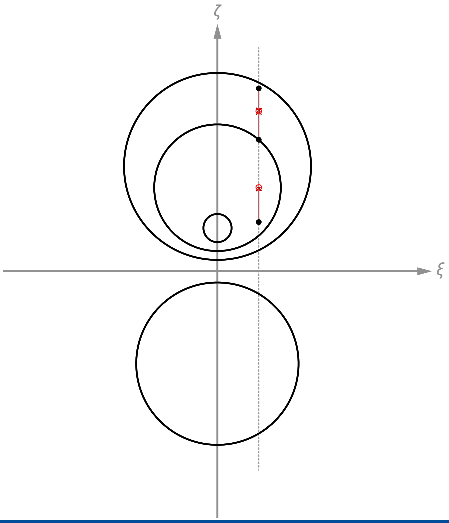
- The maximisation of $\delta\boldsymbol{\zeta}_{MOID}$ is achieved by choosing $\delta\boldsymbol{v}_d$ parallel to the eigenvector conjugated to the maximum eigenvalue of ${\boldsymbol{T}_{\delta\zeta}}^T\boldsymbol{T}_{\delta\zeta}$



Deflection

Optimal Deflection Strategy

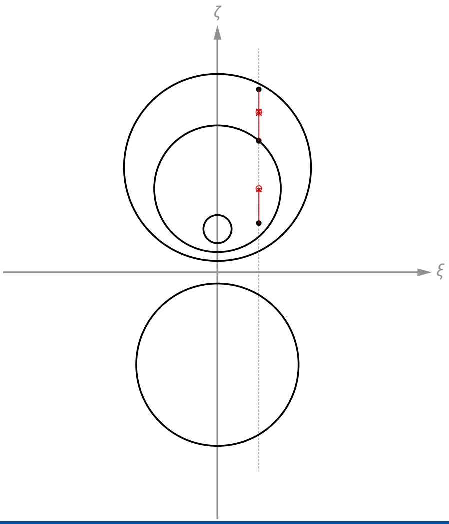
- Aimed at avoiding the keyholes
- A deviation along ζ is considered
  - Most convenient for early deflections
- Target ζ value
  - Nominal encounter within a keyhole
    - The middle point between the keyhole and the closest one
  - Nominal encounter between keyholes
    - The middle point between the considered keyholes



Deflection

Optimal Deflection Strategy

- $\boldsymbol{\delta v}$ vector determination
  - Direction of maximum $\delta\zeta$ variation through the eigenvector method
  - Modulus ${}^{\delta\zeta}/_{\delta\zeta_{eig}}$
    - $\delta\zeta_{eig}$ is the displacement obtained with a unitary $\boldsymbol{\delta v}$ vector
- Not a pure maximisation when trying to avoid a keyhole



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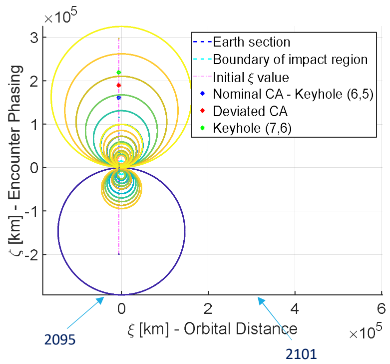
RESULTS

Results

Optimal Deflection Strategy

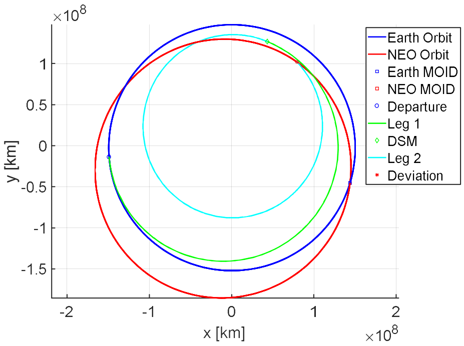
- 2095 encounter of 2010 RF_{12}-like with the Earth
  - 2-Body Problem (2BP)
  - Modified to take place in the (6,5) keyhole
- Deflection mission
  - Optimal deflection strategy to maximise the distance from the keyholes
  - Target ζ value between keyholes (6,5) and (7,6)



Results

Preliminary Deflection Mission Design

- 2095 encounter of 2010 RF_{12} with the Earth
- 2BP
- Assumed structure
  - Escape from Earth
  - Deep-Space Manoeuvre
  - Impact
- Assumed data
  - Warning time of 9 y
  - Maximum Time Of Flight (TOF) of $1 \cdot T_{NEO}$
- Multi-objective optimisation
  - Maximisation of the distance from the closest keyholes
  - Minimising the S/C initial mass



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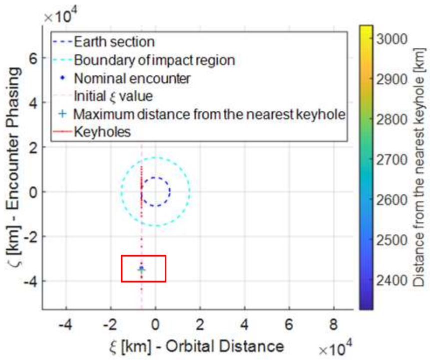
Results

Preliminary Deflection Mission Design

- Most effective strategy corresponding to a deviation along the ζ-axis away from the closest keyhole
- In this case, the deviation does not overcome the middle point between the keyholes
  - Equivalent results to maximising the distance from the closest keyhole

$$\delta v_{tnh} = \begin{Bmatrix} -1.4581 \\ -1.3736 \\ -0.0637 \end{Bmatrix} \cdot 10^{-3} \ {}^{m}/_{s}$$

- The deviation features a very significant normal component
  - The more realistic strategy cannot guarantee the ideal deflection direction



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Conclusions

Main Contributions

- An analytical correlation between the deflection and the displacement on the b-plane has been obtained
  - The eigenvector maximisation technique has been applied to each case in order to define the optimal deflection direction
- An optimal deflection technique has been devised to avoid the keyholes
  - Based on the knowledge that the deflection is most effective in the phasing (ζ-axis)
  - Aimed ad avoiding resonant returns (i.e. the keyholes)
  - A preliminary mission design supports the viability of the technique

Conclusions

Future Developments

- Consider a more refined propagation method (n-body)
  - Define the keyholes
  - Obtain more accurate propagation results
- Consider a set of initial conditions for the asteroid position
- Consider a more complex model for the deflection of the NEO
- Define alternative optimal deflection strategies
  - Account for the return time associated with each keyhole
  - Account for manoeuvre cost

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Thank you for your attention

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B-Plane

Resonances

- Reachability of resonances
  - Circles with $R < \xi_{encounter}$ are considered as unreachable, as $\xi_{encounter}$ is the minimum value that the impact parameter can reach in the case that the two orbits are perfectly phased (i.e. the MOID)
  - As the b-plane is built on the hypothesis of a two-body propagation, the circles corresponding to returns that would be very distant in time cannot be considered as representative of the real conditions
    - A limit of $h = k = 10$ can be considered as reasonable

Deflection

Gauss Planetary Equations

- $\delta a = \frac{2a^2 v_d}{\mu} \delta v_t$
- $\delta e = \frac{1}{v_d}\left[2(e + \cos\theta_d)\delta v_t - \frac{r_d}{a}\sin\theta_d\,\delta v_n\right]$
- $\delta i = \frac{r_d \cos\theta^*{}_d}{h}\delta v_h$
- $\delta\Omega = \frac{r_d \sin\ \ {}^*{}_d}{h \sin i}\delta v_h$
- $\delta\omega = \frac{1}{e v_d}\left[2\sin\theta_d\,\delta v_t + \left(2e + \frac{r_d}{a}\cos\theta_d\right)\delta v_n\right] - \frac{r_d \,\mathrm{si}\ \ {}^*{}_d\,\mathrm{co}}{h \sin i}\delta v_h$
- $\delta M_{t_d} = -\frac{b}{e a v_d}\left[2\left(1 + \frac{e^2 r_d}{p}\right)\sin\theta_d\,\delta v_t + \frac{r_d}{a}\cos\theta_d\,\delta v_n\right]$
- $\delta M_n = \delta n \Delta t = \left(\sqrt{\frac{\mu}{a^3}} - \sqrt{\frac{\mu}{(a+\delta a)^3}}\right)(t_{MOID} - t_n) = -\frac{3}{2}\frac{\sqrt{\mu}}{a^{5/2}}\Delta t \delta a$
- $\delta M = \delta M_{t_d} + \delta M_n = -\frac{b}{eav}\left[2\left(1 + \frac{e^2 r}{p}\right)\sin\theta_d\,\delta v_t + \frac{r}{a}\cos\theta_d\,\delta v_n\right] - \frac{3}{2}\frac{\sqrt{\mu}}{a^{5/2}}\Delta t \delta a$

Deflection

Proximal Motion Equations

- $\delta s_r \cong \frac{r_{MOID}}{a}\delta a + \frac{ae\,\mathrm{si}\ \ {}_{MOID}}{\eta}\delta M - a\cos\theta_{MOID}\,\delta e$
- $\delta s_\theta \cong \frac{r_{MOID}}{\eta^3}(1 + e\cos\theta_{MOID})^2 \delta M + r_{MOID}\delta\omega + \frac{r_{MOID}\sin\theta_{MOID}}{\eta^2}(2 + e\cos\theta_{MOID})\delta e + r_{MOID}\cos i\,\delta\Omega$
- $\delta s_h \cong r_{MOID}(\sin\theta^*{}_{MOID}\,\delta i - \cos\theta^*{}_{MOID}\sin i\,\delta\Omega)$

Deflection

Matrix Formulation

- $\begin{cases} \boldsymbol{\delta r}_{MOID} = \boldsymbol{A}_{MOID}\boldsymbol{\delta\alpha}_d \\ \boldsymbol{\delta\alpha}_d = \boldsymbol{G}_d\boldsymbol{\delta v}_d \end{cases} \Longrightarrow \boldsymbol{\delta r}_{MOID} = \boldsymbol{A}_{MOID}\boldsymbol{G}_d\boldsymbol{\delta v}_d = \boldsymbol{T}\boldsymbol{\delta v}_d$

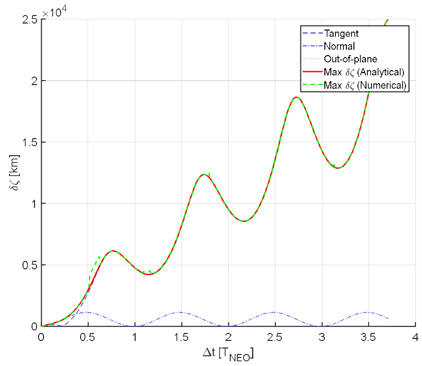
- $$\boldsymbol{A}_{MOID}{}^T = \begin{bmatrix} \frac{r_{MOID}}{a} - \frac{3}{2}\frac{e\sin\theta_{MOID}}{\eta}\frac{\sqrt{\mu}}{a^{3/2}}\Delta t & -\frac{3}{2}\frac{r_{MOID}}{\eta^3}(1+e\cos\theta_{MOID})^2\frac{\sqrt{\mu}}{a^{5/2}}\Delta t & 0 \\ -a\cos\theta_{MOID} & \frac{r_{MOID}\sin\theta_{MOID}}{\eta^2}(2+e\cos\theta_{MOID}) & 0 \\ 0 & 0 & r_{MOID}\sin\theta^*{}_{MOID} \\ 0 & r_{MOID}\cos i & -r_{MOID}\cos\theta^*{}_{MOID}\sin i \\ 0 & r_{MOID} & 0 \\ \frac{ae\sin\ \ {}_{MOID}}{\eta} & \frac{r_{MOID}}{\eta^3}(1+e\cos\theta_{MOID})^2 & 0 \end{bmatrix}$$

- $$\boldsymbol{G}_d = \begin{bmatrix} \frac{2a^2v_d}{\mu} & 0 & 0 \\ \frac{2}{v}(e+\cos\theta_d) & -\frac{r}{av_d}\sin\theta_d & 0 \\ 0 & 0 & \frac{r_d\cos\theta^*{}_d}{h} \\ 0 & 0 & \frac{r_d\sin\theta^*{}_d}{h\sin i} \\ \frac{2\sin\theta_d}{ev_d} & \frac{2e+{}^{r_d}/_a\cos\theta_d}{ev_d} & -\frac{r_d\sin\theta^*{}_d\cos i}{h\sin i} \\ -\frac{2b}{eav_d}\left(1+\frac{e^2r_d}{p}\right)\sin\theta_d & -\frac{b}{ea\ \ {}_d}\frac{r_d}{a}\cos\theta_d & 0 \end{bmatrix}$$

Deflection

Validation of the Eigenvector Method and Deflection Profile

- Apophis
- Comparison with numerical method
- Equivalent results
  - Non-convergence of the numerical method
- Cumulative effect when maximising the δb
- No cumulative effect when maximising $\delta\xi$
- Cumulative effect when maximising $\delta\zeta$



Results

Optimal Deflection Strategy

- The cost of the deflection decreases when performing the manoeuvre in advance
- A fixed-magnitude deflection yields a different effect in function of the deflection time
- The magnitude of the deflection must be controlled to avoid other keyholes