6-DOF HYDRODYNAMIC MODELLING FOR WIND TUNNEL HYBRID/HIL TESTS OF FOWT: THE REAL-TIME CHALLENGE

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ABSTRACT
This paper deals with the numerical approach and technical implementation of the 6-DoF hydrodynamic modelling, combined with the Politecnico di Milano HexaFloat robot, adopted for wind tunnel Hybrid/HIL tests floating offshore wind turbines. The hybrid testing methodology, along with its ocean-basin counterpart, is currently being considered as a valuable upgrade in the model scale experiments, for its capability to get rid of the typical scaling issues of such systems. The work reports an overview of the setup, the general testing methodology, presenting the main challenges about the deployment on the real-time hardware, summarizing the key solving choices. A set of results related to code-to-code comparison between the optimized HIL numerical model and the reference FAST computations are included, confirming the correctness of the approach.

NOMENCLATURE
DoF Degree of freedom
HIL Hardware-In-the-Loop
RT Real Time
IPC Individual Pitch Control
FOWT Floating Offshore Wind Turbines
PoliMi Politecnico Milano
SWE Stuttgart University Wind Energy
HexaFloat 6-DoF PoliMi parallel kinematic robot
CoG Center of Gravity
FAST Reference open-source aeroelastic computer-aided engineering tool for horizontal axis wind turbines
RNA Rotor Nacelle Assembly
FL Fairleads
PSD Power Spectral Density
ODE Ordinary Differential Equation
WAMIT Wave Analysis MIT
DTU Danmarks Tekniske Universitet
$\Sigma$ 6x1 platform displacements
$x_6$ 6x1 simulated platform displacements
$x, y, z$ Platform’s DoF: surge, sway, heave
$\phi, \theta, \psi$ Platform’s DoF: pitch, roll, yaw
$[M_s]$ 6x6 structural mass matrix
$[A_w]$ 6x6 infinite-frequency hydrodynamic added mass matrix
$[R_s]$ 6x6 structural damping matrix
$[K_s]$ 6x6 structural stiffness matrix
$F$ 6x1 external force vector
In this scenario model scale testing which combine the physical phenomena. The choices behind these simplifications required to make the numerical model “lighter” and compatible with RT execution, without losing the consistency with the real-time controller, [13], (c) a 6-components dynamometric balance installed between the turbine model tower base and the HexaFloat’s end effector, measures the internal actions between the floater and the turbine (i.e. aerodynamic and inertial forces). The real-time integration of the equations of motion Eq.1, combining the hydrodynamic forces $F_{wind}$ (computed) and the aerodynamic forces $F_{aero}$ (measured), provides the motion to the robot’s end effector.

The numerical model (Eq.1) is developed in Matlab/Simulink environment and it is verified against a reference FAST model. More specifically, a set of simplifications were required to make the numerical model “lighter” and compatible with RT execution, without losing the consistency with the physical phenomena. The choices behind these simplification are reported and discussed in the paper. The numerical model is then implemented in dSPACE real-time hardware; the equations of motion Eq.1, including computations and measurements, are integrated, so that the motion along the 6 platform DoF $x = \{x, y, z, \phi, \theta, \psi\}^T$ are given to the HexaFloat controller.

The experimental validation of the methodology herein reported can be found, for a 2 DoF $(x, \theta)$ similar setup, in [14].
The floating system adopted is the open-source Triple Spar concept by SWE [16], coupled with the DTU 10 MW wind turbine, [17]. In Fig.2 a sketch of the platform and the mass distribution are reported, whereas the main structural properties can be found in Tab.1. Eq.1 represents the 6-DoF dynamics of FOWT reproduced in wind tunnel through the Hybrid/HIL setup. The correction force vector comes from the measurements provided by the balance \( E_{\text{bal}} \) to which a correction force vector \( E_{\text{corr}} \) needs to be applied.

\[
E_{\text{aero}} = E_{\text{bal}} + E_{\text{corr}}
\]

More specifically, the correction force vector can be expressed as in Eq.4

\[
E_{\text{corr}} = [M]_s\ddot{x}_s + [K]_s\dot{x}_s
\]

where \([M]_s\) and \([K]_s\), reported in detail in Eq.A.4 and A.5, are respectively the mass and stiffness matrices of the physical scale model, defined under the hypothesis of rigid motion and small rotations. These are combined to the simulated DoF \( \ddot{x}_s \), to get the inertial and gravitational contributions of the model itself which are measured along with the aerodynamic forces in \( E_{\text{bal}} \), thus providing \( E_{\text{aero}} \) only. This correction is needed due to the small, although inevitable differences in the mass properties of the model with respect to the full scale target, providing inertial and gravitational contributions of the model. As a further advantage, the presented HIL methodology allows to get rid of Froude scaling [8], enhancing the quality of scale model measurements (e.g. higher wind speed, better signal/noise ratio, Reynolds number discrepancy less penalising) but leads to a scale factor of the acceleration-ancy less penalising) but leads to a scale factor of the acceleration.

Hydrodynamic forces Hydrodynamic forces \( E_{\text{hydro}} \) (Eq.5) are computed and combined with aerodynamic forces \( E_{\text{aero}} \) in real time, on right hand-side of Eq.1. The goal is making the computation as fast as possible, finding a balance between the simplification in the modelling and the consistency with the physical phenomena, object of this work.

\[
E_{\text{hydro}} = E_{\text{rad}} + E_{\text{we}} + E_{\text{stic}} + E_{\text{moor}}
\]
Radiation and wave exciting forces Radiation $E_{\text{rad}}$ and wave exciting $E_{\text{exc}}$ forces are obtained from potential flow problem solved by the 3D panel code WAMIT, [18]. More specifically, $E_{\text{rad}}$ is implemented under the form of a state-space time domain contribution [19], particularly suited for real-time implementation [20], relying on the frequency-dependent added mass and damping matrices derived from panel code computations. Wave exciting forces $E_{\text{exc}}$ are put in Eq.5 under the form of regular or irregular time histories, see [14].

Viscous Forces As reported in Eq.6, Morison’s viscous forces are obtained in the tangential, radial and axial directions, integrating the contribution of single elements along each platform cylinder, depending on the viscous drag coefficients and relative velocities between the wave particle and element’s node kinematics, Eq.A.8.

$$F_L(t) = \int_I f_L(z,t)\,dz = \int_0^1 C_D D \left| v(z)_{rel,t}\right| v(z)_{rel,t} \,dz$$

$$f_L(t) = \int_I f_L(z,t)\,dz = \int_0^1 \frac{1}{2} C_D D \left| v(z)_{rel,t}\right| v(z)_{rel,t} \,dz$$

$$F_{\text{av}}(t) = \int_I f_{\text{av}}(z,t)\,dz = \int_0^1 \frac{1}{2} C_{\text{av}} \frac{\rho}{\rho_c} \left| v(z)_{rel,ax}\right| v(z)_{rel,ax} \,dz$$

Mooring Forces Mooring lines’ forces are taken into account considering the validated formulation in [21], on which the FAST/MoorDyn module is based and whose most relevant elements are herein reported for the sake of clarity. The RT model implements the equations reported in Appendix, although performing specific simplifications, assessed in this paper, mainly due to real-time consistency. The implementation of the complete dynamics of the mooring system was preferred instead of using pre-defined look-up tables adopted for the previous 2 DoF system, [14]. Using look-up tables for the 6 DoF system requires to deploy multidimensional arrays on the HIL real-time controller and the operation turned-out to be unfeasible, both in terms of table dimensions and of look-up operations. Moreover, by using the complete mooring line model it is possible to take into account also velocity-dependent force components resulting in additional linear and quadratic damping for the floating system. The frequency-dependent component of mooring forces cannot be modeled with look-up tables due to quasi-static approach and it has a non-negligible importance in the definition of fatigue loads for the mooring system structural assessment, [22].

After having defined the static equilibrium position, [23], the dynamic response of each element of the catenary, is found by
solving the Eq.7, related to a lumped-mass mooring line model, [21]

\[
[M(\xi)] \ddot{\xi} = F(\xi, \dot{\xi})
\]

(7)

where a position-dependent overall mass matrix \([M(\xi)]\) gives rise to the related inertial forces equilibrated by the corresponding internal forces (Eq.11) of different nature.

\[
[M(\xi)] = [m] + [a(\xi)]
\]

(8)

The mass of the moorings depend both on the mass of the lines and on the hydrodynamic added mass, as in Eq.8 and Eq.A.9. The added mass is divided into transversal \([a_p]\) and tangential \([a_q]\) contributions with respect to the local reference system \((\hat{q})\) of each node, approximated through the line passing between two adjacent nodes, Eq.9 and Eq.10, [21].

\[
\begin{align*}
[m_i] &= \frac{\pi}{4} d^2 I \delta [l]_{i3 \times 3} \\
[a_i] &= [a_p_i] + [a_q_i] = \\
&= \rho \pi \frac{\pi}{4} d^2 I \left( C_{ml}(\delta [l]_{i3 \times 3} - \hat{q} \hat{q}^T) + C_{ml}(\hat{q} \hat{q}^T) \right)
\end{align*}
\]

(9)

\[
\hat{q} = \frac{\xi_{i+1} - \xi_{i-1}}{\|\xi_{i+1} - \xi_{i-1}\|}
\]

(10)

As reported in Eq.11, the internal nodes of the moorings, contribute in terms of tensile loads \(\xi\), damping \(C\), weight \(W\), contact with seabed \(D\) and viscous drag forces \(D\), see Eq.A.10-A.16.

\[
E(\xi, \dot{\xi}) = L_{i+1/2}(\xi) - L_{i-1/2}(\xi) + C_{i+1/2}(\xi, \dot{\xi}) - C_{i-1/3}(\xi, \dot{\xi}) + W + B(\xi, \dot{\xi}) + D_{pi}(\xi, \dot{\xi}) + D_{qi}(\xi, \dot{\xi})
\]

(11)

The integration of Eq.7 allows to define, at each time step, the dynamics of the nodes \((\xi, \dot{\xi})\) and to compute the overall mooring line forces \(E_{\text{moor}}(\xi, \dot{\xi})\) at the fairleads, Eq.12.

\[
E_{\text{moor}}(\xi, \dot{\xi}) = -L_N - C_N + W_N + D_{pi} + D_{qi}
\]

(12)

OPTIMIZATION FOR REAL-TIME IMPLEMENTATION

The optimization of the real time model reported in Eq.1 results in a consistent reduction of the number of computations and operations executed during the Hybrid/HIL tests with regard to the hydrodynamic forces \(E_{\text{hydro}}\) Generally not required adopting usual numerical engineering tools, [24]. This optimization can be roughly summarized in the reduction of: (I) the number of harmonics considered in the spectrum of the irregular sea state simulations \((E_{w}\)) (II) the number of elements dividing the substructure \(E_{\text{sub}}\); (III) the number of nodes composing the mooring lines and (IV) the choice of specific contributions in terms of forces to be considered from the internal nodes of the catenary and (V) general implementation issues.

(I) The number of harmonic components in the spectrum and its frequency resolution is of critical decision and it can’t be kept the same as in off-line simulations (i.e. FAST). A compromise between physical consistency and computational effort must be reached. It was found that decreasing the frequency resolution by 10 times from FAST simulations to HIL implementation was guaranteeing this balance, so that \(\Delta \omega = 0.002\) rad/s was turned to \(\Delta \omega = 0.02\) rad/s keeping satisfactory results. With regard to linear spectrum simulation a range of 0.3-3.3 rad/s was considered sufficiently representative, being the significant contribution of the adopted JONSWAP spectrum falling within this range.

(II) The viscous forces depend on the relative velocity between the platform and water (Eq.A.8). Nevertheless the wave particle speed decreases exponentially [25], and it is acceptable

\[\begin{array}{c|cc}
\hline
\text{HIL} & \text{FAST} \\
\hline
F_c & 0.0548 & 0.0548 \\
M_c & 1.8727 & 1.8657 \\
\hline
\end{array}\]

TABLE 2. Comparison (amplitudes) of the most relevant Morison’s forces for a regular sea state between the optimized model (HIL, Fig.3) and the reference one (FAST) with finer discretization. Wave height 2.2 m and period 8 s.
to reduce the number of nodes in the lower part of the platform where the velocity gradient is lower. The optimal discretization of the platform's cylinder is reported in Fig.3, showing the order of magnitude of the along-wave particle horizontal velocity $u$. The choice of considering a single element in a quite large area of substructure is consistent both with the effective wave velocity and with the low speed gradient, allowing to diminish a relevant number of operations. This is confirmed by the overall viscous forces reported in Tab.2 for a sample regular wave case, along with the results on the decay tests reported and discussed in the following (Fig.11).

(III) The number of nodes in the mooring lines were optimized considering as target the system's natural frequencies provided by decay simulation obtained with reference FAST/MoorDyn model with 100 nodes. As visible from Fig.4, an optimal number of 21 elements, including the anchor point and the fairlead, allows to reach a good compromise, resulting in natural frequencies within 2% of discrepancy with respect to the target. The corresponding optimal layout is reported in Fig.5.

(IV) Considering a combined decay with the given initial conditions $\chi = \{x, y, z, \phi, \theta, \psi\}^T = \{20 m, 20 m, 10 m, 15^\circ, 15^\circ, 15^\circ\}^T$ each contribution on internal nodes of the mooring line (Eq.11) was investigated individually to discern the most relevant ones.

In Fig.6 the mean values $\mu$ and standard deviation $\sigma$ are reported for the $[a_i]$ matrix elements of the node experiencing the largest amplitude of motion ($\#20$). It can be seen how small is the variation of these elements, allowing to consider the added mass of the mooring system in $[M]$ (Eq.A.2 and Eq.A.9) constant over the HIL tests, thus reducing the computational effort. In fact, Eq.A.9 is not required to be inverted at each time step but only during in the preliminary phase defining the mooring steady condition [23].

Fig.7-10 show statistical values (standard deviation $\sigma$ and mean $\mu$ when different from 0) for the different mooring contributions in Eq.11 every are reported for the internal nodes ($\#2-20$).

Regarding viscous forces $Q$, it must be specified that this analysis was carried out in still water (decay), consistent with the reference FAST/MoorDyn module, in which no relative velocity of the elements is considered for these computations, but only the velocity due to the elements themselves. As it is reasonable, Fig.7 shows that only the nodes off the seabed (i.e. $\#\geq 10$, Fig.5) are worth being considered in the computation of this force contribution, saving a great amount of useless operations by neglecting nodes on the seabed. On the contrary, from Fig.8 it is visible that the contribution of the seabed contact $Q$ (Eq.A.16) can be divided into constant elements, related to the very first nodes close to the anchors, then intermediate nodes with varying contribu-
FIGURE 7. Viscous transverse damping $D_p$ contribution (Eq.11) for the internal nodes (#2-20) in the combined decay tests $\mathbf{x} = \{x, y, z, \varphi, \theta, \psi\}^T = \{20 \text{ m}, 20 \text{ m}, 10 \text{ m}, 15^\circ, 15^\circ, 15^\circ\}^T$.

FIGURE 8. Seabed contact contribution $B$ (Eq.11) for the internal nodes (#2-20) in the combined decay tests $\mathbf{x} = \{x, y, z, \varphi, \theta, \psi\}^T = \{20 \text{ m}, 20 \text{ m}, 10 \text{ m}, 15^\circ, 15^\circ, 15^\circ\}^T$.

FIGURE 9. Viscous tangential damping $D_q$ contribution (Eq.11) for the internal nodes (#2-20) in the combined decay tests $\mathbf{x} = \{x, y, z, \varphi, \theta, \psi\}^T = \{20 \text{ m}, 20 \text{ m}, 10 \text{ m}, 15^\circ, 15^\circ, 15^\circ\}^T$.

FIGURE 10. $\varepsilon$ contribution (Eq.11) for the internal nodes (#2-20) in the combined decay tests $\mathbf{x} = \{x, y, z, \varphi, \theta, \psi\}^T = \{20 \text{ m}, 20 \text{ m}, 10 \text{ m}, 15^\circ, 15^\circ, 15^\circ\}^T$.

The above considerations are summarized in Tab.3, reporting the final modelling scheme for the mooring lines.

(V) Many technical/implementation issues arise in the presented real-time modelling; the most relevant ones, which have affected greatly the real time modelling approach, are herein reported. Regarding the integration time step, the mooring lines represent the most binding part of the modelling, requiring a well finer discretization compared to the rest of the model; this is reflected also in the reference FAST/MoorDyn module. An acceptable compromise was found for this floating concept to be of 0.02 s full scale, resulting in a time step around 8e-4 s at model scale, associated with an ODE45 integration scheme.
TABLE 3. Summary of the inclusion of the various mooring line’s force contributions of the internal nodes, from anchor (Δ) to fairlead (△): constant nodes (−), potentially constant (−−), varying (✓) and neglected (X).

<table>
<thead>
<tr>
<th>Node</th>
<th>Surge</th>
<th>Sway</th>
<th>Heave</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D_p</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D_q</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 4. Summary of the comparison between the real-time HIL model and the reference FAST model, including the natural frequencies f, the linear and quadratic damping parameters p and q.

<table>
<thead>
<tr>
<th></th>
<th>f (Hz)</th>
<th>f (Hz)</th>
<th>p</th>
<th>p</th>
<th>q</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIL</td>
<td>0.0052</td>
<td>0.0050</td>
<td>0.24</td>
<td>0.28</td>
<td>0.033</td>
<td>0.033</td>
</tr>
<tr>
<td>FAST</td>
<td>0.0049</td>
<td>0.0049</td>
<td>0.26</td>
<td>0.30</td>
<td>0.034</td>
<td>0.028</td>
</tr>
<tr>
<td>HIL</td>
<td>0.0628</td>
<td>0.0628</td>
<td>0.31</td>
<td>0.31</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>FAST</td>
<td>0.0360</td>
<td>0.0361</td>
<td>0.38</td>
<td>0.32</td>
<td>-0.059</td>
<td>-0.018</td>
</tr>
<tr>
<td>HIL</td>
<td>0.0380</td>
<td>0.0380</td>
<td>0.35</td>
<td>0.29</td>
<td>-0.037</td>
<td>0.001</td>
</tr>
<tr>
<td>FAST</td>
<td>0.0130</td>
<td>0.0130</td>
<td>0.10</td>
<td>0.10</td>
<td>0.014</td>
<td>0.017</td>
</tr>
</tbody>
</table>

RESULTS AND CONCLUSIONS

In Fig.11 and Fig.12 the free decay and irregular sea results (without wind) are reported to compare the HIL model to the reference FAST one, for a subset of selected DoF, that are those envisaging the most significant amplitudes. The HIL model shows an almost overlapped behaviour. The same conclusions can be drawn looking at Tab.4, which reports the corresponding natural frequencies, linear and quadratic damping p and q, respectively defined as intercepts and slope of the graph of $F_n$ vs $\frac{1}{2}(F_n + F_{n+1})$, being $F_n$ and $F_{n+1}$ the peaks of two consequent cycles of the DoF, as defined in [26]. Tab.4 confirms the correctness of the procedure reported, where very close values between HIL and FAST can be seen. This confirms that the sensitivity analysis conducted in the definition of the real-time model, with the burden of selecting the contributions that are actually relevant from an engineering point of view, can be considered satisfactory. Furthermore, following this preliminary analysis, ongoing assessments are being focused on different floating platforms and different sea-conditions contributing to make this methodology more robust; more comprehensive numerical and experimental analysis are expected to be published soon.

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knowledge the master student Lorenzo Delbene, for his help in the implementation of the model during his thesis work.

REFERENCES
Appendix: Analytics

\[ [M] = \begin{bmatrix}
M_{tot} & 0 & 0 & 0 & M_{ed} & 0 \\
0 & M_{tot} & 0 & -M_{ed} & 0 & 0 \\
0 & 0 & M_{tot} & 0 & 0 & 0 \\
0 & -M_{ed} & 0 & M_{44} & 0 & 0 \\
M_{ed} & 0 & 0 & 0 & M_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & M_{66}
\end{bmatrix} \quad (A.1) \]

\[ k_{grav} = g(M_pZ_{pl} + M_tZ_{to} + (M_{ha} + M_{to})Z_{na}) \quad (A.6) \]

\[ [K_i] = \begin{bmatrix}
0 & 0 & 0 & -mg & 0 \\
0 & 0 & mg & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -mb_zg & 0 \\
0 & 0 & 0 & 0 & -mb_xg \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad (A.5) \]

\[ [K_{grav}] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{grav} & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad (A.7) \]

\[ \begin{bmatrix}
v_{x,rel}(z,t) = u(z,t) - v_c(z,t) \\
v_{y,rel}(z,t) = v(z,t) - v_c(z,t) \\
v_{z,rel}(z,t) = w(z,t) - v_c(z,t)
\end{bmatrix} \quad (A.8) \]

\[ W_{i+1/2} = \frac{\pi}{4} d_i^2 (\rho_w - \rho) g_i W_{i+1/2} = \frac{1}{2} (W_{i+1/2} + W_{i-1/2}) \quad (A.10) \]

\[ T_{i+1/2} = E_i \frac{\pi}{4} d_i^2 \left( \frac{1}{r_i} - \frac{1}{\|E_{i+1} - E_i\|} \right) \quad (A.11) \]

\[ C_{i+1/2} = C_{iw} \frac{\pi}{4} d_i^2 \left( \frac{\|E_{i+1} - E_i\|}{\|E_{i+1} - E_i\|} \right) \quad (A.12) \]

\[ \frac{\partial}{\partial t} e_{i+1/2} = \frac{\partial}{\partial t} \left( \frac{\|E_{i+1} - E_i\|}{l_i} \right) \quad (A.13) \]
\[ D_{pi} = \frac{1}{2} \rho_s C_d \| \dot{\xi}_i \cdot \hat{q}_i \| \hat{q}_i - \vec{\xi}_i \| \left( \dot{\xi}_i \cdot \hat{q}_i \right) \hat{q}_i - \ddot{\xi}_i \] (A.14)

\[ D_{pi} = \frac{1}{2} \rho_s C_d \| \dot{\xi}_i \cdot \hat{q}_i \| \left( \dot{\xi}_i \cdot \hat{q}_i \right) \hat{q}_i \] (A.15)

\[ B_i = dl \left[ (z_{bot} - z_i) k_{bot} - \dot{z}_i c_{bot} \right] \hat{e}_z \] (A.16)