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Collaborative Eco-Drive of Railway Vehicles via Switched Nonlinear Model Predictive Control ⋆

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Abstract: A switched Nonlinear Model Predictive Control (NMPC) strategy for time efficient energy control of railway vehicles, while fulfilling constraints on velocity, journey time and driving style in a collaborative fashion (collaborative eco-drive) is proposed. More specifically, the train dynamics are modeled as discrete, switched and nonlinear, while the optimization variable is the handle position which modulates the available traction/braking force and has to belong to a set of discrete values and/or operating modes, which the human driver is able to implement. Hence the aim is to choose the optimal handle position that minimizes the cost, is implementable by the driver and also fulfills the eco-driving objective, such that the driving style is constrained by predefined driving sequences. A supervisor detects the states of the trains and subsequently modifies the weights of the cost by negotiating between constraint satisfaction and control aggressiveness, in order to share the available regenerated braking energy among the connected trains in a substation network. The efficiency of the proposed switched NMPC strategy is demonstrated using realistic simulation case study.

Keywords: Train control, predictive control, nonlinear control systems, switching algorithms.

1. INTRODUCTION

One of the most efficient sector of transport from the point of view of energy consumption is the railway sector (on average 68%-73% compared to the car and the airplane) and therefore, without any doubt, it represents a strategic sector in our society.

In past years, energy efficient techniques for trains led to the development of various numerical optimization methods. In this regard, earliest published work refers to [Ichikawa, 1968], where the problem at hand was considered as a bounded state variable problem, and the author contributed by providing a solution, which was computationally simple by assuming a simpler train model, followed by more simplified numerical optimization techniques presented in [Milroy, 1981] and [Strobel and Horn, 1973]. Later, a method which considered variable slopes was developed but only for underground trains with short station distance [Maksimov, 1971].

Apart from the development of general energy efficient techniques, additionally driving style can considerably influence the train energy consumption. The so-called "eco-drive" concept refers to the application of techniques aimed at reducing fuel consumption and emissions which are affected by the behavior of the driver, without necessarily upgrading the vehicle technology [Seewald et al., 2013]. For railway vehicles, eco-drive can be enforced for instance by constraining the operational modes of the trains in order to avoid braking immediately after acceleration.

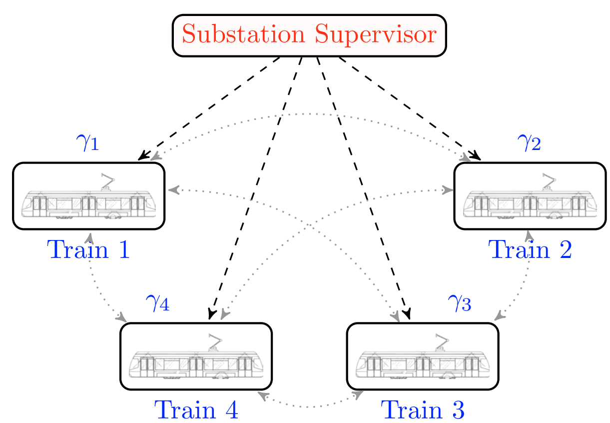

Fig. 1. Collaborative eco-drive architecture

Recently, the possibility of exploiting regenerative braking for energy efficient control and timetabling has been investigated in the literature [Scheepmaker et al., 2017; Wang and Rakha, 2017]. In this framework, collaboration among trains connected to the same substation through sharing of regenerative braking energy can be quite useful to reduce the energy consumption of the whole network. Specifically, consuming the regenerative braking energy

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instead of demanding energy from the substation not only reduces the load on the substation and overall network energy consumption but also guarantees benefits in terms of losses during the energy transfer from the substation to the trains. This idea has given rise to the paradigm of "collaborative eco-drive".

For energy efficient operation of railways, Model Predictive Control (MPC) is a suitable approach, thanks to its capability to deal with state and input constraints and economic objectives, see e.g., [Maciejowski, 2000; L.Grune and J.Pannek, 2011]. In this work, a switched Nonlinear Model Predictive Control (NMPC) for railway systems in a collaborative framework is proposed. Generally, a switched system is formed of family of subsystems together with a switching signal, which specifies at each sampling instant, the active subsystem dynamics. Important results for stability and stabilization of switched systems have been presented in [Geromel and Colaneri, 2006a,b; Colaneri et al., 2008]. As for switched systems, switched NMPC have been investigated in [Mhaskar et al., 2005; Colaneri and Scattolini, 2007]. More recently in [Zhang et al., 2016; Müller and Allgöwer, 2012; Müller et al., 2012], important stabilization results for switched MPC have been reported.

MPC has already been applied to the energy efficient operation of trains, see e.g., [Aradi et al., 2014]. The approach proposed in this paper is based on eco-drive, that goes beyond a simple driver assistance system. In this respect, the optimization variable is the input handle that decides the amount of traction/braking force and belongs to a set of discrete values or operating modes (acceleration, cruising, coasting and braking) which are realizable in practice by the driver and are constrained by predefined driving sequences to enforce eco-drive. Having in mind energy regeneration and a collaborative framework in order to allow one train to exploit the braking energy of the other trains, a substation supervisor is introduced (see Figure 1). The latter, according to a predefined state dependent triggering rule, assigns the weights of the terms constituting the cost function, related to normalized traction force, resistance force and journey time. A sensitivity study on the selection of the cost function parameter is discussed, showing the trade-off between energy minimization and fulfillment of arrival times. A realistic case study shows the effectiveness of the proposed NMPC strategy, both as a standalone eco-drive solution for a single train and as a collaborative eco-drive solution.

This work provides a possible easy-to-implement application rule to practitioners from industry willing to develop field implementations of algorithms for the energy management of a train network.

The notation adopted in the paper is mostly standard. Let $\mathbb{N}$ denote the set of natural numbers while $\mathbb{R}$ denote the set of real numbers. Let $\boldsymbol{y}$ be a vector and y_i its entry and $\boldsymbol{y}^\top$ the transpose. Given a signal w, then $w(\cdot\,|w)$ denotes its prediction trajectory with initial condition w, so that at current sampling time instant k, $w(k\,|w) = w$. Moreover, let $\mathbf{w}_{[t_1,\,t_2]}$ be the signal w defined from the instant t_1 to t_2.

The paper is organized as follows. In Section 2, some preliminaries on switched systems and switching NMPC are introduced. In Section 3 the considered single switched train model is discussed. In Section 4 the proposed NMPC based collaborative eco-drive is described in detail, while in Section 5 simulation results on a realistic scenario are illustrated. Finally, some conclusions are gathered in Section 6.

2. PRELIMINARIES

In this section, firstly the general form of switched systems essential to describe the dynamics of the considered train and secondly the basics on switching NMPC are recalled.

Consider a generic discrete time switched nonlinear system of the following form

$$\boldsymbol{x}(k+1) = \boldsymbol{f}_{\sigma(k)}(\boldsymbol{x}(k)), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{1}$$

defined for all $k \in \mathbb{N}$, where $\boldsymbol{x}(k) \in \mathbb{R}^n$ is the state, $\sigma(k)$ is the switching rule and $\boldsymbol{x}_0$ is the initial condition. The active model at the time instant k among one and M is selected by the integer $\sigma(k) \in \{1, \cdots, M\}$.

In order to design the switching NMPC controller which best minimizes a predefined prediction cost, consider the following Finite Horizon open-loop Optimal Control Problem (FHOCP):

Problem 1. At each sampling instant k with $\boldsymbol{x} := \boldsymbol{x}(k) \in \mathcal{X}$, solve the optimal control problem,

$$\min_{\boldsymbol{\chi}} J_{\boldsymbol{\chi}}(\boldsymbol{x}) = \sum_{p=k}^{k+N-1} l_{\sigma(k)}(\boldsymbol{x}(p\,|\boldsymbol{x})) + F(\boldsymbol{x}(p+N\,|\boldsymbol{x})) \tag{2}$$

subject to

$$\begin{aligned}
\boldsymbol{\chi}_{[k,\,k+N-1]} &= [\sigma(k), \ldots, \sigma(k+N-1)] \\
\boldsymbol{x}(p+1\,|\boldsymbol{x}) &= \boldsymbol{f}_{\sigma(p)}(\boldsymbol{x}(p\,|\boldsymbol{x})) \\
\boldsymbol{x}(k\,|\boldsymbol{x}) &= \boldsymbol{x} \\
\boldsymbol{x}(p\,|\boldsymbol{x}) &\in \mathcal{X}, \quad \forall\, p \in [k,\, k+N] \\
\boldsymbol{x}(k+N\,|\boldsymbol{x}) &\in \mathcal{X}_0\,.
\end{aligned}$$

In Problem 1, $\boldsymbol{x}(k\,|\boldsymbol{x}) = \boldsymbol{x}$ in turn depends on the predicted switching strategy $\boldsymbol{\chi}$ over the prediction horizon N. The set $\mathcal{X} \subset \mathbb{R}^n$ is the state constraint set. Furthermore, $\mathcal{X}$ is assumed to be compact, as well as the terminal constraint set $\mathcal{X}_0 \subseteq \mathcal{X}$.

At every sampling instant, the vector of strategies obtained by solving Problem 1 is given by the optimal switching policy $\boldsymbol{\chi}^*_{[k,\,k+N-1]} = [\sigma^*(k), \ldots, \sigma^*(k+N-1)]$, and the optimal state trajectory $[\boldsymbol{x}^*(k), \ldots, \boldsymbol{x}^*(k+N)]^\top$. In the abstract formulations of Problem 1, only the first element of the resulting optimal control switching strategy is used at each step, while the remaining entries are discarded.

3. SWITCHED TRAIN MODEL

This section presents the train model in the form of (1) to be used in the considered switched NMPC strategy. Consider an electric train controlled by a digital control unit. In this work, we consider space as the independent variable, while time will be one the system's states. Thus we denote with $k \in \mathbb{N}$ the discrete space variable used to sample the space (distance) $s \in [0, S]$, and with $D(k)$ the space dependent sampling distance. In discrete state-space, a space dependent augmented switched model $\Sigma_{\sigma(k)}$ of the train can be represented as:

$$\Sigma_{\sigma(k)}:\begin{cases} x_1(k+1) &= x_1(k)+\frac{D(k)}{Mx_1(k)}(F_{\mathrm{T}}(x_1(k),u_{\sigma(k)}(k)) \\ & \quad -F_{\mathrm{B}}(x(k),u_{\sigma(k)}(k))-F_{\mathrm{R}}(x_1(k),k)) \\ x_2(k+1) &= x_2(k)+\frac{D(k)}{x_1(k)} \end{cases} \tag{3}$$

where $\boldsymbol{x} = [x_1,\, x_2]^\top$ is the state vector, with $x_1 = v$ being the velocity and x_2 being the travel time with $u_{\sigma(k)}(k) = h(k)$ and where:

M	total mass of the train;
v	velocity of the train;
F_{T}	traction force;
F_{B}	braking force;
F_{R}	resistance force;
$h(k)$	input handle of the train as a function of space (distance).

The resistance force F_{R} is given as a combination of frictional effects due to velocity, described by the famous Davis equation, and the frictional effects due to gravity and slopes of the track, i.e.,

$$F_{\mathrm{R}}(x_1,k) = R_v(x_1) + R_g(k) \tag{4}$$

$$R_v(x_1) = A + Bx_1 + Cx_1^2 \tag{5}$$

$$R_g(k) = M_{\mathrm{s}}\left(g\tan(\alpha(k)) + \frac{\lambda}{r_{\mathrm{curve}}(k)}\right) \tag{6}$$

where

R_v	frictional forces due to velocity;
A, B, C	Davis equation parameters;
R_g	frictional forces due to slope of the track and gravity;
M_{s}	static mass of the train;
r_{curve}	radius of the curve of the track;
α	slope of the track;
λ	train dependent parameter;
g	acceleration due to gravity.

Remark 1. Notice that the resistance force is hardly known in practice and only nominal values of Davis parameters are known in advance. Hence, identification of the adherence status of the railways is an important issue for braking and traction performances. Recent contribution can be found in [Caporale et al., 2013]. In the following, these parameters are considered to be known. □

Since we are working with electric trains, the input handle $h(k) \in [1, -1]$, which represents the allowed traction (positive) and braking (negative) force that the train can use at a particular space instant, is continuous in nature. However, in this case, since the NMPC strategy has been developed keeping in mind the manual assistance scenario, that is an algorithm to assist the driver, only discrete values which are easily implementable by the driver, defining different operational modes of the train are considered and are described below:

i) Acceleration. In this mode, the handle can assume three values, i.e., $h(k) \in \{0.5,\, 0.75,\, 1\}$.
ii) Coasting. This mode means that the engine is switched off, i.e., $h(k) = 0$.
iii) Cruising. This mode means that the train maintains a constant velocity, i.e., the corresponding handle $h(k) = h_{\mathrm{cruise}}$ is chosen such that $F_{\mathrm{R}} = F_{\mathrm{T}}$ for positive slopes and $F_{\mathrm{R}} = F_{\mathrm{B}}$ for negative slopes.
iv) Braking. Due to safety reasons, whenever this mode is activated, it is preferred to use maximum braking force, i.e., the handle is chosen such that $h(k) = -1$.

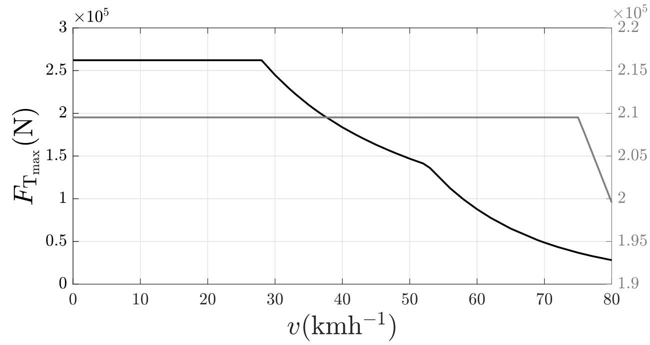

Fig. 2. Maximum traction (black line) and braking (gray line) forces as functions of velocity

Hence, this makes our system which is actually continuous switching in nature. Making reference to the formulation in (1), since the switching signal is externally updated and is a function of space, system (3) is a space-dependent switching system. Also, due to the nature of traction, braking and resistance forces described above, the considered train model is nonlinear. Finally, the model (3) has been rewritten in the form of (1), where $\sigma(k) \in \{1, \ldots, 6\}$ is the switching signal, such that $u_{\sigma(k)}(k) \in \{-1,\, 0,\, 0.5,\, 0.75,\, 1,\, h_{\mathrm{cruise}}\}$ respectively, in terms of switching input handle. For example, $\sigma(k) = 1$ implies that $u_1(k) = -1$ is chosen to be applied to the system $\Sigma_{\sigma(k)}$ at the sampling instant k.

As for the traction and braking forces, they are functions of the handle and velocity (see Figure 2) and their models can be captured by the following formulas

$$F_{\mathrm{T}} = F_{\mathrm{T_{max}}} u_{\sigma(k)}(k) \tag{7}$$

$$F_{\mathrm{B}} = F_{\mathrm{B_{max}}} u_{\sigma(k)}(k)\ , \tag{8}$$

with $F_{\mathrm{T_{max}}}$ and $F_{\mathrm{B_{max}}}$ being the maximum allowable traction and braking forces, respectively.

Furthermore, the train is subject to velocity constraints, i.e.,

$$0 \le x_1(k) \le x_{1\,\max}(k)\ , \tag{9}$$

with $x_{1\,\max}(k)$ being the maximum velocity value in k, and journey time limits, i.e.,

$$T \le T_{\max}\ , \tag{10}$$

where the journey time is given by

$$T = \sum_{k=0}^{k_f} D(k)x_1^{-1}(k)\ . \tag{11}$$

Note that the constraints on the velocity (9) are functions of space k. Hence, for the ease of the optimization, space representation of the model is more convenient and this is the reason for our model to be space dependent rather than being time dependent.

3.1 Performance Indices

For performance analysis, it is important to further define certain terms, the most important being traction energy, which is the main source of energy consumption in the train. Assuming the journey between two stops such that $v(0) = v(k_f) = 0$, with k_f being the final sampling point at the final distance S to cover, then the traction energy spent on that trip is given by

$$E_{\mathrm{T}} = \sum_{k=0}^{k_f} D(k) F_{\mathrm{T}}(x_1(k), u_{\sigma(k)}(k)) \ . \tag{12}$$

Finally the line energy to be minimized is a function of the efficiency η and is given by

$$E_{\mathrm{T}_S} = \sum_{k=0}^{k_f} \frac{F_{\mathrm{T}}(x_1(k), u_{\sigma(k)}(k))}{\eta(x_1(k), u_{\sigma(k)}(k))} \ . \tag{13}$$

Remark 2. Due to the nature of the velocity limits and the changing nature of the track characteristics such as the slopes and the curvature, there is the need to appropriately discretize the covered distance before applying the NMPC algorithm. An adaptive sampling algorithm has been developed in order to compute the switching points. Given the resistance force due to slopes, radius of the curve, velocity limits, slope changes, prediction horizon and braking dynamics, the value D is suitably adapted and appropriate switching points are selected taking into account the modes the train is going to operate in. □

4. THE PROPOSED NMPC BASED COLLABORATIVE ECO-DRIVE

This section presents the proposed switched NMPC based collaborative eco-drive approach. Furthermore, for collaboration among the trains, a triggering rule computed by a substation supervisor needs to be introduced.

4.1 Switching NMPC Algorithm

Consider now a network with M trains operating under the same substation and governed by a unique supervisor as in Figure 1. In the following, each train is indicated with the subscript $i = 1, \dots, M$, and the corresponding models $\Sigma_{i_{\sigma_i(k)}}$ as in (3).

In order to maximize the energy efficiency of the train, the choice of an appropriate cost function to fulfill our control objective is essential. The FHOCP consists in minimizing, at any sampling instant k, a suitably defined cost function with respect to the switching control sequence $\boldsymbol{\chi}_{i_{[k,\,k+N-1\,|k]}} = [\sigma_i(k), \dots, \sigma_i(k+N-1)]^\top$, with $N \in \mathbb{N}$ greater than zero being the prediction horizon. With reference to the model of the ith train $\Sigma_{i_{\sigma_i(k)}}$ as in (3), since energy efficiency as well as journey time constraints need to be ensured while adopting a driving style which is compliant with the requirements of the eco-drive, the cost function is chosen as a combination of line energy, the energy losses due to resistance and horizon time error. More specifically, with reference to Problem 1, the FHOCP consists in minimizing with respect to $\boldsymbol{\chi}_{i_{[k,\,k+N-1\,|k]}}$ the cost function

$$J_{i_{\boldsymbol{\chi}_i}}(\boldsymbol{x}_i, k_\tau, N) = \sum_{p=k}^{k+N-1} (1-\gamma_i(k_\tau)) l_{i_{\sigma_i(p)}}(\boldsymbol{x}_i(p|\boldsymbol{x}_i)) + \\ + \gamma_i(k_\tau) F_i(\boldsymbol{x}_i(p+N|\boldsymbol{x}_i)) \ , \tag{14}$$

with $i = 1, \dots, M$, where the terms are

$$l_{i_{\sigma_i(p)}}(\boldsymbol{x}_i(p)\,|\boldsymbol{x}_i) = \left(\frac{\frac{F_{\mathrm{T}}(x_{i_1}(p), u_{\sigma_i(p)})}{\eta(p)} - F_{\mathrm{R}}(x_{i_1}(p))}{F_{\mathrm{T}_{\max}}} \right)^2 \tag{15}$$

$$F_i(\boldsymbol{x}_i(p+N)\,|\boldsymbol{x}_i) = \left(\frac{T_{\mathrm{horizon}} - x_{i_2}(p+N)}{T_{\mathrm{horizon}}} \right)^2 \ , \tag{16}$$

subject to

$$0 \le x_{i_1}(k) \le x_{i_{1\max}}(k) \ . \tag{17}$$

In the cost function, T_{horizon} is the horizon time, while $0 \le \gamma_i(k_\tau) \le 1$ is a scalar weight assigned by the supervisor at the τth triggering event at k_τ with $\tau > 0$. The reason for including losses due to the resistance energy in the cost is to provide a solution which could exploit the track characteristics when the train is descending, thus reducing the energy losses and making use of the negative slopes to reduce energy consumption.

Remark 3. Note that the state x_{i_2} in this context defines the travel time, which the train needs in order to cover the horizon distance. Hence, at the beginning of each prediction horizon, it is reset to zero and the time error is steered to zero over the horizon. Furthermore, the horizon time T_{horizon} is the time needed to cover the distance, given the maximum allowed velocity limits, maximum allowed journey time and the characteristics of the track in that particular horizon in which the cost function has to be minimized. In order to compute the horizon time T_{horizon}, a heuristic approach is adopted. □

The output of the optimization problem is a sequence of constrained driving modes which respect the eco-drive driving style, i.e., trains cannot accelerate and brake in succession. According to the RH strategy, the applied switching strategy is

$$\chi_i(k) = \sigma_i^*(k) \quad [k,\, k+1) \tag{18}$$

where $\sigma_i^*(k)$ is the first value at the instant k of the optimal switching sequence of the ith train, obtained by solving the FHOCP.

4.2 The Supervisor

Assume that the supervisor has full knowledge of the current train states associated with a specific substation and is capable of enforcing a triggering rule in order to assign the weights γ_i in the cost function (14). The aim of the proposed approach is to make the trains cooperate in order to exploit the regenerative braking energy when one or more of the trains of the network are on time and in braking operation mode. This would allow other trains to accelerate, if they need, while fulfilling the constraints.

Assume γ_i, $i = 1, \dots, M$ can take two values $\{\gamma_i^{(1)}, \gamma_i^{(2)}\}$ such that $\gamma_i^{(1)} < \gamma_i^{(2)}$. So, at the triggering instant k_τ for the ith train, the triggering rule is written as

$$\gamma_j(k_\tau) = \begin{cases} \gamma_j^{(2)} & \text{if } u_i(k_\tau) = -1 \quad \text{(Braking)} \\ \gamma_j^{(1)} & \text{otherwise} \end{cases} \ , \quad \forall\, j \ne i \ . \tag{19}$$

This means that when the ith train is in braking, the other trains can exploit its regenerative energy to accelerate if they need. This is done by increasing the weight γ_j on the time term of the cost function. Otherwise, all

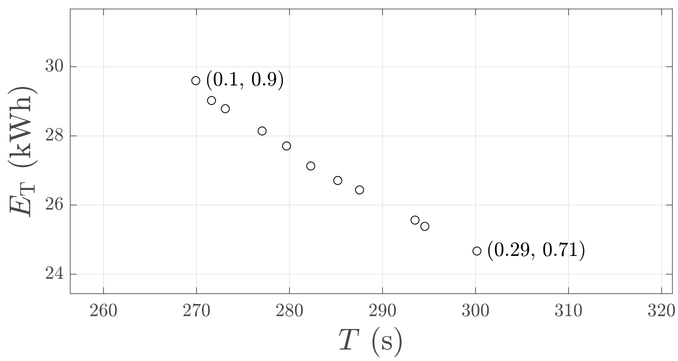

Fig. 3. Pareto graph between traction energy and journey time computed varying γ_i from $\gamma_i^{(1)} = 0.71$ to $\gamma_i^{(2)} = 0.9$

Table 1. Values of journey time, traction and braking energy as a function of γ_i

γ_i	T_i	E_{T_i} (kW h)	E_{B_i} (kW h)
0.71	324.8374	25.8686	-29.8699
0.75	315.7481	25.0872	-28.9236
0.83	287.5286	26.4371	-30.2063
0.87	277.0545	28.1431	-32.1269
0.9	269.921	29.5983	-33.6401

the trains maintain the previous value, set according to the evaluation of a Pareto graph between traction energy and journey time. For an illustrative example, Figure 3 shows the case with γ_i varying from $\gamma_i^{(1)} = 0.71$ and $\gamma_i^{(2)} = 0.9$, corresponding to the minimum and maximum energy consumption cases respectively, such that no handle oscillations are generated. The graph is obtained for a track of 807 m with r_{curve} and α almost everywhere equal to zero. Table 1 shows the corresponding values of the ith train in terms of journey time T_i, traction energy E_{T_i} and braking energy E_{B_i}.

Remark 4. Note that, the triggering rule (19) is reasonable in practice since if the train j is not on time, it can exploit the energy of train i to accelerate. On the other hand, one can assume that the train j could decide not to accept the regenerative energy and maintain its current weight γ_j for the cost function. □

5. CASE STUDY

In this section, simulation results on a realistic scenario are presented. Simulations have been carried out by considering two trains. The total track has three stops. These assumptions are realistic since in real railway networks, only nearby trains, that is trains which are connected to the same substation can share energy. In the considered scenario the consecutive trains run shifted in time by 60 s.

Table 2. Parameters of the train

Parameter	Value
M	267 464 kg
M_{s}	255 200 kg
A	3597.6 N
B	119.5 N s m^{-2}
C	6.97 N s m^{-2}

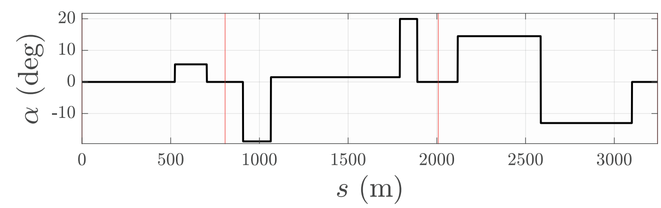

Fig. 4. Slope profile over the considered track

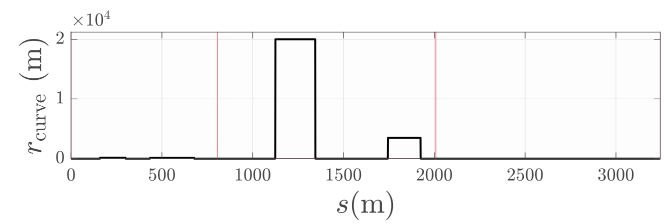

Fig. 5. Radius of the curves of the considered track

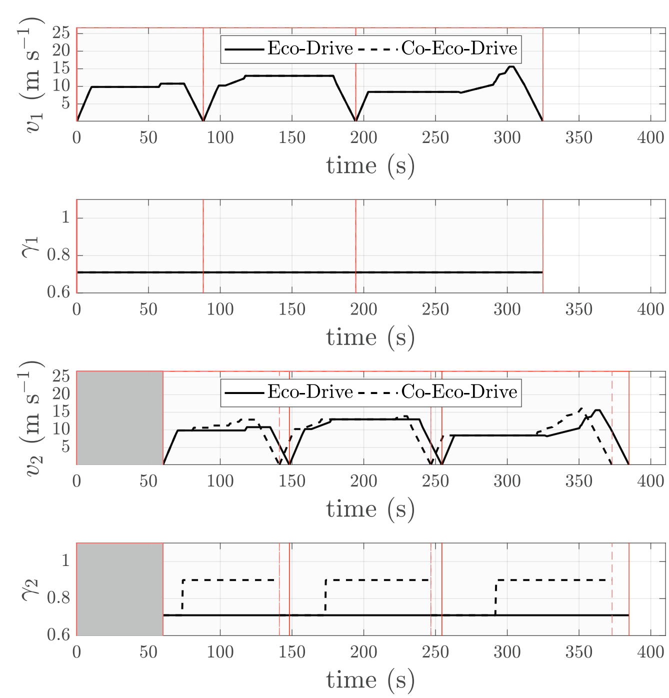

Fig. 6. Time evolution of the velocity profiles v_i, $i = 1, 2$ for both the considered trains and the corresponding weights γ_i of the cost function when the standalone eco-drive approach (solid black line) and the collaborative eco-drive method (dashed black line) are used

As discussed in the previous section, assume that when the first train is approaching the stop and needs to brake thus regenerating energy available for sharing, the second delayed train, which could be either in acceleration mode or in cruising mode can benefit from this energy rather than demanding energy from the substation.

The two trains have identical parameters which are reported in Table 2, while maximum traction and braking forces are illustrated in Figure 2. Furthermore, the considered track has curvature and slopes as illustrated in Figure 4 and Figure 5.

Table 3. Results obtained with and without collaborative eco-drive

	J_1	J_2	E_{T_1} (kW h)	E_{T_2} (kW h)	E_{B_1} (kW h)	E_{B_2} (kW h)	T_1 (s)	T_2 (s)
Eco-Drive	2083.2	2083.2	24.6698	24.6698	-28.1692	-28.1692	300.1505	300.1505
Co-Eco-Drive	2083.2	2037.2	24.6698	25.7221	-28.1692	-29.3333	300.1505	296.7020

Figure 6 shows the time evolution of the velocity v_i for both the trains and the evolution of the weights γ_i, $i = 1, 2$ when the standalone eco-drive and the collaborative eco-drive (co-eco-drive) methods are applied, respectively. As expected the supervisor verifies the status of the trains. When the first train at about 80 s starts to brake, the second train, which is in cruise mode, decides to accelerate, by changing its weight value to 0.9, thus exploiting the braking energy of the first one. After the first stop, assuming that there are no delays, the same situation is repeated at time about 180 s and then at time about 300 s. Table 3 shows the results obtained through the two strategies in terms of final value of the cost function J_i, traction energy E_{T_i}, braking energy E_{B_i}, and journey time T_i. Looking at the amount of the braking energy generated, it can be seen that it is almost equal to the traction energy used. If one shares this energy among nearby trains, it can result in a significant improvement in terms of the overall energy consumption of the network, not to forget the energy losses which can be prevented as a result of the transfer of energy from the substation to the train.

6. CONCLUSIONS

This work considered the problem of energy efficient train operation with eco-drive in a collaborative way. To address this problem, an optimal control solution to predict the velocity profile of a train by using switching NMPC algorithm in a collaborative fashion was proposed. For the purpose, a supervisor was introduced to manage all the trains governed by the same substation and tune the NMPC law in order to use the braking energy, while taking into account velocity limits and constraints on journey times. The proposed NMPC is able to minimize the traction energy, which depends on the input handle and the characteristics of the track while fulfilling all the constraints. One of the key points of the algorithm is the implementation of an eco-drive based approach, where the set of modes which could be implemented is restricted to certain operation sequences.

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