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Phase shift measurements between intensity and frequency modulations of a self-mixing interferometer

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We present a simple and straightforward method to determine the phase shift between the amplitude modulation (AM) and the frequency modulation (FM) of a laser diode. The approach is based on the measurement of AM and FM signals produced by the dynamical response of a laser diode exposed to weak optical feedback. The AM signal is measured directly on a photodiode detecting the emitted power and the FM signal is demodulated to intensity modulation through the edge filtering techniques using (a) a volumetric Bragg grating or (b) a Mach-Zehnder interferometer as filters prior to the signal detection. In this work, we show experimentally that the phase shift determination unveils the linewidth enhancement factor of semiconductor lasers as predicted by the Lang and Kobayashi theory.

1. Introduction to Self-mixing interferometry

Semiconductor lasers exhibit strong variation of their active medium properties such as the gain and the refractive index when the laser cavity is feedback with the laser light reflected from a moving target. The variation of the active medium produces amplitude modulation (AM) and frequency modulation (FM) of the laser output. When the target displacement is recovered from the modulated signal carried in the output power (AM), the approach is known as optical feedback interferometry or self-mixing interferometry (SMI) [1-3]. The theory of SMI predicts that the AM modulation (amplitude of the electric field ΔE) is proportional to $\cos(2ks)$ at small C, where k, s and C represent the wave-vector ($k = 2\pi/\lambda$, and λ denotes the laser wavelength), the optical path and the so-called feedback parameter that describes the strength of the optical feedback [2], respectively. On the other hand, the FM modulation, Δν, is proportional to $sin[2ks + tan^{-1}(\alpha)]$, where α stand for the linewidth enhancement factor of the laser [4]. The optical phase of the signal can be written in terms of the frequency since $2ks = \omega_0\tau_{ext}$, where ω_0 and τ_{ext} represent the angular frequency of the unperturbed laser and the external laser roundtrip time, respectively [5].

The AM and FM signals can be obtained from the Lang and Kobayashi equations [5] by letting C<<1 and developing the field amplitude and phase in the small signal regime as $E \approx E_0 + \Delta E$ and $\omega = d\phi/dt \approx \omega_0 + \Delta\omega$, where E0 and ω0 are the quiescent values obtained for dE/dt = 0 and dϕ/dt = 0. With these approximations, the field amplitude and frequency modulations can be expressed as [6]:

$$\Delta E = E_0 \kappa cos(2ks) \qquad (1),$$

$$\Delta\nu = -(C/\tau_{ext})\sin(2\text{ks} + (\pi/2 - \Delta\varphi)) \qquad (2).$$

In Eq. (1), κ represents the fraction of the field coupled into the laser and it is proportional to the C factor [1]. Thus, for C<<1, both modulations are sinusoidal, and they are phase-shifted by the difference $\Delta\varphi = \arg(\Delta E) - \arg(\Delta\nu)$ of the sine and cosine functions. In the Eq. (6), the phase difference can be expressed in terms of the alpha factor of the laser, α, as: $\Delta\varphi = \pi/2 - tan^{-1}(\alpha)$.

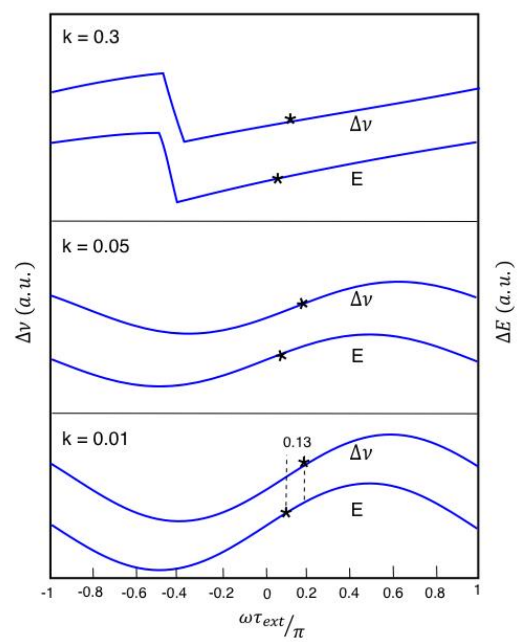

Fig. 1. Diagram of amplitude E and frequency difference $\Delta\nu$ of the optical spectrum plotted vs phase $2ks = \omega_0\tau_{ext}$ for some values of the coupling factor κ (proportional to C-factor) [7] and α =3. Vertical scale is shifted for ease of comparison. At small C=0.3 both Δν and E are sinusoidal waveforms, and there is a 0.13π phase shift between them, as indicated by dotted lines. At larger coupling, C=6 and 15, the phase shift decreases, and the waveform is saw-tooth distorted [7].

Fig. 1 shows the calculated amplitude modulation ΔE and the frequency modulation Δν as a function of the phase 2ks for various C-values [2,7]. For a small C value ΔE and Δν exhibit a sine-dependence on the phase 2ks, whereas at larger C the waveform becomes progressively distorted, like a sawtooth waveform. From the waveforms of Fig. 1, we can also read the phase delay between AM and FM modulations as about

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0.13π, which is substantially in agreement with the low-C theoretical value of $\pi/2 - tan^{-1}(\alpha) = 0.11\pi$ for $\alpha = 3$ [7]. Plotting ΔE and $\Delta\nu$ on Fig. 1 with a slightly different C-value would reduce the small deviation between 0.13π and $0.11\,\pi$. The phase shift measurements between AM and FM outputs of semiconductor lasers are not straightforward. The reason is that FM detection is out of reach by direct means since the modulation is carried in the optical frequency. Recently, Contreras et. al. proposed a method to demodulate the FM into intensity modulation (IM) that can be easily detected by a photodiode [8]. The demodulation method is based on the edge filtering technique based on optical absorption means [9]. Since then, diverse types of edge filters have been proposed to recover the FM signal on the SMI approach [8,10,11]. The proposed demodulating methods are based on different physical properties, such as absorption and interference of light, achieving different performances on each method. In few words: the steeper the filter edge, the higher the demodulation factor providing a higher FM amplitude, however, the noise related to the frequency is also increased and the bandwidth gets more restricted (since the free spectral range is reduced). For example, in [11] the use of a Mach-Zehnder interferometer as filter (MZI filter) provides an outstanding conversion factor due to the steepness or narrowness in the transmission profile of this interferometric approach. Also, the volume Bragg gratings (VBGs) have been used as edge filters [10]. In the case of the VBGs, besides the FM demodulation performance, the filters allow the use of inexpensive multimode longitudinal laser diodes due to the dependence of the wavelength selectivity with the input angle of the beam, letting to spatial discrimination of the longitudinal modes [10]. The VBG filters can be custom-designed enabling the use of lasers emitting at any wavelength [12]. However, a specific VBG is needed for a specific wavelength.

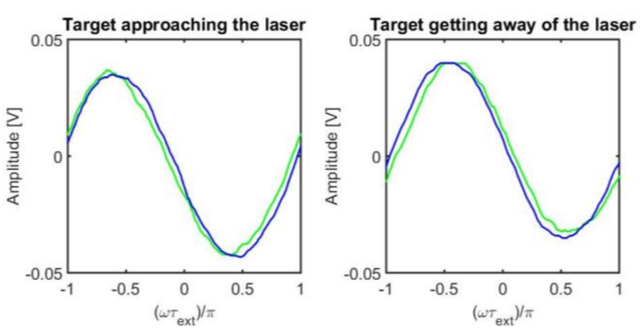

Fig. 2. AM and FM signals in the time domain of a laser diode under weak optical feedback. The AM and FM signals showed a phase shift as predicted by the self-mixing theory. Besides the phase shift magnitude, comparing the AM and FM signals provides information about the direction of the target.

Since the edge filtering techniques allow a straightforward method to detect the FM signal, monitoring simultaneously both signals (the demodulated FM and the AM signal) enables a direct method to determine the phase shift between the intensity and frequency modulations of the self-mixing signal. Moreover, measuring simultaneously the AM and FM signals has the additional benefit of determining the motion direction of the target directly in the time domain. That is, if the longitudinal mode of the laser is fixed at a frequency of the right edge of the filter, target moving to the laser will shift the laser frequency slightly to higher frequency. So, Eq. (2) can be written as:

$$\sin(\omega_0\tau_{ext} + (\pi/2 - \Delta\varphi)) = cos(\omega_0\tau_{ext} - \Delta\varphi) \quad (3).$$

If the target moves to the opposite direction, the frequency shift changes direction and the slope of the filter inverts the phase. The argument in Eq. (2) can be written as:

$$\sin(\omega_0\tau_{ext} - (\pi/2 - \Delta\varphi)) = cos(\omega_0\tau_{ext} + \Delta\varphi) \quad (4).$$

Therefore, in the time domain the FM signal goes before or after the AM signal if the object moves away or to the laser cavity, respectively as shown in Fig. 2. This is important for metrology applications where direction of the motion is of interest and the feedback strength is not enough to produce a sawtooth SMI signal to provide this information [13].

Based on the FM signal demodulation, this paper presents a method to measure the phase difference between AM and FM outputs of semiconductor lasers. To the best of our knowledge, this is the first time this method is applied for the phase shift determination between the AM-FM signals on lasers exposed to optical feedback. For the FM signal demodulation, we used two different edge filters, the VBG and the MZI filters. The following text describes the performance of both edge filters on the AM-FM phase shift determination.

2. FM conversion using the VBG as edge filter

The first method we present in this work, to simultaneously acquire the AM and FM signals, is based on the FM demodulation through a VBG filter. Since the goal is to compare the AM and the FM signals for the phase shift determination, filters with high conversion factors are not required and the VBG technology provides a conversion factor enough to compare the FM signal with the AM signal in a similar amplitude scale. The laser used in this approach (laser 1) emits multiple longitudinal modes around 405 nm and the wavelengths can be tuned slightly by tuning the operation current and temperature.

Fig. 3 shows schematically the experimental set up for AM and FM signal detection from self-mixing interferometry. The collimated beam of laser 1 emitting multiple longitudinal modes is directed to a moving target. The beam goes through a silica wedge window (ww) placed in front of the diode laser to get two reflected beams for the AM and the FM signal detection. For the AM signal detection, one of the reflected beams is simply directed to a photodiode (DET1). For the FM signal demodulation, the other beam reflected from the ww is directed to the volume Bragg grating, VBG, at an appropriate angle, i.e., the laser wavelength of the most intense longitudinal mode coincides with the wavelength at one of the

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edges of the VBG filter as described in [10]. To keep spectral stability in the experiments, the laser was driven at a fixed temperature of 26 °C and a fixed laser current of 37 mA. The FM demodulation, obtained with the VGB as the edge filter, provides a signal enhancement factor about 10 if compared with the conventional AM signal

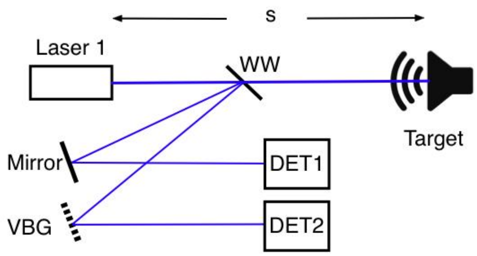

Fig. 3. Schematic experimental set up for AM and FM phase shift determination using a VBG as edge filter. A collimated beam is directed to a moving target, placed at a distance s, to produce the self-mixing interference. A wedge window (ww) is placed in between the laser and the target to reflect two beams. One of them is directed into a detector (DET1) for AM signal detection and the other is directed to a VBG filter to demodulate the FM signal before its detection by DET2.

Fig. 2 shows the AM and FM signals in the time domain over one cycle when (a) the loudspeaker is approaching the laser and when (b) the loudspeaker is moving away to the laser. To compare signals with similar amplitudes, we compensate the enhancement factor with the amount of light arriving on DET1 and DET2. That is, the light coupled to the AM channel (DET1) is around 10 times higher than the light coupled to DET2. The measurement was performed in a weak-coupled regime to obtain sinusoidal waveforms. The waveforms where analysed when the movement of the speaker was in its linear behaviour. Moreover, the average of signals with the phase shifts in each direction allows to compensate any possible error due to different electronic frequency response of the two detectors. For the phase shift determination, high frequencies were filtered out to reduce the noise. A phase shift of 0.11 π was found for the laser 1.

3. FM conversion using the MZI as edge filter

The second method we present to demodulate the FM signal is based on a MZI filter. This method has the practical advantage of being implemented at any wavelength and it was used in this work with a laser diode (laser 2) operating at 70 mA and emitting a single longitudinal mode at 1550 nm.
Fig. 4 shows the schematic experimental set up for AM and FM signal detection from self-mixing interferometry. The collimated beam of laser 2 emitting a single longitudinal mode is directed to a wedge window (ww). The transmitted power hits the target under test. One of the beams reflected from the ww is directed to the MZI for further FM-to-IM conversion. The MZI is built with 50-50 beam splitters and silver coated mirrors. The sensitivity of the MZI is controlled by varying the distance ΔL since the mirrors are mounted on a translational stage plate. For these measurements we set ΔL=45 cm, allowing a very good sensitivity for the FM channel. The phase shift between AM and FM was measured in two ways: by measuring on a rotating disk (Target 1) and on a loudspeaker (Target 2). Surface of the rotating disk is angled with respect to the orthogonal line of sight to produce a velocity component I this direction. Varying the angle varies the velocity component. The measurement on a rotating disk shows a stable Doppler frequency, modulated by the speckle-effect. By analysing the signal in the frequency domain, we can evaluate the phase of the main tone, after some averages on the spectrum. Using a simultaneous acquisition of AM and FM modulation, we directly get the phase shift as phase difference between the main tones, equal to 0.05 π.

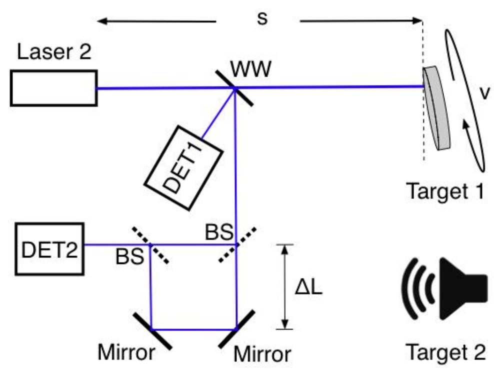

Fig. 4. Schematic experimental set up for AM and FM phase shift determination using a Mach-Zehnder Interferometer (MZI) as edge filter. A collimated beam is directed to a moving target (1 or 2) to produce the self-mixing interference. A wedge window (ww) is placed in between the laser and the target to reflect one of the beams into the MZI for FM demodulation signal. The FM-to-AM conversion factor can be controlled with varying the distance ΔL [11]. One of the MZI output beams is directed to a detector (DET1). For AM detection a DET1 is used to extract directly de AM signal. The photocurrents of DET1 and DET2 are coupled into an oscilloscope for further signal analysis.

On the loudspeaker, the acquisition was realized as described for FM conversion using the VBG filter with the advantages of cancelling electronic filters effect, but realizing a single measurement, not averaged as for the spectral acquisition.
This last measurement, on laser 2 with a loudspeaker, highlights an unexpected variability in the AM-FM phase shift. It always leads to a mean value well in agreement with previous measurement (0.05 π), but it indicates for this laser an unpredicted dependence on other feedback parameters, to be further studied in future works.

Laser	FM-demodulation method	AM-FM phase shift	Alpha factor
Laser 1	VBG filter	0.11 π	2.7
Laser 2	MZI filter	0.05 π	6.3

Measuring simultaneously AM and FM self-mixing signals unveils a straightforward method to measure the linewidth enhancement factor of a laser source, an important parameter of the semiconductor lasers [4, 14-18]. This is possible since it suffices to measure the phase shift of the waveforms to find out the α-factor. We would like to highlight that different methods to determine the alpha factor based on SMI have been proposed; however, they require specific experimental conditions [14-18]. For instance, in [16] the interferometric signal in the AM channel must present a sawtooth shape, a condition that is achieved only in a moderate feedback regime. Though the method proposed in [16] is a simple and direct way to determine alpha factor, it is restricted to experimental conditions where $C > 1$. Moreover, this method cannot be applied to Fabry-Perot laser cavities due to the multiple interference between longitudinal modes that produce too complex signal. In contrast, the methodology presented in this work represents an alternative way to get the phase shift between AM and FM laser outputs and the corresponding alpha factor. Table 1 shows AM-FM phase shift and the respective alpha factor measured in this work by two of the filters we used on two different lasers. In the case of the laser 2 a similar value for alpha factor was obtained with the methodology presented on [16] for the same device.

In summary, we have presented a new method to measure the phase shift between the AM and the FM outputs of semiconductor lasers under optical feedback. This method complements the existing methods based on self-mixing interferometry and other techniques [15,17,19] that determine the AM-FM phase shift and the alpha factor of semiconductor lasers since it is not restricted to any optical feedback regime. Moreover, the proposed method can be applied to both, laser diodes emitting single and multiple longitudinal modes, depending on the selected filter to demodulate the FM channel.

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