



**POLITECNICO**  
MILANO 1863

DIPARTIMENTO DI MECCANICA



## Optimization of cutting conditions using an evolutive online procedure

Del Prete, Antonio; Franchi, Rodolfo; Cacace, Stefania; Semeraro, Quirico

This is a post-peer-review, pre-copyedit version of an article published in JOURNAL OF INTELLIGENT MANUFACTURING. The final authenticated version is available online at:  
<http://dx.doi.org/10.1007/s10845-018-01460-x>

This content is provided under [CC BY-NC-ND 4.0](https://creativecommons.org/licenses/by-nc-nd/4.0/) license



## **Title: Optimization of cutting conditions using an evolutive online procedure**

**Authors: Antonio Del Prete <sup>(1)</sup>, Rodolfo Franchi <sup>(1)</sup>, Stefania Cacace<sup>(2)</sup>, Quirico Semeraro <sup>(2)</sup>**

**<sup>(1)</sup> Università del Salento, Dipartimento di Ingegneria dell'Innovazione - Via per Monteroni, Centro Ecotekne, Edificio "O"- 73100 Lecce (Italy)**

**<sup>(2)</sup> Politecnico di Milano, Dipartimento di Meccanica - Via La Masa 1,- 20156 Milano (Italy)**

**Corresponding Author – mail: [rodolfo.franchi@unisalento.it](mailto:rodolfo.franchi@unisalento.it), phone: 00390832297810.**

### **Abstract**

This paper proposes an online evolutive procedure to optimize the Material Removal Rate in a turning process considering a stochastic constraint. The usual industrial approach in finishing operations is to change the tool insert at the end of each machining feature to avoid defective parts. Consequently, all parts are produced at highly conservative conditions (low levels of feed and speed), and therefore, at low productivity.

In this work, a framework to estimate the stochastic constraint of tool wear during the production of a batch is proposed.

A simulation campaign was carried out to evaluate the performances of the proposed procedure. The results showed that it was possible to improve the Material Removal Rate during the production of the batch and keeping the probability of defective parts under a desired level.

**Keywords: Tool wear, Stochastic constraint, Machining, Optimization.**

### **1 Introduction**

Superalloys are heat resistant alloys of nickel, iron–nickel and cobalt which frequently operate at temperatures exceeding 550 °C. They exhibit high strength, good fatigue and creep resistance, good corrosion resistance and the ability to operate at elevated temperatures for a long time.

Unfortunately, the characteristics that convey Superalloys good high-temperature materials are responsible for their poor machinability and, consequently, their machining is still a challenge, see the works by Devillez et al. (2007), Schornik (2015) and Zhu et al. (2013) to cite a few. As for all materials, the tool wear rate depends on the cutting parameters; for turning they are cutting speed, feed rate and depth of cut. The machining production time also depends on the cutting parameters and practitioners select them balancing the tool change time, which depends on the number of tools necessary to complete a batch, with the machining time, which depends on the cutting parameters. As a matter of fact, by increasing the Material Removal Rate MRR, which is roughly the product of cutting speed, feed rate and depth of cut, the number of tools increases, and the machining time decreases, creating the conditions for the existence of an optimal solution.

A more structured approach is to define an optimization problem characterized by an objective function depending on the cutting parameters (machining cost or machining time are the two classical objective functions; the profit rate is also proposed, but rarely used), and some associated constraints (e.g. power, deformation, roughness etc.).

On this topic many research papers were published since the Taylor's ASME seminal paper in the early twentieth century (Taylor 1907). The published papers differ in many respects (for brevity, only some papers are mentioned):

- the technique used (analytical optimization or heuristic optimizations such GA or PSO) (Ganesan et al. 2011; Zainal et al. 2016);
- the framework considered: statistical or deterministic optimization (Rao 2009);

- the constraints: considered or not considered (Venkata Rao and Pawar 2010; Zhang et al. 2006);
- the specific machining operation with its specific process parameters (Venkata and Pawar 2010; Costa et al. 2011);
- the production framework considered: objective function, batch size, cost and time constant parameters (Yildiz 2013; Klocke et al 2012).

However, the above-mentioned techniques often appear more as mathematical exercises than effective industrial methods and their diffusion at the shop floor is limited to the simplest cases.

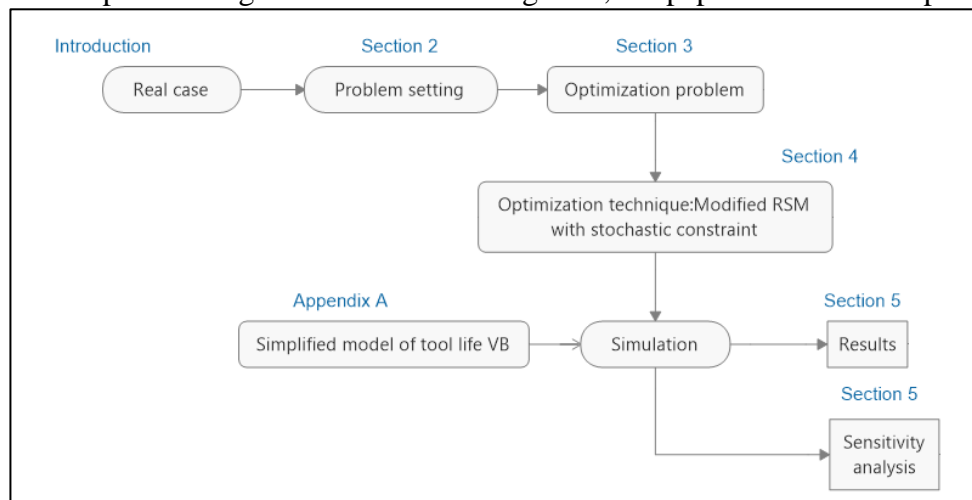
As a matter of fact, only unconstrained optimization problems with a single decision variable (usually the cutting speed) are proposed by tool sellers and used in real industrial cases. Moreover, to our best knowledge the published papers assume that the relationship between the tool wear rate and the cutting parameters is perfectly known. However, this is a condition which is scarcely found in practice if we exclude the most common machining operations and part materials. In literature the prediction of the tool wear has been widely investigated, for example in Wang and Cui (2013), Wang et al. (2014) and D'Addona et al. (2017). However, the approach used in these works employs large dataset to train neural networks. As a result, these techniques cannot be applied to real industrial cases.

For Superalloys it is difficult to find the relationship which links the tool wear rate with cutting parameters since the cost of the material is high, and the batch size is usually moderate-to-small, a preliminary experimentation to estimate the tool wear rate relationship is not a reasonable industrial option.

The objective of the present work is to propose an evolutive online minimization of the machining time having as a constraint the probability of not exceeding a maximum tool wear level. The proposed approach does not require the *a priori* knowledge of the relationship linking the tool wear rate to the cutting parameters as we propose to estimate this relationship using an empirical model fitted during the machining of the batch.

In this paper we propose a new procedure to carry out an online optimization of cutting conditions in a turning operation without any knowledge about the tool wear function. In the optimization problem the objective function is the MRR and a stochastic constraint is considered since we want to maintain the probability of producing defective parts under a desired level.

The real case motivating this work could not be solved through physical experimentation due to time and cost constraints. So, to validate the proposed procedure, a simulation campaign was carried out based on a simplified tool wear model. This simplified case was used to sample tool wear values at different feed and speed during the simulation. In Figure 1, the paper framework is presented.



**Fig. 1.** The framework of the present paper

The paper is organized as follows:

- In Section 2, a detailed problem statement is given based on the motivating industrial case;
- In Section 3, the optimization problem is described;
- In Section 4, the proposed evolutive on line procedure is presented and discussed;
- In Section 5, the proposed procedure is validated with a simulation campaign using a real tool wear function obtained through an experimental campaign (which is described in Appendix A). Later, a sensitivity analysis is performed, and the results are discussed.
- Eventually, in Section 6, the conclusions and future works are discussed.

## 2 Problem statement

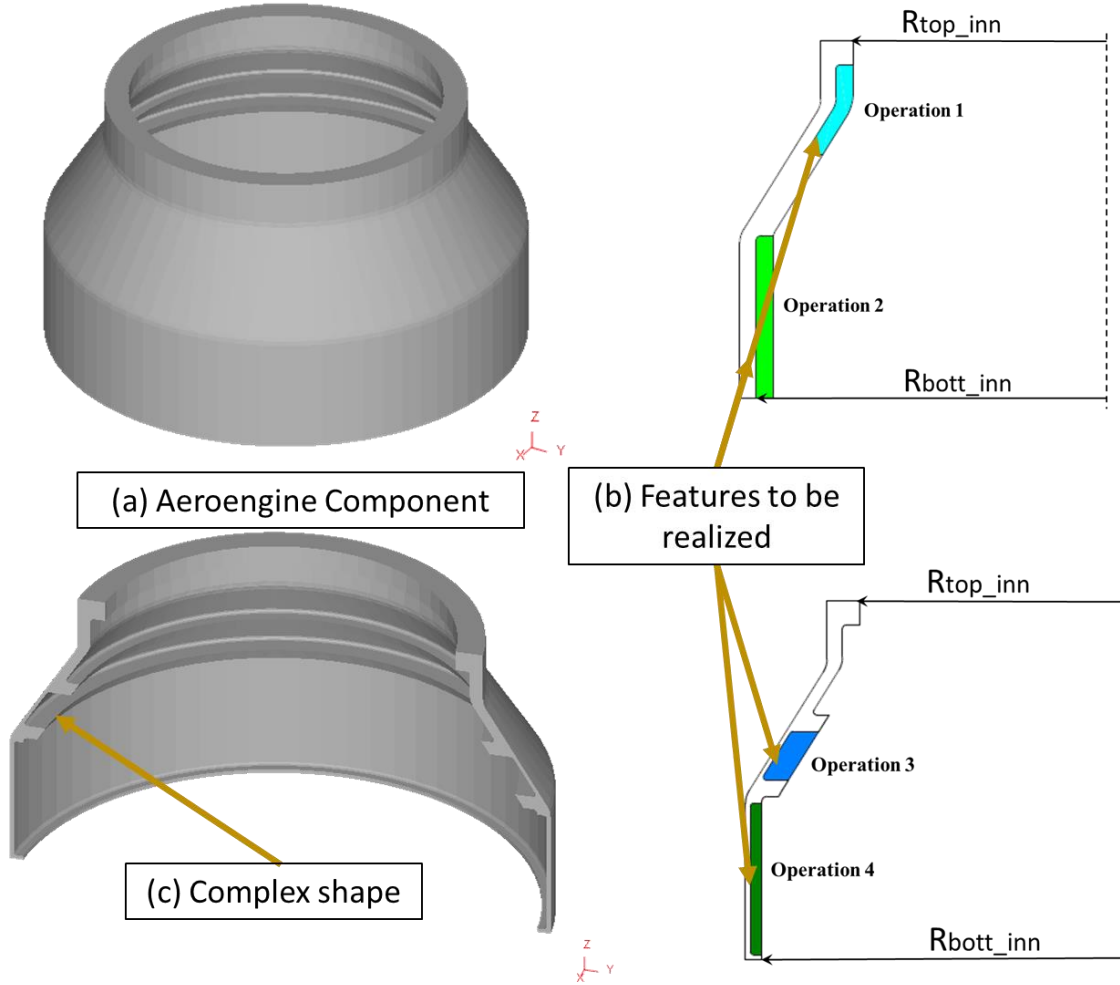
The aeronautic industry growth imposes a deep revision of the design and management processes in accordance with the continuous decrease of profit margins. For this reason, cost saving methods are applied more and more frequently. Moreover, in this sector, hard to cut materials such as Nickel superalloys are commonly used, and parts are produced in moderate – to–small batches.

In the present work, the turning operation of a generic aero-engine component is proposed as the motivating problem and its production is characterized by the above-mentioned criticalities.

A simplified example of these components is shown in Fig.2a. As shown in the Figure 2, the overall cutting process is split into simpler operations called *features* (Figure 2b) with a diameter whose size can vary from large (even larger than 400 mm) to small (approximately 100 mm).

For each *feature*, a process set is defined: tool type, cutting speed (from now on speed), feed rate (from now on feed) and tool path. The section of the removed material instantly changes due to the complex shape of the final part (Fig.2c).

It is important to remark that each *feature* must be machined without interruptions. A tool change during a *feature* execution is not allowed, since cut interruptions can cause material alteration thus leading to discard the part. At the end of a *feature*, the tool should not exceed a certain value set in advance. The tool wear in the industrial context considered is the flank wear VB, it was measured according to ISO 3685 and its maximum value was set at 0.3 mm.



**Fig.2** A 3D view of a typical industrial component geometry (a) description of the particular *features* to be realized (b) for comparison with the complex shape of the part (c)

We have empirically observed that quite often the removed tool inserts show a rather low wear level. These empirical observations convinced us to search for a method to use more efficiently the tool inserts. In particular, it could be possible to set higher values for the cutting parameters reducing the processing time and the production cost and satisfying the tool wear constraint.

The goal of this paper is to propose an evolutive online method to minimize the machining time for each *feature*, with a constraint on the maximum allowable flank wear VB on the tool insert.

The optimization problem could be easily solved if the relationship between the VB and the cutting parameters were known (Appendix B). Unfortunately, this relationship is not known in our case.

As previously stated, the estimate of the VB relationship cannot be carried out on the actual industrial components before the start of production for the following two reasons:

- A) Due to the complex geometry of each *feature* and the complex shape of the removed volume within the same *feature* (Figure 1c), the tool works in continuously variable instantaneous chip section (even if it maintains constant values of speed and feed). Therefore, a correct estimate of the relationship between tool wear and cutting parameters could be obtained only for each cutting path which is specific to each single *feature*.
- B) To analyse the Wear function for speed and feed values different from those used in the industrial certified process, it would be necessary to use a costly large instrumented Machine tool normally used for the ordinary production, and also to make Tool Wear tests on real industrial components. The real industrial components probably should be scrapped after the experimentation and, due to their high value, this eventuality cannot be considered.

The high number of *features* per component, the cost considerations previously stressed, the need to fit the relationship between the VB and the cutting parameters for each *feature* cutting path involve the practical impossibility to use an experimental approach with the real industrial components before the production of the batch.

Even if in literature several authors studied the effects of the cutting parameters on process performances and developed different optimization strategies, none of them, as far as we know, tried to minimize the machining time by using the information acquired online, from part to part, considering the risk of exceeding a defined value of the tool wear.

The idea proposed in this work is an evolutive online optimization methodology whereby:

- After a real *feature* machining, a dedicated system or the operator measures the VB (this is the actual industrial practice to assess the tool wear level);
- The VB information is used to estimate a tool wear model;
- A set of decision rules selects a new set of cutting conditions, and the new parameter set is used to machine the next component until the batch is completed.

The proposed procedure is based on the well-known Repose Surface Methodology proposed by Box and Wilson (1992). In its original form, RSM solved on-line and unconstrained optimization problems with an unknown stochastic objective function that could be measured through well-designed experiments. Readers interested in this method can refer to Myers et al. (2009) and Del Castillo (2007). In our case the objective function is deterministic, but a stochastic constraint was considered. As far as we know, this type of problems has not been solved yet. The unique related paper is by Angün et al. (2009) where the topic of the problem was simulation experiments. As a consequence, the stochastic constraint could be estimated through sampling techniques and not through real experiments.

### 3 The optimization problem

In cutting condition optimization, common objective functions are: the machining time per part, the machining cost per part and the profit rate per part. The first two objective functions are the most popular ones; the third one is rarely implemented since it requires the definition of the revenue associated with the single machining operation, therefore we shall not consider it.

The objective functions “cost” or “time” share the same function structure and are the sum of three terms.

- The first term does not depend on the cutting parameters and it is of no interest in the optimization.
- The second term is the machining time (or cutting contact time) which is the volume of material to be removed divided by the material removal rate MRR. If we want to consider the machining cost, we must multiply the machining time by the constant hourly machine cost.
- The third term is the time to change the insert and it depends on the number of times we must change the worn insert to complete the batch. In machining time optimization problem, this number must be multiplied by the tool change time. In machining cost optimization, we must consider the cost of the insert. It could be useful to remind that the number of times we change the insert depends on the tool life which depends on the MRR.

If we increase the material removal rate the second term decreases, but the third term increases (the tool life decreases) and the conditions for a possible optimum are realized. The interested reader can consult a standard manufacturing book (Kalpakjian and Schmidh 2001, Davim 2008) for further details. However, our problem has a distinctive characteristic to consider since the tool insert is changed every time we complete the machining of a *feature*. This implies that the third term vanishes in the objective function formulation.

The Objective functions becomes:

$$\text{Machining time} = C + \frac{\text{Material volume}}{MRR}$$

$$\text{Machining time} = C^* + \frac{\text{Hourly machine cost} \times \text{Material volume}}{MRR}$$

It is easy to see that the minimization of these objective functions is equivalent to the maximization of the MRR which is roughly proportional to the product of the cutting parameters.

Before describing our method, it is appropriate to define the optimization problem in greater detail. Without loss of generality we shall simplify the *feature* by referring to a simple cylindrical turning operation. The material removal rate is the product  $fvp$  where  $f$  is the feed,  $v$  is the speed and  $p$  is the depth of cut. The volume of material to be removed per operation is  $V=Yp$  where  $Y$  is the tool path. The time to complete a feature is  $V/MRR$  or  $Y/fv$ . With an appropriate definition of the constant  $Y$ , this result holds even for more complex *features* as the ones we are considering.

The objective function considered is to minimize  $Y/fv$  where  $Y$  is a constant depending on the tool path and the specific geometry of the *feature* and  $f$  and  $v$  are the decision variables. Considering that  $Y$  is constant, the minimization is equivalent to the maximization of  $fv$ . In the real case study considered for this work, the decision variables are constrained to have a wear level  $VB_0$  less than 0.3 mm at the end of the *feature* machining.

During the evolutive online procedure we need a function to predict the wear level when changing the cutting conditions. To do that we use a simple first-degree polynomial with a multiplicative term

$$VB(v, f) = \beta_0 + \beta_1 v + \beta_2 f + \beta_{12} vf + \varepsilon \text{ with } \varepsilon \sim \text{NID}(0, \gamma_\varepsilon^2) \quad (1)$$

To estimate the function (1) we measure the  $VB$  at the end of the machining of each *feature* and with a linear regression approach, the  $VB$  function is estimated before deciding the cutting conditions of the procedure next step. The fitted model  $VB(v, f)$  is used to predict the wear level when changing the decision variables (note that the **hat on** is a standard convention to signal that the quantity under the hat is estimated from experimental data). However, since the  $VB(v, f)$  is a random variable both because the model (1) has the stochastic component  $\varepsilon$  and because the deterministic part is estimated by experimental data, the stochastic constraint on the wear level is expressed by:

$$\text{Prob}(VB(v, f) \leq VB_0) \leq \alpha \quad (2)$$

where

- $VB(v, f)$  is model (1) fitted using experimental data.
- $VB_0$  is the wear limit;
- $\alpha$  is the risk we accept to violate the constraint, which is the desired average percentage of defective parts.

The probability statement (2) can be formulated as:

$$VVB(v, f) - VB_0 \leq 0 \quad (3)$$

where  $VVB(v, f)$  is the inferior limit of the unilateral flank wear prediction interval with confidence coefficient  $1-\alpha$  and it is defined as:

$$VVB(v, f) = \hat{\beta}_0 + \hat{\beta}_1 v + \hat{\beta}_2 f + \hat{\beta}_{12} vf + t_{1-\alpha}(df_E) \sqrt{\left(1 + \mathbf{x}^T (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{x}\right) \hat{\gamma}_\varepsilon^2} \quad (4)$$

Note that in (4):

- $\mathbf{x}^T = \{1, v, f, vf\}$  is the point where we want to estimate the future tool wear according to the fitted model;
- $\mathbf{X}$  is the design matrix;
- $df_E$  are the error degrees of freedom in the fitting procedure;

- $t_{1-\alpha}(df_E)$  is the  $1-\alpha$ -quantile of t-Student distribution with  $df_E$  degrees of freedom;
- $\hat{\gamma}_\varepsilon^2$  is the estimated variance of the regression model (1);
- The quantities with the hat on are the estimated model parameters at the current step.

Equation (4) is a standard result in linear regression models. The interested reader can consult a quite popular book (Draper and Smith 2005) on linear regression topics and the development of the prediction intervals.

Two different algorithms are proposed and investigated to estimate  $VB(v, f)$ . The first one is the local algorithm (**L**), which fits the function (1) using the results of the last full factorial design. In local algorithm, the function (1) is always estimated with the same number of data, i.e.  $2^2 + n_C$ .

The second algorithm is called historical (**S**) and in this case equation (1) is fitted using all the results obtained so far during the batch manufacturing.

Eventually, the optimization problem is expressed as:

$$\begin{aligned} \min_{v, f} \frac{Y}{v \cdot f} \\ VVB(v, f) - VB_0 \leq 0 \\ v_{\min} \leq v \leq v_{\max} \\ f_{\min} \leq f \leq f_{\max} \end{aligned} \quad (5)$$

#### 4 Proposed evolutive online methodology

The proposed evolutive optimization procedure works through the following steps:

##### Step 1. Machining.

- Select a starting point  $(v_0, f_0)$ , e.g. the current operating conditions;
- Build a  $2^2$  full factorial experiment centered in the starting point, the central point is replicated  $n_C$  times;
- Machine and measure the tool wear for each experimental condition.

##### Step 2. First order model.

Fit a first order model with interaction for the flank wear  $VB$  using equation (1).

##### Step 3. Optimization problem.

Solve the optimization problem (5)

Let us indicate the optimal solution  $(v^*, f^*)$ .

##### Step 4. Steepest ascent direction.

- The line joining the center point  $(v_0, f_0)$  to  $(v^*, f^*)$  is the steepest ascent direction;
- A new point is set at a step Delta ( $\Delta$ ) from the previous center point along the steepest direction. The step  $\Delta$  is defined as a percentage of the total distance between the two points,  $(v_0, f_0)$  and  $(v^*, f^*)$  in order to have a conservative approach to the violation of the wear constraint;
- Let us indicate  $(v_1, f_1)$  the point at distance  $\Delta$  from the previous center point.

##### Step 5. Check of the part produced.



- Check the total number of good parts (i.e. satisfying the constraint) produced;
- If the total number of parts produced is less than the Batch size and we have enough parts to complete the full factorial design, go back to step 1 after choosing the working conditions  $(v_1, f_1)$  as the new  $(v_0, f_0)$ ;
- Otherwise produce the remaining parts using the cutting conditions  $(v_1, f_1)$ .

## 5 Procedure Validation

The procedure should be validated in a real context. However, as discussed in the Section 2, this is unfeasible given the high costs of the aeronautical parts we are considering. One could argue that the procedure could be applied for a simpler process and for materials that are easier to cut, thus validating the procedure in a simpler case. This is true but, even in this case, to have enough data to give the validation process a statistical significance the cost would be prohibitive for our budget.

The only possibility left is to use a numerical simulation fed by realistic data and this was the choice we opted to. In particular, we consider a simple cylindrical turning operation with a constant depth of cut while feed  $f$  and speed  $v$  are the decision variables.

To carry out the simulations, a tool wear equation is required to mimic the real wear behaviour. To have a realistic function, a physical experimental campaign was carried out and the  $VB$  was measured at different levels of feed and speed. Data were analysed and a statistical model linking  $VB$  with feed  $f$ , speed  $v$  and tool contact time  $t$  was obtained where, as mentioned previously, the tool contact time  $t$  depends on the process parameters as:

$$t = \frac{Y}{vf}$$

where  $Y$ , in the simple example considered, is the volume to remove divided by the constant depth of cut. The  $VB$  empirical model used for all the simulations is:

$$\ln(VB(v, f)) = 76.6 - 1.763 \ln t - 40 \ln v - 9.25 \ln f + 0.0892 (\ln t)^2 + 5.03 (\ln v)^2 + 0.549 \ln v * \ln t + 0.549 \ln t * \ln f + 2.095 \ln v * \ln f + \theta \quad \text{where } \theta \sim NID(0, \sigma^2) \quad (6)$$

and  $\sigma^2 = 0.02922$

The empirical model was fitted using the maximization of the **R-sq(adj)** parameter.

See Appendix A for the relevant details.

The simulator implementing the solution scheme is written in Matlab®. The simulation input parameters are changed to study their influence on the performance of the two proposed evolutionary algorithms.

### 5.1 Simulation results

The goal of the first analysis is to evaluate the performance of the proposed evolutive procedure considering the main problem parameters: the number of replicates of the center point  $n_C$ , the batch size  $B$ , the two algorithm versions, *local L* and *historical S*.

A standard full factorial design has been used. The factors and their levels are:

- Number of replicates of the center point  $n_C$ : [2,3,4];
- Batch size  $B$ : [30, 50, 100];
- Algorithm version: [**L**, **S**].

The 18 different conditions were replicated 100 times. The other input parameters of the simulations are in Table 1.

**Table 1** Values of parameters used through the simulation runs

Parameter	Value
$\alpha$	0.05
$\Delta$	30%

$(v_0, f_0)$	(60 m/min, 0.22 mm/rev)
$VB_0$	0.3 mm
$Y$	8000 mm <sup>2</sup>

The goodness of the algorithm was evaluated comparing the total simulated batch production time  $t_s$  with the total production time of the theoretical ideal condition  $t_{ott}$ . If the tool wear equation had been known in advance, the theoretical average minimum time to produce a batch  $B$  would have been:

$$t_{ott} = t_u B(1 + \alpha) \quad (7)$$

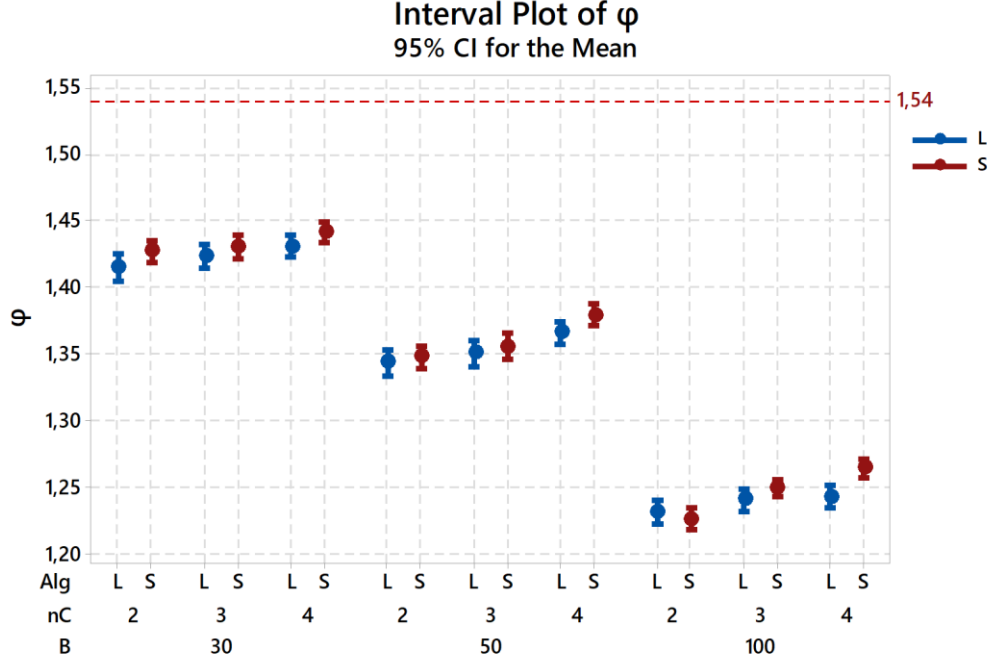
where  $t_u$  is the unitary theoretical optimal production time [Appendix B] and  $\alpha$  is the maximum allowed expected scrap piece percentage. The simulated machining time  $t_s$  is given by the sum of the production time for each part of the batch computed during the simulation.

The performance index  $\varphi = t_s / t_{ott}$  is used to evaluate the validity of the proposed procedure. The simulated time  $t_s$  is higher than the ideal condition, because the procedure starts from a non-optimal combination  $(v, f)$ . The results of the simulation campaign are in Table 2.

**Table 2** Average  $\varphi$  and, in brackets, the standard deviation over 100 replicates.

<b>Local algorithm</b>			
	$n_C=2$	$n_C=3$	$n_C=4$
B=30	1.4159 (0.0512)	1.4242 (0.0443)	1.4315 (0.0421)
B=50	1.3437 (0.0466)	1.3506 (0.0476)	1.3661 (0.0423)
B=100	1.2308 (0.0461)	1.2407 (0.0419)	1.2428 (0.0427)
<b>Historical algorithm</b>			
	$n_C=2$	$n_C=3$	$n_C=4$
B=30	1.4257 (0.0067)	1.4306 (0.0449)	1.4419 (0.0387)
B=50	1.3482 (0.0417)	1.3558 (0.0489)	1.3796 (0.0428)
B=100	1.2263 (0.0400)	1.2492 (0.0325)	1.2644 (0.0366)

In Figure 3 the  $\varphi$  simulation results and its value in the starting condition, about 1.54, are reported.



**Fig. 3** Simulation results

The simulation results in Table 2 and Figure 3 suggest that:

1. In all the tested conditions, the proposed procedure improves compared to the initial conditions. For example, when the batch is equal to 30 the average improvement is about 10%, while if  $B=100$  the average improvement is about 25%.
2. As the batch size  $B$  increases, the proposed procedure performance improves. As a matter of fact, as the batch size increases the algorithm is able to move the cutting conditions towards the theoretical optimal one (perfect *a priori* knowledge). When the batch size is small (i.e. 30) the number of experiments needed to estimate the  $VB$  function prevents the algorithm to move too far from the starting point and its performances get worse;
3. The **Local** algorithm performs comparably or slightly better than the *historical* one;
4. As the number of center points  $n_C$  increases, the performance gets worse. To explain this, let us consider the factorial design  $2^2$  with  $n_C$  central points, in this case the points with the lower production time (or highest MRR) are the top right corner of the design while the central points have a lower value of MRR. A high number of central points implies the production of parts with a combination of process parameters  $(v, f)$  which increases the total production time.

In conclusion, the proposed procedure allowed to improve from the initial combination of parameters and the introduction of the stochastic constraint in a RSM framework was proven to be beneficial from a production point of view. Moreover, it was showed that even for small batches it was possible to improve the machining time. The simulation results suggest that the best experimental design to be used for the estimation of the tool wear is a  $2^2$  factorial with only two center points. Eventually, the local algorithm should be used and therefore for the estimate of the tool wear only the last  $2^2+n_C$  results should be used.

## 5.2 Sensitivity Analysis

To study the influence of the fixed parameters on the new proposed methodology, a sensitivity analysis was carried out. We focused on the following parameters:

- $\sigma^2$ , i.e. **VB** true model variance;
- $Y$ , volume to be removed per piece, divided by the constant depth of cut;
- $\alpha$ , type I error;
- $B$ , batch size;
- $\Delta$ , step size along the steepest ascent direction.

Previously, we have shown that the proposed algorithm performs better with two replicates of central points ( $n_C$ ) and that the local algorithm has more consistent results for all the different conditions. The simulations carried out for the sensitivity analysis are based on these results.

It is important to notice that factors  $\alpha$  and  $\sigma^2$  influence the tool wear constraint in equation (4). In particular, as  $\alpha$  decreases, the constraint becomes tighter, preventing the procedure to get close to the theoretical optimal point. For this reason, if we change the type I error, a variation in the theoretical optimal process conditions occurs. On the other hand, as  $\sigma^2$  increases, the lower limit of the prediction interval (4) changes, influencing negatively the capability of the algorithm to reach the theoretical optimal point.

Moreover,  $Y$  also influences the theoretical optimal point. As  $Y$  increases, speed and feed must be changed accordingly to complete the turning operation without exceeding the tool wear constraint. By inspecting equation (3) we see that  $Y$  is related to the machining time  $t$  and to the theoretical optimal conditions  $(v_{opt}, f_{opt})$ . Since different parameter combinations generate different theoretical optimal conditions  $(v_{opt}, f_{opt})$ , a new index is used to compare the results.

We are interested in quantifying the improvement given by the procedure in respect to the basic case, i.e. no procedure applied, accounting also for the distance of the starting conditions from the theoretical optimal conditions of the problem.

The used new index is  $\eta = \frac{t_s - t_i}{t_{opt} - t_i}$  where:

- $t_s$  is the batch production time;
- $t_i$  is the batch production time at the starting conditions;
- $t_{opt}$  is the batch theoretical optimal production time, calculated according to equation (6) and the procedure described in Appendix B.

A  $3^5$  full factorial design consisting of 243 different combination of input is used. The details are reported in Table 3. Each parameter combination is simulated 100 times and the number of the experiments used to estimate the tool wear function at each procedure step is constant and equal to six (as the number of center point,  $n_C$ , is set to 2).

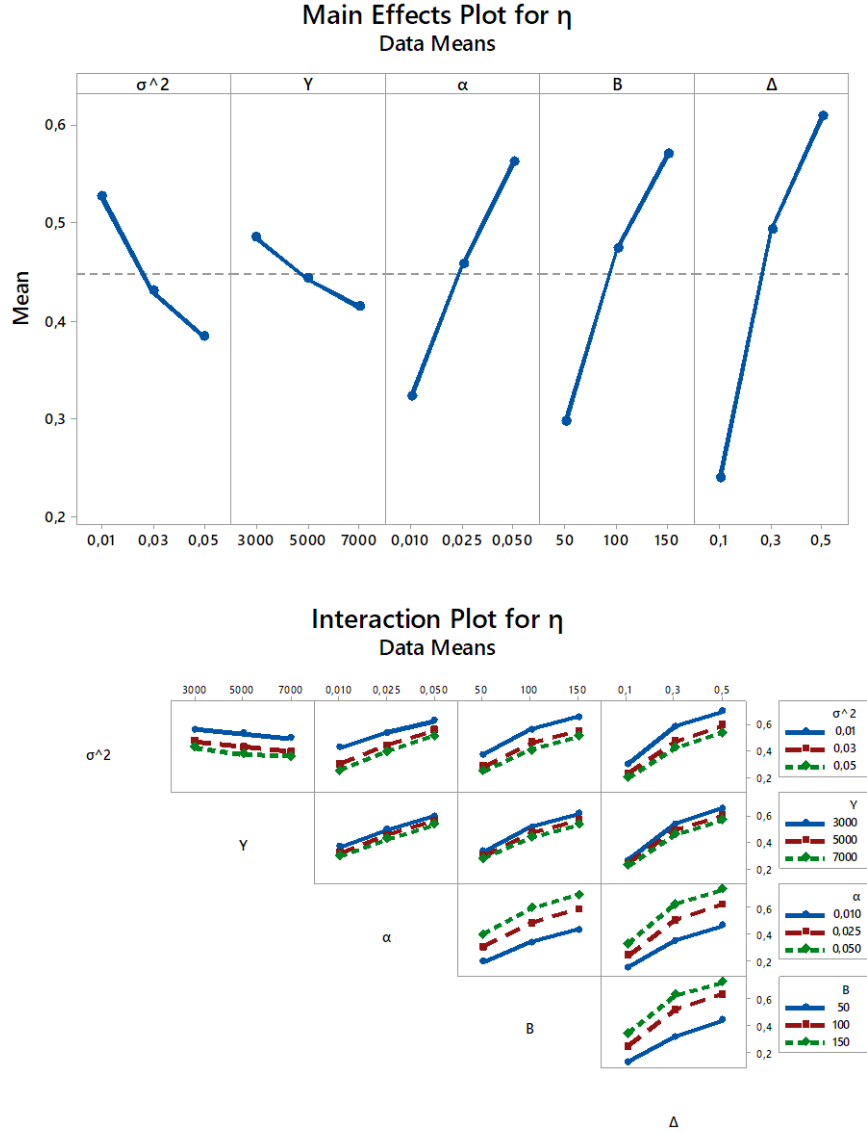
**Table 3** Factorial design for sensitivity analysis

	Low Level	Medium level	High level
$\sigma^2$	0.01	0.03	0.05
$Y$	3000	5000	7000
$\alpha$	0.05	0.025	0.01
$B$	50	100	150
$\Delta$	0.1	0.3	0.5

The two followings quantities are recorded and analyzed:

- the index  $\eta$ ;
- $std_{\eta}$ , the standard deviation of the index  $\eta$ , computed over the 100 replicates for each parameter combination.

The results for index  $\eta$  are reported in Figure 4.



**Fig. 4 Main Effect Plot and Interaction Plot for index  $\eta$ .**

As shown in the Main Effects plot, the most significant parameters are the batch size  $B$ , the step size  $\Delta$  and the type I error  $\alpha$ . As it was expected, the performance of the procedure decreases as  $\alpha$  decreases, since the improvement reduces when the wear constraint is more stringent. When the step  $\Delta$  along the steepest ascent direction is increased, the procedure will get close to the theoretical optimal point faster. An increase in the batch size  $B$  results in a better performance of the algorithm, since it has more attempts to get closer to the theoretical optimal condition. As expected, when the process noise  $\sigma^2$  increases, the procedure performance decreases. The interaction plot does not exhibit an interaction pattern as the lines are nearly parallel and we expect that the two factor interaction model components have a modest influence on the response variable.

The ANOVA Table is reported in Table 4.

**Table 4 F-values and p-values of the ANOVA table for the sensitivity analysis on index  $\eta$** 

Factor	Type	Levels	Values
$\sigma^2$	Fixed	3	0.01; 0.03; 0.05
Y	Fixed	3	3000; 5000; 7000
B	Fixed	3	50; 100; 150
$\Delta$	Fixed	3	0.1; 0.3; 0.5
$\alpha$	Fixed	3	0.010; 0.025; 0.050

**Analysis of Variance**

Source	DF	Adj SS	Adj MS	F-Value	p-Value
$\alpha$	2	23.450	117.251	2002.92	0.000
B	2	31.173	155.863	2662.51	0.000
$\Delta$	2	58.117	290.585	4963.90	0.000
$\sigma^2$	2	0.8701	0.43506	743.19	0.000
Y	2	0.2027	0.10136	173.15	0.000
B* $\alpha$	4	0.0220	0.00550	9.39	0.000
$\Delta$ * $\alpha$	4	0.0860	0.02149	36.71	0.000
B* $\Delta$	4	0.0890	0.02224	38.00	0.000
$\sigma^2$ *Y	4	0.0061	0.00153	2.61	0.037
$\sigma^2$ *B	4	0.0054	0.00136	2.32	0.058
$\sigma^2$ * $\Delta$	4	0.0294	0.00736	12.57	0.000
$\sigma^2$ * $\alpha$	4	0.0305	0.00763	13.04	0.000
Y*B	4	0.0048	0.00120	2.05	0.089
Y* $\Delta$	4	0.0152	0.00379	6.48	0.000
Y* $\alpha$	4	0.0011	0.00027	0.46	0.766
Error	192	0.1124	0.00059		
Total	242	127.487			

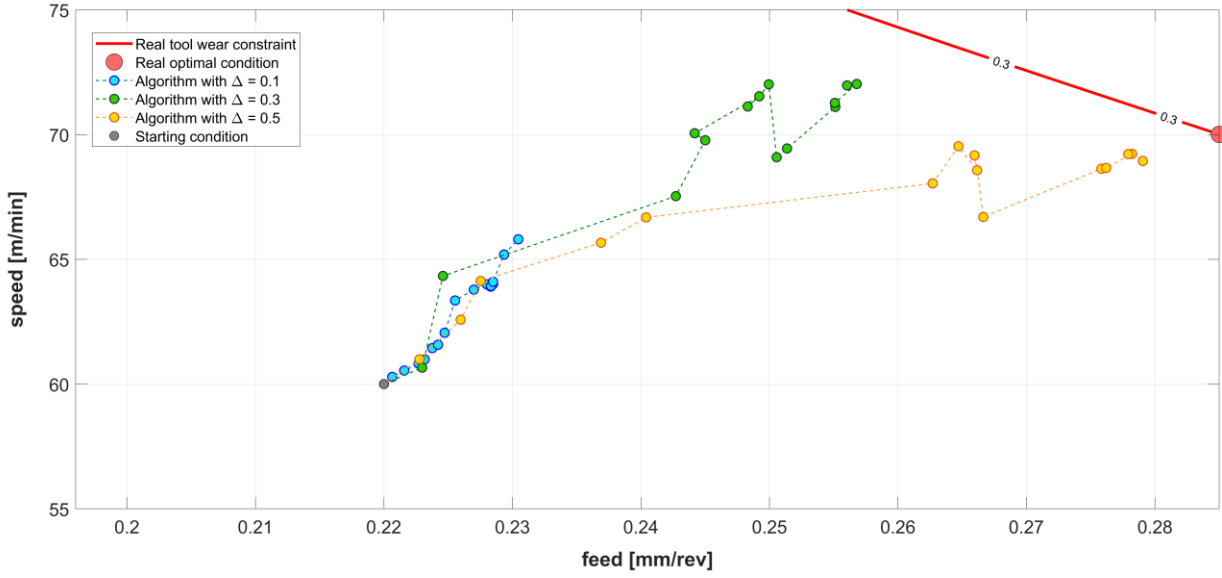
**Model Summary**

S	R-sq	R-sq(adj)	R-sq(pred)
0.0241950	99.12%	98.89%	98.59%

The ANOVA results in Table 4 confirm the qualitative remarks discussed above. Two interactions are statistically significant, but the low F-values suggest that they are not important from a practical point of view.

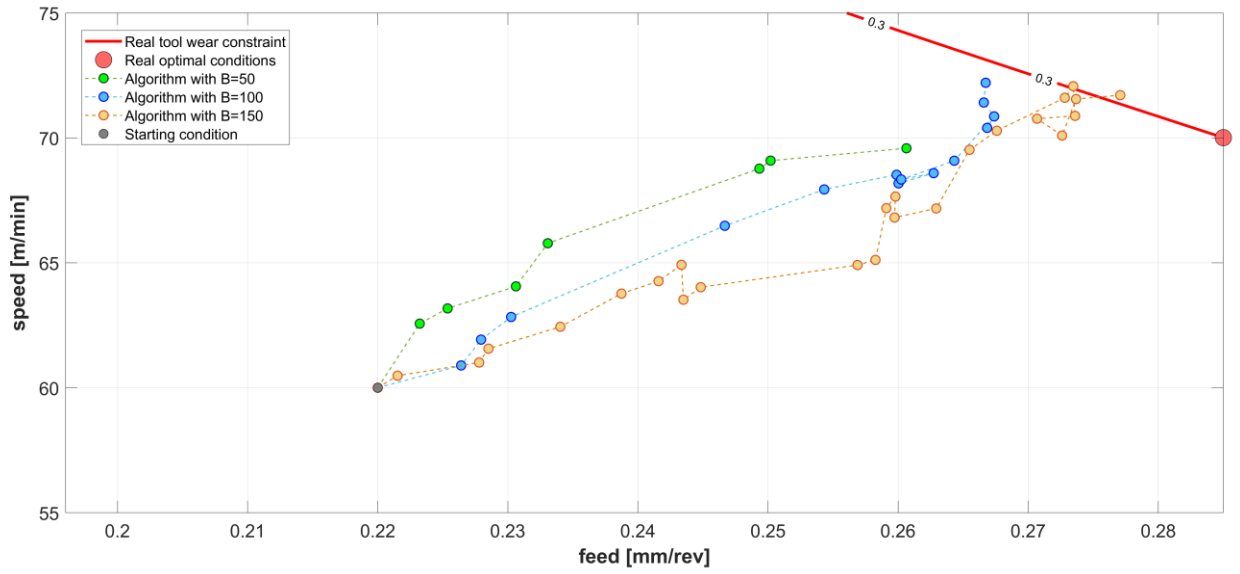
The behaviour of the evolutionary algorithm in different conditions for demonstration purposes only is now discussed. For a better readability, we will plot only one of the 100 replicates for each condition. In Figures 5-7 only the centers of the experimental designs are shown to simplify the reading of the plot.

In Figure 5, the behaviour of the evolutionary algorithm as the step size changes is shown. The theoretical constraint is in red and the theoretical optimal condition (red circle) is placed on it. When we say theoretical, we mean that we are using the true tool wear equation, not the ones estimated at each algorithm step. This is a key point to understand why the centre points does not get close to the theoretical optimal points in some circumstances.

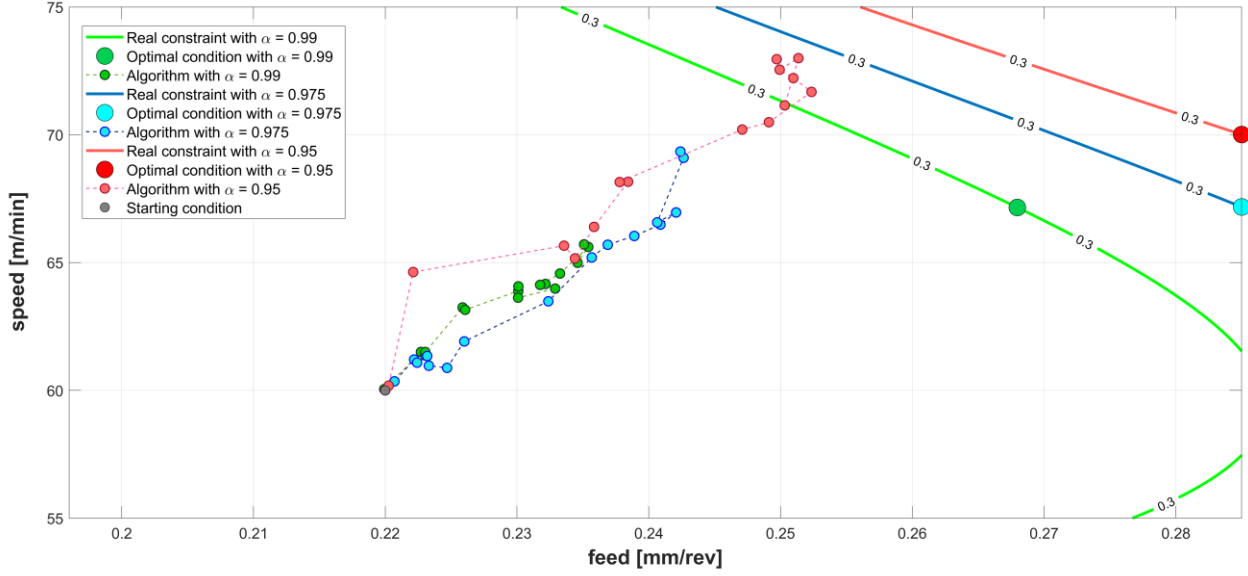


**Fig. 5** Proposed procedure behaviour with different step sizes  $\Delta$  ( $\sigma^2 = 0.03$  Local,  $n_c=2$ ,  $B=100$ ,  $\alpha=0.025$ ,  $Y=5000$ )

By inspecting the Figure 5, it is possible to observe the great influence of the step size  $\Delta$  in going quickly to the VB constraint. This result confirmed the ANOVA analysis in Table 4 where the F-value is the maximum observed (509.41).



**Fig. 6** Proposed procedure behavior with different batch sizes  $B$  ( $\sigma^2 = 0.03$ ,  $\Delta = 0.3$ , Local,  $n_c=2$ ,  $\alpha=0.025$ ,  $Y=5000$ )



**Fig. 7** Proposed procedure behavior with different batch sizes  $B$  ( $\sigma^2 = 0.03$ ,  $\Delta = 0.3$ , *Local*,  $n_c = 2$ ,  $\alpha = 0.025$ ,  $Y = 5000$ )

The influence of the batch size on the proposed procedure performances is evaluated in Figure 6. The path in blue ( $B=150$ ) approaches the theoretical optimum while when the batch size is small ( $B=50$ ) or ( $B=100$ ) the procedure stops far away from the true constraint and the theoretical optimal condition.

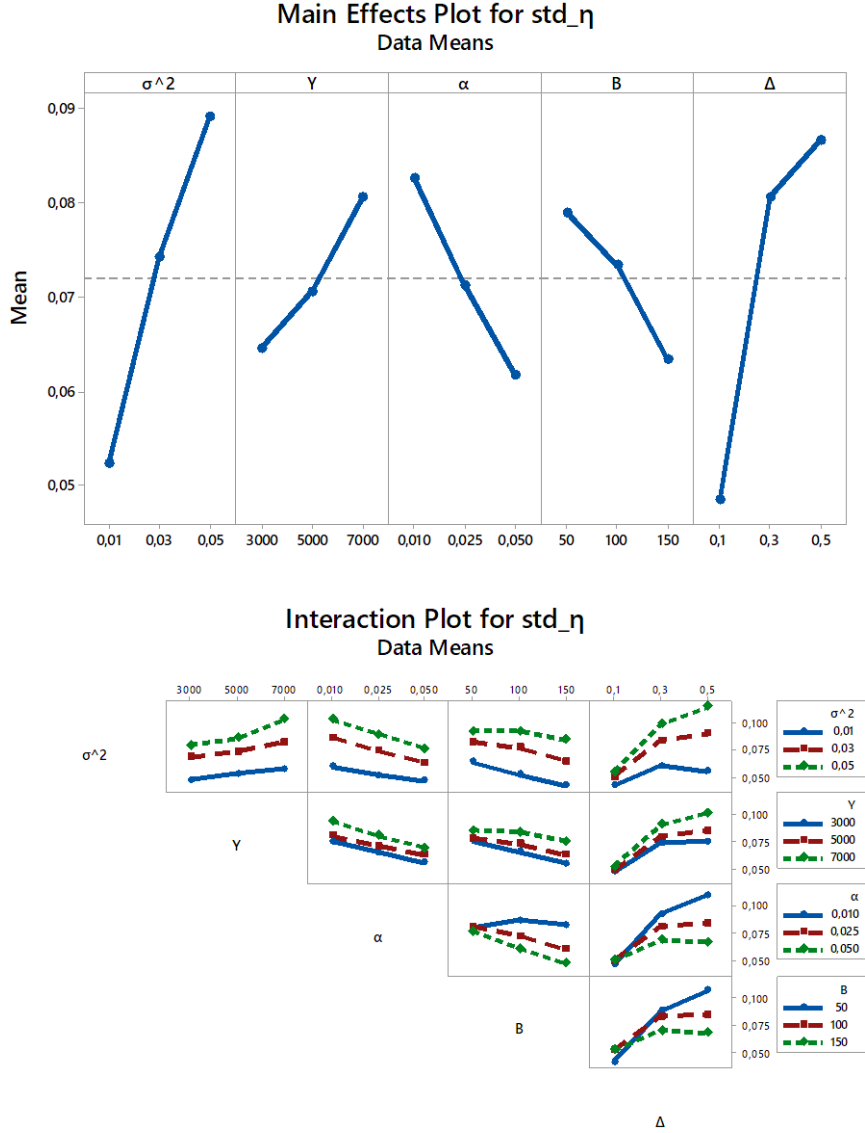
The other significant factor is type I error,  $\alpha$ . In this case, the position of the theoretical optimal point changes because the constraint in equation (2) depends directly on  $\alpha$ . Figure 7 shows how different values of  $\alpha$  affect the theoretical constraints, the theoretical optimal conditions (coloured circles on the constraints) and the algorithm search. As  $\alpha$  is increased, the constraint estimation reduced the available region of parameters. This means that by reducing the probability of accepting a tool wear higher than 0.3, we reduce the available combinations of feed and speed, reducing the potential improvement of productivity. As a matter of fact, the performances with  $\alpha=0.99$  (in green) are much worse compared to the case of  $\alpha=0.95$  (in red). The choice of the confidence level  $\alpha$  depends on the cost of the scrapped part, machining time, batch size etc.

This consideration is also valid for high values of  $\Delta$ , by increasing its value there is also a higher probability to exceed the constraint, see Figure 6. One must keep in mind that  $\Delta$  is lower than 1, so the procedure does not move close to the estimate constraint, but a conservative approach is used. However, as the estimation of the constraint is subjected to error, sometimes the experimental data exceed the constraint. Exceeding the constrain of a small amount does not mean that the parts produced are rejected as the constraint is only the inferior limit of the prediction interval and not the expected value.

To understand the variability of the simulation results, the standard deviation of the response  $\eta$  has been computed using the 100 replicates and indicated as  $std_\eta$ . In this case, since no replicates are available for the response  $std_\eta$ , we limited the analysis to two factor interactions only.

The main effects and the interaction plots are reported in Figure 8.





**Fig. 8** Main Effect Plot and Interaction Plot for index  $sd_{\eta}$

From Figure 8 we conclude that the response standard deviation  $sd_{\eta}$  is affected mainly by the step size  $\Delta$  and the variance of the wear model  $\sigma^2$ ; then the type I error  $\alpha$  with a modest influence, eventually  $Y$  and  $B$  with the same and small importance. The three factor interactions  $B * \Delta$ ,  $\sigma^2 * \Delta$  and  $\alpha * \Delta$  seem to affect the standard deviation more than the others.

A large step size  $\Delta$  has a large impact on the results of the evolutive procedure. The evolutive procedure estimates the tool wear constraint at each step and moves towards the estimated optimum with a speed that is proportional to the step size. If the constraint estimation is not effective, the evolutive procedure moves towards a non-optimal region. On the contrary, when the constraint estimate is effective, the results of the simulation are close to the theoretical optimal condition. At a small step size, the differences between an effective and a non-effective estimate of the constraint has a lower impact on the choice of the new condition for the next step of the evolutive procedure. This is the same reason why the step size is also important in terms of interaction on the final performance of the evolutive procedure, as it is shown by the Interaction plot.

As an example, let us consider the interaction  $B * \Delta$ . With a small batch size and a small step size: there are few steps available for moving towards a new region and, moreover, these steps are limited in length. Therefore, the results in terms of variability are small. Now consider the case where the

batch is still small, but the step size is large. In this case, there are few experiments available to reach the optimal point, so an incorrect estimate of the tool wear in one step leads to non-optimal regions very fast and there might be insufficient experiments to move back to optimal parameters. Hence, this case is the worst condition in terms of result variability. On the other hand, a large step size combined with a large batch reduces the probability of completing the batch production away from an optimal region, as an incorrect estimate of the constraint can be solved with the remaining experiments. This is highlighted in the right-bottom box in interaction plot of Figure 8.

Another important factor for the response standard deviation is the variance of the tool wear model,  $\sigma^2$ . Intuitively, as  $\sigma^2$  increases, the process noise increases and therefore it is more difficult to satisfy the stochastic constraint on the tool wear (see Equation (2) and constraint (3)). With a greater process noise, the algorithm follows a different direction leading to different results in terms of  $t_s$ . The less effective estimate of the constraint is worsened by a large step size. On the contrary, a highly precise estimate of the constraint (small  $\sigma^2$ ) combined with a large step size, allow the procedure to move faster towards the theoretical optimum, reducing the final production time of the batch.

The evolutive procedure variability is affected in a small part by the type I error. According to the main effect plot in Figure 8, the smaller the type I error the larger the variability. As the constraint moves closer to the initial condition due to a smaller type I error  $\alpha$ , the evolutive procedure becomes increasingly conservative and therefore small changes in the constraint estimation result in a higher variability of the total production time.

The same consideration made for the interaction  $B*\Delta$  can be made for the batch size. A large batch size  $B$  increases the number of experiments available to move towards the constraint and the theoretical optimal condition, reducing the influence of other factors such as  $\Delta$  and  $\sigma^2$ .

Eventually, when the volume of material to be removed  $Y$  is high, it is easier for the procedure to evaluate the constraint using the Wear model of Equation (5), as more data are obtained for the estimate. For this reason, the large information acquired during the processing of one part reduces the influence of other factors on the estimate of the steepest ascent direction.

The ANOVA Table is reported in Table 5.

**Table 5 F-values and p-values of the ANOVA table for the sensitivity analysis on index  $st_\eta$**

Factor	Type	Levels	Values
$\sigma^2$	Fixed	3	0.01; 0.03; 0.05
$Y$	Fixed	3	3000; 5000; 7000
$B$	Fixed	3	50; 100; 150
$\Delta$	Fixed	3	0.1; 0.3; 0.5
$\alpha$	Fixed	3	0.010; 0.025; 0.050

Analysis of Variance					
Source	DF	Adj SS	Adj MS	F-Value	p-Value
$\alpha$	2	0.017871	0.008936	155.43	0.000
$B$	2	0.010094	0.005047	87.79	0.000
$\Delta$	2	0.068758	0.034379	598.00	0.000
$\sigma^2$	2	0.055990	0.027995	486.96	0.000
$Y$	2	0.010532	0.005266	91.60	0.000
$B*\alpha$	4	0.007864	0.001966	34.20	0.000
$\Delta*\alpha$	4	0.015921	0.003980	69.23	0.000
$B*\Delta$	4	0.018662	0.004666	81.16	0.000
$\sigma^2*Y$	4	0.001702	0.000426	7.40	0.000

$\sigma^2*B$	4	0.001653	0.000413	7.19	0.000
$\sigma^2*\Delta$	4	0.015145	0.003786	65.86	0.000
$\sigma^2*\alpha$	4	0.001353	0.000338	5.88	0.000
$Y*B$	4	0.000854	0.000213	3.71	0.006
$Y*\Delta$	4	0.003249	0.000812	14.13	0.000
$Y*\alpha$	4	0.000366	0.000091	1.59	0.178
Error	192	0.011038	0.000057		
Total	242	0.241051			

### Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0075822	95.42%	94.23%	92.67%

Some terms showed a low p value ( $< 0.05$ ), however their F-values are small ( $< 70$ ) compared to the other factors such as  $B$  or  $\Delta$  and they can be considered negligible. From an industrial point of view, the higher the uncertainty in the estimation of the tool wear the lowest the improvement in terms of MRR generated by the procedure. If the process is in control, the variability of the tool wear should not be excessively high, and the proposed procedure was proven able to provide improvements in respect to the starting condition in terms of productivity, reducing the production of defective parts. The step size is another important factor to be considered when applying this evolutionary framework. Increasing  $\Delta$  allows to improve in terms of production time, as showed in Table 4, however the practitioner should also consider that this improvement comes with the cost of higher variability in the results. Batch size, material to be removed and type I error have a small influence on the variability of the production time; consequently, the level of these factors could be chosen based on their effect on the expected results.

### Conclusions and future developments

In the present paper, an evolutive procedure for the online optimization of machining operations has been proposed. The motivation comes from the machining of very expensive and hard to cut materials such as Inconel 718 Nickel Superalloy often used in aerospace applications where the batch sizes are moderate-to small. The main feature of the developed evolutive procedure is that it is based on the online measurement of the tool wear at the end of each machining *feature*, a procedure used currently by our Company partner.

To search the optimal solution, the evolutive procedure uses a small experimental design centred in the current machining conditions. The design estimates the tool wear relationship which links the tool wear with the cutting parameters, and it estimates the wear constraint in terms of maximum allowable wear. Eventually, it moves the current machining conditions along the steepest ascent direction with a certain step size  $\Delta$ .

The evolutive procedure stops when the number of total good parts (satisfying the tool wear constraint) is equal to the batch size  $B$ . Two different versions of the algorithm (*Local L* and *Historical S*) have been proposed and investigated.

The evaluation of the proposed evolutive online procedure has been carried out through an extensive simulation to have statistically significant results. To carry out the simulations, a tool wear equation has been fit through an experimental campaign to mimic the real tool wear behaviour. The simulation input parameters have been varied to study their influence on the performance of the proposed evolutionary algorithms.

The main results of this study can be summarized as follows:

1. In all the tested conditions, machining with the proposed evolutive procedure is better than machining with the starting conditions.

2. The algorithm **L** (Local) shows a comparable if not better performance than **S** (Historical) one;
3. As the number of experiment central points increases, the evolutionary procedure performance gets worse;
4. The effectiveness of the evolutive procedure decreases as the safety coefficient  $1-\alpha$  decreases;
5. As the tool wear constraint is more stringent, the potential improvement reduces;
6. As the step along the steepest ascent direction is increased, the procedure quickly gets close to the theoretical optimal point;
7. When the volume to be removed or the batch size **B** increases, the evolutive procedure has more information to find the theoretical optimal condition and improves its performances;
8. The importance of the step size reduces as the batch size increases.

However, from a scientific point of view, some issues about the procedure are still open and could be investigated, e.g.

- Dynamic setting of the step size, perhaps decreasing it when approaching the constraints;
- The role of a higher number of constraints, considering for example power or surface roughness;
- Application of the evolutive procedure to other types of machining operations such as rough turning and milling.
- Study of the potentiality of the procedure in case the inserts can machine several different *features* of the same part or several equal *features* belonging to consecutive parts.

From an industrial point of view the proposed evolutive procedure is economically interesting because it can be applied online, directly on the manufacturing process without additional costs since the Wear measurement is currently made. As a matter of fact, the procedure is currently assessed by the Company partner for application on the shop floor; one crucial point is to understand if the certification issues can be overcome, considering that they are of paramount importance in the aeronautical context.

Symbol	Measurement Units	Definition	Value
$v$	m/min	Speed.	
$f$	mm/rev	Feed.	
$v_{\min}$	m/min	Minimum speed.	55
$v_{\max}$	m/min	Maximum speed.	75
$f_{\min}$	mm/rev	Minimum feed.	0.196
$f_{\max}$	mm/rev	Maximum feed.	0.285
$(v_0, f_0)$	(m/min, mm/rev)	Starting point.	(60; 0.22)
$(v^*, f^*)$	(m/min, mm/rev)	Online solution of the optimization problem with a stochastic constraint.	
$(v_{ott}, f_{ott})$	(m/min, mm/rev)	Theoretical optimal condition, if the real tool wear constraint was perfectly known.	
$Y$	mm <sup>2</sup> /part	Constant which depends on the <i>feature</i> tool path and geometry. In parallel turning $Y$ is the volume to be removed divided by the depth of cut.	8000; Sensitivity analysis: {3000, 5000, 7000}
$X$	--	Design matrix.	
$t_\alpha(df_E)$	--	$\alpha$ -quantile of a t-Student distribution with $df_E$ degrees of freedom.	
$VB$	mm	Tool flank wear width according to the ISO standard [19].	
$VB_0$	mm	Maximum tool wear allowed.	0.3
$VB(v, f)$	mm	Tool wear model estimated at each procedure step.	
$VVB(v, f)$	mm	Inferior limit of the unilateral flank Wear Prediction Interval with confidence coefficient $1-\alpha$ estimated at each procedure step.	
$\Delta$	%	Step size in steepest ascent direction.	30; Sensitivity analysis: {10, 30, 50}
$B$	part	Batch size.	{50, 100, 150}
$n_C$	--	Number of central points in the experimental design.	{2;3;4};
$\alpha$	--	Type I error.	0.05; Sensitivity analysis: {0.01;0.025;0.05}
$t$	s/part	Contact time $t = Y / vf$	
$t_i$	s	Batch production time at the starting conditions	
$t_s$	s	Simulated batch machining time.	
$t_u$	s/part	Unit theoretical optimal machining time.	
$t_{ott}$	s	Batch theoretical optimal machining time knowing the unit theoretical optimum conditions.	
$\hat{\gamma}_\varepsilon^2$	mm <sup>2</sup>	Estimated variance of the tool wear relationship at each procedure step.	

$\hat{\sigma}_{\varepsilon}^2$	mm <sup>2</sup>	Estimated variance of the true tool wear relationship used in the first simulation campaign.	0.02922
$\sigma^2$	mm <sup>2</sup>	Variance of the true Tool Wear relationship in Sensitivity analysis.	Sensitivity analysis: {0.01, 0.03, 0.05}
$\varphi$		Performance index $\varphi = t_s / t_{ott}$	
$\eta$	--	Index $\eta = \frac{t_s - t_i}{t_{opt} - t_i}$	
$sd_{\eta}$	--	Standard deviation of the index estimated through the simulation on 100 replicates.	

## Appendix A – Tool wear equation

An experimental activity was performed to fit the Tool wear equation by changing speed  $v$  and feed  $f$ , keeping the depth of cut  $p$  constant. Specifically, the experimental tests were carried out on cylindrical bars in Inconel 718 Nickel Superalloy (hardness equal to 43 HRC).

The dimensions of the bars used were:

- Diameter = 102.6 mm;
- Length = 500 mm.

The tests were carried out on a lathe (nominal power equal to 22 KW) in dry cooling conditions.

The tool used in the experimental activity was a coated VBMT, with a tool tip radius equal to 1.6 mm. Its bulk chemical composition is:

- 89.3% WC;
- 10.2% Co;
- 0.2% TaC.

The coating consisted of three layers, as below reported:

- TiCN internal layer (thick 2.2  $\mu\text{m}$ );
- $\text{Al}_2\text{O}_3$  central layer (thick 1.5  $\mu\text{m}$ );
- TiN external layer (thick 0.5  $\mu\text{m}$ ).

A Dinolite Pro AM413T microscope (230x magnification) was used to measure the flank wear width VB during the test execution. A full factorial experiment was designed and carried out and the investigated levels of the cutting parameters were:

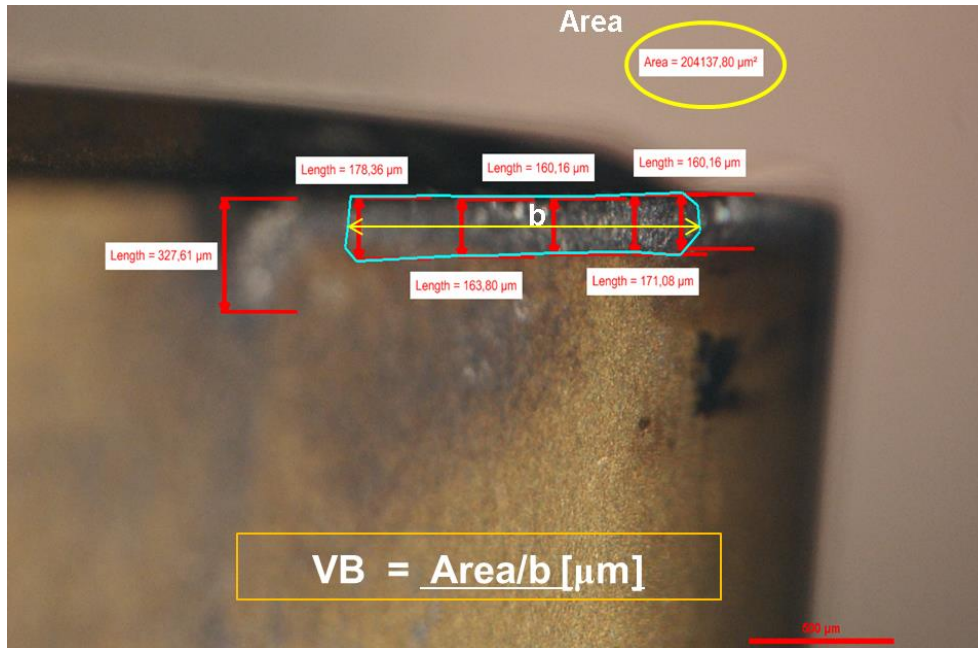
- $f = 0.196 - 0.214 - 0.249 - 0.285$  [mm/rev];
- $v = 55, 65, 75$  [m/min].

As previously mentioned, a constant depth of cut  $p$ , equal to 1.5 mm, was set. Subsequently, twelve combinations of  $f$  and  $v$  were considered. One replication was performed for the vertices points of the design resulting in 16 runs.

The flank wear width VB was measured in accordance with the ISO 3685 Standard.

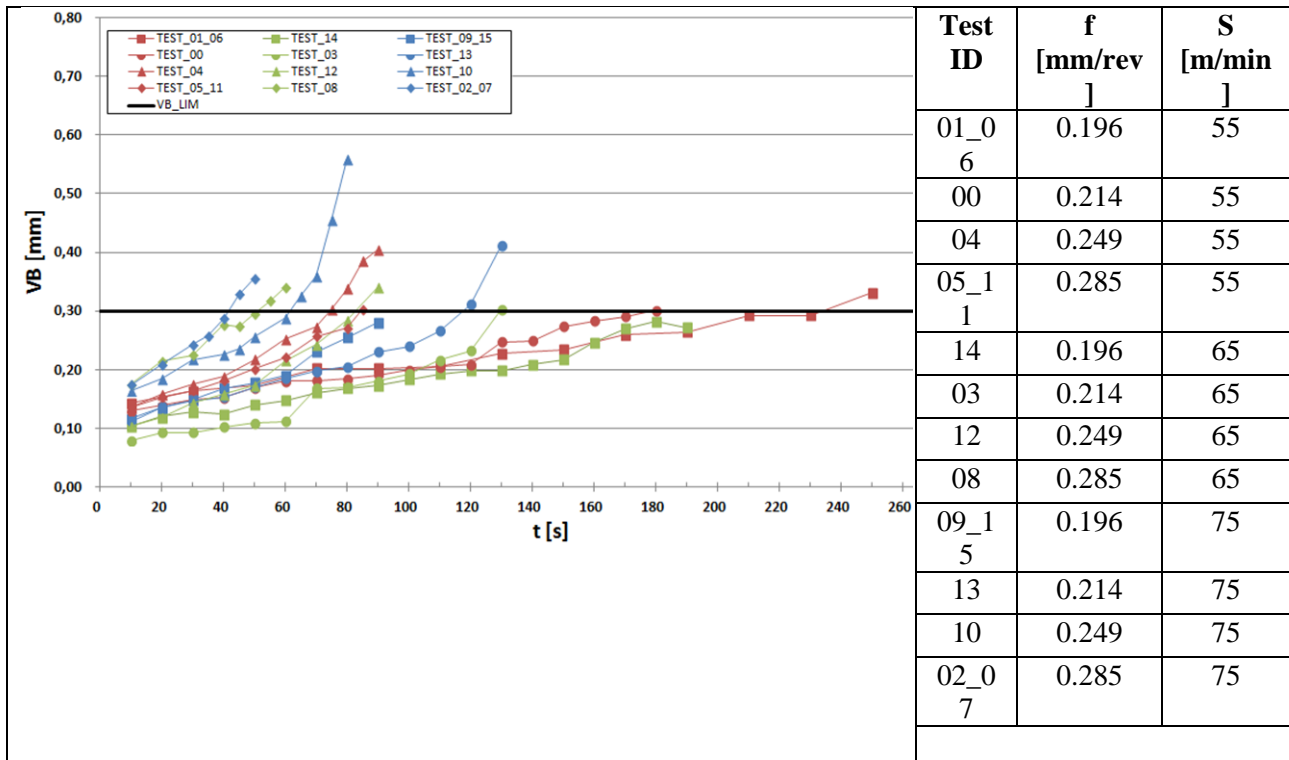
For each run, the measurements of the VB width were made at regular time intervals, depending on the actual values of the cutting parameters. Hence, small time intervals correspond to high cutting parameters, as in these conditions tool wear is faster. The sequence for each test is described as follows:

- Step 0: turning is executed for a fixed time interval;
- Step 1: the tool is removed from the tool holder and then positioned below the microscope lens; the operator captures the picture (focused on the tool wear region) and measures the VB (see Figure A1) in accordance with the ISO 3685 Standard; after the Wear measurement the tool is placed again on the tool holder and then used for a new time interval, repeating the Step 1. This operation is repeated several times until the default VB limit of 0.30 mm is reached or exceeded.



**Fig. A1 Example for the tool Flank wear detection**

In Figure A2, the  $VB$  versus time trends are shown for the investigated conditions.



**Fig. A2 Tool Flank wear (VB) vs time for the experimental conditions**

The experimental data were used to estimate the function  $VB = VB(f, v, t)$ , where  $t$  represents the tool contact time  $t = Y / f v$  ( $Y$  is a constant depending on the volume of the material to be removed and other technological parameters, e.g. the depth of cut). The empirical equation found by Linear Regression is:



$$\ln(VB(v, f)) = 76.6 - 1.763 \ln t - 40 \ln v - 9.25 \ln f + 0.0892 (\ln t)^2 + 5.03 (\ln v)^2 + 0.549 \ln v * \ln t + 0.549 \ln t * \ln f + 2.095 \ln v * \ln f + \theta \quad \text{where } \theta \sim NID(0, \sigma^2) \quad (A1)$$

and  $\sigma^2 = 0.02922$

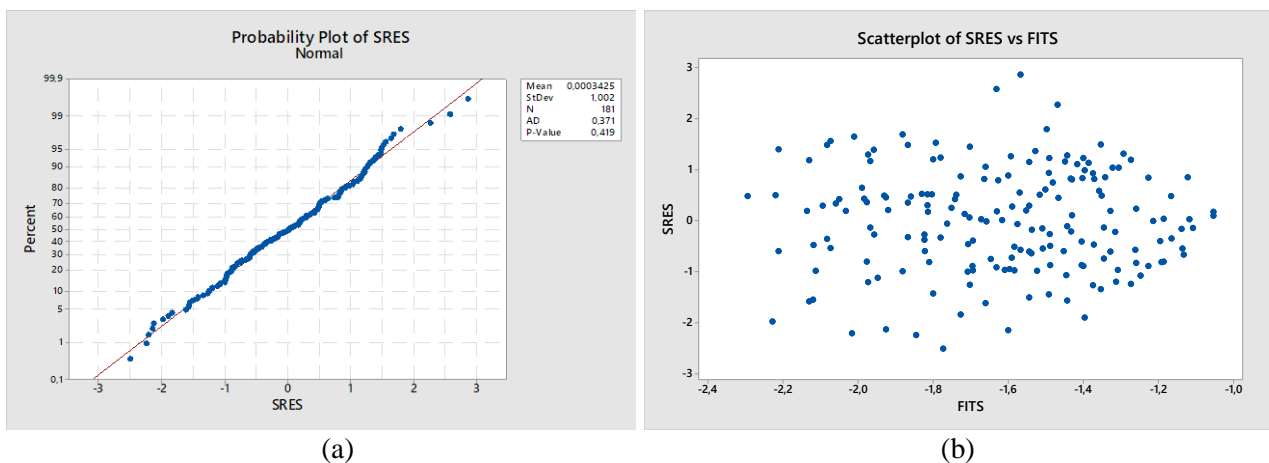
Note that the empirical equation is estimated from the experimental data, however we will not use the hat notation because we consider it as if it were perfectly known. This is not an issue because we use the equation A1 to sample VB to mimic the real process. Some regression details are reported in the following tables (Table A1 and Table A2) and the analysis of the residuals is showed in Figure A3.

**Table A1 ANOVA Table for Regression Analysis: Ln VB versus Ln t; LnSpeed; LnFeed**

Source	DF	Adj SS	Adj MS	F-Value	p-Value
<b>Regression</b>	8	15.2346	1.90432	65.16	0.000
<b>Ln t</b>	1	0.2308	0.23075	7.90	0.006
<b>LnSpeed</b>	1	0.4383	0.43834	15.00	0.000
<b>LnFeed</b>	1	0.2488	0.24880	8.51	0.004
<b>Ln t*Ln t</b>	1	0.6016	0.60160	20.59	0.000
<b>LnSpeed*LnSpeed</b>	1	0.4870	0.48704	16.67	0.000
<b>Ln t*LnSpeed</b>	1	0.4382	0.43817	14.99	0.000
<b>Ln t*LnFeed</b>	1	0.4664	0.46638	15.96	0.000
<b>LnSpeed*LnFeed</b>	1	0.2483	0.24829	8.50	0.004
<b>Error</b>	172	5.0266	0.02922		
<b>Lack-of-Fit</b>	137	2.9841	0.02178	0.37	1.000
<b>Pure Error</b>	35	2.0425	0.05836		
<b>Total</b>	180	20.2612			

**Table A2 Model Summary**

S	R-sq	R-sq(adj)	R-sq(pred)
0.170952	75.19%	74.04%	72.60%



**Fig A3 Standardized Residuals Probability (a) and Scatter Plots (b)**

## Appendix B – Theoretical optimal conditions

If the Tool wear equation was perfectly known the theoretical optimal solution could be easily derived. Let us consider that the tool wear is a random variate  $VB \sim D(\mu(v, f), \sigma_\varepsilon^2)$  where  $D$  is a known probability distribution with  $\text{Expected}(VB) = \mu(v, f)$  and  $\text{Variance}(VB) = \sigma_\varepsilon^2$ . The stochastic optimization problem can be transformed into a deterministic one as follows:

$$\begin{aligned} \min_{v, f} \frac{Y}{v \cdot f} \\ \Pr\{VB(v, f) \geq VB_0\} \leq \alpha \\ v_{\min} \leq v \leq v_{\max} \\ f_{\min} \leq f \leq f_{\max} \end{aligned} \quad (B1)$$

Where  $VB_0$  is the maximum tool wear allowed, in our case it is 0.3 mm. The solution of the problem (B1) is the theoretical optimum  $(v_{ott}, f_{ott})$ , and the corresponding unit optimum tool contact time is

equal to  $t_u = \frac{Y}{v_{ott} \cdot f_{ott}}$ . The average batch optimal production time can be derived as follows:

$$t_{opt} = t_u B(1 + \alpha) \quad (B2)$$

Note that Equation B2 accounts for the expected proportion of defective parts  $\alpha$  (i.e. scraps generated by a tool wear measured at the end of the machining of a *feature* greater than  $VB_0$ ).

## Acknowledgements

The authors sincerely thank the reviewers for their very helpful comments on earlier drafts of this manuscript, for their time and for their encouragement.

## Bibliography

Angün, E., Kleijnen, J., den Hertog, D., & Gürkan, G. (2009). Response surface methodology with stochastic constraints for expensive simulation. *Journal of the operational research society*, 60(6), 735-746.

Box G. E. P. and Wilson K. B., "On the experimental attainment of optimum conditions." *Breakthroughs in Statistics*. Springer New York, 1992. 270-310.

Costa A., Celano G., Fichera S., "Optimization of multi-pass turning economies through a hybrid particle swarm optimization technique", *International Journal of Advanced Manufacturing Technology* 53, pp. 421–433, 2011.

D'Addona, D. M., Ullah, A. S., & Matarazzo, D. (2017). Tool-wear prediction and pattern-recognition using artificial neural network and DNA-based computing. *Journal of Intelligent Manufacturing*, 28(6), 1285-1301.

Davim J.P. Editor, "Machining: Fundamentals and Recent advances", *Springer-Verlag London Limited*, 2008.

Del Castillo E., "Process optimization: a statistical approach", *Vol. 105, Springer Science & Business Media*, 2007.

Devillez A., Schneider F., Dominiak S., Dudzinski D. and Larrouquere D., "Cutting forces and wear in dry machining of Inconel 718 with coated carbide tools," *Wear*, no. 262, pp. 931-942, 2007.

Draper N., Smith H., "Applied Regression Analysis", 3<sup>rd</sup> ed., *John Wiley & Sons*, 2005.

Ganesan H., Mohankumar G., Ganesan K., Ramesh Kumar K., 2011, "Optimization of machining parameters in turning process using genetic algorithm and particle swarm optimization with experimental verification", *International Journal of Engineering Science and Technology (IJEST)*, Vol. 3 No. 2, pp. 1091-1102, 2011.

Kalpakjian S., Schmidt S. R., "Manufacturing Engineering and Technology" 4<sup>th</sup> ed., *Prentice Hall International*, 2001.

Klocke F., Zeis M., Klink A., Veselovac D., "Technological and Economical Comparison of Roughing Strategies via Milling, EDM and ECM for Titanium- and Nickel-based Blisks", *Procedia CIRP Volume 2*, pp. 98-101, 2012.

Myers, R.H., Montgomery D. C. and Anderson-Cook C. M., "Response surface methodology: process and product optimization using designed experiments", 3<sup>rd</sup> Edition, *John Wiley & Sons*, 2009.

Rao S. S., "Engineering Optimization Theory and Practice", *John Wiley and Sons*, 2009.

Schorník V., Zetek M. and Daňa M., "The Influence of Working Environment and Cutting Conditions on Milling Nickel-Based Super Alloys with Carbide Tools," *Procedia Engineering*, no. 100, pp. 1262-1269, 2015.

Taylor F. W. “On the Art of Cutting Metals”, *American society of mechanical engineers*, 1907.

Venkata Rao R., “Teaching Learning Based Optimization Algorithm And Its Engineering Applications”, *Springer*, 2016.

Venkata Rao R., Pawar P.J., “Parameter optimization of a multi-pass milling process using non-traditional optimization algorithms”, *Applied Soft Computing* 10, pp. 445–456, 2010.

Wang, G., & Cui, Y. (2013). On line tool wear monitoring based on auto associative neural network. *Journal of Intelligent Manufacturing*, 24(6), 1085-1094

Wang, G., Guo, Z., & Qian, L. (2014). Online incremental learning for tool condition classification using modified Fuzzy ARTMAP network. *Journal of Intelligent Manufacturing*, 25(6), 1403-1411.

Yildiz A. R., “A new hybrid differential evolution algorithm for the selection of optimal machining parameters in milling operations”, *Applied Soft Computing Volume 13, Issue 3*, pp. 1561-1566, 2013.

Zainal, N., Zain, A. M., Radzi, N. H. M., & Othman, M. R. (2016). Glowworm swarm optimization (GSO) for optimization of machining parameters. *Journal of Intelligent Manufacturing*, 27(4), 797-804.

Zhang, J., Liang, S. Y., Yao, J., Chen, J. M., & Huang, J. L. (2006). Evolutionary optimization of machining processes. *Journal of Intelligent Manufacturing*, 17(2), 203-215.