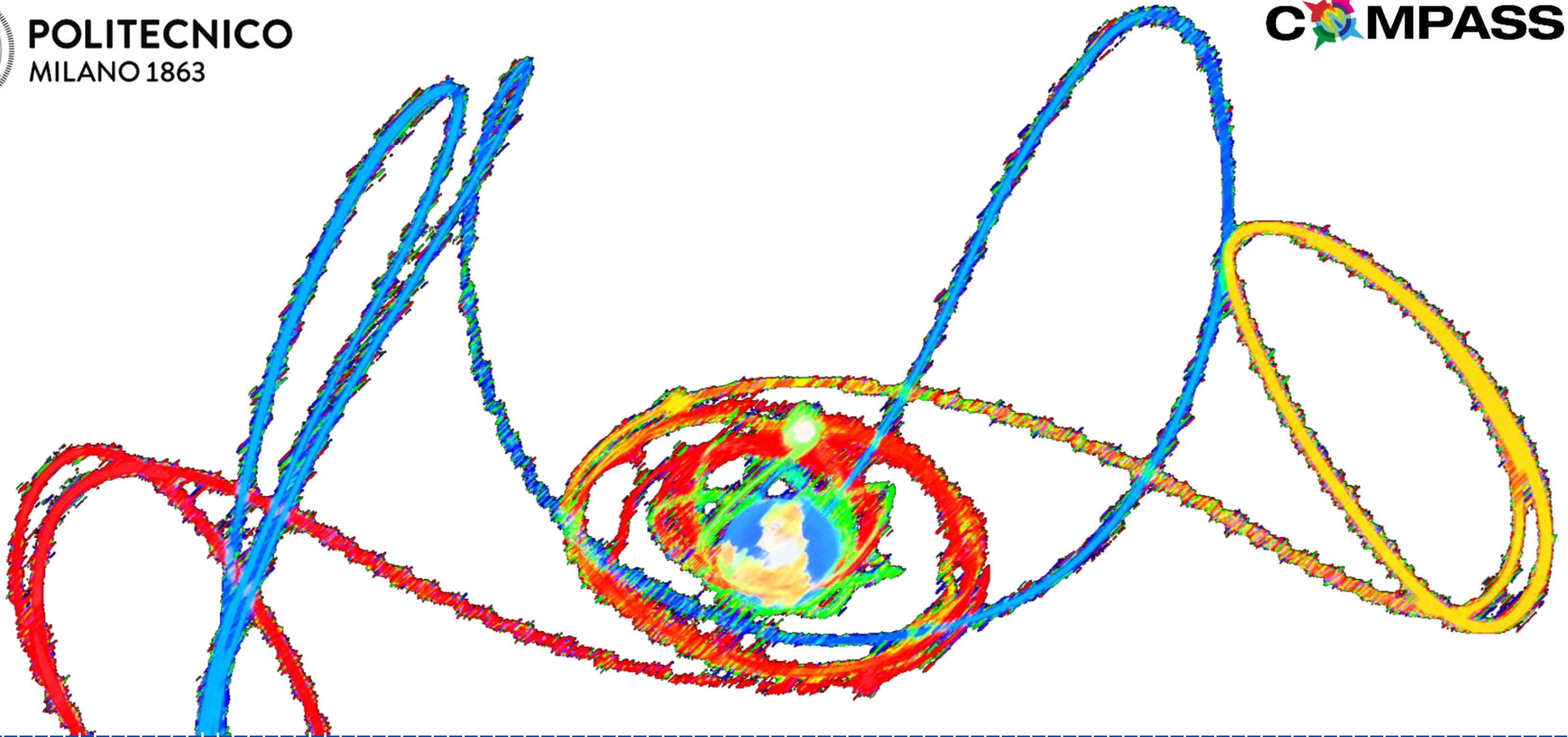
POLITECNICO MILANO 1863



Planetary protection analysis for interplanetary missions

SDSM 2017 Summer School, San Martino al Cimino (VT)

Matteo Romano & Camilla Colombo

Contents

PLANETARY PROTECTION

Planetary Protection

Motivations

- During interplanetary missions, spacecraft and debris may impact with other planets over long times
  - Impacts from man-made objects can cause **biological contamination**
  - Sensible targets for scientific research (Mars, Europa, Enceladus) impose very stringent **planetary protection requirements**[1]
  - Space missions must satisfy these requirements during design phase

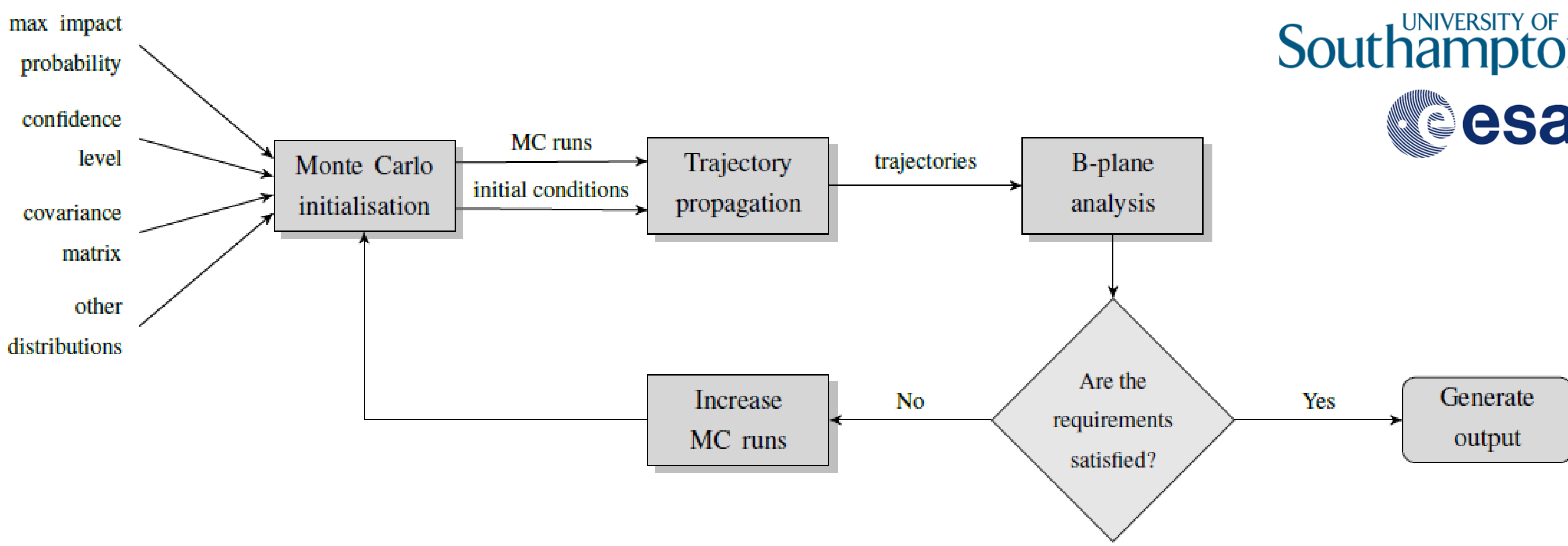
- Driving factors
  - Uncertainty over the initial state of the launcher injection
  - Uncertainty over spacecraft design parameters (e.g. area/mass ratio)
  - Random failures of spacecraft propulsion system

[1] G. Kminek, *ESA planetary protection requirements*, Technical Report ESSB-ST-U-001, European Space Agency, February 2012

Planetary Protection

Suite for Numerical Analysis of Planetary Protection

SNAPPshot (Suite for the Numerical Analysis of Planetary Protection)[2], developed by the University of Southampton under a study for the European Space Agency



[2] Letizia F., Colombo C., Van den Eynde J., Armellin R., Jehn R., *SNAPPSHOT: Suite for the numerical analysis of planetary protection*, ICATT 2016
[2] Colombo C., Letizia F., Van den Eynde J., *SNAPPSHOT: ESA planetary protection compliance verification software, Final report*, ESA contract, Jan 2016

Planetary Protection

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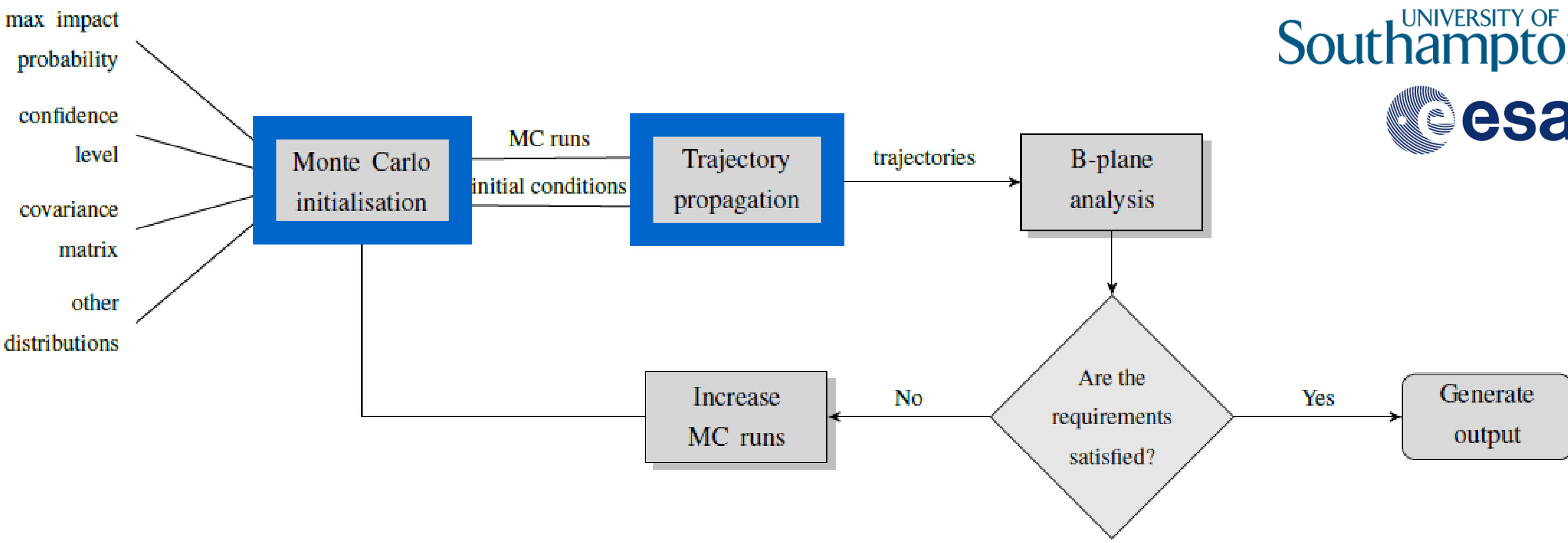

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Planetary Protection

Our approach

The main goal is to **improve the accuracy and the efficiency** of the Planetary Protection analysis:

- **Numerical integration**
  Understand how the errors in a single propagation may affect planetary protection verification
  - RK schemes (current)
  - Symplectic and energy-preserving methods (in development)
  - Other (future)
- **Sampling techniques**
  Efficient methods to sample the initial dispersion
  - Monte Carlo (current)
  - Line Sampling & Subset Simulation (in development)
  - Other (future)

NUMERICAL INTEGRATION

Numerical integration

Introduction

- Numerical methods accumulate errors during long-term integrations
  - **Fictitious dissipation** of total energy of the system
  - **Large errors** in the propagation introduced by fly-bys
  - Effect of numerical errors in single propagations on the overall planetary protection has to be determined
- Alternative numerical approaches may improve the accuracy of the orbital propagation
  - Symplectic schemes preserve constants of motion exactly or with bounded oscillations
  - Additional numerical techniques can help in maintaining the correct qualitative behaviour of the solution (no energy dissipation)

Numerical integration

Selection of numerical methods

Method	Explanation	Pros	Cons
RK	Generic Runge-Kutta schemes can become symplectic when applied to Hamiltonian systems under **conditions on their coefficients**	• Step adaptation is possible	• Symplectic schemes have to be implicit
GLRK	Implicit method based on **Gauss-Legendre quadrature points**	• Symplectic • Numerically stable	• Implicit • Fixed step
RKN	**Runge-Kutta-Nystrom** methods for **separable Hamiltonian**, use different schemes to schemes to integrate coordinates and momenta of the Hamiltonian	• Symplectic • Can be explicit	• High number of evaluations • Fixed step
SY	Derived from the Hamiltonian formulation, make use of successive **canonical transformations**	• Symplectic • Explicit	• High number of evaluations • Fixed step

Numerical integration

Selection of numerical methods

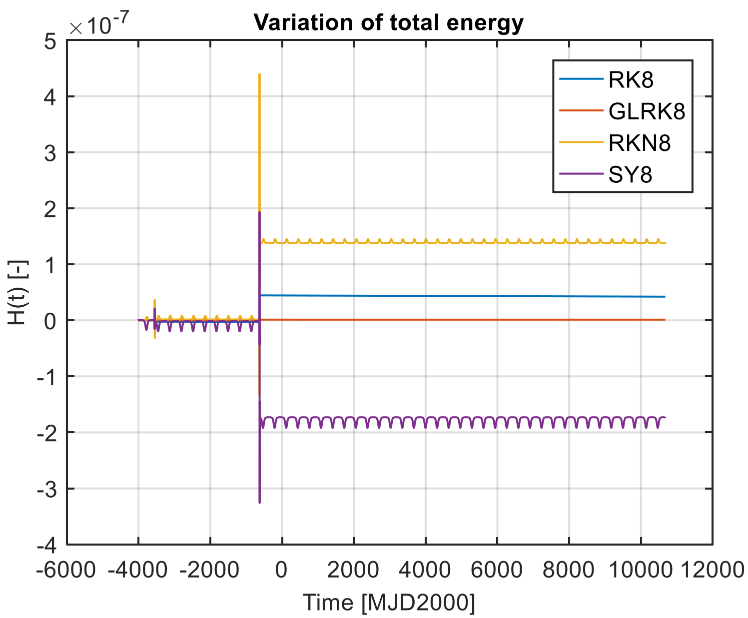
Method	(Order, Stages)	Type	Time step	Property	Reference
RK	(4,4) **(8,13)**	Explicit	Fixed step		Dormand and Prince, 1980
	(5/4,7) **(8/7,13)**	Explicit	**Adaptive step**		Prince and Dormand, 1981
GLRK	(4,2) (6,3) **(8,4)**	**Implicit**	Fixed step	**Symplectic**	Butcher, 1964 Jones et al., 2012 Aristoff et al., 2012
RKN	(6,6) **(8,26)**	Explicit	Fixed step	**Symplectic**	Dormand et al., 1987 Calvo et al., 1993
SY	(4,4) (6,8) **(8,16)**	Explicit	Fixed step	**Symplectic, canonical**	Yoshida et al., 1990 Neri, 1988

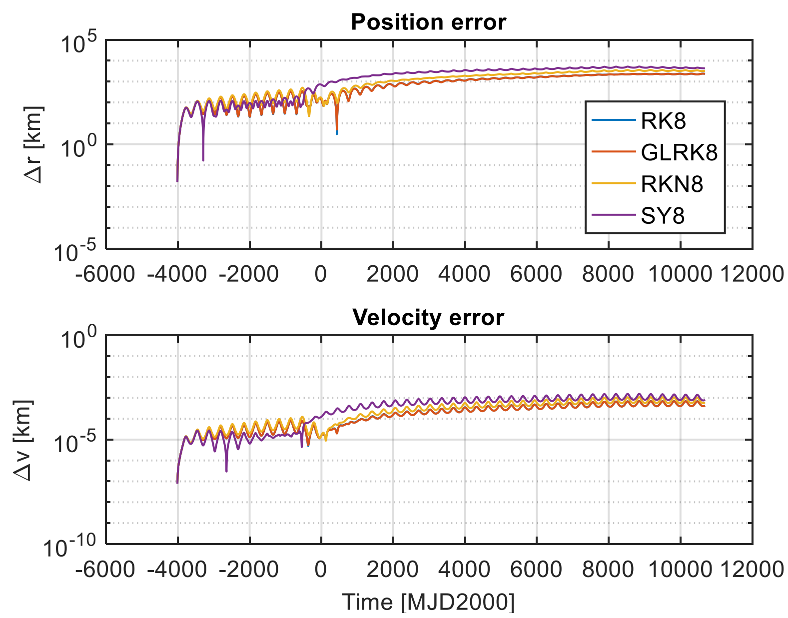
Numerical integration

Tests

Propagation of Apophis 1989-2029 (reference ephemeris from JPL SPICE)

8th order, fixed-step methods
(initial step determined according to RK8(7) with relative tolerance 1e-12)





*Propagations performed in Matlab on processor Intel® Core™ i7-6500U CPU @ 2,50GHz

Numerical integration

Additional techniques

- **Step regularisation**
  Step is rescaled during the integration according to the behaviour of the dynamics
  - Maximum eigenvalue Λ of Jacobian matrix taken as reference value[4]

    $$h_{n+1} = h_n \, \Lambda(t_n)/\Lambda(t_{n+1})$$

  - More efficient tracking of dynamics, but change in time step during the integration **breaks down the conservation properties** of symplectic methods
- **Projection methods**
  Correct the numerical solution according to the **gradient of the energy function**[5]
  - Energy error is minimised
  - **Implicit non-linear problem** has to be solved

[4] F. Debatin, A. Tilgner, F. Hechler, *Fast numerical integration of interplanetary orbits*. In *Second International Symposium on Spacecraft Flight Dynamics*, 1986

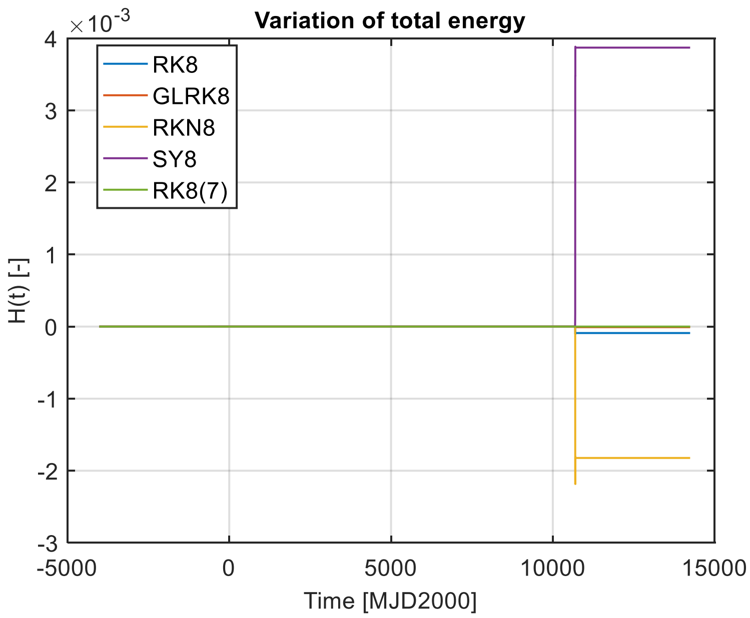
[5] Hairer, *Geometric Numerical Integration Structure-preserving algorithms for ordinary differential equations* (1990)

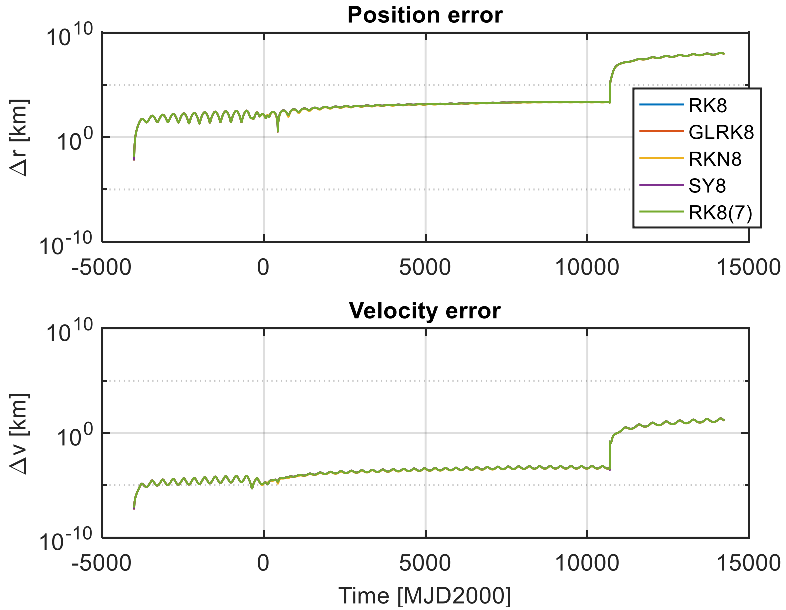
Numerical integration

Tests

Propagation of Apophis 1989-2039 (reference ephemeris from JPL SPICE)
Fly-by of 2029 is included

8th order, Regularised step
(initial step determined according to RK8(7) with relative tolerance 1e-12)





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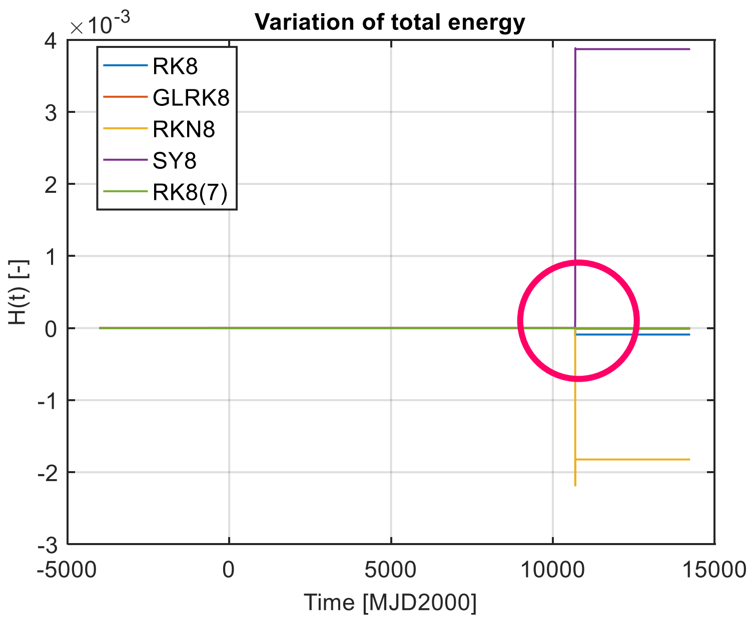

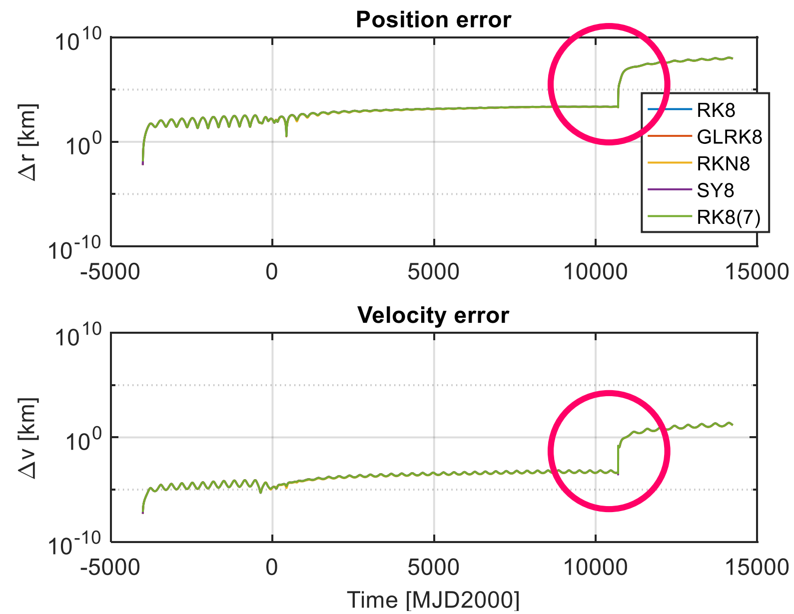

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Numerical integration

Additional techniques

Fly-bys heavily affect the numerical solution, **magnifying small numerical errors** due to a strong non-linearity of the dynamics

- Possible solution: apply other techniques only during the fly-by
  - Switch the centre of the propagation[6]
  - Add projection
  - other
- Problem: identification of the fly-by condition
  - Set distance from the planet (arbitrary or SOI definition)
  - **Alternative approach**: detection of fly-by according to the behaviour of the dynamics (Jacobian)

[6] D. Amato, G. Baù, C. Bombardelli, *Accurate orbit propagation in the presence of planetary close encounters*, *Monthly Notices of the Royal Astronomical Society*, Volume 470, Issue 2, 11 September 2017, Pages 2079–2099

Physical model update

Fly-by detection through Jacobian

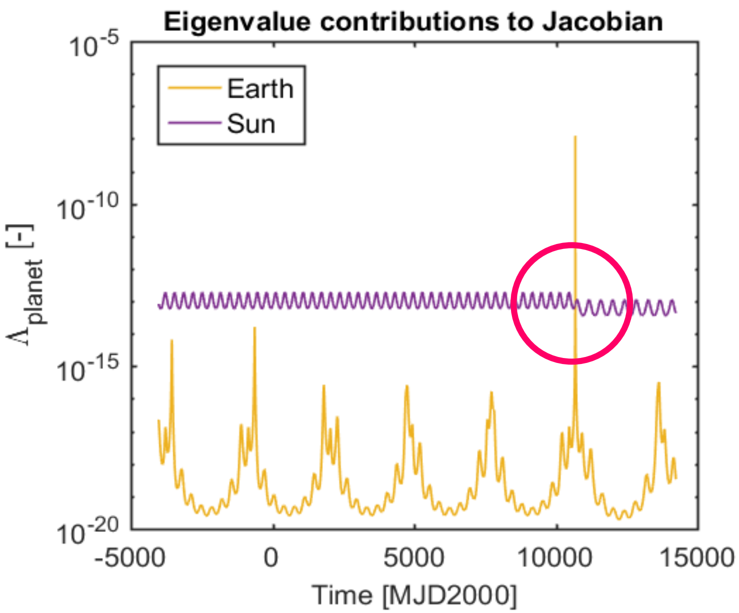
Considering the singular planet contributions to the Jacobian

- Value of planet contribution

$$\Lambda_j = \frac{2\mu_j}{\|\underline{r}-\underline{r}_j\|^3}$$

- Time variation of planet contribution

$$\dot{\Lambda}_j = -2\mu_j \frac{3(\underline{r}-\underline{r}_j)(\underline{v}-\underline{v}_j)}{\|\underline{r}-\underline{r}_j\|^5}$$



Fly-by detection criteria (approximation)

- Relative value w.r.t. main attractor: $\Lambda_j/\Lambda_0 \geq \mathrm{tol}$ (similar to SOI definition)
- Relative variation w.r.t. main attractor: $\dot{\Lambda}_j/\dot{\Lambda}_0 \geq \mathrm{tol}$

Physical model update

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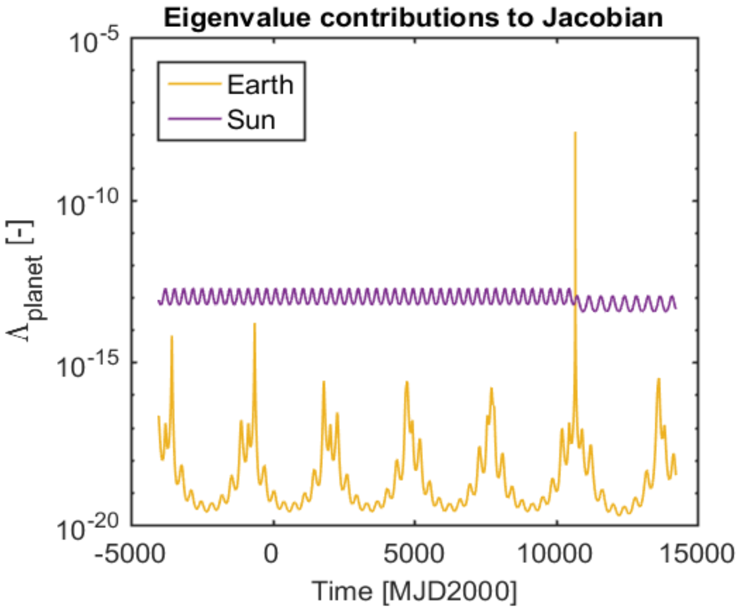

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SAMPLING TECHNIQUES

Sampling techniques

Explored ideas

- Monte Carlo
  - Number of runs selected to ensure the desired confidence level is respected (SNAPPShot)
- Other approaches to sampling
  Methods compared in previous work[7]:
  - Line Sampling
    Aimed to increase accuracy of probability estimation
  - Subset Simulation
    Aimed to increase efficiency by reducing number of propagations

[7] M. Romano, M. Losacco, C. Colombo, P. Di Lizia, *Estimation of impact probability of asteroids and space debris through Monte Carlo Line Sampling and Subset Simulation*, KePASSA 2017 Workshop

Sampling techniques

Line Sampling

The Line Sampling (LS) is a **Monte Carlo sampling** method that probes the uncertainty domain by using **lines** instead of random points

- The lines are used to **identify the boundaries of the impact region** inside the coordinate space
  - This can be done **independently from the initial uncertainty** and the probability estimation
  - The lines follow a reference direction pointing toward the impact subdomain
- The estimation of impact probability is reduced to a number of 1D integration problems along each line
  - **Analytical integration** results into a more accurate solution[3]

[3] Enrico Zio, Nicola Pedroni, *Subset Simulation and Line Sampling for Advanced Monte Carlo Reliability Analysis*, Proceedings of the European Safety and RELiability (ESREL) 2009 Conference, 2009, pp.687-694. <hal-007210>

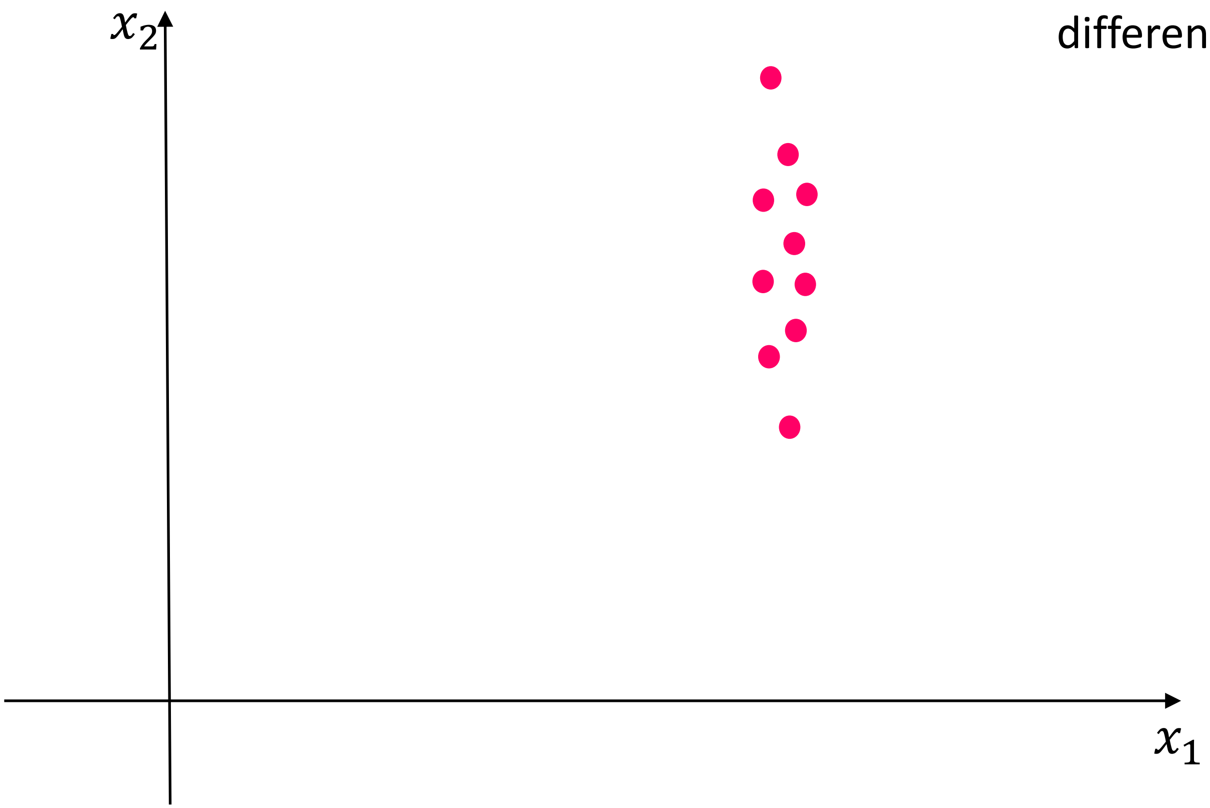
Sampling techniques

Line Sampling

1. **Determination of the "reference direction"**
   a. Impact region not known a priori

Sampling techniques

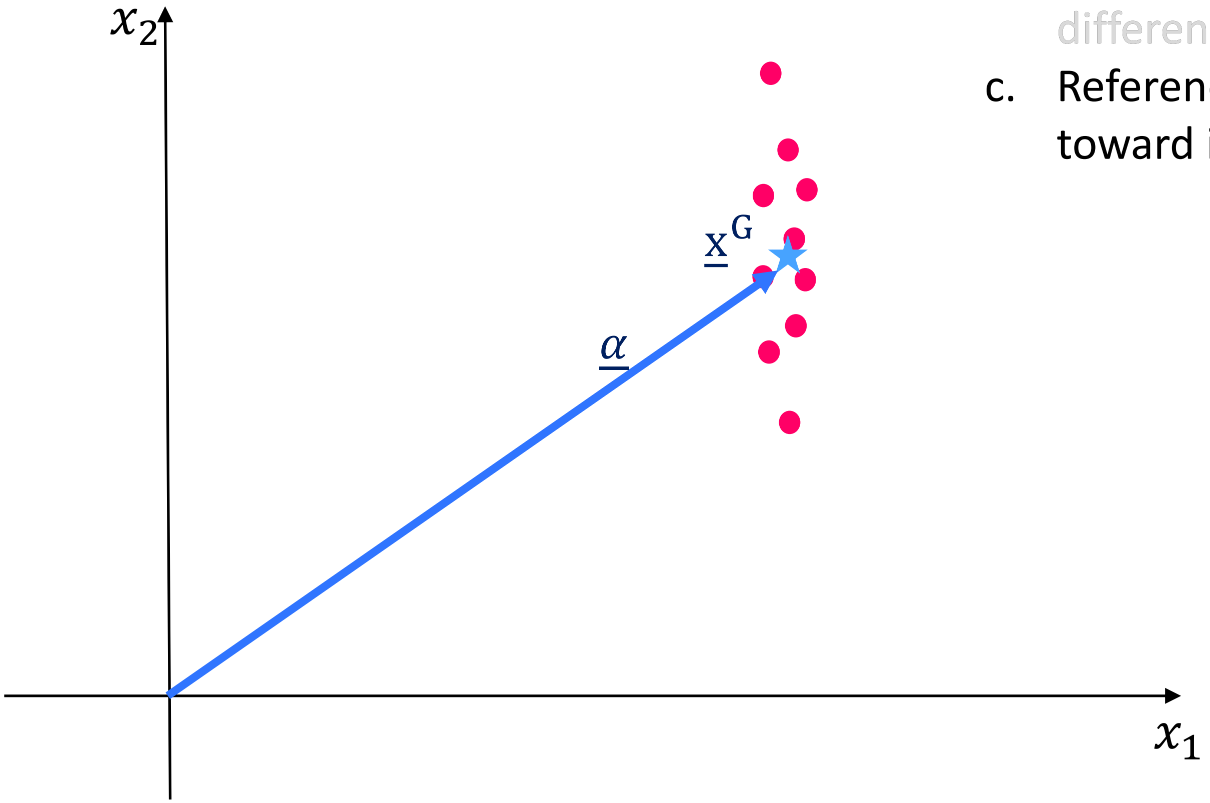
Line Sampling



1. **Determination of the "reference direction"**
   a. Impact region not known a priori
   b. Information can be obtained in different ways

Sampling techniques

Line Sampling



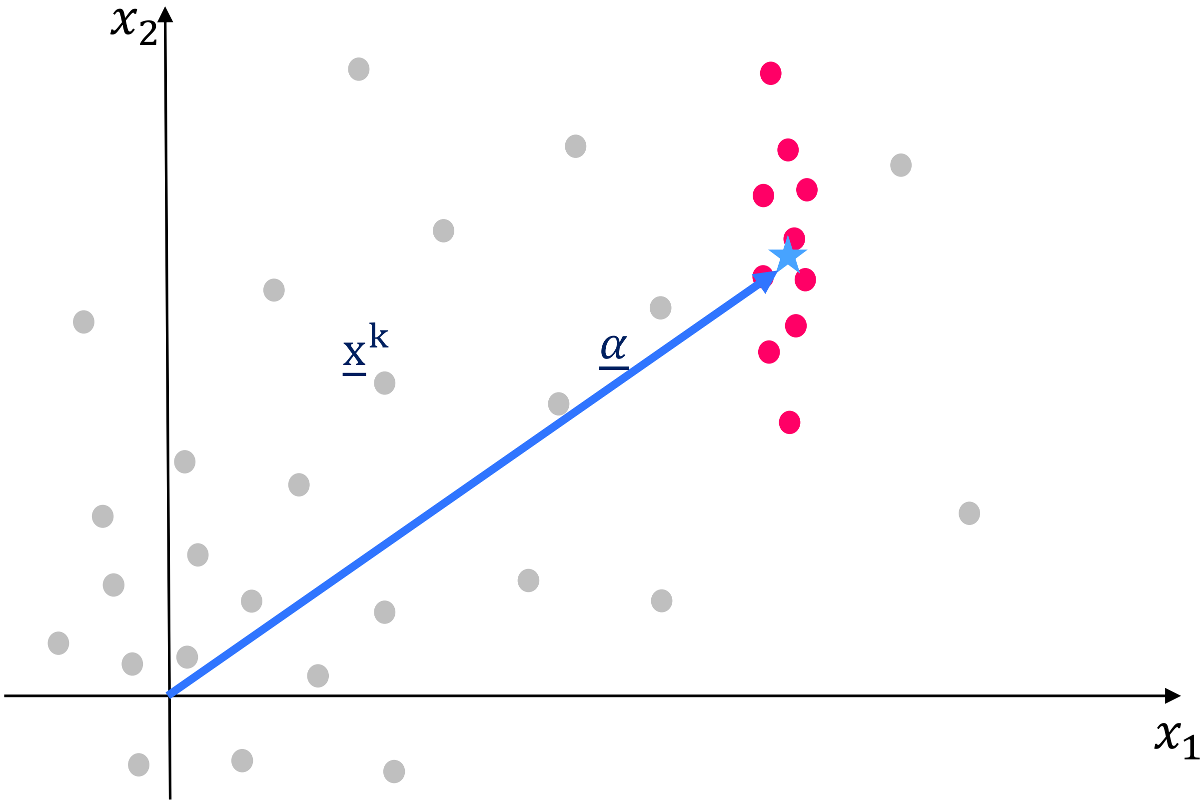
1. **Determination of the "reference direction"**
   a. Impact region not known a priori
   b. Information can be obtained in different ways
   c. Reference direction generally pointing toward impact region

Sampling techniques

Line Sampling

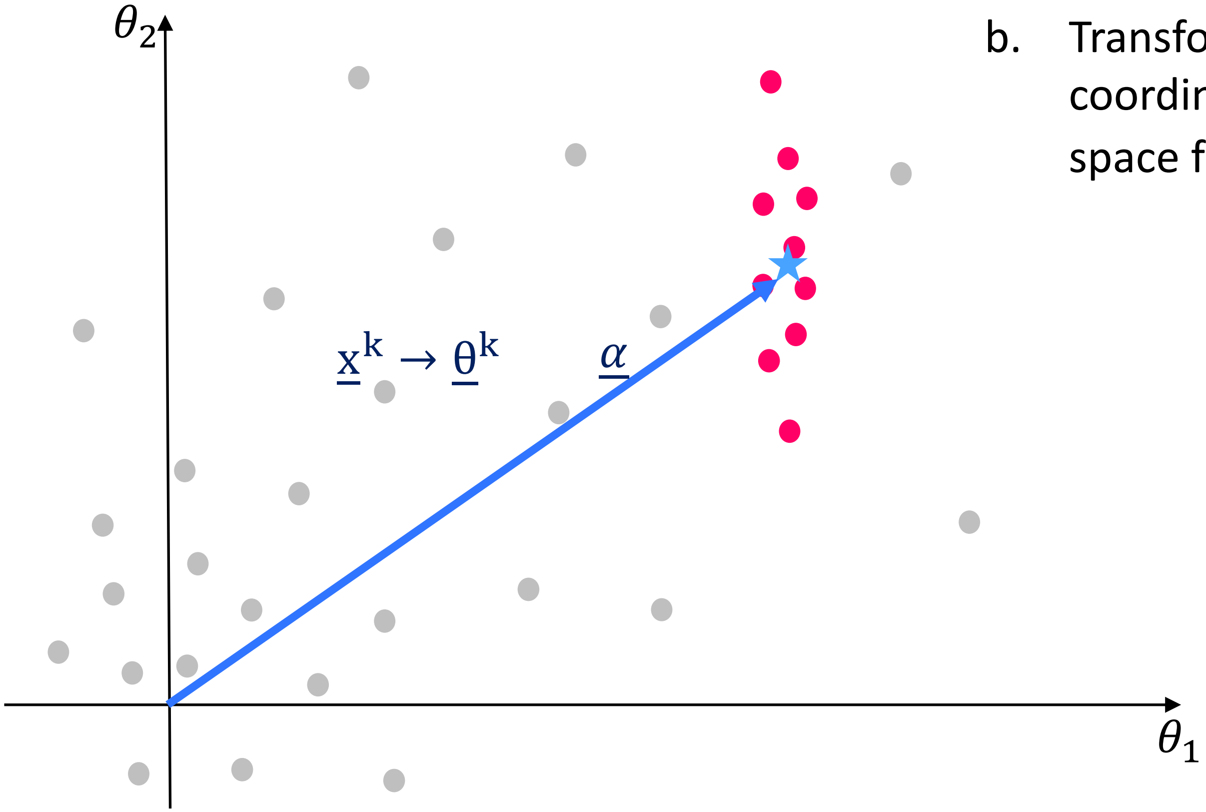
2. Mapping onto the standard normal space

a. Generation of random samples according to given distribution



Sampling techniques

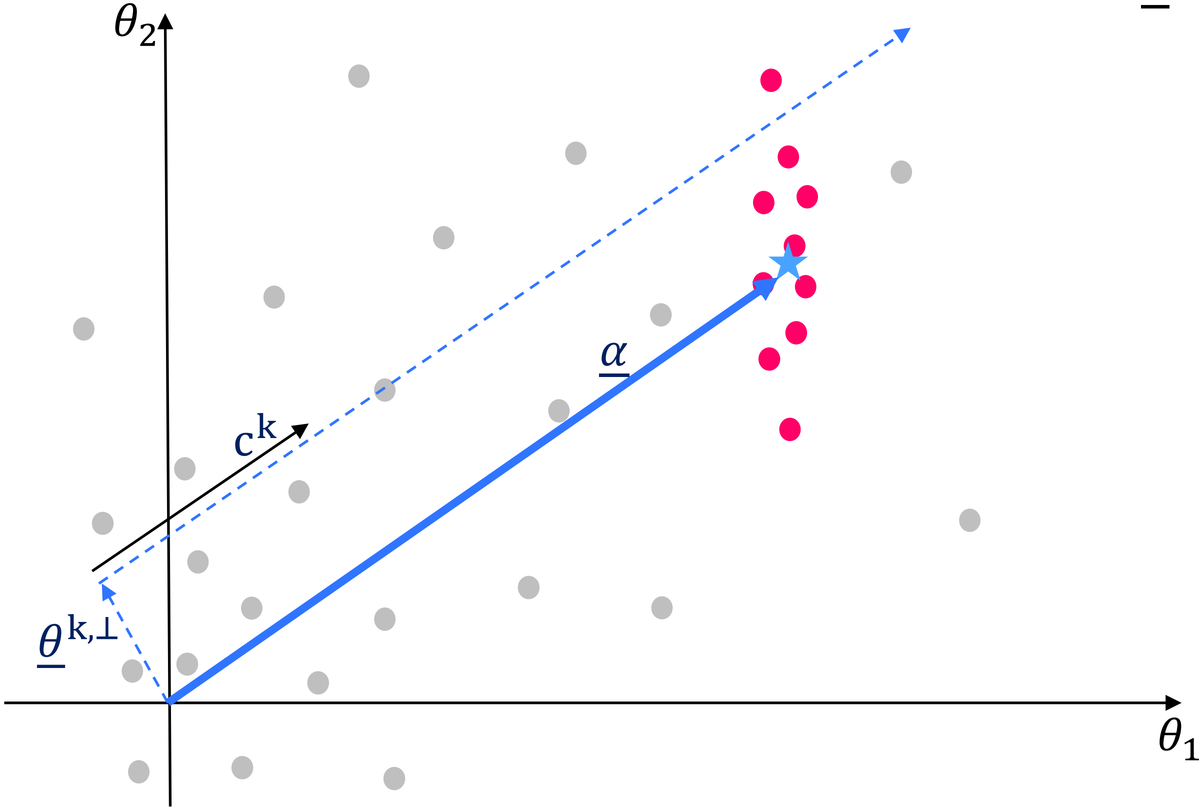
Line Sampling



2. **Mapping onto the standard normal space**
   a. Generation of random samples according to given distribution
   b. Transformation from physical coordinates into normalised standard space following $\Phi(\underline{\theta}^k) = F(\underline{x}^k)$

Sampling techniques

Line Sampling



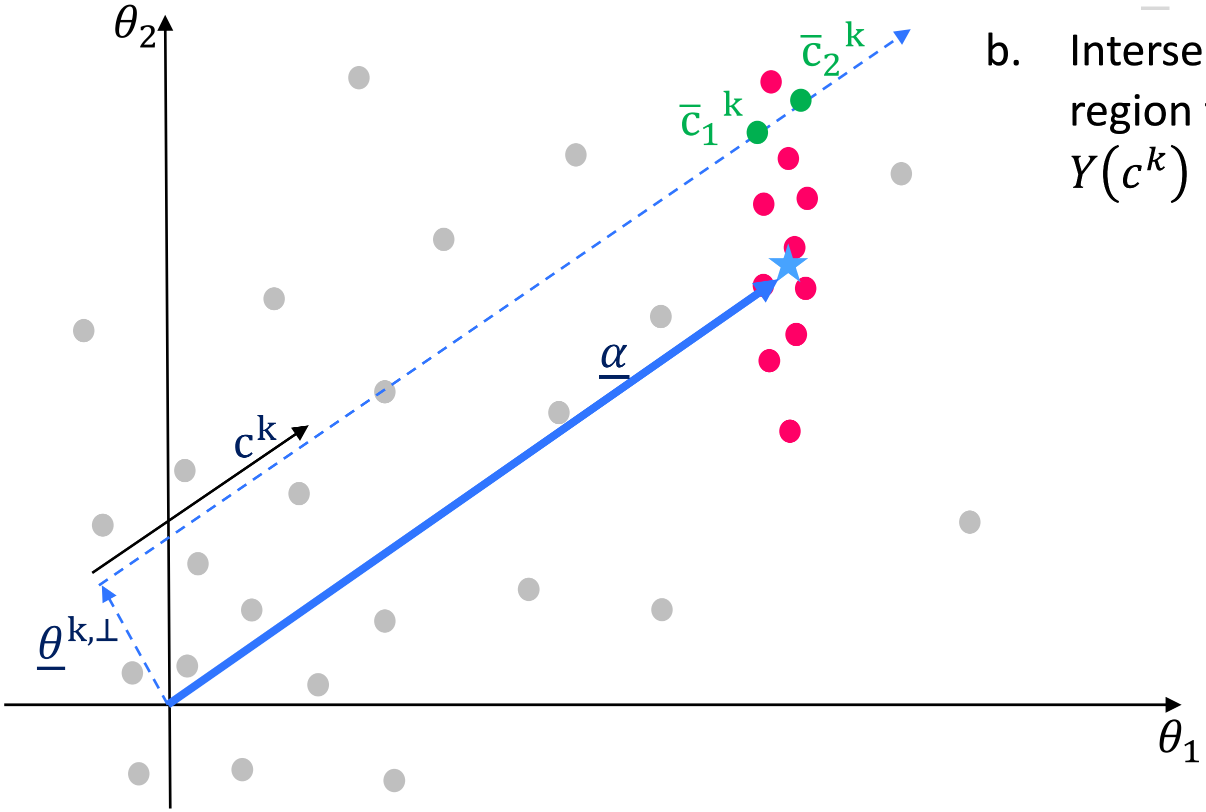
3. Sampling along the lines

a. Lines defined in normalised space

$$\underline{\tilde{\theta}}^k = c^k \underline{\alpha} + \underline{\theta}^{k,\perp}$$

Sampling techniques

Line Sampling



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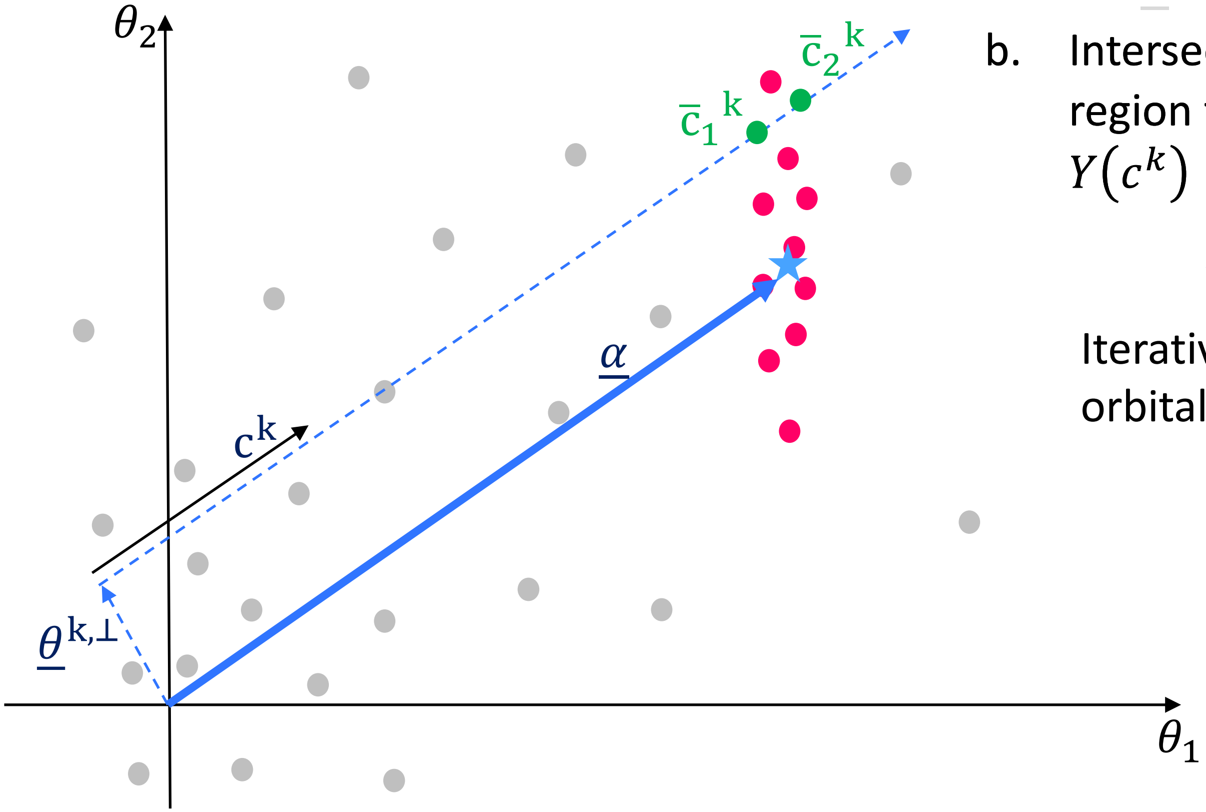
   $$\underline{\tilde{\theta}}^k = c^k \underline{\alpha} + \underline{\theta}^{k,\perp}$$
   b. Intersections $\left(\bar{c}_1{}^k, \bar{c}_2{}^k\right)$ with impact region found where objective function $Y\left(c^k\right) = 0$

Sampling techniques

Line Sampling



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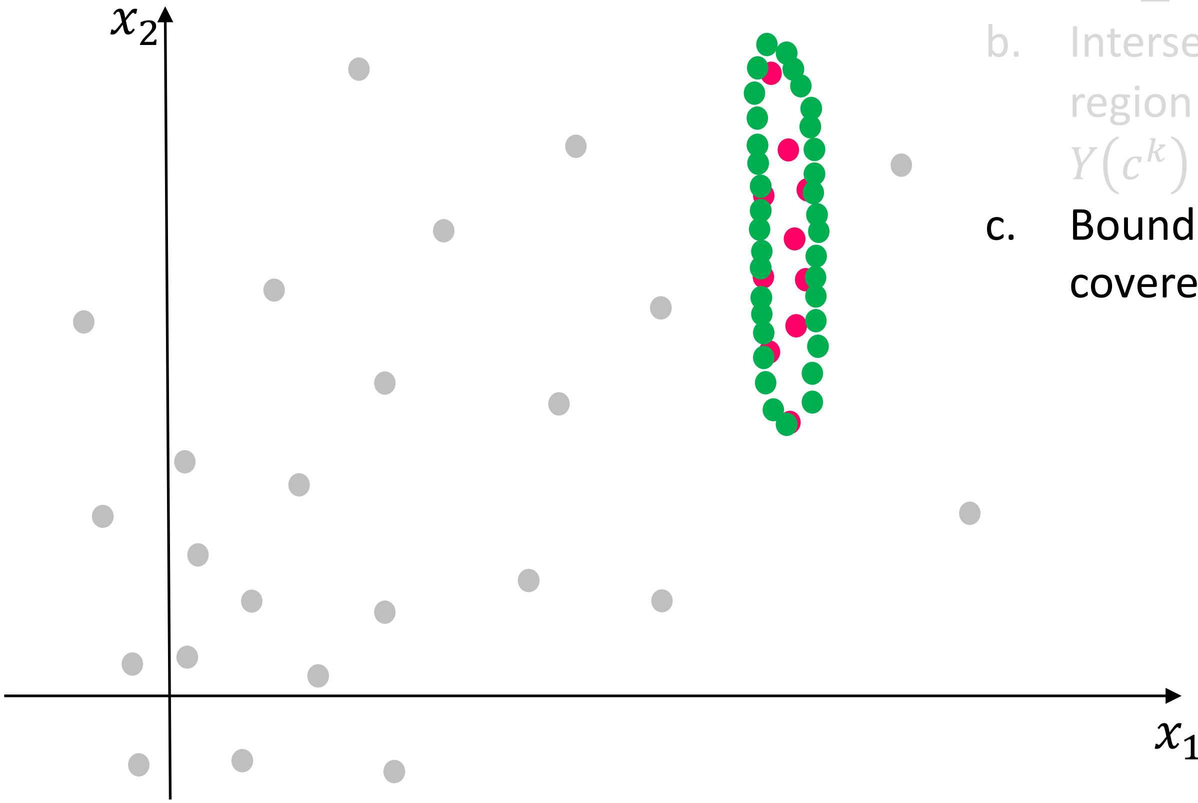
Iterative procedure requires extra orbital propagations

Sampling techniques

Line Sampling



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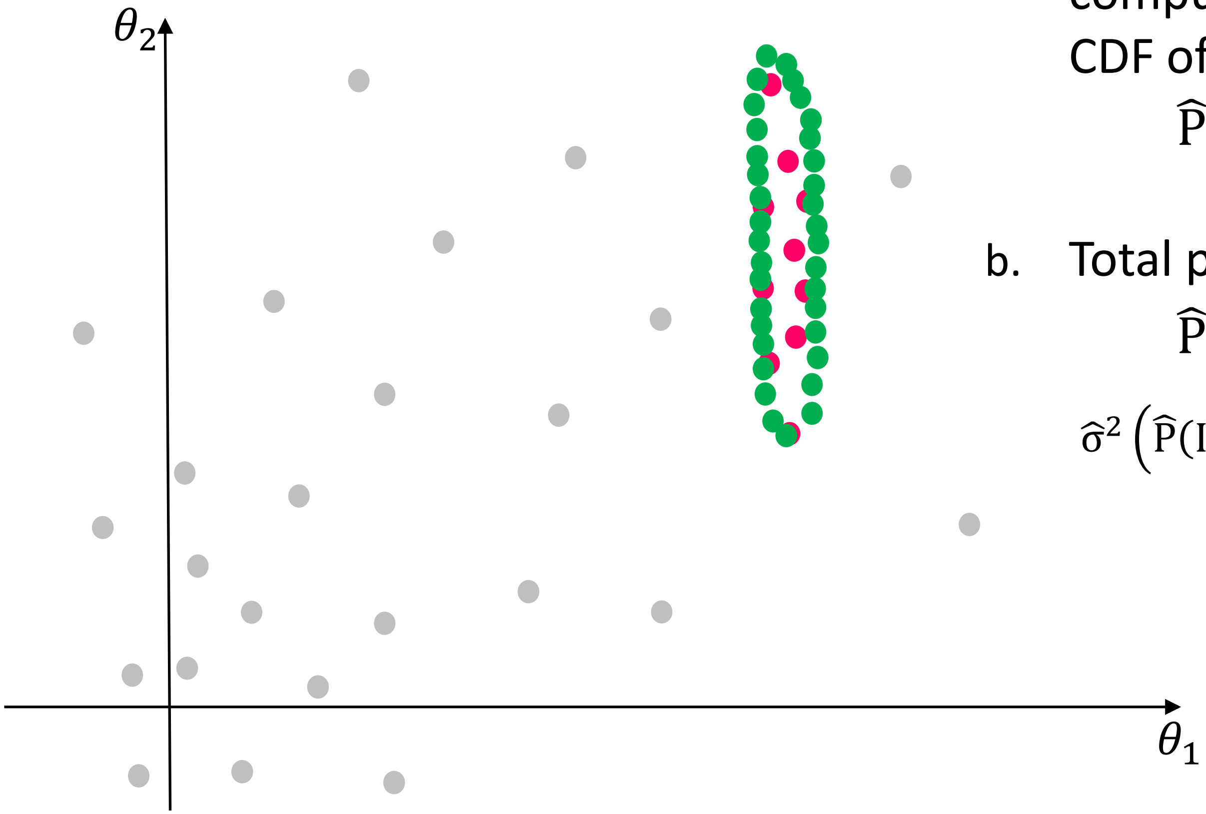
   b. Intersections $\left(\bar{\mathbf{c}}_1{}^{\mathbf{k}}, \bar{\mathbf{c}}_2{}^{\mathbf{k}}\right)$ with impact region found where objective function $Y\left(c^k\right) = 0$
   c. Boundaries of the impact region are covered

Sampling techniques

Line Sampling



4. Estimation of impact probability

a. Partial probability estimates are computed along each line using the CDF of the unit gaussian:

$$\widehat{P}^k(I) = \Phi\left(\bar{c}_2{}^k\right) - \Phi\left(\bar{c}_1{}^k\right)$$

b. Total probability and variance

$$\widehat{P}(I) = \frac{1}{N_T}\sum_{k=1}^{N_T} \widehat{P}^k(I)$$

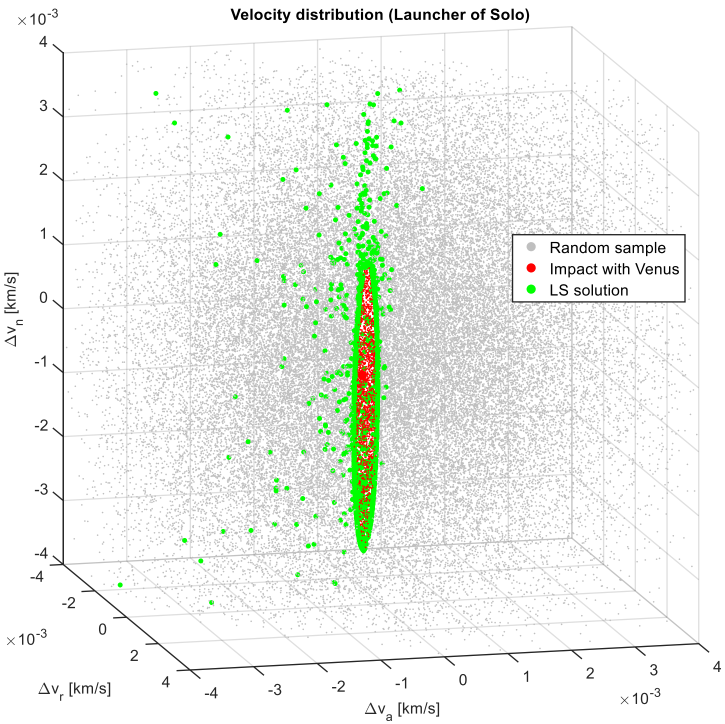
$$\widehat{\sigma}^2\left(\widehat{P}(I)\right) = \frac{1}{N_T(N_T-1)}\sum_{k=1}^{N_T}\left(\widehat{P}^k(I) - \widehat{P}(I)\right)^2$$

Sampling techniques

Test: Launcher of Solo

Solar Orbiter (SolO) is a planned Sun-observing satellite, under development by ESA.

Analysed event: fly-by of Venus



	$N_{Samples}$	N_{Prop}	$\hat{P}(I)$	$\hat{\sigma}$
MCS	54114	54114	4.2e-2	8.6e-4
LS	~54000	~200000	4.3e-2	5.5e-4

LS identifies well the boundaries of the impact region

but

Large expected probability and **compact impact region** make the method less efficient for the same confidence level

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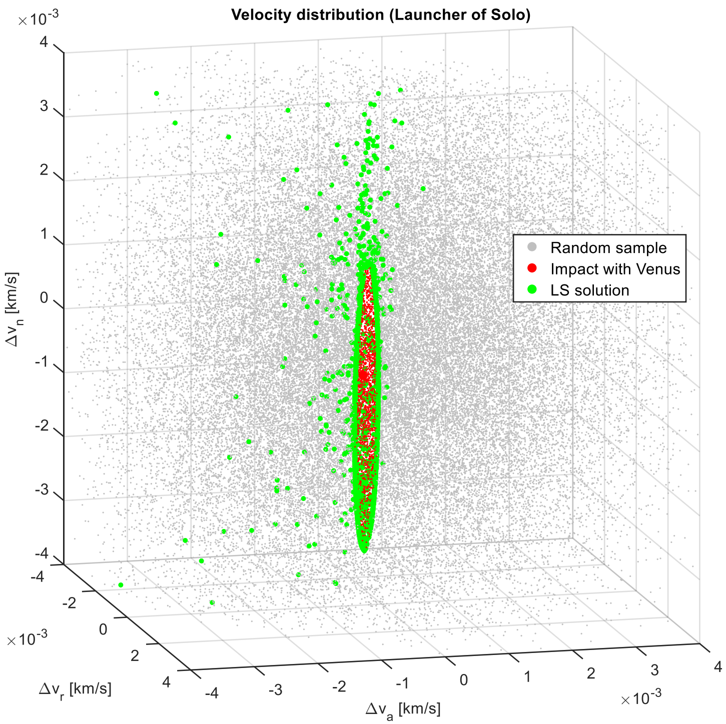

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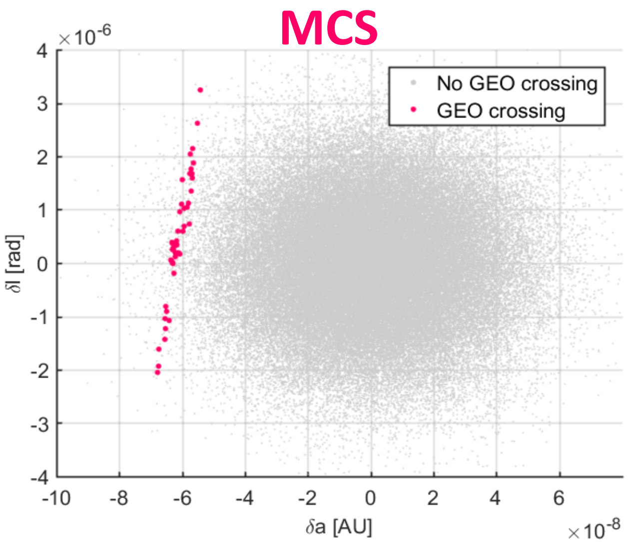
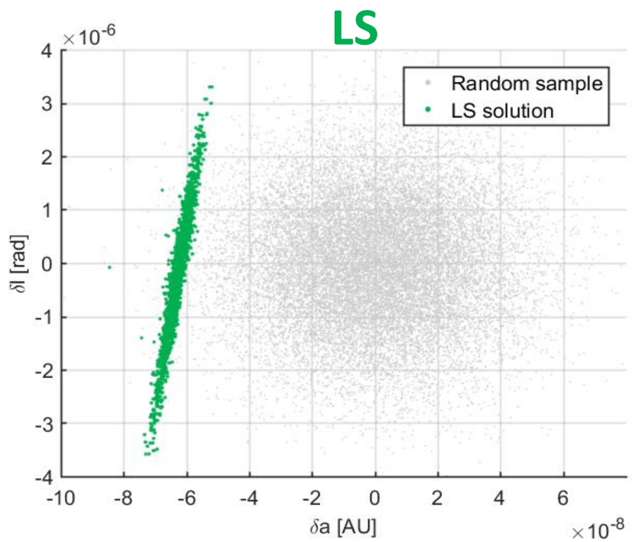
Sampling techniques

Test: Apophis

Analysed event: return in 2036 (according to observations in 2009)[9]





Small expected probability
Distributed impact region

	$N_{Samples}$	N_{Prop}	$\hat{P}(I)$	$\hat{\sigma}$
MCS	1e6	1e6	5.00e-5	6.86e-6
LS	1e4	~1e5	5.38e-5	1.18e-6
	1e5	~1e6	5.32e-5	3.45e-7

Similar confidence level as MC

Similar number of orbital propagations as MC

[9] http://newton.dm.unipi.it/neodys

Sampling techniques

Test: Apophis

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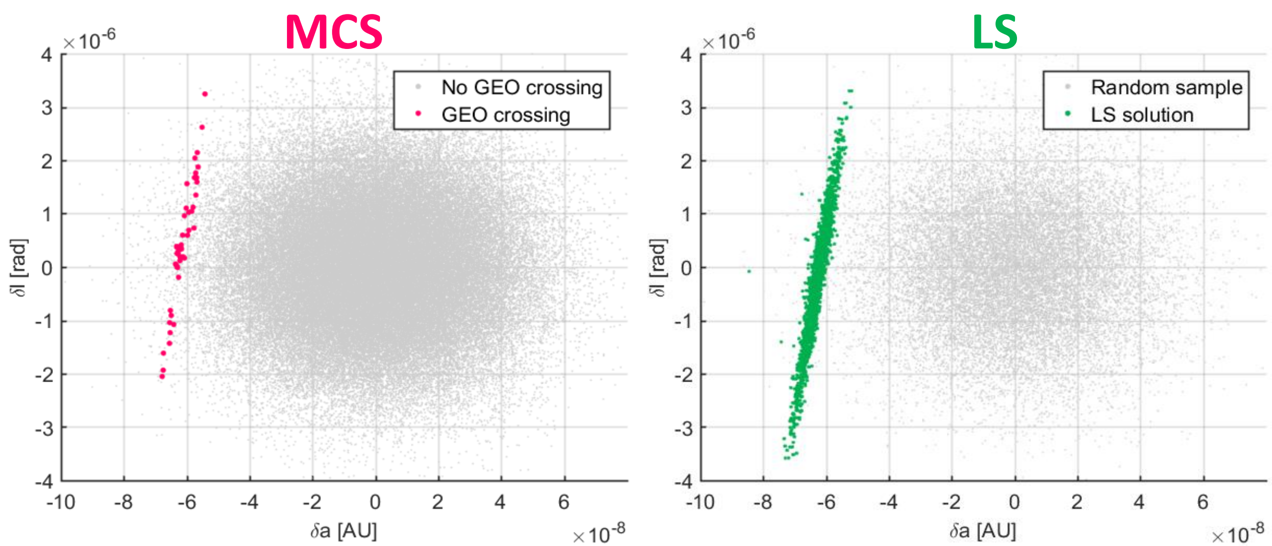

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CONCLUSIONS

Conclusions

Final considerations

- Integration
  - Symplectic methods show good performance in cases of regular dynamics
  - Very close **fly-bys** introduce very large numerical **errors** in the integration
  - Techniques to cancel the effect of a fly-by on the propagation exist and are being investigated

- Sampling
  - LS can achieve a lower variance of the solution (**higher accuracy**) with the same number of random samples, with larger efficiency as the impact probability gets lower
  - Current implementation supposes a unique impact region with a regular shape, during a given time window
  - Current implementation uses **extra evaluations** to probe each line, thus decreasing the efficiency of LS

Conclusions

Future work

- Numerical integration
  - Explore alternative methods (symplectic and non)
  - Explore alternative formulations of the dynamics (Keplerian/equinoctial parameters, Delaunay parameters, universal variables, etc.)
  - Develop other numerical schemes (symplectic and non)
- Uncertainty sampling
  - Obtain an analytical expression for the confidence interval in LS
  - Improve the computation of zeros
  - Explore other techniques to improve efficiency
- **Final goal**: apply the two approaches (efficient sampling and conservative integration) to planetary protection analysis

THANK YOU FOR YOUR ATTENTION
ANY QUESTION?

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