Assessment of alternative adsorption models and global sensitivity analysis to characterize Hexavalent chromium loss from soil to surface runoff

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Abstract

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32   We investigate our ability to assess transfer of hexavalent chromium, Cr(VI), from

33   the soil to surface runoff by considering the effect of coupling diverse adsorption

34   models with a two-layer solute transfer model. Our analyses are grounded on a set of

35   two experiments associated with soils characterized by diverse particle size

36   distributions. Our study is motivated by the observation that Cr(VI) is receiving much

37   attention for the assessment of environmental risks due to its high solubility, mobility,

38   and toxicological significance. Adsorption of Cr(VI) is considered to be at equilibrium

39   in the mixing layer under our experimental conditions. Four adsorption models, i.e., the

40   Langmuir, Freundlich, Temkin, and the linear model, constitute our set of alternative

41   (competing) mathematical formulations. Experimental results reveal that the soil

42   samples characterized by the finest grain sizes is associated with the highest release of

43   Cr(VI) to runoff. We compare the relative abilities of the four models to interpret

44   experimental results through Maximum Likelihood model calibration and four model

45   identification criteria (i.e., the information criteria AIC and AIC_C, and the Bayesian

46   criteria BIC and KIC). Our study results enable us to rank the tested models on the basis

47   of a set of posterior weights assigned to each of them. A classical variance-based global

48   sensitivity analysis is then performed to assess the relative importance of the uncertain

49   parameters associated with each of the models considered, within sub-regions of the

50   parameter space. In this context, the modeling strategy resulting from coupling the

51   Langmuir isotherm with a two-layer solute transfer model is then evaluated as the most

52   skillful for the overall interpretation of both sets of experiments. Our results document

that (*a*) the depth of the mixing layer is the most influential factor for all models tested, with the exception of the Freundlich isotherm, and (*b*) that the total sensitivity of the adsorption parameters varies in time, with a trend to increase as time progresses for all of the models. These results suggest that adsorption has a significant effect on the uncertainty associated with the release of Cr(VI) from the soil to the surface runoff component.

INTRODUCTION

According to the soil assessment results of China nationwide Multi-Purpose Regional Geochemical Survey (MPRGS) project, considerable portions of lands used for agriculture purpose in China show pollution signatures by metals. Chemical transfer from the soil to surface runoff is a key process that contributes to non-point source pollution. Metals (e.g., Cd, Pb and Cr) originating from a contaminated soil can then contaminate surface water and groundwater, with a negative impact on human health and various compartments of the ecosystem (Krishna and Govil, 2008). Among these pollutants, Cr(VI) has received much attention because of its high solubility, mobility, and toxicological significance in the environment.

Numerous studies focus on efficient methods (*a*) to reduce the amounts of Cr(VI) in the environment, (*b*) to investigate the fate of Cr(VI) in soil and groundwater, and (*c*) to monitor space-time distributions of Cr(VI) in surface runoff. He et al. (2004) monitored metal (i.e., Cd, Co, Cr, Cu, Fe, Ni, Pb, Zn, Mn, and Mo) concentrations in surface runoff at 11 sites in Florida and documented a positive correlation between concentrations of the analyzed metals in runoff and soil. Ghosh et al. (2012) found that Cr(VI) could be successfully adsorbed onto the fine-grained soil used as a liner material in a landfill. Núñez-Delgado et al. (2015) found that both pine sawdust and oak wood ash could be used (as biosorbents) to reduce the concentration of Cr(VI) released from soil to water.

Migration of Cr(VI) through a soil system is typically described by advection, dispersion, adsorption and reaction processes. Advection is controlled by the mean flow rate of water, and dispersion is characterized by molecular diffusion and mechanical

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83   dispersion. Langmuir (1918), Freundlich (1907), and linear isotherm models are often

84   used to simulate Cr(VI) adsorption experimental results under equilibrium assumption

85   (e.g., Ghosh et al., 2012; Núñez-Delgado et al., 2015; Sangiumsak and Punrattanasin,

86   2014; Mendonca et al., 2013; Fifi et al., 2013; Li et al., 2014). Gupta and Bahu (2009)

87   successfully simulated breakthrough curves of Cr(VI) by combining the Langmuir

88   model with a mathematical transport model. Chakraborty et al. (2015) embedded the

89   Langmuir and linear isotherms, respectively, in a one-dimensional advection-

90   dispersion-reaction-equation to estimate Cr(VI) transport parameters. Such isotherms

91   have also been integrated in well-known numerical codes (e.g., Hydrus-1D (Šimůnek

92   et al., 2009) and MT3DMS (Zheng et al., 1998)) to simulate reactive solute migration.

93   Transport of Cr(VI) is affected by the redox reaction of Cr, associated with Cr(III)

94   and/or Cr(VI) in the environment. Based on thermodynamics, Cr(III) oxidation should

95   be a spontaneous process. However, it needs to be catalyzed to take place in a natural

96   system, due to its very slow kinetics (Apte et al., 2005). In this context, one can note

97   that while chromium is highly mobile with flow in the hexavalent form, its migration

98   would be strongly retarded in the trivalent form because of the typically strong

99   adsorption capacity of soil to Cr(III). It is therefore worth noting that detectable

100   amounts of chromium found in natural waters are usually in the hazardous hexavalent

101   form, manganese oxides essentially being the materials that can oxidize Cr(III) to Cr(VI)

102   in a natural system (Fendorf and Zasoski, 1992). Organic materials, such as sulfides,

103   and ferrous iron, can reduce Cr(VI) to Cr(III) (Fendorf, 1995). We refer to Fendorf

104   (1995) for a comprehensive review on this aspect.

105   Transfer of a chemical from soil to surface runoff is a complex process. Most of the

106   experimental studies on this process are conducted at the laboratory scale (sand boxes).

107   Various chemical transport models have been developed to explain experimental results.

108   Modeling methods based on the diffusion (Wallach and van Genuchten, 1990) and the

109   mixing-layer theory (Donigian et al., 1977) were the two approaches generally used to

110   simulate the process. The diffusion theory assumes that chemical exchange between

111   runoff and soil is controlled by an accelerated diffusion process. The theory has been

112   used in a variety of studies (Ahuja et al., 1981, 1983; Sharpley et al., 1980), even though

113   the function to describe the accelerated diffusion process is somewhat arbitrarily chosen

114   (Gao et al., 2004). Note that using an accelerated diffusion coefficient in the simulations

115   may aptly capture a set of experimental data, while providing no explanation to the

116   physical mechanism of the diffusion process. Because the physical mechanism in the

117   accelerated diffusion theory remains unclear, the diffusion theory has limited ability for

118   prediction.

119   The mixing-layer theory was first proposed by Donigian et al. (1977). It is based on

120   the assumption that surface runoff water mixes entirely and instantaneously with soil

121   water in a thin layer on the soil surface with no contribution from the soil below such a

122   mixing layer. However, Zhang et al. (1997) found that the diffusion flux from the soil

123   underlying the mixing layer could not be neglected under poor drainage conditions.

124   Therefore, Zhang et al. (1999) used the convection-diffusion equation to simulate the

125   diffusion flux from the underlying soil.

126   A series of mechanistic models have been developed in recent years. Soil erosion

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127   was characterized by the Rose model (Rose et al., 1994; Hairsine and Rose, 1991), and

128   solute transfer between the runoff and the exchange layers was determined by the water

129   transfer rate. Gao et al. (2004) assumed that the solute transfer from soil to surface

130   runoff was affected by both drop-liquid and drop-liquid-solid interactions, and the

131   process was conceptualized as the model of the three layers/compartments, i.e., a runoff,

132   an exchange, and the underlying soil compartment. Tong et al. (2010) proposed a new

133   model to integrate the runoff and the mixing layers into a unique mixing region. In the

134   model, solute concentration in the runoff was calculated as αC_w, where α $(0 < \alpha \leq 1)$

135   is an incomplete mixing coefficient and C_w is the aqueous-phase solute concentration

136   in the mixing layer [M L^{-3}]. The net chemical flux from the mixing layer to the

137   underlying soil is calculated as $i\gamma C_w$, and i and γ $(0 < \gamma \leq 1)$ are, respectively, the

138   infiltration rate [L T^{-1}] and another incomplete mixing coefficient. All of these models

139   assume that the soil surface is (nearly) horizontal, the thickness of the mixing layer is

140   regarded as stable or constant, and lateral and return flows in the soil are negligible.

141   Dong and Wang (2013) considered an inclined soil surface and relied on the conceptual

142   model of Gao et al. (2004). They assumed solute concentrations in the runoff and the

143   mixing layer were to be identically affected by raindrop splash. In addition, the model

144   allows the mixing layer to have a variable depth, and takes into account the effects of

145   raindrop splash, lateral flow, and return flow. Notably, solute concentrations in the

146   mixing and/or exchange layers are considered as uniform.

147   The main considered factors affecting Cr(VI) transfer from soil to surface runoff in

148   the above referenced models include rain intensity (Gao et al., 2004), ponding water

149 depth (Gao et al., 2004), soil characteristics (Tong et al., 2010), soil slope (Dong and

150 Wang 2013), and drainage conditions (Walker et al., 2007; Zhang et al., 1999).

151 Adsorption of a chemically active solute is another important factor that affects solute

152 loss. Gao et al. (2004) integrated the linear adsorption model with a solute loss model

153 to evaluate phosphorus concentrations in runoff. In this broad context, it is still lack of

154 a detailed study on the influence of the adsorption model choice on Cr(VI) loss

155 simulation from soil to surface runoff.

156   An objective of our study is to evaluate the impact of a model selected from a set of

157 differing adsorption models (i.e., the Langmuir, Freundlich, Temkin and linear models)

158 on the Cr(VI) loss simulation. We do so by relying on a set of two experiments and

159 grounding our study on rigorous model identification criteria (Neuman, 2003; Ye et al.,

160 2004, 2008; Neuman et al., 2012; Bianchi-Janetti et al., 2012). These enable us to (*a*)

161 compare the ability of each of the tested models to interpret the target experimental

162 results and (*b*) rank the models through the evaluation of probabilistic weights assigned

163 to each of them. We then provide model-averaged estimates (and associated uncertainty

164 bounds) of Cr(VI) concentrations in runoff by leveraging on the diverse interpretive

165 skills of all models analyzed. Since each of the models is associated with a set of

166 typically unknown/uncertain parameters, we also perform a classical variance-based

167 global sensitivity analysis to assess the relative contribution of the uncertain parameters

168 associated with each model to the variability of Cr(VI) released from the soil to surface

169 runoff.

170   The rest of the study is structured as follows. We first provide descriptions for the

171 experiments performed and for the modeling approaches. We then present and discuss

172 our results, in terms of model ranking and multi-model analysis as well as global

173 sensitivity analysis. We finally make our conclusions.

174

175   MATERIALS AND METHODS

176 *Soil Chemical characteristics*

177   Representative silt soil samples obtained from the region surrounding the city of

178 Wuhan, China, were dried, ground and passed through 2 mm and 1 mm sieves, to form

179 the two subsamples, respectively termed soil sample 1 and 2, employed in our

180 experimental investigations. Table 1 lists the main chemical characteristics of soil 2.

181 The latter is seen to be characterized by a very low initial Cr(VI) concentration, the

182 overall picture suggesting that the soil is oxic. As such, we do not consider Cr(VI)

183 reduction in our experimental investigations.

184 *Experiments*

185   We leverage on the experimental set-up used by Tong et al. (2010), to which we refer

186 for additional details. In summary, a steel sandbox (with length, width and depth

187 respectively of 100, 30, and 40 cm) with rustproof paint and equipped with two drainage

188 holes at the bottom was used (see Fig. 1). A 5-cm-thick layer of gravel is packed at the

189 bottom of the sand box to allow for water drainage. The elevations of the drainage

190 outlets can be modified to achieve diverse drainage conditions. The gravel layer is

191 covered with a nylon screen to prevent loss of soil particles, the soil subject to the

192 experiments being packed above the screen. A rectangular hatch that opens into a V-

193 shaped trench and located 30 cm above the bottom of the box is used to collect surface

194   runoff water. The height difference between the soil surface and the runoff hatch

195   corresponds to the depth of ponded water. A rainfall simulator (formed by 8 hypodermic

196   needles) is placed at an elevation of 120 cm above the soil layer.

197   Both soil samples were purposely set at the same Cr(VI) concentration of 300 mg kg^{-1}

198   in both the liquid and soild phases, taking into account differences of their initial

199   volumetric water contents (respectively equal to 0.28 and 0.30 for soil 1 and 2) and the

200   preset saturated volumetric water contents (0.49 for both soils), the initial Cr(VI)

201   concentrations (equal to 0 mg kg^{-1} for both soils), and the soil bulk density (1.35 g cm^{-3}

202   ). The soils were then packed gradually to a depth of 18.5 cm. The depth of the runoff

203   layer was 1 cm for both experiments. The soil surfaces were covered with plastic films

204   and allowed to incubate for 12 hours. During this time, the system was kept at a constant

205   room temperature of $25°C$. Thus, evaporation from the soil surfaces were considered

206   as negligible. Before the beginning of the rainfall simulation, the outlets of the drainage

207   holes were set to a height of 0 cm for both experiments, i.e., both experiments were

208   conducted under free drainage conditions.

209   The simulated rainfall was set to an intensity of 0.092 cm min^{-1} and 0.100 cm min^{-1},

210   respectively for experiments 1 (soil 1) and 2 (soil 2). We denote t_p as the time at which

211   water began ponding on the surface of the soil, t_r and t_s respectively as the times at

212   which runoff first occurred and attained stationarity. Table 2 lists the key parameters

213   characterizing the experimental conditions. Collection of runoff samples was initially

214   performed at 4-10 minutes intervals, progressively longer sampling intervals being used

215   as time elapsed. Dissolved Cr(VI) in the runoff samples were measured by the atomic

216 flame method using an atomic flame spectrophotometer. Minute quantities of sediments

217 eventually found in the collected water samples were neglected.

218 *Two-layer mathematical model*

219 We rely on the two-layer model for its computational efficiency and because it is

220 grounded on assumptions that are consistent with our experimental conditions. We

221 briefly describe the main characteristics of the two-layer model used in this study,

222 additional details being found in Tong et al. (2010). As shown in Fig. 2, the conceptual

223 model comprised two vertical layers, i.e., the entire mixing zone, that includes the soil

224 mixing layer and the runoff layer, and the underlying soil.

225 Without considering adsorption, the dissolved chemical mass per unit area, M_w [M

226 L^{-2}], is given by:

227 $$M_w = C_w \left[\alpha \left(h_w - h_{mix}\theta_s \right) + h_{mix}\theta_s \right] \tag{1}$$

228 Here, C_w is the aqueous-phase solute concentration in the mixing layer [M L^{-3}]; α

229 ($0 < \alpha \leq 1$) is an incomplete mixing coefficient [-]; h_w is the net water depth across the

230 entire mixing zone [L], h_{mix} is the mixing layer thickness [L], and θ_s is saturated

231 volumetric water content in the soil system [L^3 L^{-3}].

232 In the presence of adsorption, Eq. (1) becomes:

233 $$M_w = \alpha \left(h_w - h_{mix}\theta_s \right) C_w + h_{mix} \left[\theta_s C_w + \rho_b S \right] \tag{2}$$

234 where S [M M^{-1}] and ρ_b [M L^{-3}] respectively are the solute concentration adsorbed

235 onto the soil and the soil bulk density.

236 The dynamic behavior of the system can then be characterized as:

237   $$\frac{d\left[M_w(t)\right]}{dt} = -\gamma i C_w(t) - \alpha q C_w(t) \tag{3}$$

238   where γ ($0 < \gamma \le 1$) is an incomplete mixing coefficient [-] (different from α); q [L

239   T^{-1}] and t [T] respectively denoting the specific discharge rate of the overland flow and

240   time.

241   *Adsorption isotherm models*

242   We consider three nonlinear models (i.e., the Langmuir, Freundlich, and Temkin

243   equations) and a linear model to interpret solute adsorption. Given the experimental

244   setting, adsorption is assumed to take place under isothermal conditions.

245   The Langmuir isothermal nonlinear equation (Langmuir et al., 1918) is:

246   $$S = S_{\max} \frac{K_{eq} C_w}{1 + K_{eq} C_w} \tag{4}$$

247   where $S_{\max}$ [M M^{-1}] and K_{eq} [L^3 M^{-1}] are (typically unknown and uncertain) model

248   parameters, respectively representing the theoretical maximum adsorption capacity of

249   the soil and the equilibrium adsorption coefficient.

250   Freundlich et al. (1906) proposed the following empirical nonlinear equation:

251   $$S = K_f C_w^{\frac{1}{n}} \tag{5}$$

252   model parameters being the adsorption coefficient K_f [L$^{3/n}$ M$^{-1/n}$] and the exponent n

253   [-]. When $n = 1$, Eq. (5) reduces to the linear model:

254   $$S = K_d C_w \tag{6}$$

255   where K_d [L^3 M^{-1}] is the adsorption coefficient.

256   The Temkin isotherm model was first proposed by Temkin and Pyzhev (1940) and

257   can be expressed in the form:

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$$258 \quad S = \frac{RT_e}{f} \ln(KC_w) \tag{7}$$

259 where R [J mol^{-1} K^{-1}] is the universal gas constant, T_e [K] is absolute temperature, f [J

260 mol^{-1}] is the Temkin isotherm constant, and K [L^3 M^{-1}] is the Temkin isotherm

261 equilibrium binding constant.

262 *Numerical modeling*

263 Embedding each of the adsorption isotherm models (4)-(7) into equations (2)-(3)

264 leads to multiple mathematical models whose relative skill to interpret the observed

265 system behavior is evaluated here. Hereinafter, we denote (*a*) Langmuir, (*b*) Freundlich,

266 (*c*) linear adsorption, or (*d*) Temkin coupled models as the formulations respectively

267 arising by coupling (2)-(3) with (4), (5), (6), or (7). The numerical solution of these

268 solute transport models relies on direct observation of water flow characteristics from

269 the experimental campaign. The observed flow dynamics are then used as input to each

270 transport model and are characterized by segmenting the overall flow process onto the

271 five sequential temporal windows described in the following, each corresponding to a

272 well-defined hydrological manifestation.

273 *Period* 0. It comprises observation times ranging from the beginning of the simulated

274 rainfall to the saturation of the mixing layer. During this period, i.e., before the start of

275 water ponding, the water infiltration rate i_{up} [L T^{-1}] coincides with rainfall intensity,

276 p, i.e., $i_{up} = p$ (corresponding to $i = 0$, and $q = 0$). Infiltration from the mixing layer

277 to the underlying soil is assumed to be negligible because the mixing layer is very thin

278 and the time t_{sa} required to attain saturation of the mixing layer can be approximated

279 as:

280 $$t_{sa} = \frac{h_{mix}\left(\theta_s - \theta_0\right)}{i_{up}} \qquad (8)$$

281 where θ_0 is the initial volumetric water content in the soil system [$M^3 \, M^{-3}$].

282 *Period* 1. During this period, ranging from saturation of the mixing layer

283 (corresponding to time t_{sa} given by Eq. (8)) to the onset of water ponding, water has

284 not yet ponded on the soil surface, and the infiltration rate of soil equals the rainfall

285 intensity, i.e.:

286 $$h_w = h_{mix}\theta_s ; \qquad\qquad i_{up} = i = p ; \qquad\qquad q = 0 \qquad (9)$$

287 Hereinafter, we illustrate our derivations using the Langmuir isotherm as a test bed, the

288 corresponding derivations associated with the other isotherm models being directly

289 inferable from this. Considering Eqs. (2) and (4) leads to:

290 $$M_w = \alpha\left(h_w - h_{mix}\theta_s\right)C_w + h_{mix}\left[\theta_s C_w + \rho_s \frac{C_w}{BC_w + C}\right] \qquad (10)$$

291 where $B = 1/S_{max}$, and $C = 1/\left(K_{eq}S_{max}\right)$. The initial concentration of the solute in the

292 mixing layer, $C_w\left(t_{sa}\right)$, is obtained from Eq. (10). Replacing Eq. (10) into Eq. (3) yields:

293 $$\begin{aligned}&\left(h_{mix}\theta_s + \frac{h_{mix}\rho_s}{C}\right)\ln\frac{C_w\left(t\right)}{C_w\left(t_{sa}\right)} - \frac{h_{mix}\rho_s}{C}\ln\frac{BC_w\left(t\right)+C}{BC_w\left(t_{sa}\right)+C} - \\ &\frac{h_{mix}\rho_s}{BC_w\left(t\right)+C} + \frac{h_{mix}\rho_s}{BC_w\left(t_{sa}\right)+C} = \gamma p\left(t_{sa}-t\right)\end{aligned} \qquad (11)$$

294 We solve Eq. (11) via a fourth-order Runge-Kutta method with a uniform time step of

295 0.01 seconds.

296 *Period* 2. During this window, spanning from the onset of water ponding to the

297 beginning of runoff, respectively observed at experimental times t_p and t_r, the

298 infiltration rate is assumed to decrease linearly in time according to

299     $i(t) = p - a_0(t - t_p)$, the value of $a_0 > 0$ characterizing the rate of increase of the

300   ponding depth and being evaluated via $i(t_r) = p - a_0(t_r - t_p)$. As no surface runoff is

301   observed, $q = 0$ during this period and the rate of ponding depth increase equals to

302     $p - i(t)$. The temporal variation of the net water depth of the mixing zone can then be

303   obtained as:

304     $$h_w = h_p + h_{mix}\theta_s = \frac{1}{2}a_0(t - t_p)^2 + h_{mix}\theta_s \qquad (12)$$

305   where we recall that h_p is the depth of the ponding layer. Concentration $C_w(t_p)$ of

306   the solute in the mixing layer is calculated through Eq. (11) evaluated at time t_p.

307   Combining Eqs. (12), (10), and (3), leads to the following equation describing $C_w(t)$

308   across this time period:

309     $$\frac{1}{2}a_0\alpha(t - t_p)^2 \frac{dC_w(t)}{dt} + a_0\alpha C_w(t)(t - t_p) + h_{mix}\theta_s \frac{dC_w(t)}{dt} + \\ \frac{h_{mix}\rho_s C}{\left[BC_w(t) + C\right]^2} \frac{dC_w(t)}{dt} = \gamma\left[a_0(t - t_p) - p\right]C_w(t) \qquad (13)$$

310   Similar to Eq. (11), we solve Eq. (13) via a fourth-order Runge-Kutta method with a

311   uniform time step of 0.01 seconds.

312   *Period* 3. During this interval, ranging from the onset of runoff to its stabilization (at

313   time t_s), the temporal decrease of infiltration is approximated via:

314     $$i(t) = i(t_r) - b(t - t_r); \qquad\qquad q = p - i \qquad (14)$$

315   where b is a parameter characterizing the infiltration decrease rate and is evaluated via

316     $i(t_s) = i(t_r) - b(t_s - t_r)$. The initial solute concentration in the runoff fluid, $\alpha C_w(t_r)$,

317   can be obtained from the solution of Eq. (13), evaluated at time $t = t_r$. Substituting Eqs.

318   (14) and (10) into Eq. (3) yields:

319

$$\left[\alpha h_p + h_{mix}\theta_s\right]\ln\frac{C_w(t)}{C_w(t_r)} + \frac{h_{mix}\rho_s}{C}\ln\frac{C_w(t)}{C_w(t_r)} - \frac{h_{mix}\rho_s}{C}\ln\frac{BC_w(t)+C}{BC_w(t_r)+C} - \frac{h_{mix}\rho_s}{BC_w(t)+C} +$$
$$\frac{h_{mix}\rho_s}{BC_w(t_r)+C} = \left\{(\alpha-\gamma)\left[i(t_r)+bxt_r\right]-\alpha p\right\}t-t_r - \frac{b}{2}(\alpha-\gamma)\left(t^2-t_r^2\right)$$

(15)

320   A fourth-order Runge-Kutta method is employed to solve Eq. (15) with the same time

321   step as in the previous time periods.

322   *Period* 4. The infiltration rate is stationary during this last period and can be obtained

323   by evaluating Eq. (14) at time $t = t_s$. The initial concentration of the runoff fluid,

324    $\alpha C_w(t_s)$, is obtained by evaluating Eq. (15) at time $t = t_s$. The resulting format of Eq.

325   (3), i.e.:

326

$$\left[\alpha h_p + h_{mix}\theta_s\right]\ln\frac{C_w(t)}{C_w(t_r)} + \frac{h_{mix}\rho_s}{C}\ln\frac{C_w(t)}{C_w(t_r)} - \frac{h_{mix}\rho_s}{C}\ln\frac{BC_w(t)+C}{BC_w(t_r)+C} -$$
$$\frac{h_{mix}\rho_s}{BC_w(t)+C} + \frac{h_{mix}\rho_s}{BC_w(t_r)+C} = \left[i(t_s)(\alpha-\gamma)-\alpha p\right](t-t_s)$$

(16)

327   is then solved by a fourth order Runge-Kutta method, as described above.

328   For brevity, the solute transport models obtained by considering the Langmuir,

329   Freundlich, Temkin and linear adsorption models are respectively denoted as L, F, T

330   and H models.

331   *Maximum Likelihood Model calibration*

332   We consider the vector $\mathbf{Y}^* = [Y_{(1)}^*, Y_{(2)}^*, \cdots, Y_{(N_C)}^*]$ collecting N_C measurements of

333   dissolved Cr(VI) concentration observed in surface the runoff at sampling time $i = 1$,

334   2, ..., N_C. The general relationship expressing Cr(VI) concentration in runoff through

335   a mathematical model $f(.)$ associated with the vector $\mathbf{X} = [x_1, x_2, \cdots, x_{N_p}]$ of N_P

336   unknown parameters is here represented as $\mathbf{Y} = f(\mathbf{X})$, vector $\mathbf{Y} = [Y_{(1)}, Y_{(2)}, \cdots, Y_{(N_C)}]$

337   including N_C simulated Cr(VI) concentration values in the runoff at time $i = 1$, 2, ...,

338   N_C. Experimental measurement errors are considered to be independent (e.g., Carrera

339   and Neuman, 1986; Bianchi-Janetti et al., 2012), the corresponding error covariance

340   matrix, $\mathbf{B}_C$, being diagonal, σ_i^2 (i = 1, 2, ..., N_C) representing observation error

341   variance. The Maximum Likelihood (ML) estimate $\hat{\mathbf{X}}$ of the vector of the N_P

342   uncertain model parameters can be obtained by minimizing with respect to $\mathbf{X}$ the

343   negative log likelihood criterion (e.g., Carrera and Neuman 1986):

344   $$\text{NLL} = \sum_{i=1}^{N_C} \frac{Y_{(i)}^* - \hat{Y}_{(i)}}{\sigma_i^2} + \ln|\mathbf{B}_C| + N_C \ln(2\pi) \tag{17}$$

345   where $\hat{Y}_{(i)}$ is the output provided by a given interpretive model at the ith observation

346   time. We note that minimizing Eq. (17) corresponds to minimization of the least square

347   criterion (Carrera and Neuman, 1986; Bianchi-Janetti et al., 2012 and references

348   therein):

349   $$J = \sum_{i=1}^{N_C} \frac{Y_{(i)}^* - \hat{Y}_{(i)}}{\sigma_i^2} \tag{18}$$

350   Here, minimization of (18) is obtained upon relying on the iterative Levenberg-

351   Marquardt algorithm as embedded in the well documented computational framework

352   PEST (Doherty, 2002). We consider minimizing Eq. (18) with uniform (and generally

353   unknown) measurement error variance, i.e., $\sigma_i^2 = \sigma^2$ for i = 1, 2, ..., N_C (see, e.g.,

354   Bianchi-Janetti et al., 2012).

355   *Model Identification Criteria*

356   To evaluate the performance of the four alternative models considered (i.e., L, F, T

357   and H), we rely on the four criteria:

358   $$\text{AIC} = \text{NLL} + 2N_P \tag{19}$$

17

$$359 \quad \text{AIC}_C = \text{NLL} + 2N_P + \frac{2N_P\left(N_P + 1\right)}{N_C - N_P - 2} \tag{20}$$

$$360 \quad \text{BIC} = \text{NLL} + N_P \ln(N_C) \tag{21}$$

$$361 \quad \text{KIC} = \text{NLL} + N_P \ln\left(\frac{N_C}{2\pi}\right) - \ln|\mathbf{Q}| \tag{22}$$

362   where $\mathbf{Q}$ represents the Cramer-Rao lower-bound approximation for the covariance

363   matrix of the parameter estimates, i.e., the inverse expected Fisher information matrix,

364   which renders a quantitative appraisal of the quality of parameter estimates and of the

365   information content carried by data about model parameters (see, e.g., Ye et al., 2008

366   for details). Here, Eq. (19) is proposed by Akaike (1974), Eq. (20) by Hurvich and Tsai

367   (1989), Eq. (21) by Schwartz (1978) and Eq. (22) by Kashyap (1982). It is noted that

368   the lowest value of a given model identification criterion indicates the most favored

369   model (according to the criterion itself) at the expense of the other ones.

370   *Maximum Likelihood Bayesian Modeling averaging*

371   The discrimination criteria (19)-(22) can also be considered to assign posterior

372   probability weights quantifying uncertainty associated with each of the tested isotherm

373   models. The posterior probability linked to model M_k (k = 1, 2, ..., N_M, N_M, which is

374   equal to 4 in our study, being the number of interpreting models assessed) is evaluated

375   as (Ye et al., 2008):

$$376 \quad P\left(M_k \mid \mathbf{Y}^*\right) = \frac{\exp\left(-\frac{1}{2}\left(IC_k - IC_{\min}\right)\right)P(M_k)}{\sum_k^{N_M} \exp\left(-\frac{1}{2}\left(IC_k - IC_{\min}\right)\right)P(M_k)} \tag{23}$$

377   where IC_k is either AIC (19), AIC$_C$ (20), BIC (21), or KIC (22), IC_{min} = min$\{IC_k\}$

378   being its minimum value calculated across the set of the four models examined; and

379    $P(M_k)$ is the prior probability associated with the kth alternative model. Because no

380   prior information is available, we set $P(M_k) = 1/N_M$.

381   Averaging across the moments provided by all alternative models renders the

382   following (Bayesian-averaged) expressions for the leading moments (Draper, 1995;

383   Hoeting et al., 1999):

384    $$E\left(\mathbf{Y} \mid \mathbf{Y}^*\right)=\sum_{k=1}^{N_M} E\left(\mathbf{Y} \mid \mathbf{Y}^*, M_k\right) P\left(M_k \mid \mathbf{Y}^*\right) \qquad (24)$$

385
$$V\left(\mathbf{Y} \mid \mathbf{Y}^*\right)=\sum_{k=1}^{N_M} Var\left(\mathbf{Y} \mid \mathbf{Y}^*, M_k\right) P\left(M_k \mid \mathbf{Y}^*\right)+$$
$$\sum_{k=1}^{N_M}\left(E\left(\mathbf{Y} \mid \mathbf{Y}^*, M_k\right)-E\left(\mathbf{Y} \mid \mathbf{Y}^*\right)\right)^2 P\left(M_k \mid \mathbf{Y}^*\right) \qquad (25)$$

386   Here, $E\left(\mathbf{Y} \mid \mathbf{Y}^*\right)$ and $V\left(\mathbf{Y} \mid \mathbf{Y}^*\right)$ respectively are model-averaged estimate and

387   variance of $\mathbf{Y}$ conditional on the set of Cr(VI) observations collected in $\mathbf{Y}^*$; and

388    $E\left(\mathbf{Y} \mid \mathbf{Y}^*, M_k\right)$ and $V\left(\mathbf{Y} \mid \mathbf{Y}^*, M_k\right)$ respectively are the mean and variance of $\mathbf{Y}$

389   conditional on $\mathbf{Y}^*$ and model M_k.

390

391   RESULTS AND DISCUSSION

392   Here, we start by illustrating the available Cr(VI) observations and discuss the results

393   of ML-based calibration of the four models analyzed. We then quantify posterior model

394   weights according to the selection criteria considered and use these to (*a*) rank the

395   models in terms of their relative skill to interpret the available data and (*b*) compute

396   model-averaged estimates and corresponding uncertainty bounds. We resort to a

397   classical variance-based global sensitivity analysis (GSA) to quantify the relative

398   contribution of the uncertain parameters characterizing each of the models tested to the

399 variability of Cr(VI) concentration in the runoff water. We then discuss the implication

400 of adsorption function by relying on the most skillful amongst the models tested to

401 characterize Cr(VI) loss from soil to surface runoff.

402 *Model ranking*

403 Observed Cr(VI) concentrations in the runoff for experiments 1 and 2 are

404 respectively depicted in Fig. 3a and 3b. These figures depict the corresponding

405 concentration values obtained after ML model calibration for the four models analyzed.

406 Table 3 lists ML parameter estimates together with the associated standard deviation

407 (SD).

408 Simulated concentrations are generally in good agreement with experimental

409 evidences, a remarkable exception being model T in experiment 1. The high values of

410 the estimated standard deviations listed in Table 3 can be partially due to linearity

411 assumptions upon which the employed parameter optimization strategy is implemented

412 and/or a trade-off between the information content associated with data and the number

413 and nature of model parameters considered (Doherty 2002). The latter observation is

414 consistent with the observed high values of the entries of the cross-correlation matrix

415 associated with parameter estimates. These are listed in Table 4 and suggest that the

416 available data are not conducive to unique estimates of model parameters.

417 Model calibration results indicate that the soil used in experiment 2 has a higher

418 adsorption capacity than the soil in experiment 1. This result is consistent with the

419 experimental setting, which comprises a finer soil texture in experiment 2. One can also

420 note that experiment 2 is associated by a higher depth of the mixing layer than

421 experiment 1. This finding is consistent with our experiment setting, according to which

20

422   soil 1 can form a much stronger shield (Heilig et al., 2001) against rain drop erosion on

423   surface soil 1 than soil 2, because of its sedimentological composition. This partially

424   supports the higher Cr(VI) concentrations obtained for experiment 2, which are

425   consistent with the observation that the depth of the mixing layer directly influences

426   the total mass of solute that can be transferred from soil to surface runoff. One should

427   also notice that a higher infiltration rate occurs in experiment 1 than in experiment 2,

428   thus indirectly suggesting that less Cr(VI) mass is lost in the mixing layer through

429   surface runoff in the former set-up than in the latter.

430   Calculated values for each of the model identification criteria considered are listed

431   in Table 5. We can see that AIC, AIC_C and BIC values are close, because of their similar

432   structures. For experiment 1, the lowest AIC, AIC_C, BIC and KIC values are -140.79, -

433   137.46, -134.90, and -156.22, respectively, their corresponding counterparts for

434   experiment 2 being -36.04, -30.79, -29.23, and -61.81, respectively. Posterior model

435   weights of the alternative models analyzed are listed in Table 6. For experiment 1, the

436   highest AIC-, AIC_C- and BIC-based posterior model weights are 47.09%, 66.54% and

437   61.60%, respectively, all of them being associated with the H model. Otherwise, the

438   highest KIC-based posterior model weight (i.e., 53.97%) is linked to the L model. For

439   experiment 2, the highest AIC-, AIC_C- and BIC-based posterior model weights are

440   90.39%, 90.30% and 90.34%, respectively, all of them being connected to the L model.

441   The highest KIC-based posterior model weight is 77.87% and is linked to the T model.

442   One can then conclude that the H and/or L model are identified as the most skillful ones

443   for experiment 1, respectively according to the BIC- and/or KIC-based posterior model

weights, the L and/or T models being favored to the interpretation of experiment 2.

According to Ye et al. (2008), AIC and AIC$_C$ are based on the premise that the true model is comprised in the set of alternative models tested, a constraint which is not shared by BIC or KIC. Additionally, it can be noted that KIC imbues a balancing effect between expected value of information and model parsimony. At the same time, it has been observed that KIC tends to favor models which can lead to potentially controversial results, due to the effect of **Q** (e.g., Tsai and Li, 2008; Li and Tsai, 2009). On the basis of this discussion, and for the purpose of our application, we base our model selections on KIC. By further noting that the T model displays an unsatisfactory pattern in the interpretation of experiment 1 (see Fig. 3a), we conclude that the L model can be considered as the most skillful amongst the alternatives considered for the purpose of the overall interpretation of both experimental datasets.

We now illustrate the results of the MLBMA multi-model analysis. The latter is performed through a numerical Monte Carlo (MC) framework structured according to the following steps: (*a*) $N = 100,000$ MC samples of each uncertain model parameter x_i ($i = 1, 2, \cdots, N_P$) are randomly generated for each candidate model M_k ($k = 1, 2, \cdots, N_M$) through the Latin Hypercube sampling technique considering each x_i as independent and identically distributed (i.i.d.) random variables, uniformly distributed within the support $[\hat{x}_i - 30\% \hat{x}_i, \ \hat{x}_i + 30\% \hat{x}]$, with mean value equal to $\hat{x}_i$ (i.e., the ML-based parameter estimate); (*b*) calculating N MC realizations of dissolved Cr(VI) concentrations in the surface runoff (collected in vector **Y**) via each candidate model M_k ; (*c*) computing (ensemble) mean $E\left(\mathbf{Y} \mid \mathbf{Y}^*, M_k\right)$ and variance

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466 $V\left(\mathbf{Y}\,|\,\mathbf{Y}^{*},M_{k}\right)$; (*d*) and evaluating model-averaged estimates $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*}\right)$ and

467 $Var\left(\mathbf{Y}\,|\,\mathbf{Y}^{*}\right)$ through (25) and (26).

468 Fig. 4 depicts scatterplots of model-averaged estimates $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*}\right)$ and

469 corresponding uncertainty bounds (of width equal to $\pm\sqrt{V\left(\mathbf{Y}\,|\,\mathbf{Y}^{*}\right)}$) versus Y

470 measurements for experiment 1 on the basis of IC = AIC (Fig. 4a), AIC$_C$ (Fig. 4b), BIC

471 (Fig. 4c), and KIC (Fig. 4d). For completeness, each subplot also depicts

472 $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*},\tilde{M}_{k}\right)$ and $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*},\tilde{M}_{k}\right)\pm\sqrt{V\left(\mathbf{Y}\,|\,\mathbf{Y}^{*},\tilde{M}_{k}\right)}$, as computed by the most

473 skillful model, $\tilde{M}_{k}$, identified by the corresponding IC. Maximum Likelihood

474 estimates of Y obtained through model $\tilde{M}_{k}$ (with corresponding model parameter

475 set $\hat{\mathbf{X}}$) are also depicted. Values of the highest posterior model weights, $P\left(\hat{M}_{k}\,|\,\mathbf{Y}^{*}\right)$,

476 are also included in each subplot. Corresponding depictions for experiment 2 are shown

477 in Fig. 5.

478 These results suggest that: (*i*) $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*}\right)$ and $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*}\right)\pm\sqrt{V\left(\mathbf{Y}\,|\,\mathbf{Y}^{*}\right)}$ are

479 respectively very close to $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*},\tilde{M}_{k}\right)$ and $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*},\tilde{M}_{k}\right)\pm\sqrt{V\left(\mathbf{Y}\,|\,\mathbf{Y}^{*},\tilde{M}_{k}\right)}$

480 when $P\left(\hat{M}_{k}\,|\,\mathbf{Y}^{*}\right)\geq 90\%$ (see Fig. 5); (*ii*) $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*}\right)$ can provide better estimates

481 than $E\left(\mathbf{Y}\,|\,\mathbf{Y}^{*},\tilde{M}_{k}\right)$ when the individual models of the set considered yield very

482 different results (see Fig. 4); (*iii*) the KIC-based $Var\left(\mathbf{Y}\,|\,\mathbf{Y}^{*},\tilde{M}_{k}\right)$ is always smaller

483 than its counterparts based on the other IC considered for a given experiment (see Figs.

484 4 and 5). These results are consistent with observations by Ranaee et al. (2016), who

485 noted that model-averaged estimates were virtually coinciding with those associated

486 with the most skillful model in their study when the latter was characterized by

487 $P\left(\hat{M}_{k}\,|\,\mathbf{Y}^{*}\right)\geq 95\%$. They are also consistent with the results of Winter and Nychka

488   (2010), who note that a model average can only be more skillful than the model

489   identified as best solely when the individual models in the collection produce very

490   different forecasts.

491   Figs. 6 and 7 complement the results illustrated above by depicting the Monte Carlo

492   based probability density functions (*pdfs*) of Cr(VI) concentration in the water runoff

493   evaluated through the candidate models, respectively for experiments 1 and 2. Results

494   are illustrated for early (T_1), median (T_2), and late (T_3) simulation times, respectively

495   corresponding to sampling times when observations $Y^*_{(i)}$ ($i = 1, 5$, and 23) are collected.

496   One can see that the densities calculated by the L model generally peak at a value

497   closely corresponding to the measured Cr(VI) concentration, an exception being given

498   by the late times results of Fig. 6c, where the *pdfs* associated with the H and L models

499   resemble a Delta function. The *pdfs* associated with F and H models generally show

500   higher skewness and heavier tails, which partially indicate increased probability of

501   extreme values, in comparison with L and T models.

502   *Variance-based global sensitivity analysis*

503   We provide further insights on the way that model uncertain parameters can

504   contribute to the variability of model responses through a sensitivity analysis. In this

505   context, local sensitivity analysis approaches (*i*) quantify the sensitivity of a model

506   output to small perturbations of parameter values and (*ii*) provide an appraisal of the

507   behavior of the modeled system in regions of the parameter space close to the perturbed

508   value of the parameter (e.g., Razavi and Gupta, 2015 and references therein). GSA

509   techniques enable us to evaluate sensitivities of model outputs across the overall

24

510   support defining the space of variability of model parameters (e.g., Formaggia et al.,

511   2013; Ciriello et al., 2013, 2015; Riva et al., 2015; Razavi and Gupta, 2015, Dell'Oca

512   et al., 2017 and references therein). The latter is based on the evaluation of the Sobol'

513   indices, which are based on a classical decomposition of variance and provide

514   information on the relative contribution of each uncertain model parameter to the

515   variance of a target model output. Here, we apply the variance-based GSA approach

516   (Saltelli et al., 2008; 2010).

517   Evaluation of the variance-based sensitivity indices is performed by considering

518   uncertain model parameters as independently random variables, within the space of

519   variability described above. The total sensitivity indices for the ith model parameter

520   (or factor) corresponding to the jth Cr(VI) measurement in the runoff is evaluated as

521   (Saltelli et al., 2008, 2010)

522
$$
\begin{aligned}
S_{T(ij)} &= 1 - \frac{V(E(Y_{(j)} \mid x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{N_P}))}{V(Y_{(j)})} \\
&= \frac{E(V(Y_{(j)} \mid x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{N_P}))}{V(Y_{(j)})}
\end{aligned}
\tag{25}
$$

523   Here, $V(Y_{(j)})$ is the variance of model response at the time corresponding to the jth

524   Cr(VI) observation in the runoff; $V(E(Y_{(j)} \mid x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{N_P}))$ represents the

525   variance of model responses expectation conditioned to all factors, excluding factor x_i;

526    $E(V(Y_{(j)} \mid x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{N_P})) = 1 - V(E(Y_{(j)} \mid x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{N_P}))$ is

527   the expected variance conditioned on all factors, excluding factor x_i. In our study,

528    $E(V(Y_{(j)} \mid x_1, x_2, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{N_P}))$ is evaluated through Eq. (19) in Saltelli et al.

529   (2010). We note that Sobol' total sensitivity indices are informative of the relative

530   importance of each model input to the variance of model output and are not amenable

25

531   to interpretations leading to ranking of the relative interpretive skill of the collection of

532   models considered.

533   We illustrate our results of GSA for the setting of experiment 2. Figs. 8-11 depict the

534   temporal pattern displayed by $S_{T(ij)}$ as a function of the size, N, of the collection of

535   random parameter values for the models tested. These results suggest that $N = 5000$

536   samples yields sufficiently stable results.

537   Fig. 12 depicts the temporal variation of the contribution (quantified by

538     $S_{T(ij)} / \sum_{i=1}^{N_P} S_{T(ij)} \times 100\%$) of each input factor to model output for all candidate models.

539   These results indicate that model output Y is most sensitive to h_{mix} for all alternative

540   models, and are consistent with the observation that the depth of the mixing layer

541   directly determines the mass of Cr(VI) that could be transported from soil to surface

542   runoff. An exception to this pattern is given by model F whose model output is very

543   sensitive to parameter n, which is mainly due to the importance of n to drive the power

544   law behavior in Eq. (5). Results in Fig. 12 suggest that the sum of the contributions

545   associated with the adsorption parameters consistently increases as simulation time

546   progresses for all models. Considering model L as an example, the contributions of

547   K_{eq} and S_{max} are respectively 0.47% and 1.38% at the beginning simulation, and

548   increase up to 6.61% and 24.02% at the end of simulation. This corresponds to an

549   increase of the combined total contributions of K_{eq} and S_{max} to σ_Y^2 from 1.85% to

550   30.63% during the simulation period and denote the significant effect of the adsorption

551   process on Cr(VI) loss from soil to the surface runoff, especially at late times.

552   Fig. 12 also suggests that the parameters with the lowest (in an average sense)

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553   contribution to σ_Y^2 are K_{eq} (for model L), α (for models F and H), and K_t (for

554   model T). We denote here as $C_{T(K_{eq})} = 5.41\%$, $C_{T(\alpha_F)} = 2.33\%$, $C_{T(\alpha_H)} = 10.41\%$,

555   and $C_{T(K_t)} = 0.00\%$, the average of the temporal contributions evaluated for each of

556   these parameters over the set of 23 observation times in experiment 2. It is interesting

557   to note that the ranking $C_{T(K_t)} < C_{T(\alpha_F)} < C_{T(K_{eq})} < C_{T(\alpha_H)}$ is somehow consistent with the

558   ranking of posterior model weights based on KIC (i.e., 77.87%, 17.76%, 4.37% and

559   0.00%, respectively for models T, F, L and H; see Table 6).

560   As a complement to these results, Fig. 13 depicts the temporal variation of the

561   quantity $\frac{1}{N_p}\sum_{i=1}^{N_p}\left|S_{T(ij)} - S_{1(ij)}\right|$, representing the mean of absolute difference between the

562   total ($S_{T(ij)}$) and first-order ($S_{1(ij)}$) sensitivity indices. As a remark, one can see that low

563   values of this quantity typically correspond to high KIC-based posterior model weights,

564   with the exception of model T in experiment 1. This finding might be considered as an

565   indication that low values of $\frac{1}{N_p}\sum_{i=1}^{N_p}\left|S_{T(ij)} - S_{1(ij)}\right|$ can be consistent with the low

566   expected Fisher information. Additional theoretical developments are needed to fully

567   explore possible implications of these results and will be the subject of future

568   investigations.

569   *Effect of adsorption on Cr(VI) loss from soil to surface runoff*

570   While global sensitivity analyses of the kind we illustrate provide global measures

571   quantifying the contribution of uncertain input parameters to the variance of a model

572   output *pdf* across the entire investigated parameter space, they do not yield a

573   straightforward assessment of (*a*) the actual values attained by model outputs within

574 the parameter space or (*b*) the direction of variation of model results as a function of

575 parameter variation. These features can be readily visualized through a scatterplot

576 analysis. We illustrate the results of the latter by considering the L model, which has

577 been ranked as best in our prior analyses.

578   Figs. 14 and 15 depict the data clouds associated with the scatterplots of Cr(VI)

579 concentrations in the water runoff resulting from the L model at sampling times

580 respectively corresponding to the collection of the first and last concentration

581 measurement (i.e., Y_1^* and $Y_{N_C}^*$) in experiment 2. Linear regression curves are also

582 depicted for completeness.

583   These results generally indicate that *Y* is positively correlated to α and negatively

584 correlated to γ. They are consistent with the definition of $Y = \alpha C_w$ and with the

585 observation that a strong incomplete mixing (associated with high values of the mixing

586 coefficient γ) results in decreased values of Cr(VI) in the water runoff (i.e., an

587 increased transfer of Cr(VI) from the mixing layer to the underlying soil). The positive

588 correlation of *Y* with the depth of the mixing layer h_{mix} (Figs. 14e, and 15e) stems

589 from the physical effects of the mixing layer.

590   The correlation between *Y* and the adsorption parameter S_{max} changes from

591 negative to positive with elapsing time (compare Figs. 14d and 15d). At the beginning

592 of the simulation period, high values of S_{max} tend to increase the total Cr(VI) mass

593 adsorbed onto the solid phase with a decrease of the mass released to runoff. As time

594 progresses, the dissolved Cr(VI) concentration in the mixing layer decreases, promoting

595 desorption from the solid and subsequent transfer to runoff. These results are also

consistent with the tailing observed for the breakthrough curve of Cr(VI) for experiment 1.

CONCLUSIONS

The results from two experiments are used to investigate Cr(VI) losses from soils with diverse grain sizes to water runoff. Four solute transport models (denoted as L, F, T and H), coupling a two-layer solute transfer model, respectively, with Langmuir, Freundlich, Temkin, and linear adsorption isotherms, are assessed to simulate Cr(VI) transfer from soil to surface runoff. Each of the analyzed models is calibrated against experimental data through Maximum Likelihood (ML) parameter estimation. Four model identification criteria (i.e., AIC, AIC_C, BIC and KIC) are evaluated, and posterior probabilistic weights are then calculated to identify the most skillful model to interpret the available data. The classical variance-based global sensitivity and scatterplot sensitivity analyses are then performed in the context of both experimental settings. Our study leads to the following key conclusions.

(1) Cr(VI) concentrations in the runoff are much higher in experiment 2 than in experiment 1. The most significant difference between the two experimental scenarios is the soil grain size (i.e., soils were sieved through 2 mm and 1 mm sieves for experiments 1 and 2, respectively). This result can be explained by two mechanisms: (*a*) larger grain sizes would increase infiltration rates; and (*b*) a mixing layer depth tends to be decreased in a soil with larger grain size, resulting in a

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617   decreased Cr(VI) loss through runoff.

618 (2) For experiment 1, the largest AIC, AIC$_C$ and BIC-based posterior model weights

619   are associated with model H, respectively with values of 47.09%, 66.54% and

620   61.60%. Model L is ranked highest by KIC, with a posterior weight of 53.97%. For

621   experiment 2, the highest AIC, AIC$_C$ and BIC-based posterior model weights are

622   90.39%, 90.30% and 90.34%, respectively, and are linked to model L; Model T is

623   favored by KIC, with a posterior weight equal to 77.87%. Model L is then evaluated

624   as the most skillful for the overall interpretation of both experimental results.

625 (3) Variance-based global sensitivity results suggest that the thickness of the mixing

626   layer, h_{mix}, is the most sensitive parameter for all models, an exception being the

627   Freundlich model, where n is the uncertain parameter with the highest contribution

628   to the model output variance. The total sensitivity of the adsorption parameters

629   tends to increase with simulation time. For example, in the case of model L the sum

630   of the K_{eq} and S_{max} contributions (see Fig. 12b) to model output variance

631   increases from 1.85% to 30.63% across the temporal window spanned in

632   experiment 2. This result suggests that adsorption has a significant effect on the

633   uncertainty of prediction for Cr(VI) loss from soil to runoff.

634 (4) The scatterplot analysis results from model L suggest that the incomplete mixing

635   coefficient (α) and the depth of mixing layer (h_{mix}) are positively correlated with

636   Cr(VI) concentration in the runoff, and the incomplete mixing coefficient (γ) and

637   Cr(VI) concentration are negatively correlated. The correlation between Cr(VI)

638   concentration in the runoff and the maximum adsorption capacity (S_{max}) changes

639   from negative to positive during the simulation process. These results are obtained

640   from the finding that (*a*) large values of S_{max} tend to increase the total Cr(VI) mass

641   adsorbed onto the solid phase at the beginning of the simulation period; and (*b*) the

642   dissolved Cr(VI) concentration in the mixing layer decreases as time progresses,

643   promoting Cr(VI) transfer from soil to runoff.

644   It is remarked that the mathematical formulation for the adsorption process plays a

645   significant role on prediction of solute loss from soil to surface water runoff, and the

646   equilibrium adsorption assumption is not accurate in some field scenarios. Non-

647   equilibrium adsorption models should be further explored in future studies when they

648   are used in the two-layer model to describe chemical transport from soil to water runoff.

649   Additional elements of future study should also include the use of global sensitivity

650   techniques that allow exploring the relative importance of each uncertain model

651   parameter through the evaluation of (statistical) Moment-based Metrics of the kind

652   proposed by Dell'Oca et al. (2017). The latter are not confined to a description of the

653   feedback between uncertain model inputs and outputs via variance-based metrics (such

654   as the Sobol' indices) and aim at providing a comprehensive picture, quantifying the

655   impact of model parameter uncertainties on the statistical moments driving the main

656   features of the probability density function of model outputs.

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822

1   **Tables**

2

3   Table 1. Main chemical characteristics of soil sample 2 used in the experiments.

Cr(VI)	Fe_2O_3	Al_2O_3	MnO	Eh	pH
≤0.015 (mg kg^{-1})	0.0377 (g kg^{-1})	0.0094 (g kg^{-1})	0.0051 (g kg^{-1})	497.31 (mV)	7.6

4

5   Table 2. Key parameters and quantities characterizing the experimental conditions.

Experiment	Rainfall intensity, p /(cm min^{-1})	Stable infiltration rate, i_s /(cm min^{-1})	The time start to produce ponding water, t_p /(min)	The time runoff occur, t_r /(min)	The time of infiltration rate reached stable, t_s /(min)
1	0.092	0.028	6.5	27	40
2	0.100	0.024	5.75	28.7	39

6

Table 3. Maximum Likelihood model parameter estimates and associated standard deviation (SD).

model		L					F				
parameter		α	γ	K_{eq}	S_{max}	h_{mix}	α	γ	K_f	n	h_{mix}
Experiment 1	estimate	0.28	0.78	0.02	69.00	0.37	0.40	0.69	6.61	1.31	0.13
	SD	34.34	12.89	0.44	2753.87	5.61	162.21	96.57	1162.71	5.19	2.84
Experiment 2	estimate	0.15	0.21	0.51	111.00	0.46	0.22	1.0	17.50	2.89	0.88
	SD	10.87	2.16	12.81	1722.21	11.74	76.13	18.52	590.91	50.92	28.85

model		T					H			
parameter		α	γ	K	f	h_{mix}	α	γ	K_d	h_{mix}
Experiment 1	estimate	0.19	0.70	1.13	291.64	0.35	0.13	0.66	0.45	0.38
	SD	3749	988.1	295223.64	1542530.39	1863.25	359.57	69.39	424.10	1.81
Experiment 2	estimate	0.28	0.92	0.004	124.41	0.67	0.035	0.52	0.82	0.70
	SD	98.29	31.00	4.30	13784.77	73.28	390.06	25.67	843.01	5.57

Table 4. Correlation coefficients between model parameters

L model

	α	γ	K_{eq}	S_{max}	h_{mix}
α	1	0.99	0.61	0.77	0.65
γ	0.99	1	0.62	0.76	0.64
K_{eq}	-0.99	-0.99	1	-0.02	-0.19
S_{max}	0.98	0.98	-0.97	1	0.98
h_{mix}	0.99	0.99	-0.99	0.99	1

F model

	α	γ	K	n	h_{mix}
α	1	1	0.99	-0.99	0.98
γ	0.99	1	0.99	-0.99	0.97
K	-0.99	-0.98	1	-0.99	0.97
n	-0.95	-0.97	0.95	1	-0.95
h_{mix}	0.97	0.95	-0.97	-0.87	1

T model

	α	γ	K_t	f	h_{mix}
α	1	1	0.99	1	1
γ	0.99	1	0.99	1	1
K_t	0.91	0.94	1	1	1
f	0.98	0.99	0.96	1	1
h_{mix}	0.99	0.99	0.95	0.99	1

H model

	α	γ	K_d	h_{mix}
α	1	1	1	-0.97
γ	0.99	1	1	-0.97
K_d	0.99	1	1	-0.97
h_{mix}	-0.43	-0.44	-0.44	1

Note: the upper and lower triangular regions respectively represent correlation coefficients for experiment 1 and 2.

15 Table 5. Results of model identification criteria for both experiments.

Model	L				F			
Index	AIC	AICC	BIC	KIC	AIC	AICC	BIC	KIC
Experiment 1	-139.29	-134.34	-132.22	-156.22	-139.94	-135.00	-132.87	-155.78
Experiment 2	-36.04	-30.79	-29.23	-56.05	-30.38	-25.13	-23.57	-58.85
Model	T				H			
Index	AIC	AICC	BIC	KIC	AIC	AICC	BIC	KIC
Experiment 1	-80.22	-75.27	-73.15	-135.52	-140.79	-137.46	-134.90	-150.26
Experiment 2	-29.92	-24.67	-23.10	-61.81	-21.71	-18.18	-16.03	-38.45

16

17

18 Table 6. Posterior model weights (%) for the set of alterative models tested.

Model	L				F			
Index	AIC	AIC_C	BIC	KIC	AIC	AIC_C	BIC	KIC
PMW 1	22.18	14.03	16.10	53.97	30.7	19.43	22.30	43.29
PMW 2	90.39	90.30	90.34	4.37	5.32	5.32	5.32	17.76
Model	T				H			
Index	AIC	AIC_C	BIC	KIC	AIC	AIC_C	BIC	KIC
PMW 1	0.00	0.00	0.00	0.00	47.09	66.54	61.60	2.74
PMW 2	4.22	4.22	4.22	77.87	0.07	0.16	0.12	0.00

19 **Note: PMW 1 (or 2) denotes posterior model weights for experiment 1 (or 2)**

1   **Figures**
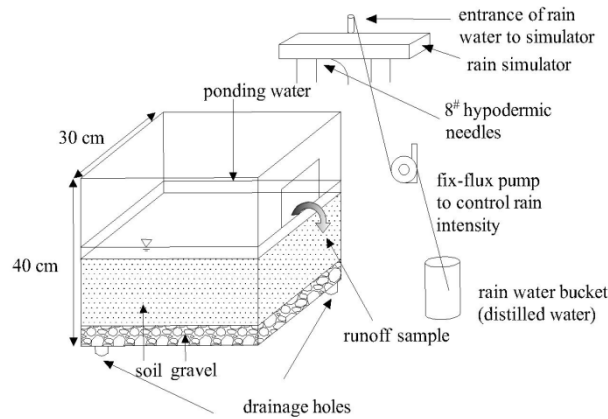
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3   Figure 1. Sketch of the experimental set-up

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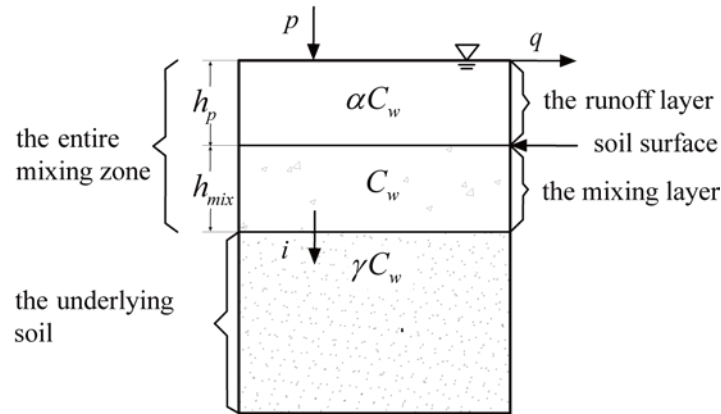
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6 Figure 2. Conceptual depiction of the two-layer model. Notations are: the rainfall

7   intensity p; the specfic discharege rate of overland flow q; the depth of the

8   ponding layer h_p; the depth of the mixing layer h_{mix}; infiltration rate i; the solute

9   concentration in the mixing layer C_w; the solute concentration in the ponding layer

10   /the runoff layer αC_w; the solute concentration in the underlying soil γC_w.

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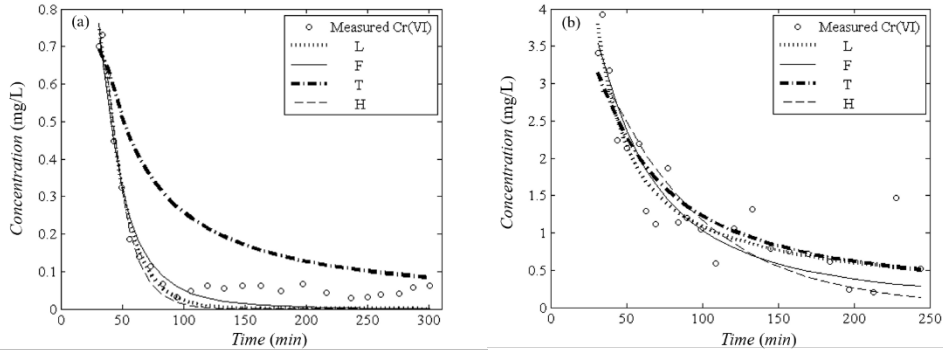
12

13 Figure 3. Temporal variation of measured Cr(VI) concentrations in the runoff together

14 with their simulated counterparts based on optimized parameters (L, F, T and H

15 models) for experiments (a) 1 and (b) 2.
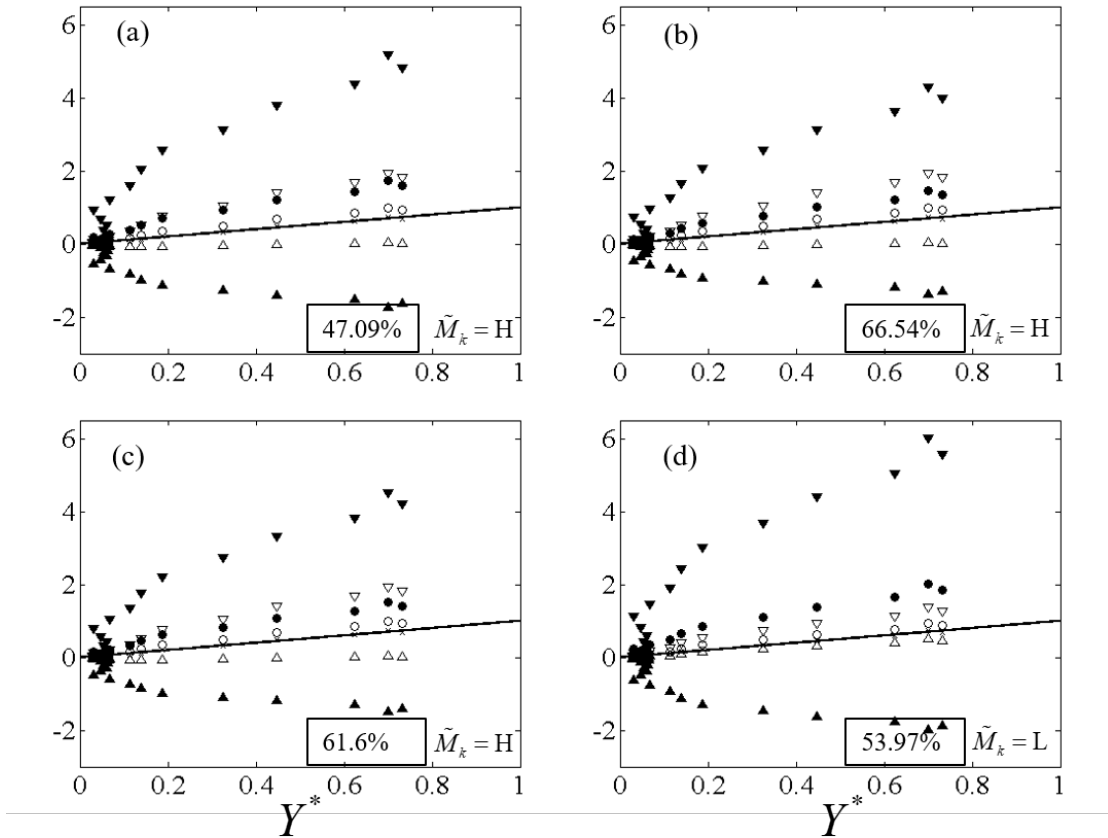
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17 Figure 4. Scatterplots of estimates of Cr(VI) concentration in the surface runoff

18 (collected in vector $\mathbf{Y}$) versus data (collected in vector $\mathbf{Y}^*$) of experiment 1. When

19 used, posterior model weights are evaluated through model identification criterion (a)

20 AIC, (b) AIC$_C$, (c) BIC, and (d) KIC; $\tilde{M}_k$ represents the most skillful model, as

21 identified by the corresponding criterion.

$$\bullet \; E(\mathbf{Y}\,|\,\mathbf{Y}^*) \qquad \blacktriangledown\!\!\blacktriangle \; E(\mathbf{Y}\,|\,\mathbf{Y}^*) \pm \sqrt{Var\left(\mathbf{Y}\,|\,\mathbf{Y}^*\right)} \qquad \times \; \mathrm{ML}$$

$$\circ \; E(\mathbf{Y}\,|\,\mathbf{Y}^*,\tilde{M}_k) \; \triangledown\!\!\vartriangle \; E(\mathbf{Y}\,|\,\mathbf{Y}^*,\tilde{M}_k) \pm \sqrt{Var\left(\mathbf{Y}\,|\,\mathbf{Y}^*,\tilde{M}_k\right)}$$

22
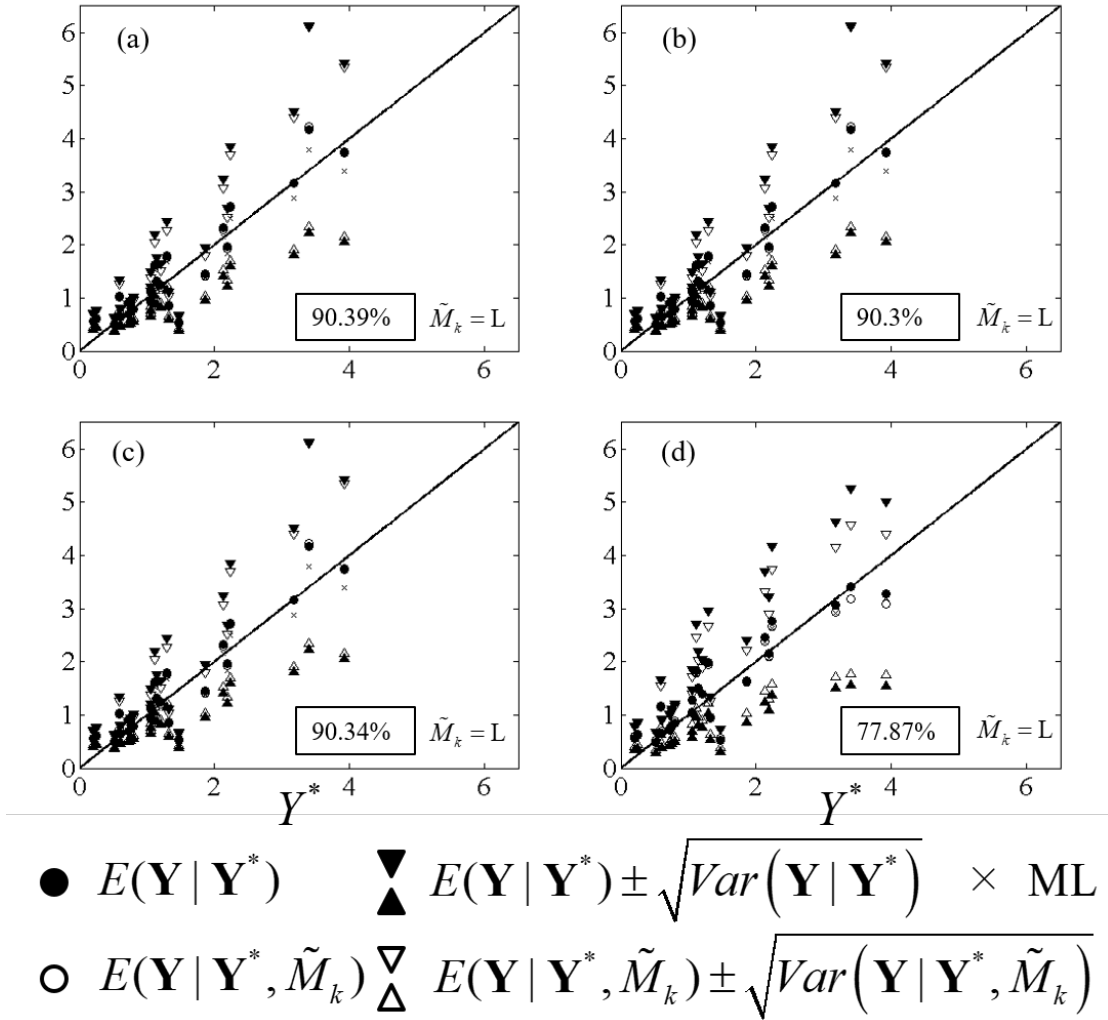
23 Figure 5. Scatterplots of estimates of Cr(VI) concentration in the surface runoff

24 (collected in vector $\mathbf{Y}$) versus data (collected in vector $\mathbf{Y}^*$) of experiment 2. When

25 used, posterior model weights are evaluated through model identification criterion (a)

26 AIC, (b) AIC$_C$, (c) BIC, and (d) KIC; $\tilde{M}_k$ represents the most skillful model, as

27 identified by the corresponding criterion.

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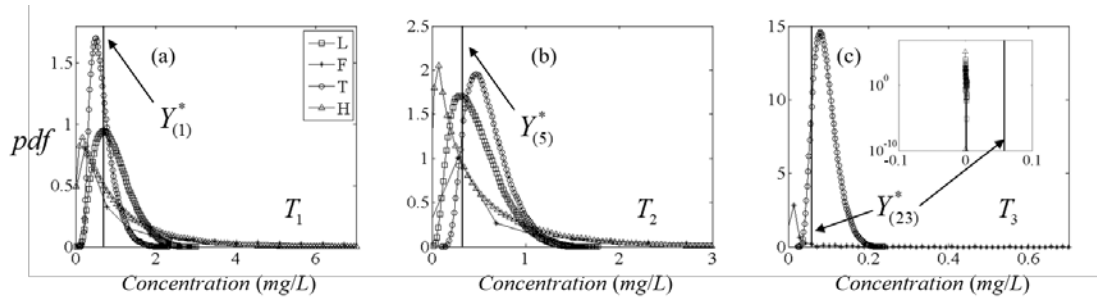
Figure 6. Sample probability density functions (*pdfs*) of Cr(IV) in runoff water (*Y*) based on the Monte Carlo simulations performed for each model at (a) early (T_1), (b) median (T_2), and (c) late (T_3) simulation times, respectively corresponding to sampling times where observations $Y_{(i)}^*$ (i = 1, 5, and 23), are collected in experiment 1. Vertical lines correspond to measured values $Y_{(i)}^*$.

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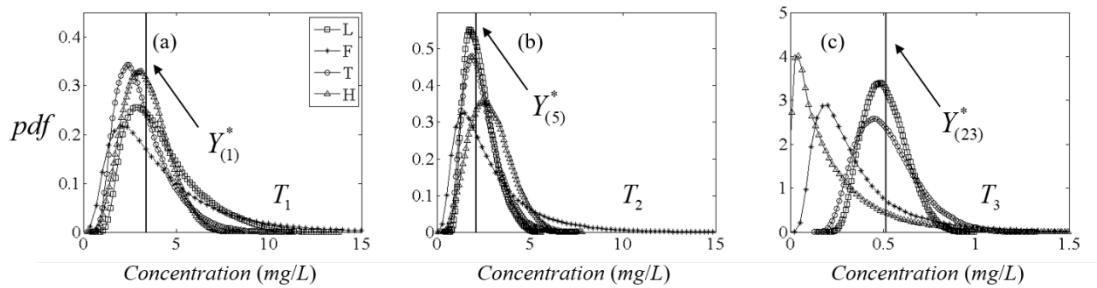
Figure 7. Sample probability density functions (*pdfs*) of Cr(IV) in runoff water (*Y*) based on the Monte Carlo simulations performed for each model at (a) early (T_1), (b) median (T_2), and (c) late (T_3) simulation times, respectively corresponding to sampling times where observations $Y_{(i)}^*$ (i = 1, 5, and 23), are collected in experiment 2. Vertical lines correspond to measured values $Y_{(i)}^*$.

44
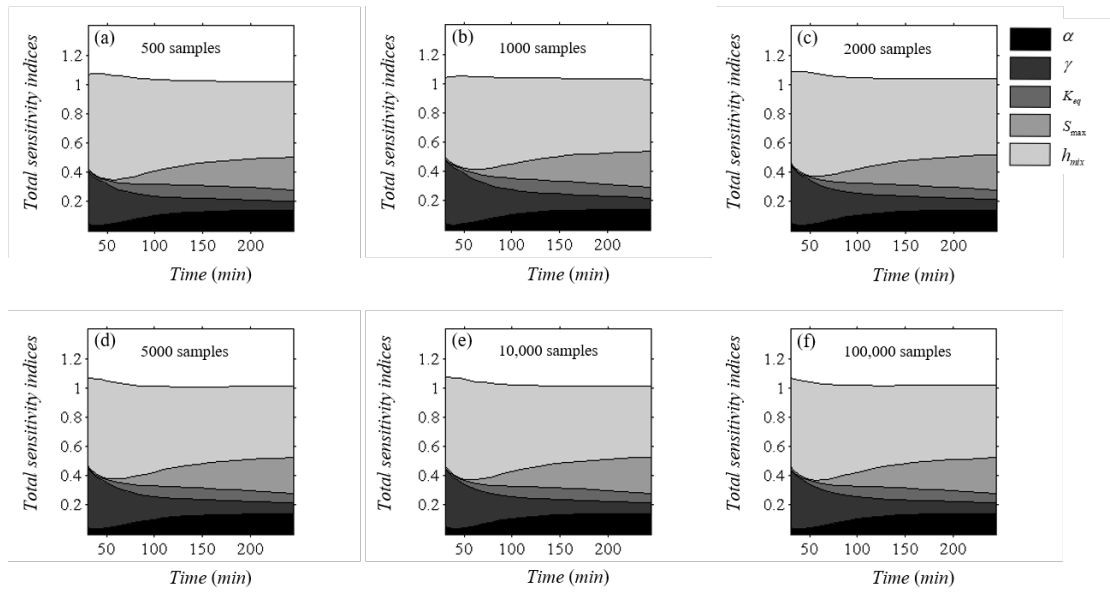
Figure 8. Total sensitivity indices of L model versus time and for diverse sample size,

i.e., N = (a) 500, (b) 1000, (c) 2000, (d) 5000, (e) 10,000, and (f) 100,000.
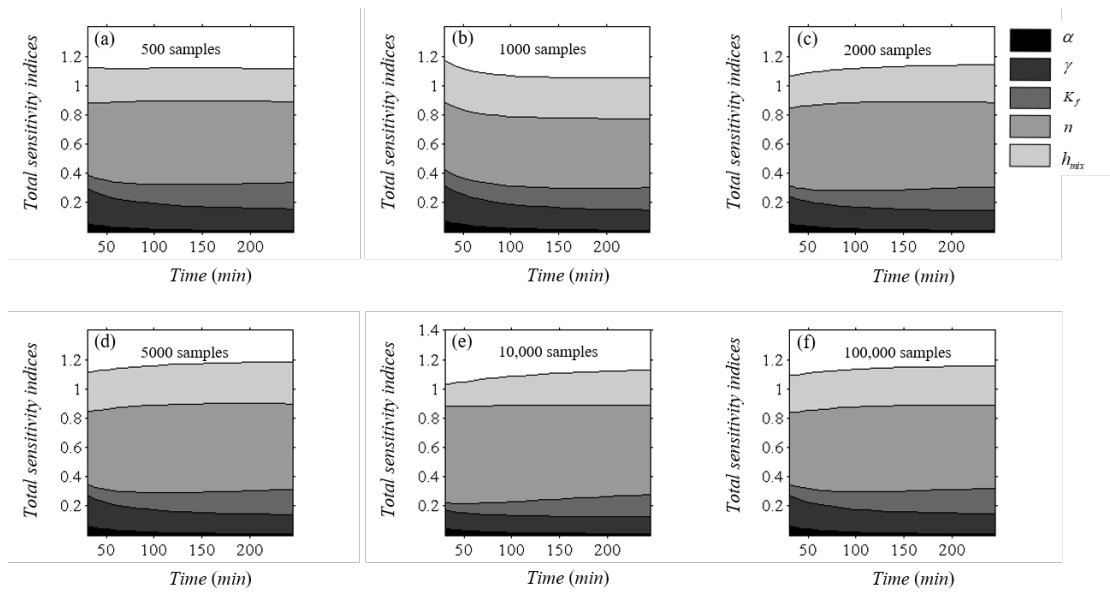
Figure 9. Total sensitivity indices of F model versus time and for diverse sample size,

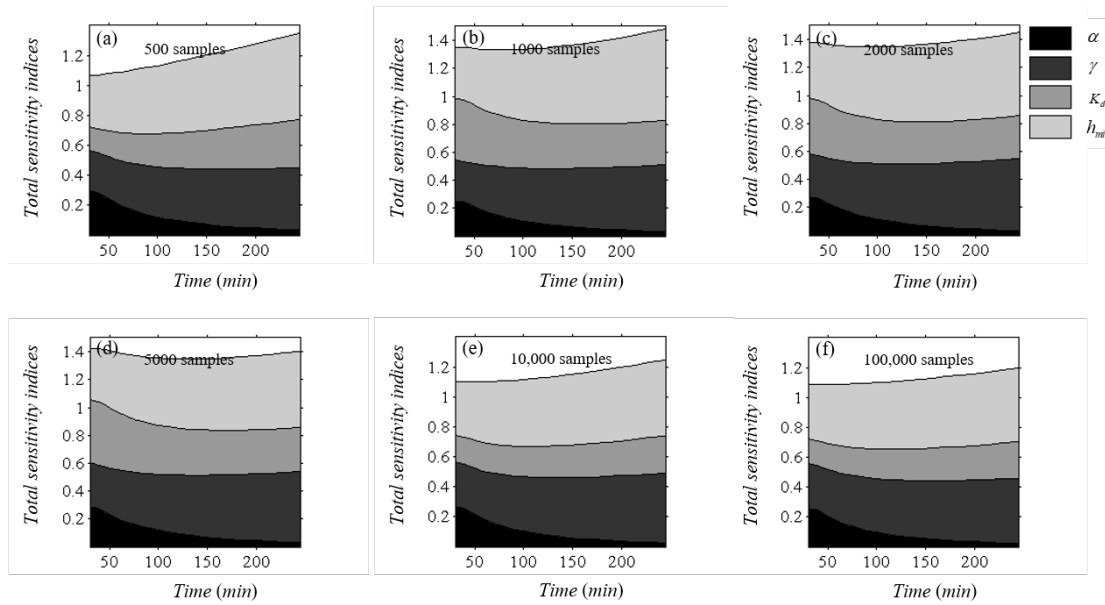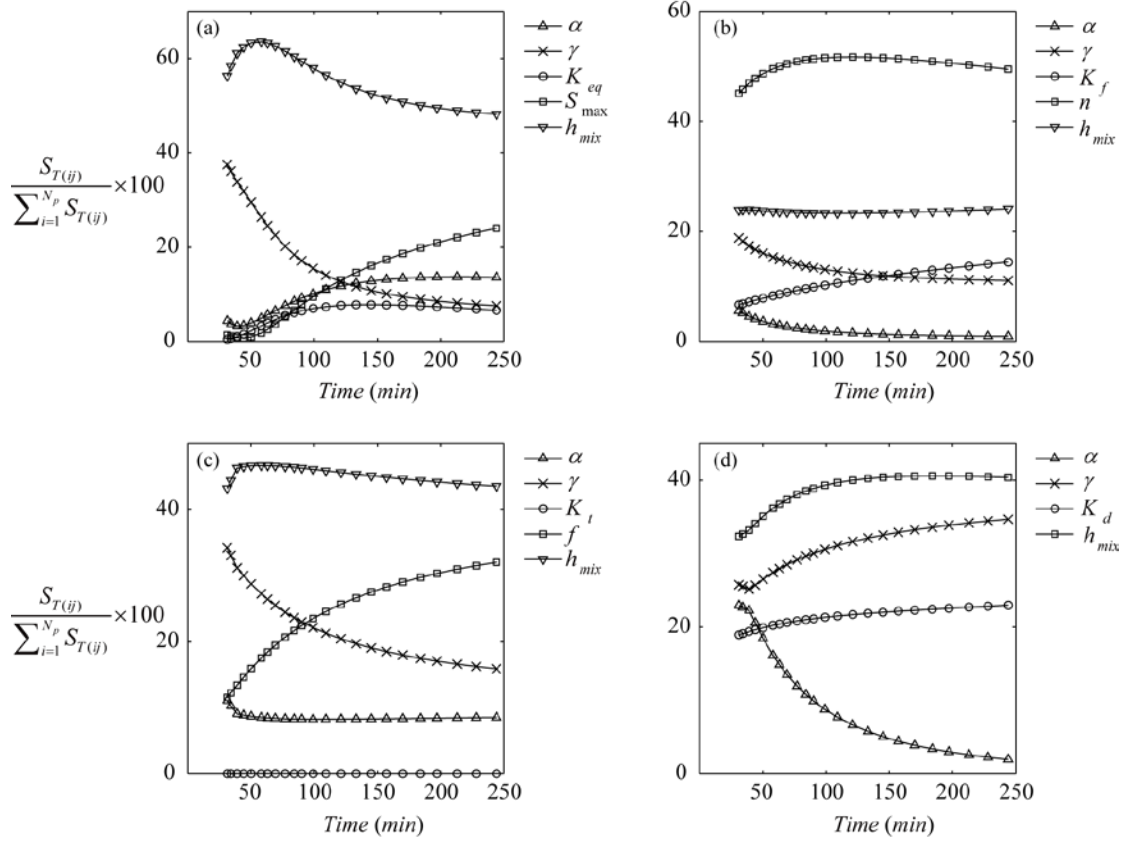i.e., N = (a) 500, (b) 1000, (c) 2000, (d) 5000, (e) 10,000, and (f) 100,000.

53
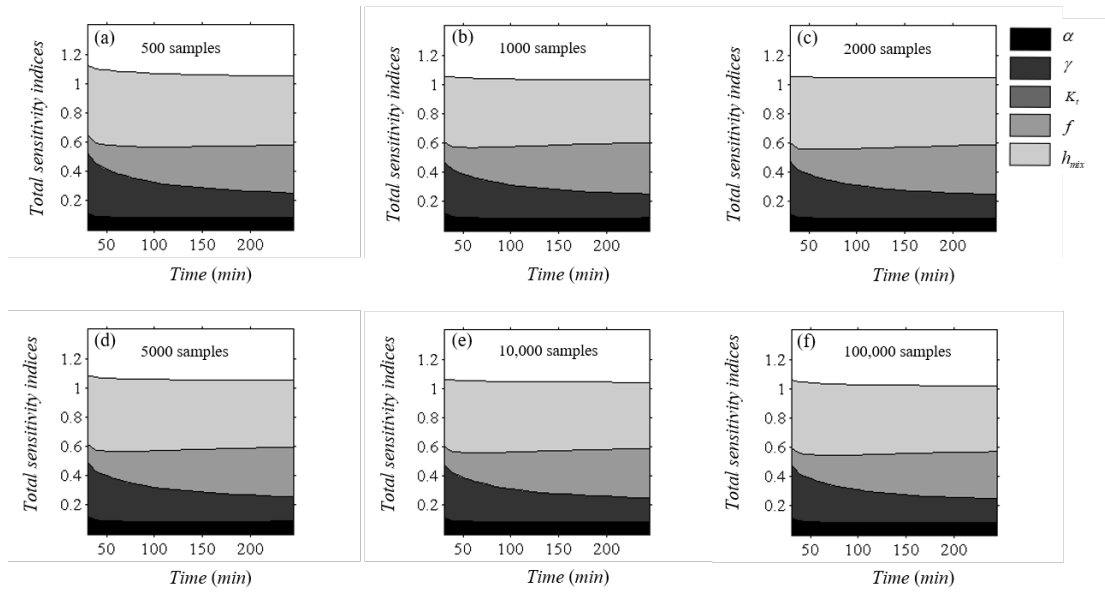
Figure 10. Total sensitivity indices of T model versus time and for diverse sample size,

i.e., N = (a) 500, (b) 1000, (c) 2000, (d) 5000, (e) 10,000, and (f) 100,000.

56

57

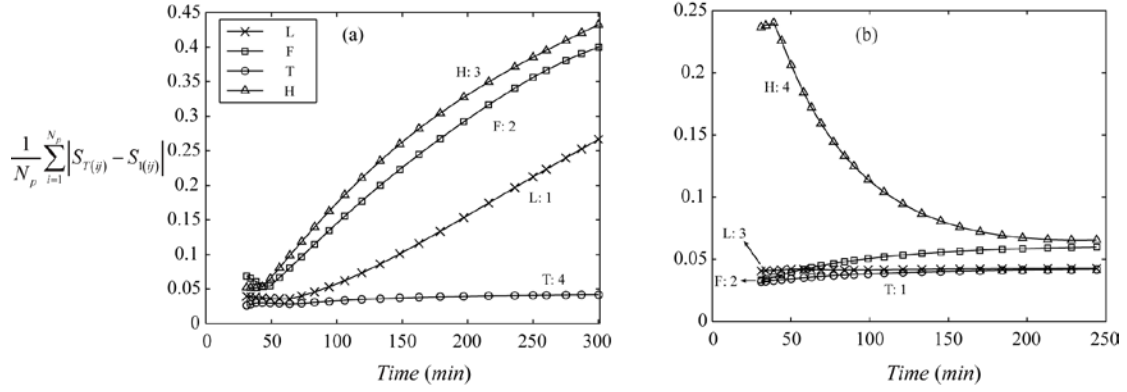Figure 11. Total sensitivity indices of H model versus time and for diverse sample size,

i.e., N = (a) 500, (b) 1000, (c) 2000, (d) 5000, (e) 10,000, and (f) 100,000.

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61   Figure 12. Contributions of each input factor to variation of Cr(VI) in surface runoff

62   versus time for experiment 2 according to model (a) L, (b) F, (c) T, and (d) H.
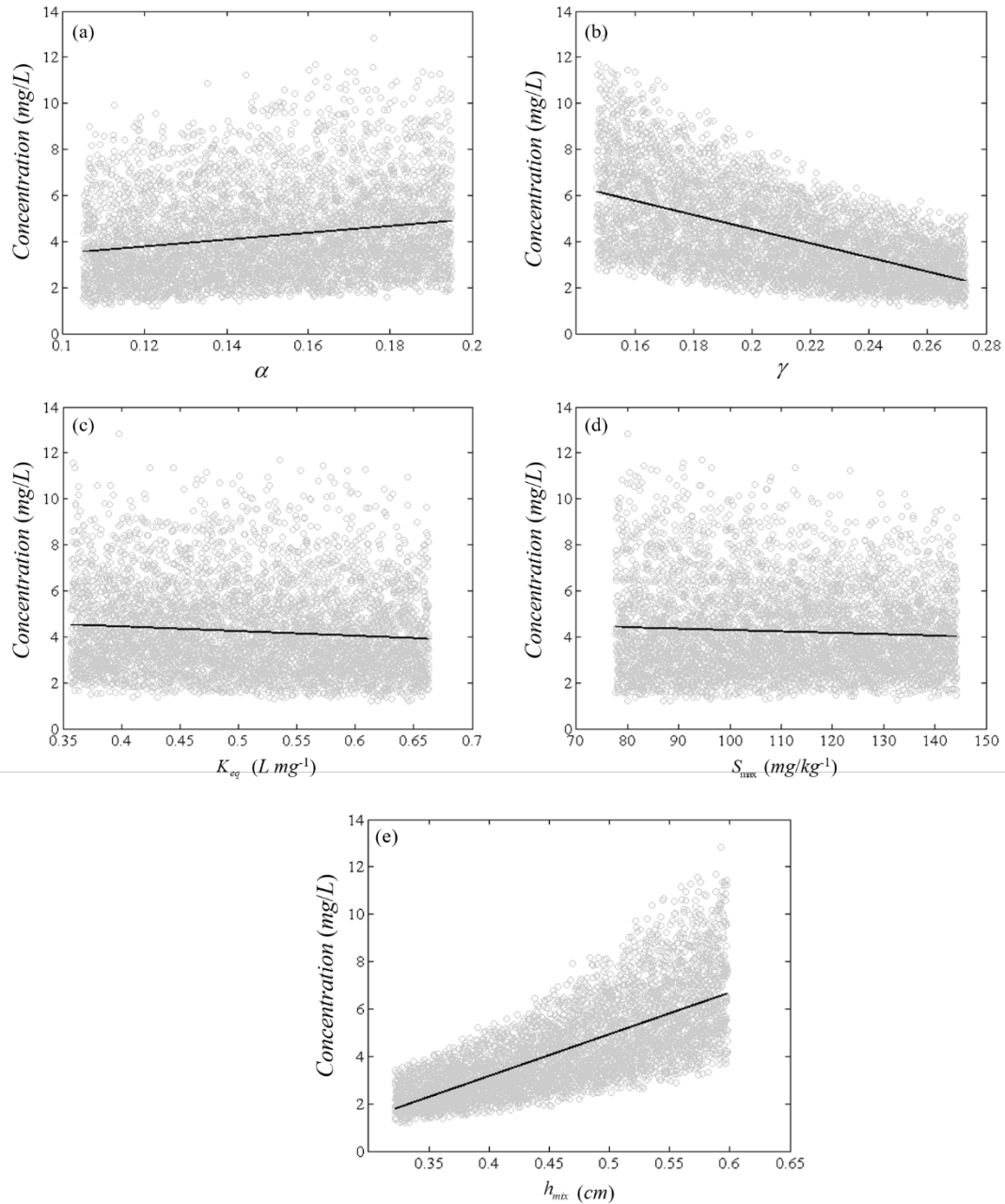
63

64

Figure 13. Temporal variation of the mean of the absolute difference between the total ($S_{T(ij)}$) and first-order ($S_{1(ij)}$) sensitivity indices for experiment (a) 1 and (b) 2; the order of model ranking according to the KIC-based posterior model weights are indicated.
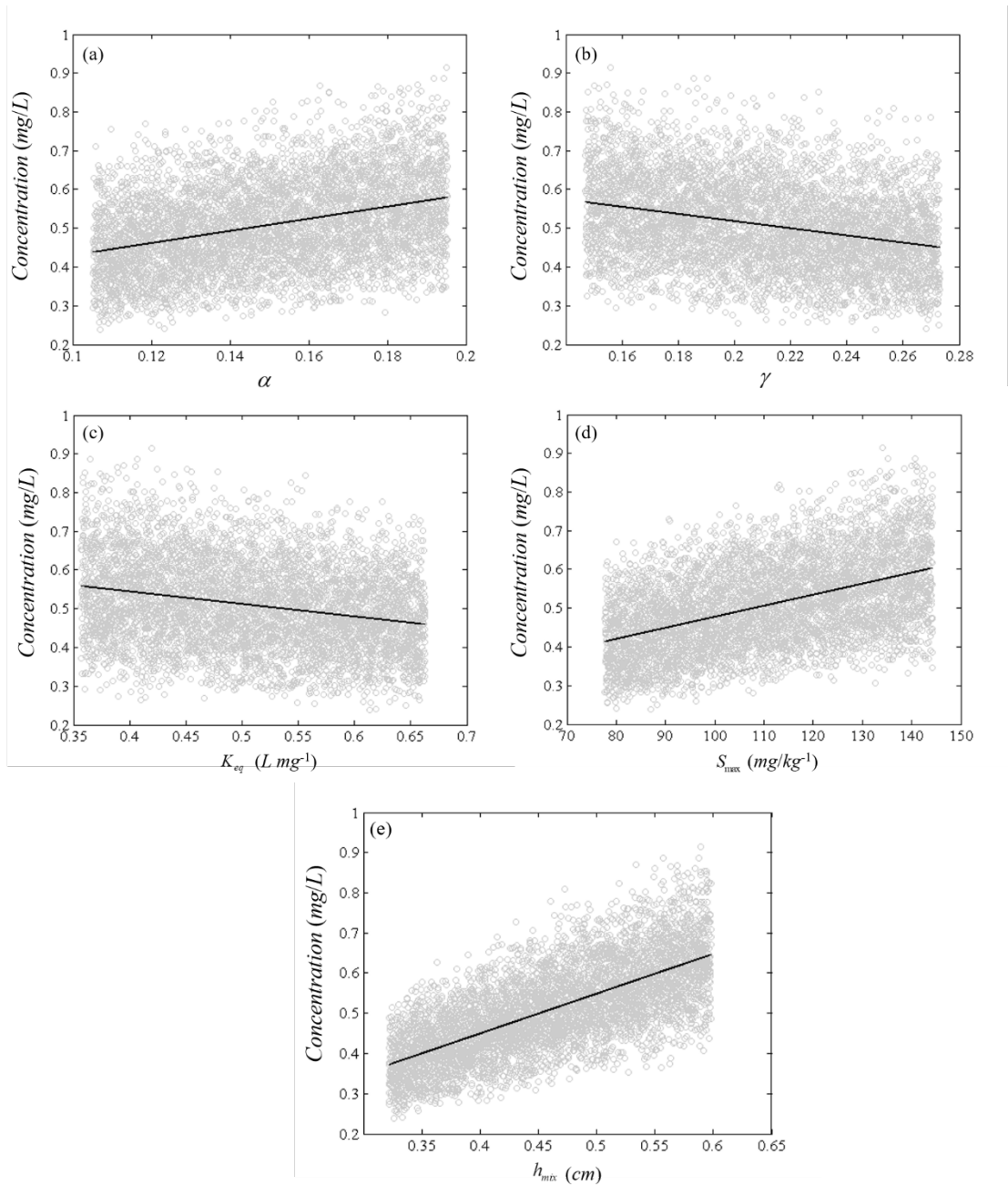
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Figure 14. Scatterplots depicting the dependence of model output Y and parameters (a) α, (b) γ, (c) K_{eq}, (d) S_{max}, and (e) h_{mix} of the Langmuir coupled tow-layer (L) for experiment 2 at observation time corresponding to the first sampling time. Linear regression curves (solid lines) are included.

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Figure 15. Scatterplots depicting the dependence of model output Y and parameters (a) α, (b) γ, (c) K_{eq}, (d) S_{max}, and (e) h_{mix} of the Langmuir coupled tow-layer (L) for experiment 2 at observation time corresponding to the last sampling time. Linear regression curves (solid lines) are included.

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