Adaptive reliable output tracking of networked control systems against actuator faults

R. Sakthivel[a,*], P. Selvaraj[b], Yongdo Lim[a], H.R. Karimi[c]

[a]*Department of Mathematics, Sungkyunkwan University, Suwon 440-746, South Korea*
[b]*Department of Mathematics, Anna University Regional Campus, Coimbatore 641046, India*
[c]*Department of Mechanical Engineering, Politecnico di Milano, 20156 Milan, Italy*

This paper investigates the reliable adaptive observer-based output tracking control problem for a class of networked control systems subject to actuator faults and external disturbances via equivalent-input disturbance technique. Notably, the reliable control design based on adaptive mechanism is implemented to compensate the on-line actuator faults automatically and an observer-based controller is introduced through communication networks to drive the output of controlled plant to track the output of a reference model. Moreover, due to the effect of network-induced delays and packet dropouts in the controller-to-actuator channel, the inputs of controlled plant and observer-based tracking controller are updated in an asynchronous way. Then, based on the asynchronous characteristic, the resulting closed-loop networked control system is formulated with two interval time-varying delays for obtaining the required result. In particular, the equivalent-input disturbance approach improves the disturbance rejection performance and it does not require any prior knowledge of the disturbances. By constructing a suitable Lyapunov–Krasovskii functional and using free-weighting matrix approach, a new set of sufficient conditions for the solvability of the addressed problem is derived in terms of linear matrix inequalities. At last, the proposed result is validated through two numerical examples and also a comparison study is presented which shows the effectiveness of the developed control scheme over some existing conventional control schemes.

Received 24 January 2016;
received in revised form 25 April 2016;
accepted 29 June 2016
Available online 9 July 2016

*Corresponding author.
E-mail addresses: krsakthivel@yahoo.com (R. Sakthivel), hamidreza.karimi@polimi.it (H.R. Karimi).

Published Journal Article available at: http://dx.doi.org/10.1016/j.jfranklin.2016.06.022

1. Introduction

Networked control systems (NCSs) are control systems in which the signal transmission between sensors, actuators, and controllers is implemented through communication networks [3,4,37–39]. Due to its important advantages such as low cost, simple installation, reduced system wiring and high reliability, NCSs have successful applications in a wide range of areas such as in environmental monitoring, autonomous robots, industrial automation, smart grids, mobile communications and so on (see [5,6,42,43] and references therein). In recent years, NCSs have attracted much attention and many important results are available on NCSs about stability analysis [41], quantization [43], packet dropouts and network-induced delays [11], stabilization via sampled-data control [47], reliable control [50], H_∞ control [48,49] and so on. An observer-based fault detection filter design criterion has been obtained for a continuous-time NCSs with packet dropouts and network-induced delays in [1], where the designed fault detection filter can guarantee the sensitivity of the residual signal to faults. The quantized stabilization for event-triggered NCSs with packet losses has been addressed in [36], where a new set of sufficient conditions for the stabilization is derived by using the Lyapunov functional approach and control synthesis of event-triggered NCSs is established in terms of linear matrix inequalities. The H_∞ control problem for a class of wireless NCSs with time delays and packet losses has been studied in [42].

In many practical applications, the NCSs are often required to have high reliability, especially for safety critical systems such as aircraft systems, autonomous mobile robots and medical systems [31–33]. In particular, control of any plant depends on the availability and quality of sensor measurements and the performance of system relies heavily on the quality of sensor for feedback. Due to broken or bad communication in feedback control problems, sensor characteristics may change over time so there may be partial or complete failure occurs in controller which can degrade the performance and even destroy the stability of the overall system [34,40]. Also, the actuators may subject to failures in real processes due to sudden environmental disturbances [17,18]. Therefore, in order to increase NCSs reliability, it is necessary and important to consider the actuator failures in control input. Reliable control has been considered as one of the most important and promising control approaches for maintaining certain prespecified safety performance in NCSs in the presence of unexpected faults [16,29]. The robust H_∞ reliable control problem for a class of switched neutral systems with distributed delays which involve uncertainties and unknown disturbances has been reported in [44], where the uncertainties under consideration are norm-bounded and the disturbances are bounded in energy. The fault estimation and accommodation problem for a class of NCSs with nonuniform uncertain sampling periods has been investigated in [2], where a novel fault estimation is developed to observe both continuous-time faults and system states by using non-uniformly discrete-time sampled outputs. Moreover, adaptive technique is one of the significant practical approaches for fault compensation in dynamical control systems (see [12,19] and references therein). Adaptive control design technique is used for a more general actuator fault model which covers the cases of normal operation, loss of effectiveness and outage [13,14]. The problem of sliding mode control has been studied in [30] for uncertain switched systems subject to actuator faults, where an adaptive sliding mode controller is designed for on-line estimating the loss of effectiveness of the actuators. The adaptive fault-tolerant control problem for a class of switched Takagi–Sugeno fuzzy systems containing unmeasured states and actuator failures has been examined in [35]. In [51], the authors addressed the H_∞ reliable control problem for linear systems based on adaptive mechanism. Further, the adaptive reliable control mechanism is discussed for both linear and nonlinear systems with time-delay and actuator saturation in [12].

It is well known that the unknown external disturbances strongly affect the control performances in NCSs. Also from the practical view point of the NCSs, it is more reasonable to estimate a disturbance on the control input channel than to estimate the disturbance itself because the usage of control input is to improve the disturbance rejection performance in NCS [25]. Also, it is noted that the trade-off between reference tracking and disturbance rejection always exists in the controller design of NCSs. Therefore, we should pay great attention in controller design for obtaining satisfactory disturbance rejection performance and high requirement of output tracking performance even in the presence of huge amount of disturbances in NCSs. The equivalent-input disturbance (EID) strategy is an active disturbances rejection approach which was proposed in [24] and it can effectively reject both matched and unmatched disturbances. It should be pointed out that EID does not require any prior knowledge of the disturbance still the estimated signal on the control input channel produces the same effect on the output as an actual disturbance does, thus the external disturbances can be compensated effectively in the EID approach [25]. For the past few years, based on EID approach, a considerable number of studies have been devoted to estimation and rejection of an unknown disturbance in various control systems (see [26,27] and the references therein). The disturbance rejection for a modified repetitive control system that contains a strictly proper plant with time-varying uncertainties has been studied in [28], where an EID-based modified repetitive control system compensates for all types of disturbances and also guarantees tracking of a periodic reference input. Thus the incorporation of an EID estimator in the controller enables rejection of unknown periodic and aperiodic disturbances. Nevertheless, most of the existing results have been focused on the estimation of faults and disturbances separately in the derivation of reliable control law for NCSs. In addition, it should be pointed out that, no work has been reported dealing both estimation of faults and disturbances simultaneously. Therefore, the present study focuses on the derivation of controller for the system model under consideration in the presence of estimation of faults and disturbances simultaneously.

Tracking control is one of the most fundamental problems in control engineering and is widely used in industries, which can be classified into two categories namely, state tracking and output tracking controls [20–22,46], in which output tracking control has wide applications in real world problems. The main advantage of output tracking control is that it can significantly minimize the error between the output of the plant and the output of the given reference model [23]. Recently, several authors have proposed various techniques and some useful results on output tracking control problems for dynamical systems with time delay [9,10,15]. Network-based output tracking control problem for T–S fuzzy system which cannot be stabilized by a non-delayed fuzzy output feedback controller, but it can be stabilized by a delayed fuzzy output controller has been studied in [7], where a new delay-dependent criterion for tracking performance is derived by using the deviation bounds of asynchronous normalized membership functions and Lyapunov technique. Zhai et al. [8] investigated the output tracking control problem for a class of switched nonlinear systems with multiple time-varying delays by using the average dwell-time technique together with free-weighting matrix method and in which a state feedback tracking controller is obtained by solving a set of linear matrix inequalities to satisfy the H_∞ model reference tracking performance. Based on average dwell-time approach and Gronwall–Bellman inequality technique, a new stability criterion has been established for a class of switched nonlinear systems with time-varying delay in [45], where a state feedback controller and a switching signal are proposed to satisfy the H_∞ model reference tracking performance.

More precisely, controllers that ignore actuator faults may cause undesirable responses and even closed-loop system instability. Therefore, it is necessary and highly desirable to develop an effective controller that can tackle actuator failures at the design outset. However, to the authors' best knowledge, up to now, no work has been reported in output tracking adaptive reliable control problem for NCSs in the presence of actuator faults and external disturbances via EID technique which motivates this study. The main contributions of this paper can be summarized as follows.

(i) Based on EID approach and a communication network channel consisting packet dropout and network induced delays, a novel output feedback adaptive reliable control strategy is proposed to the NCSs under consideration.
(ii) The proposed approach estimates the disturbances directly by using observer and the estimated disturbances are injected through control input channel which makes easy to deal with the disturbance rejection performance without knowing prior knowledge about disturbances.
(iii) The proposed adaptive reliable controller not only compensates the external disturbances but also tolerates the on-line actuator faults so that the proposed control scheme is more significant.
(iv) The developed adaptive reliable control algorithm is very simple and disturbances are also considered in its design. According to this control algorithm, some new delay-dependent stability criteria are derived for the addressed NCSs.

Finally, two numerical examples including a moving mobile robot model are provided with simulation results to illustrate the effectiveness of the proposed control design.

2. EID formulation for an NCS and its preliminaries

In this paper, we consider a class of networked control systems (NCSs) described by the continuous-time systems with actuator faults and external disturbances as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu^F(t) + B_1 w(t), \\ y(t) = Cx(t), \\ x(t_0) = x_0, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u^F(t) \in \mathbb{R}^m$ is the control input vector; $w(t) \in \mathbb{R}^r$ is the bounded disturbance input; $y(t) \in \mathbb{R}^q$ is the measured output, A, B, B_1 and C are system matrices with compatible dimensions; x_0 is an initial condition.

Also, it is assumed that the external disturbance $w(t)$ is converted into the input disturbance $w_e(t)$. Therefore, the system (1) can be rewritten as

$$\begin{cases} \dot{x}(t) = Ax(t) + B(u^F(t) + w_e(t)), \\ y(t) = Cx(t), \\ x(t_0) = x_0, \end{cases}$$

In particular, the fault type considered in this work is the loss of actuator effectiveness. In order to formulate the fault tolerant control problem, the following actuator fault model is adopted as follows [51]:

$$u_{ij}^F(t) = (I - \rho_i^j)u_i(t), \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, N,$$

where $0 \le \rho_i^j \le 1$ is an unknown actuator fault constant; the index j denotes the jth faulty mode and N is the total faulty modes. Let $u_{ij}^F(t)$ represent the signal from the ith actuator that has failed in the jth faulty mode. For every faulty mode, $\underline{\rho}_i^j$ and $\overline{\rho}_i^j$ represent the lower and upper bounds of ρ_i^j, respectively. It should be noted that when $\underline{\rho}_i^j = \overline{\rho}_i^j = 0$, there is no fault for the ith actuator u_i in the jth faulty mode. When $\underline{\rho}_i^j = \overline{\rho}_i^j = 1$, the ith actuator u_i is outage in the jth faulty mode. When $0 < \underline{\rho}_i^j = \overline{\rho}_i^j < 1$, the ith actuator has partial failure in the jth faulty mode.

Define the following sets for lower and upper bounds $(\underline{\rho}_i, \overline{\rho}_i)$:

$$\mathcal{S} = \left\{ \rho : \rho = \mathrm{diag}\{\rho_1, \rho_2, \ldots, \rho_m\}, \ \rho_i \in [\underline{\rho}, \overline{\rho}], \ i = 1, 2, \ldots, m \right\}$$

$$\mathcal{N}_\rho = \left\{ \rho : \rho = \mathrm{diag}\{\rho_1, \rho_2, \ldots, \rho_m\}, \ \rho_i = \underline{\rho}_i, \ \rho_i = \overline{\rho}_i, \ i = 1, 2, \ldots, m \right\}.$$

Thus, the set $\mathcal{N}_\rho$ contains a maximum of 2^m elements.

For all possible faulty modes N, a uniform actuator model is defined as follows:

$$u^F(t) = (I - \rho)u(t), \quad \rho \in \{\rho^1, \ldots, \rho^N\}. \tag{2}$$

In this paper, we will consider a reliable output tracking control for the NCSs (1). The tracking control purpose is to drive the output of the NCSs (1) via a reliable output feedback controller to track a reference signal as close as possible. Further, in parallel to Eq. (1), we consider the reference output vector $y_r(t)$ given by the reference model

$$\begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r r(t), \\ y_r(t) = C_r x_r(t), \\ x_r(t_0) = x_{r0}, \end{cases} \tag{3}$$

where $x_r(t) \in \mathbb{R}^n$ is the reference state vector which is to be used for control signals; $r(t) \in \mathbb{R}^r$ is the energy bounded reference input vector; $y_r(t) \in \mathbb{R}^q$ is the reference output; A_r is a Hurwitz constant matrix, B_r and C_r are known constant matrices with appropriate dimensions; x_{r0} is the initial condition of the reference state.

On the other hand, the above-mentioned networked control system (1) consisting of sensor and actuator nodes which are connected to the controller through a communication network. For system analysis and controller design, we assume that the following hypotheses are hold [11]:

(A1) The sensor is clock-driven with the sampling period h, and the controller and the actuator are event-driven.

(A2) The time-delay is unavoidable one when the signals transmit through network. Thus the sensor to controller delay denoted as τ^{sc} and the controller to actuator delay represented as τ^{ca}. The total time delay in the control system is $\tau^{sa} = \tau^{sc} + \tau^{ca}$ which is bounded.

It is assumed that the states of the NCSs (1) are not completely measurable, so that an observer-based controller should be constructed to estimate the states and perform output tracking control task. Therefore, in order to estimate immeasurable states, we consider the following observer-based controller:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B_i\hat{u}(t) + L(y(t) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(t), \\ \hat{u}(t) = K_0(\hat{x}(t) - x_r(t)) \\ \hat{x}(t) = 0, \quad t \le t_0, \end{cases}$$

where $\hat{x}(t)$, $\hat{y}(t)$, $u^F(t)$, L and K_0 are state estimate, observer output, control input, observer gain and controller gain, respectively.

Based on Assumption (A1), the measurements $y(kh)$ and $x_r(kh)$ ($k \in \mathbb{Z}$) (h is a sampling period) are augmented as a single packet with a time stamp and transmitted to the controller in the sensor-to-controller channel [11]. During the data transmission from sensor to controller, there may be packet dropouts due to the unreliability of the network. Considering this fact, in this paper, it is assumed that the data are successfully transmitted at instants $y(b_ih)$ and $x_r(b_ih)$ ($i \in \mathbb{N}$) and the packet dropouts happen at instants other than $y(b_ih)$ and $x_r(b_ih)$ ($i \in \mathbb{N}$). Moreover, the time delay which arises between sensor and controller is denoted as τ^{sc}_{bi}. Then, the observer-based controller on $[b_ih + \tau^{sc}_{b_i}, b_{i+1}h + \tau^{sc}_{b_{i+1}})$ can be expressed as follows:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + B_i\hat{u}(t) + L(y(b_ih) - \hat{y}(t)), \\ \hat{y}(t) = C\hat{x}(b_ih), \\ \hat{u}(t) = K_0(\hat{x}(b_ih) - x_r(b_ih)) \\ \hat{x}(t) = 0, \quad t \le t_0, \end{cases} \tag{4}$$

where b_i ($\forall i \in \mathbb{N}$) represent some nonnegative integers that indicate the packets that successfully update the controller, $\{b_1, b_2, \ldots\} \subseteq \mathbb{Z}$, and the sequence $\{b_i\}$ is strictly increasing.

Similar to controller, the actuator also has a hardware that can actively drop outdated packets. In consequence, $\overline{u}(s_kh)$ ($k \in \mathbb{N}$) is available to update the actuator after the controller-to-actuator delay τ^{ca}. The actuator holds the signal until next update. Then the control input (2) of the plant (1) with actuator fault can be described by

$$u^F(t) = (I - \rho)\overline{u}(s_kh)$$

Considering the system (4) with actuator faults, the design problem under consideration in this paper is to find an adaptive fault-tolerant control scheme. When some actuator faults occur, namely some actuators loss partial control effectiveness, a reliable control scheme based on adaptive compensation control should be considered. To handle the actuator fault, we choose the following adaptive tracking controller [51]:

$$\begin{aligned} u(t) &= K(\hat{\rho}(t))(\hat{x}(t) - x_r(t)) \\ &= (K_0 + K_a(\hat{\rho}(t)) + K_b(\hat{\rho}(t)))(\hat{x}(s_kh) - x_r(s_kh)), \quad t \in [s_kh + \tau^{sa}_{s_k}, s_{k+1}h + \tau^{sc}_{s_{k+1}}),\ k \in \mathbb{N}, \end{aligned}$$

where K_0 is the fixed feedback control gain, $\hat{\rho}(t)$ is the estimation of ρ, $K_a(\hat{\rho}(t)) = \sum_{i=1}^{m} \hat{\rho}_i(t)K_{ai}$ and $K_b(\hat{\rho}(t)) = \sum_{i=1}^{m} \hat{\rho}_i(t)K_{bi}$.

Now, as discussed in [11], the discrete-time samples b_{ih} will be equivalently converted into continuous time-varying delays. Let $\tau_1(t) = t - b_ih$ for $t \in [b_ih + \tau^{sc}_{b_i}, b_{i+1}h + \tau^{sc}_{b_{i+1}})$, $\tau_2(t) = t - s_kh$ for $t \in [s_kh + \tau^{ca}_{s_k}, s_{k+1}h + \tau^{ca}_{s_{k+1}})$ $k \in \mathbb{N}$ and define $\tau_{1M} = \max_{i \in \mathbb{N}}\left\{(b_{i+1} - b_i)h + \tau^{sc}_{b_{i+1}}\right\}$, $\tau_{1m} = \min_{i \in \mathbb{N}}\left\{\tau^{sc}_{bi}\right\}$ $\tau_{2M} = \max_{k \in \mathbb{N}}\left\{(s_{k+1} - s_k)h + \tau^{ca}_{s_{k+1}}\right\}$, $\tau_{2m} = \max_{k \in \mathbb{N}}\left\{\tau^{ca}_{sk}\right\}$. Therefore, time delays

$\tau_1(t)$ and $\tau_2(t)$ satisfy the following conditions:

$$\begin{aligned} &0 < \tau_{1m} \le \tau_1(t) \le \tau_{1M}, \quad t \in [b_i h + \tau^{sc}_{b_i}, b_{i+1} h + \tau^{sc}_{b_{i+1}}), \ \forall i \in \mathbb{N} \\ &0 < \tau_{2m} \le \tau_2(t) \le \tau_{2M}, \quad t \in [s_k h + \tau^{ca}_{s_k}, s_{k+1} h + \tau^{ca}_{s_{k+1}}), \ \forall k \in \mathbb{N} \end{aligned}$$

Based on the above discussion the sampled control inputs can be rewritten as delayed continuous inputs that are given by:

$$\hat{u}(t) = K_0(\hat{x}(t-\tau_1(t)) - x_r(t-\tau_1(t))) \quad \text{and} \quad u(t) = K(\hat{\rho}(t))(\hat{x}(t-\tau_2(t)) - x_r(t-\tau_2(t)))$$

Further, the observer system can be rewritten as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + BK_0(\hat{x}(t-\tau_1(t)) - x_r(t-\tau_1(t))) + LC(x(t-\tau_1(t)) - \hat{x}(t-\tau_1(t))). \tag{5}$$

It should be noted that the bounded external disturbance cannot be compensated by equivalent input disturbance in the control channel. Further, the EID signals on the control input channel produce the same effect on the control output as an actual disturbance does. As explained in [25], we obtain an estimate of the EID by

$$w_e(t) = \frac{B^T}{B^T B} LC(x(t-\tau_1(t)) - \hat{x}(t-\tau_1(t))) - u^F(t) + \hat{u}(t). \tag{6}$$

In general, the estimated disturbance $w_e(t)$ contains the output measurement noise of the system. In order to reject this noise, the estimated disturbance $w_e(t)$ passes through the low pass filter $F(s)$ with cut-off frequency ω_f and obtain the noiseless disturbance $\tilde{w}(t)$. Moreover, the filter $F(s)$ satisfies the following relation:

$$|F(j\omega)| \approx 1, \quad \forall \omega \in [0, \omega_r],$$

where ω_r is the highest angular frequency of the disturbance estimation. The state-space form of the low pass filter $F(s)$ is given by

$$\begin{aligned} \dot{x}_F(t) &= A_F x_F(t) + B_F w_e(t) \\ \tilde{w}(t) &= C_F x_F(t). \end{aligned} \tag{7}$$

Combining the disturbance estimate $\tilde{w}(t)$ together with original reliable feedback control law yields the following controller:

$$u^F(t) = u(t) - \tilde{w}(t). \tag{8}$$

Based on the aforementioned description and considering Eqs. (3)–(8), closed-loop form of networked control system (1) can be described by

$$\begin{aligned} \dot{x}(t) = Ax(t) + B(I-\rho)\left(K_0 + \sum_{i=1}^{m} \hat{\rho}_i(t) K_{ai} + \sum_{i=1}^{m} \hat{\rho}_i(t) K_{bi} \right)(\hat{x}(t-\tau_2(t)) - x_r(t-\tau_2(t))) \\ - BC_F x_F(t) + B_1 w(t). \end{aligned} \tag{9}$$

In order to obtain the required result, it is assumed that the exogenous signals $r(t)$ and $w(t)$ to be zero. Define the tracking error between state of the plant (9) and the state of the observer (5) as

$$\Delta x(t) = x(t) - \hat{x}(t) \tag{10}$$

From Eqs. (6), (8) and (10), Eq. (7) can be rewritten as

$$\dot{x}_F(t) = (A_F + B_F C_F) x_F(t) + \frac{B_F B^T}{B^T B} LC \Delta x(t-\tau_1(t)). \tag{11}$$

By combining Eqs.(3), (5), (9), (11) and using controller (8), the augmented state-space representation of the closed-loop system is described by the states $\hat{x}(t), \Delta x(t), x_F(t)$ and $x_r(t)$ such that

$$\psi^T(t) = \left[\hat{x}^T(t) \;\; \Delta x^T(t) \;\; x_r^T(t) \;\; x_F^T(t)\right]$$

and the corresponding state-space equation can be written as

$$\dot{\psi}(t) = \overline{\mathcal{A}}\psi(t) + \overline{\mathcal{B}}_{\tau_1}\psi(t-\tau_1(t)) + \overline{\mathcal{B}}_{\tau_2}\psi(t-\tau_2(t)) \tag{12}$$

where

$$\overline{\mathcal{A}} = \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & A & 0 & -B_F C_F \\ 0 & 0 & A_r & 0 \\ 0 & 0 & 0 & A_F + B_F C_F \end{bmatrix}, \quad \overline{\mathcal{B}}_{\tau_1} = \begin{bmatrix} 0 & BK_0 + LC & -BK_0 & 0 \\ 0 & -BK_0 - LC & BK_0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & B_F B^+ LC & 0 & 0 \end{bmatrix},$$

$$\overline{\mathcal{B}}_{\tau_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (I-\rho)BK(\hat{\rho}(t)) & -(I-\rho)BK(\hat{\rho}(t)) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Before proceeding further, we provide the following lemmas which are needed to derive main results.

Lemma 2.1 ([52]). *Given constant matrices Ξ_1, Ξ_2 and Ξ_3 with appropriate dimensions, where $\Xi_1 = \Xi_1^T > 0$ and $\Xi_2 = \Xi_2^T > 0$ then $\Xi_1 + \Xi_3^T \Xi_2^{-1} \Xi_3 < 0$, if and only if, $\begin{bmatrix} \Xi_1 & \Xi_3^T \\ * & -\Xi_2 \end{bmatrix} < 0$, or $\begin{bmatrix} -\Xi_2 & \Xi_3 \\ * & \Xi_1 \end{bmatrix} < 0$.*

Lemma 2.2 ([28]). *For a given matrix $M \in \mathbb{R}^{p\times n}$, where* rank$(M) = p$, *there exists a matrix $\overline{X} \in \mathbb{R}^{p\times p}$ such that $MX = \overline{X}M$ holds for any $X \in \mathbb{R}^{n\times n}$ if and only if it can be decomposed as*

$$X = W\overline{X}W^T, \quad \overline{X} = diag\{\overline{X}_{11}, \overline{X}_{22}\},$$

where $W \in \mathbb{R}^{n\times n}$ is a unitary matrix, $\overline{X}_{11} \in \mathbb{R}^{p\times p}$, and $\overline{X}_{22} \in \mathbb{R}^{(n-p)\times(n-p)}$.

Lemma 2.3 ([51]). *If there exists a symmetric matrix Θ with*

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{bmatrix},$$

where $\Theta_{11}, \Theta_{22} \in R^{sn\times sn}$ such that the following inequalities hold:

$$\Theta_{22ii} \leq 0, \quad i = 1, 2, \ldots, s$$

with $\Theta_{22} \in R^{sn\times sn}$ is the (i, i) block of Θ_{22}. For any $\delta \in \mathcal{S}$

$$\Theta_{11} + \Theta_{12}\Delta(\delta) + (\Theta_{12}\Delta(\delta))^T + \Delta(\delta)\Theta_{22}\Delta(\delta) \geq 0$$

and

$$\begin{bmatrix} Q & E \\ E^T & F \end{bmatrix} + U^T U + G^T \Theta G < 0 \quad \textit{for all } \delta_i \in \{\underline{\delta}_i, \overline{\delta}_i\}$$

then for all $\delta_i \in [\underline{\delta}_i\ \overline{\delta}_i]$,

$$W(\delta) = Q + \sum_{i=1}^{s} \delta_i E_i + \left(\sum_{i=1}^{s} \delta_i E_i \right)^T + \sum_{i=1}^{s} \sum_{j=1}^{s} \delta_i \delta_j F_{ij}$$
$$+ \left(U_0 + \sum_{i=1}^{s} \delta_i U_i \right)^T \left(U_0 + \sum_{i=1}^{s} \delta_i U_i \right) < 0$$

where

$$Q = Q^T, F_{ij} = F_{ji}^T, \quad \Delta(\delta) = \mathrm{diag}\{\delta_1 I\ \dots\ \delta_s I\},$$
$$E = [E_1\ E_2\ \dots\ E_s], \quad U = [U_0\ U_1\ \dots\ U_s],$$
$$F = \begin{bmatrix} F_{11} & F_{12} & \dots & F_{1s} \\ F_{21} & F_{22} & \dots & F_{2s} \\ \vdots & \vdots & \dots & \vdots \\ F_{s1} & F_{s2} & \dots & F_{ss} \end{bmatrix}, \quad G = \begin{bmatrix} \begin{bmatrix} I \\ \vdots \\ I \end{bmatrix} & 0 \\ 0 & I \end{bmatrix}.$$

3. Main results

In this section, we will develop a new set of sufficient conditions for the solvability of the EID based adaptive reliable augmented NCSs which contains the actuator faults and external disturbances. In particular, the required controller will be obtained based on an Lyapunov technique together with EID approach. More precisely, a new set of sufficient conditions in terms of linear matrix inequalities is obtained which ensures that closed-loop augmented NCSs (12) to be asymptotically stable.

Before presenting main result of this paper, we assume that the output matrix $C \in \mathbb{R}^{q \times n}$ has a full row rank and its singular value decomposition is defined as

$$C = U[S\ \ 0]V^T,$$

where U, V are unitary matrices and S is the semi-positive definite diagonal matrix.

Moreover, it should be noted that the main task of the adaptive reliable controller is to estimate the fault of the actuator in the control precessing time. For this purpose, we have to split the controller expression as follows:

$$\begin{aligned}(I-\rho)K(\hat{\rho}(t))e(t-\tau_2(t)) &= (I-\rho)[K_0 + K_a(\hat{\rho}(t)) + K_b(\hat{\rho}(t))]e(t-\tau_2(t)) \\ &= (K_1(\rho) + K_2(\hat{\rho}(t)) + K_3(\hat{\rho}(t)))e(t-\tau_2(t), \end{aligned} \tag{13}$$

where $e(t-\tau_2(t)) = x(t-\tau_2(t)) - x_r(t-\tau_2(t))$, $\tilde{\rho}(t) = \hat{\rho}(t) - \rho$, $K_1(\rho) = [(I-\rho)K_0 + K_a(\rho)]$ $K_2(\hat{\rho}(t)) = [-\rho K_a(\hat{\rho}(t)) + (I-\hat{\rho}(t))K_b(\underline{\hat{\rho}(t)})]$ and $K_3(\hat{\rho}(t)) = [K_a\tilde{\rho}(t) + \tilde{\rho}(t)K_b(\hat{\rho}(t))]$.

With the use of Eq. (13), the matrix $\overline{\mathcal{B}}_{\tau_2}$ can be written as

$$\overline{\mathcal{B}}_{\tau_2} = \overline{\mathcal{B}}_{\tau_2}^1 + \overline{\mathcal{B}}_{\tau_2}^2 + \overline{\mathcal{B}}_{\tau_2}^3, \tag{14}$$

where

$$\overline{\mathcal{B}}_{\tau_2}^{1}=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & BK_1(\rho) & BK_1(\rho) & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix},\quad \overline{\mathcal{B}}_{\tau_2}^{2}=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & BK_2(\hat{\rho}(t)) & BK_2(\hat{\rho}(t)) & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix},$$

$$\overline{\mathcal{B}}_{\tau_2}^{3}=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & BK_3(\hat{\rho}(t)) & BK_3(\hat{\rho}(t)) & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}.$$

The following theorem plays a key role in the design of the adaptive reliable output tracking controller.

Theorem 3.1. *Consider the closed-loop augmented NCSs (12). For the given scalars ϵ_i $(i=1,2,3,4)$, $0\le\tau_{1m}\le\tau_{1M}$ and $0\le\tau_{2m}\le\tau_{2M}$, the closed-loop augmented NCSs (12) is asymptotically stable if there exists symmetric matrices $P>0,\ Q_i>0,\ R_i>0\ (i=1,2,3,4)$ and Θ with*

$$\Theta=\begin{bmatrix}\Theta_{11} & \Theta_{12}\\ \Theta_{12}^T & \Theta_{22}\end{bmatrix}$$

and $\Theta_{11}\in R^{11m(2n+2)\times 11m(2n+2)}, \Theta_{22}\in R^{mn\times mn}$ such that the following LMIs hold:

$$\Theta_{22ii}\le 0,\quad i=1,\ldots,m$$

where $\Theta_{22ii}\in R^{mn\times mn}$ is the (i,i) block of Θ_{22}.

$$\Theta_{11}+\Delta(\hat{\rho}(t))\Theta_{12}+(\Delta(\hat{\rho}(t))\Theta_{12})^T+\Delta(\hat{\rho}(t))\Theta_{22}\Delta(\hat{\rho}(t))\ge 0,\quad \text{for } \hat{\rho}(t)\in\Delta_{\hat{\rho}} \tag{15}$$

$$\frac{1}{\tau_{M1}-\tau_{m1}}\hat{R}<\epsilon_1 X,\quad \frac{1}{\tau_{M2}-\tau_{m2}}\hat{R}<\epsilon_2 X,\quad \frac{1}{\tau_{m1}}\hat{R}<\epsilon_3 X,\quad \frac{1}{\tau_{m2}}\hat{R}<\epsilon_4 X, \tag{16}$$

$$\begin{bmatrix}\tilde{\Pi}+G\Theta G & \Gamma_1\\ \Gamma_1^T & -\Gamma_2\end{bmatrix}<0\quad \text{for } \rho\in\{\rho^1\ldots\rho^L\},\quad \rho^j\in N_{p^j}, \tag{17}$$

where

$$\tilde{\Pi}=\begin{bmatrix}[\tilde{\Pi}_0]_{11\times 11} & \tilde{\Pi}_1\\ * & \tilde{\Pi}_2\end{bmatrix}$$

with

$$\begin{bmatrix} \tilde{\Pi}_{1,1} & \tilde{\Pi}_{1,2} & \overline{\mathcal{B}}_{\tau_1}X & 0 & \tilde{\Pi}_{1,5} & \overline{\mathcal{B}}^1_{\tau_2}X & 0 & X\overline{\mathcal{A}}^T & 0 & 0 & \tilde{\Pi}_{1,11} \\ * & \tilde{\Pi}_{2,2} & \tilde{\Pi}_{2,3} & \tilde{\Pi}_{2,4} & \tilde{\Pi}_{2,5} & 0 & 0 & 0 & \tilde{\Pi}_{2,9} & 0 & \tilde{\Pi}_{2,11} \\ * & * & \tilde{\Pi}_{3,3} & \tilde{\Pi}_{3,4} & 0 & 0 & 0 & X\overline{\mathcal{B}}^T_{\tau_1} & \tilde{\Pi}_{3,9} & 0 & 0 \\ * & * & * & \tilde{\Pi}_{4,4} & 0 & 0 & 0 & 0 & \tilde{\Pi}_{4,9} & 0 & 0 \\ * & * & * & * & \tilde{\Pi}_{5,5} & \tilde{\Pi}_{5,6} & \tilde{\Pi}_{5,7} & 0 & 0 & \tilde{\Pi}_{5,10} & \tilde{\Pi}_{5,11} \\ * & * & * & * & * & \tilde{\Pi}_{6,6} & \tilde{\Pi}_{6,7} & 0 & 0 & \tilde{\Pi}_{6,10} & 0 \\ * & * & * & * & * & * & \tilde{\Pi}_{7,7} & 0 & 0 & \tilde{\Pi}_{7,10} & 0 \\ * & * & * & * & * & * & * & \tilde{\Pi}_{8,8} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\epsilon_1 X & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\epsilon_2 X & 0 \\ * & * & * & * & * & * & * & * & * & * & -\epsilon_3 X - \epsilon_4 X \end{bmatrix}$$

$$\tilde{\Pi}_{1,1} = \overline{\mathcal{A}}X + X\overline{\mathcal{A}}^T + \hat{Q}_3 + \hat{Q}_4 + 2\hat{Z}_{11}, \quad \tilde{\Pi}_{1,2} = \hat{Z}^T_{12} - \hat{Z}_{11} + \hat{Z}_{21},$$

$$\tilde{\Pi}_{2,2} = \hat{Q}_1 - \hat{Q}_3 + 2\hat{M}_{11} - 2\hat{Z}_{12} + 2\hat{Z}_{22}, \quad \tilde{\Pi}_{2,3} = \hat{M}^T_{12} - \hat{M}_{11} + \hat{M}_{21},$$
$$\tilde{\Pi}_{2,4} = \hat{M}^T_{13} - \hat{M}_{21}, \quad \tilde{\Pi}_{3,3} = -2\hat{M}_{12} + 2\hat{M}_{22}, \quad \tilde{\Pi}_{3,4} = -\hat{M}^T_{13} + \hat{M}^T_{23} - \hat{M}_{22}$$
$$\tilde{\Pi}_{4,4} = -\hat{Q}_1 - 2\hat{M}_{23}, \quad \tilde{\Pi}_{1,5} = \hat{Z}^T_{13} - \hat{Z}_{21}, \quad \tilde{\Pi}_{2,5} = -\hat{Z}^T_{13} + \hat{Z}^T_{23} - \hat{Z}_{22},$$
$$\tilde{\Pi}_{5,5} = \hat{Q}_2 - \hat{Q}_4 - 2\hat{N}_{11} - 2\hat{Z}_{23}, \tilde{\Pi}_{5,6} = \hat{N}^T_{12} - \hat{N}_{11} + \hat{N}_{21}, \quad \tilde{\Pi}_{5,7} = \hat{N}^T_{13} - \hat{N}_{21},$$
$$\tilde{\Pi}_{6,6} = -2\hat{N}_{12} + 2\hat{N}_{22}, \quad \tilde{\Pi}_{6,7} = -\hat{N}^T_{13} + \hat{N}^T_{23} - \hat{N}_{22}, \quad \tilde{\Pi}_{7,7} = -\hat{Q}_2 - 2\hat{N}_{23},$$
$$\tilde{\Pi}_{8,8} = \epsilon_1 X + \epsilon_2 X + \epsilon_3 X + \epsilon_4 X, \quad \tilde{\Pi}_{2,9} = \hat{M}_{11} + \hat{M}_{21}, \quad \tilde{\Pi}_{2,11} = \hat{Z}_{11} + \hat{Z}_{21},$$
$$\tilde{\Pi}_{3,9} = \hat{M}_{12} + \hat{M}_{22}, \quad \tilde{\Pi}_{4,9} = \hat{M}_{12} + \hat{M}_{22}, \quad \tilde{\Pi}_{5,10} = \hat{N}_{11} + \hat{N}_{21},$$
$$\tilde{\Pi}_{5,11} = \hat{Z}_{13} + \hat{Z}_{23}, \quad \tilde{\Pi}_{6,10} = \hat{N}_{12} + \hat{N}_{22}, \quad \tilde{\Pi}_{7,10} = \hat{N}_{12} + \hat{N}_{22},$$
$$\tilde{\Pi}_1 = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ [-B\rho Y_{a1} + BY_{b1}\ \ \ldots\ \ -B\rho Y_{am} + BY_{bm}]\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0],$$
$$\tilde{\Pi}_2 = \begin{bmatrix} -B^1Y_{b1} - (B^1Y_{b1})^T & \ldots & -B^1Y_{bm} - (B^mY_{b1})^T \\ \vdots & \vdots & \vdots \\ -B^mY_{b1} - (B^1Y_{bm})^T & \ldots & -B^mY_{bm} - (B^mY_{bm})^T \end{bmatrix},$$
$$G = \begin{bmatrix} \begin{bmatrix} I_{n\times n} \\ \vdots \\ I_{n\times n} \end{bmatrix} & 0 \\ 0 & I_{n\times n} \end{bmatrix},$$
$$\Gamma_1 = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ [C_1X + D_{12}(I-\rho)Y_0\ \ \Xi_1\ \ \ldots\ \ \Xi_m]\ \ 0\ \ 0\ \ 0\ \ 0]$$
$$\Gamma_2 = \epsilon_1 X + \epsilon_2 X + \epsilon_3 X + \epsilon_4 X, \quad \Xi_i = D_{12}(I-\rho)(Y_{ai} + Y_{bi}),$$
$$\Delta(\hat{\rho}) = diag\{\hat{\rho}_1 I_{n\times n}\ \ldots\ \hat{\rho}_m I_{n\times n}\}, \quad X = diag\{X_1, X_2, X_3, X_4\},$$
$$\hat{Q}_i = diag\{\hat{Q}_{i1}, \hat{Q}_{i2}, \hat{Q}_{i3}, \hat{Q}_{i4}\}, \quad \hat{R}_i = diag\{\hat{R}_{i1}, \hat{R}_{i2}, \hat{R}_{i3}, \hat{R}_{i4}\}$$
$$\hat{M}_{kl} = diag\{\hat{M}_{kl1}, \hat{M}_{kl2}, \hat{M}_{kl3}, \hat{M}_{kl4}\}, \quad \hat{N}_{kl} = diag\{\hat{N}_{kl1}, \hat{N}_{kl2}, \hat{N}_{kl3}, \hat{N}_{kl4}\},$$
$$\hat{Z}_{kl} = diag\{\hat{Z}_{kl1}, \hat{Z}_{kl2}, \hat{Z}_{kl3}, \hat{Z}_{kl4}\}, \quad (k = 1, 2,\ \ l = 1, 2, 3),$$

$$\overline{\mathcal{B}}_{\tau_1}=\begin{bmatrix}0 & BY_0+CW & -BY_0 & 0\\ 0 & -BY_0-CW & BY_0 & 0\\ 0 & 0 & 0 & 0\\ 0 & B_FB^{+}CW & 0 & 0\end{bmatrix},$$

$$\overline{\mathcal{B}}^1_{\tau_2}X=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & \left((I-\rho)Y_0+\sum_{i=1}^{m}\rho_iY_{ai}\right) & \left((I-\rho)K_0+\sum_{i=1}^{m}\rho_iY_{ai}\right) & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}.$$

Further, $\hat{\rho}_i(t)$ *can be determined according to the following adaptive estimation algorithm*:

$$\dot{\hat{\rho}}_i(t)=\underset{[\min_j\{\underline{\rho}_i^j\},\max_j\{\overline{\rho}_i^j\}]}{Proj}\{L_{1i}\}=\begin{cases}0, & if \quad \begin{array}{l}\hat{\rho}=\min\limits_j\{\underline{\rho}_i^j\} \text{ and } L_{1i}\le 0\\ \text{or } \hat{\rho}=\max\limits_j\{\overline{\rho}_i^j\} \text{ and } \{L_{1i}\}\ge 0;\end{array}\\ L_{1i}, & \text{otherwise}\end{cases}\tag{18}$$

where $L_{1i}=-l_ix^T(t)[PB^iK_b(\hat{\rho})+PBK_{ai}]x(t)$, *here* $P=X^{-1}$, $K_{ai}=Y_{ai}X^{-1}$, $K_{bi}=Y_{bi}X^{-1}$ *and* $l_i>0$ $(i=1\ldots m)$ *is the adaptive learning gain to be determined according to the lower and upper bounds of actuator faults*; $\mathrm{Proj}\{\cdot\}$ *denotes the projection operator*, *whose role is to project the estimates* $\hat{\rho}_i(t)$ *in the interval* $[\min_j\{\underline{\rho}_i^j\},\max_j\{\overline{\rho}_i^j\}]$. *Moreover*, *the adaptive controller gain can be calculated by*

$$K(\hat{\rho})=Y_0X^{-1}+\sum_{i=1}^{m}\hat{\rho}_i(t)Y_{ai}X^{-1}+\sum_{i=1}^{m}\hat{\rho}_i(t)Y_{bi}X^{-1}$$

and the state observer gain can be determined by $L=Y_2USX_{11}^{-1}S^{-1}U^T$.

Proof. Consider the Lyapunov–Krasovskii functional (LKF) candidate for the closed-loop NCSs (12) in the following form:

$$V(\psi(t))=\sum_{i=1}^{4}V_k(\psi(t)),\tag{19}$$

where

$$\begin{aligned}V_1(\psi(t))&=\psi^T(t)P\psi(t),\\ V_2(\psi(t))&=\int_{t-\tau_{1M}}^{t-\tau_{1m}}\psi^T(s)Q_1\psi(s)\mathrm{d}s+\int_{t-\tau_{2M}}^{t-\tau_{2m}}\psi^T(s)Q_2\psi(s)\mathrm{d}s+\int_{t-\tau_{1m}}^{t}\psi^T(s)Q_3\psi(s)\mathrm{d}s\\ &\quad+\int_{t-\tau_{2m}}^{t}\psi^T(s)Q_4\psi(s)\mathrm{d}s,\end{aligned}$$

$$V_3(\psi(t))=\int_{-\tau_{1M}}^{-\tau_{1m}}\int_{t+\theta}^{t}\dot{\psi}^T(s)R_1\dot{\psi}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\tau_{2M}}^{\tau_{2m}}\int_{t+\theta}^{t}\dot{\psi}^T(s)R_2\dot{\psi}(s)\mathrm{d}s\mathrm{d}\theta$$

$$+\int_{-\tau_{1m}}^{0}\int_{t+\theta}^{t}\dot{\psi}^T(s)R_3\dot{\psi}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\tau_{2m}}^{0}\int_{t+\theta}^{t}\dot{\psi}^T(s)R_4\dot{\psi}(s)\mathrm{d}s\mathrm{d}\theta$$

$$V_4(\psi(t))=\sum_{i=1}^{m}\frac{\tilde{\rho}_i^2(t)}{l_i}.$$

By taking time derivative of the LKF (19) along the trajectories of the augmented closed-loop system (12), we obtain

$$\begin{aligned}\dot{V}(x(t))=&\,\psi^T(t)P\dot{\psi}(t)+\dot{\psi}^T(t)P\psi(t)+\psi^T(t-\tau_{1m})Q_1\psi(t-\tau_{1m})-\psi^T(t-\tau_{1M})Q_1\psi(t-\tau_{1M})\\&+\psi^T(t-\tau_{2m})Q_1\psi(t-\tau_{2m})-\psi^T(t-\tau_{2M})Q_1\psi(t-\tau_{2M})+\psi^T(t)(Q_3+Q_4)\psi(t)\\&-\psi^T(t-\tau_{1m})Q_3\psi(t-\tau_{1m})-\psi^T(t-\tau_{2m})Q_4\psi(t-\tau_{2m}))+\dot{\psi}^T(t)[(\tau_{M1}-\tau_{m1})R_1\\&+(\tau_{M2}-\tau_{m2})R_2+\tau_{m1}R_3+\tau_{m2}R_4]\dot{\psi}(t)\\&-\int_{t-\tau_{1M}}^{t-\tau_{1m}}\dot{\psi}^T(s)R_1\dot{\psi}(s)\mathrm{d}s+\int_{t-\tau_{2M}}^{t-\tau_{2m}}\dot{\psi}^T(s)R_2\dot{\psi}(s)\mathrm{d}s\end{aligned}\tag{20}$$

$$-\int_{t-\tau_{1m}}^{t}\dot{\psi}^T(s)R_3\dot{\psi}(s)\mathrm{d}s-\int_{t-\tau_{2m}}^{t}\dot{\psi}^T(s)R_4\dot{\psi}(s)\mathrm{d}s+2\sum_{i=1}^{m}\tilde{\rho}_i\frac{\dot{\tilde{\rho}}_i}{l_i}.\tag{21}$$

On the other hand, by Newton–Leibniz formula for any arbitrary matrices $M_i=\left[M_{i1}^T\ M_{i2}^T\ M_{i3}^T\right]^T$, $N_i=\left[N_{i1}^T\ N_{i2}^T\ N_{i3}^T\right]^T$, $N_i=\left[Z_{i1}^T\ Z_{i2}^T\ Z_{i3}^T\right]^T$, $i=1,2,\ k=1,2,3$ with compatible dimensions, we have the following equalities:

$$2\alpha_1^T(t)M_1\left[\psi(t-\tau_{1m})-\psi(t-\tau_1(t))-\int_{t-\tau_1(t)}^{t-\tau_{1m}}\dot{\psi}(s)ds\right]=0,\tag{22}$$

$$2\alpha_1^T(t)M_2\left[\psi(t-\tau_1(t))-\psi(t-\tau_{M1})-\int_{t-\tau_{M1}}^{t-\tau_1(t)}\dot{\psi}(s)ds\right]=0,\tag{23}$$

$$2\alpha_2^T(t)N_2\left[\psi(t-\tau_{2m})-\psi(t-\tau_2(t))-\int_{t-\tau_2(t)}^{t-\tau_{2m}}\dot{\psi}(s)ds\right]=0,\tag{24}$$

$$2\alpha_2^T(t)N_2\left[\psi(t-\tau_2(t))-\psi(t-\tau_{M2})-\int_{t-\tau_{M2}}^{t-\tau_2(t)}\dot{\psi}(s)ds\right]=0,\tag{25}$$

$$2\alpha_3^T(t)Z_1\left[\psi(t)-\psi(t-\tau_{1m})-\int_{t-d_m}^{t}\dot{\psi}(s)ds\right]=0,\tag{26}$$

$$2\alpha_3^T(t)Z_2\left[\psi(t)-\psi(t-\tau_{2m})-\int_{t-d_{2m}}^{t}\dot{\psi}(s)ds\right]=0,\tag{27}$$

$$(\tau_1(t)-\tau_{1m})\alpha_1^T(t)M_1R_1^{-1}M_1^T\alpha_1(t)-\int_{t-\tau_1(t)}^{t-\tau_{1m}}\alpha_1^T(t)M_1R_1^{-1}M_1^T\alpha_1(t)\mathrm{d}s=0,\tag{28}$$

$$(\tau_{1M}-\tau_1(t))\alpha_1^T(t)M_2R_1^{-1}M_2^T\alpha_1(t)-\int_{t-\tau_{1M}}^{t-\tau_1(t)}\alpha_1^T(t)M_2R_1^{-1}M_2^T\alpha_1(t)\mathrm{d}s=0,\tag{29}$$

$$(\tau_2(t)-\tau_{2m})\alpha_2^T(t)N_2R_2^{-1}N_2^T\alpha_2(t)-\int_{t-\tau_2(t)}^{t-\tau_{2m}}\alpha_2^T(t)N_2R_2^{-1}N_2^T\alpha_2(t)\mathrm{d}s=0, \tag{30}$$

$$(\tau_{2M}-\tau_2(t))\alpha_2^T(t)N_2R_2^{-1}N_2^T\alpha_2(t)-\int_{t-\tau_{2N}}^{t-\tau_2(t)}\alpha_2^T(t)N_2R_2^{-1}N_2^T\alpha_2(t)\mathrm{d}s=0, \tag{31}$$

$$\tau_{1m}\alpha_3^T(t)Z_1R_3^{-1}Z_1^T\alpha_3(t)-\int_{t-\tau_{1m}}^{t}\alpha_3^T(t)Z_1R_3^{-1}Z_1^T\alpha_3(t)\mathrm{d}s=0, \tag{32}$$

$$\tau_{2m}\alpha_3^T(t)Z_2R_3^{-1}Z_2^T\alpha_3(t)-\int_{t-\tau_{2m}}^{t}\alpha_3^T(t)Z_2R_3^{-1}Z_2^T\alpha_3(t)\mathrm{d}s=0, \tag{33}$$

where $\alpha_1^T(t)=\left[\psi^T(t-\tau_{1m})\ \psi^T(t-\tau_1(t))\ \psi(t-\tau_{1M})\right]$, $\alpha_2^T(t)=\left[\psi^T(t-\tau_{2m})\ \psi^T(t-\tau_2(t))\ \psi(t-\tau_{2M})\right]$ and $\alpha_3^T(t)=\left[\psi^T(t)\ \psi^T(t-\tau_{1m})\ \psi(t-\tau_{2m})\right]$.

It follows from Eqs. (20)–(33) that

$$\begin{aligned}
\dot V(x(t))\leq{}& \psi^T(t)P\dot\psi(t)+\dot\psi^T(t)P\psi(t)+\psi^T(t-\tau_{1m})Q_1\psi(t-\tau_{1m})-\psi^T(t-\tau_{1M})Q_1\psi(t-\tau_{1M})\\
&+\psi^T(t-\tau_{2m})Q_2\psi(t-\tau_{2m})-\psi^T(t-\tau_{2M})Q_2\psi(t-\tau_{2M})+\psi^T(t)(Q_3+Q_4)\psi(t)\\
&-\psi^T(t-\tau_{1m})Q_3\psi(t-\tau_{1m})-\psi^T(t-\tau_{2m})Q_4\psi(t-\tau_{2m}))+\dot\psi^T(t)[(\tau_{M1}-\tau_{m1})R_1\\
&+(\tau_{M2}-\tau_{m2})R_2+\tau_{m1}R_3+\tau_{m2}R_4]\dot\psi(t)+2\alpha_1^T(t)M_1[\psi(t-\tau_{1m})-\psi(t-\tau_1(t))]\\
&+2\alpha_1^T(t)M_2[\psi(t-\tau_1(t))-\psi(t-\tau_{M1})]+2\alpha_2^T(t)N_1[\psi(t-\tau_{2m})-\psi(t-\tau_2(t))]\\
&+2\alpha_2^T(t)N_2[\psi(t-\tau_2(t))-\psi(t-\tau_{2M})]+2\alpha_3^T(t)Z_1[\psi(t)-\psi(t-\tau_{1m})]\\
&+2\alpha_3^T(t)Z_2[\psi(t)-\psi(t-\tau_{2m})]+2\sum_{i=1}^{m}\tilde\rho_i\frac{\dot{\tilde\rho}_i}{l_i}+(\tau_1(t)-\tau_{1m})\alpha_1^T(t)M_1R_1^{-1}M_1^T\alpha_1(t)\\
&+(\tau_{1M}-\tau_1(t))\alpha_1^T(t)M_2R_1^{-1}M_2^T\alpha_1(t)+(\tau_2(t)-\tau_{2m})\alpha_2^T(t)N_2R_2^{-1}N_2^T\alpha_2(t)\\
&+(\tau_{2M}-\tau_2(t))\alpha_2^T(t)N_2R_2^{-1}N_2^T\alpha_2(t)+\tau_{1m}\alpha_3^T(t)Z_1R_3^{-1}Z_1^T\alpha_3(t)\\
&+\tau_{2m}\alpha_3^T(t)Z_2R_3^{-1}Z_2^T\alpha_3(t)\leq\psi^T(t)P\left[\overline{\mathcal{A}}\psi(t)+\overline{\mathcal{B}}_{\tau_1}\psi(t-\tau_1(t))+\overline{\mathcal{B}}_{\tau_2}\psi(t-\tau_2(t))\right]\\
&+\left[\overline{\mathcal{A}}\psi(t)+\overline{\mathcal{B}}_{\tau_1}\psi(t-\tau_1(t))+\overline{\mathcal{B}}_{\tau_2}\psi(t-\tau_2(t))\right]^TP\psi^T(t)+\psi^T(t-\tau_{1m})Q_1\psi(t-\tau_{1m})\\
&-\psi^T(t-\tau_{1M})Q_1\psi(t-\tau_{1M})+\psi^T(t-\tau_{2m})Q_2\psi(t-\tau_{2m})\\
&-\psi^T(t-\tau_{2M})Q_2\psi(t-\tau_{2M})+\psi^T(t)(Q_3+Q_4)\psi(t)\\
&-\psi^T(t-\tau_{1m})Q_3\psi(t-\tau_{1m})-\psi^T(t-\tau_{2m})Q_4\psi(t-\tau_{2m}))\\
&+\left[\overline{\mathcal{A}}\psi(t)+\overline{\mathcal{B}}_{\tau_1}\psi(t-\tau_1(t))+\overline{\mathcal{B}}_{\tau_2}\psi(t-\tau_2(t))\right]^T[(\tau_{M1}-\tau_{m1})R_1+(\tau_{M2}-\tau_{m2})R_2\\
&+\tau_{m1}R_3+\tau_{m2}R_4]\left[\overline{\mathcal{A}}\psi(t)+\overline{\mathcal{B}}_{\tau_1}\psi(t-\tau_1(t))+\overline{\mathcal{B}}_{\tau_2}\psi(t-\tau_2(t))\right]\\
&+2\alpha_1^T(t)M_1[\psi(t-\tau_{1m})-\psi(t-\tau_1(t))]\\
&+2\alpha_1^T(t)M_2[\psi(t-\tau_1(t))-\psi(t-\tau_{M1})]+2\alpha_2^T(t)N_1[\psi(t-\tau_{2m})-\psi(t-\tau_2(t))]\\
&+2\alpha_2^T(t)N_2[\psi(t-\tau_2(t))-\psi(t-\tau_{2M})]+2\alpha_3^T(t)Z_1[\psi(t)-\psi(t-\tau_{1m})]\\
&+2\alpha_3^T(t)Z_2[\psi(t)-\psi(t-\tau_{2m})]+2\sum_{i=1}^{m}\tilde\rho_i\frac{\dot{\tilde\rho}_i}{l_i}+(\tau_{1M}-\tau_{1m})\alpha_1^T(t)M_1R_1^{-1}M_1^T\alpha_1(t)\\
&+(\tau_{1M}-\tau_{1m})\alpha_1^T(t)M_2R_1^{-1}M_2^T\alpha_1(t)+(\tau_{2M}-\tau_{2m})\alpha_2^T(t)N_2R_2^{-1}N_2^T\alpha_2(t)\\
&+(\tau_{2M}-\tau_{2m})\alpha_2^T(t)N_2R_2^{-1}N_2^T\alpha_2(t)+\tau_{1m}\alpha_3^T(t)Z_1R_3^{-1}Z_1^T\alpha_3(t)
\end{aligned}$$

$$+\tau_{2m}\alpha_3^T(t)Z_2R_3^{-1}Z_2^T\alpha_3(t)$$

By using Lemma 2.1 and Eq. (14) into the above inequality, we can obtain an inequality in the following form:

$$\dot{V} \le \zeta^T(t)[\Pi]_{11\times 11}\zeta(t) + \psi^T(t)P\overline{\mathcal{B}}^3_{\tau_2}\psi(t-\tau_2(t)) + \left(\psi^T(t)P\overline{\mathcal{B}}^3_{\tau_2}\psi(t-\tau_2(t))\right)^T + 2\sum_{i=1}^{m}\frac{\tilde{\rho}_i\dot{\tilde{\rho}}_i}{l_i},$$

where $\zeta(t) = \left[\psi^T(t)\ \psi(t-\tau_{1m})\ \psi(t-\tau_1(t)\ \psi(t-\tau_{1M})\ \psi(t-\tau_{2m})\ \psi(t-\tau_2(t))\ \psi(t-\tau_{2M})\right]$

$$\begin{bmatrix}
\Pi_{1,1} & \Pi_{12} & P\overline{\mathcal{B}}_{\tau_1} & 0 & \Pi_{1,5} & P\overline{\mathcal{B}}^1_{\tau_2}+P\overline{\mathcal{B}}^2_{\tau_2} & 0 & \overline{\mathcal{A}}^T & 0 & 0 & \Pi_{1,11}\\
* & \Pi_{2,2} & \Pi_{2,3} & \Pi_{2,4} & \Pi_{2,5} & 0 & 0 & 0 & \Pi_{2,9} & 0 & \Pi_{2,11}\\
* & * & \Pi_{3,3} & \Pi_{3,4} & 0 & 0 & 0 & \overline{\mathcal{B}}^T_{\tau_1} & \Pi_{3,9} & 0 & 0\\
* & * & * & \Pi_{4,4} & 0 & 0 & 0 & 0 & \Pi_{4,9} & 0 & 0\\
* & * & * & * & \Pi_{5,5} & \Pi_{5,6} & \Pi_{5,7} & 0 & 0 & \Pi_{5,10} & \Pi_{5,11}\\
* & * & * & * & * & \Pi_{6,6} & \Pi_{6,7} & \overline{\mathcal{B}}^T_{\tau_2} & 0 & \Pi_{6,10} & 0\\
* & * & * & * & * & * & \Pi_{7,7} & 0 & 0 & \Pi_{7,10} & 0\\
* & * & * & * & * & * & * & \Pi_{8,8} & 0 & 0 & 0\\
* & * & * & * & * & * & * & * & -\frac{1}{\tau_{M1}-\tau_{m1}}R_1 & 0 & 0\\
* & * & * & * & * & * & * & * & * & -\frac{1}{\tau_{M2}-\tau_{m2}}R_2 & 0\\
* & * & * & * & * & * & * & * & * & * & -\frac{1}{\tau_{m1}}R_3-\frac{1}{\tau_{m2}}R_4
\end{bmatrix}$$

with

$$\begin{aligned}
&\Pi_{1,1} = P\overline{\mathcal{A}} + \overline{\mathcal{A}}^TP + Q_3 + Q_4 + 2Z_{11}, \quad \Pi_{1,2} = Z_{12}^T - Z_{11} + Z_{21},\\
&\Pi_{2,2} = Q_1 - Q_3 + 2M_{11} - 2Z_{12} + 2Z_{22}, \quad \Pi_{2,3} = M_{12}^T - M_{11} + M_{21},\\
&\Pi_{2,4} = M_{13}^T - M_{21}, \quad \Pi_{3,3} = -2M_{12} + 2M_{22}, \quad \Pi_{3,4} = -M_{13}^T + M_{23}^T - M_{22}\\
&\Pi_{4,4} = -Q_1 - 2M_{23}, \quad \Pi_{1,5} = Z_{13}^T - Z_{21}, \quad \Pi_{2,5} = -Z_{13}^T + Z_{23}^T - Z_{22},\\
&\Pi_{5,5} = Q_2 - Q_4 - 2N_{11} - 2Z_{23}, \quad \Pi_{5,6} = N_{12}^T - N_{11} + N_{21}, \quad \Pi_{5,7} = N_{13}^T - N_{21},\\
&\Pi_{6,6} = -2N_{12} + 2N_{22}, \quad \Pi_{6,7} = -N_{13}^T + N_{23}^T - N_{22}, \quad \Pi_{7,7} = -Q_2 - 2N_{23},\\
&\Pi_{8,8} = \frac{R_1^{-1}}{\tau_{M1}-\tau_{m1}} + \frac{R_2^{-1}}{\tau_{M2}-\tau_{m2}} + \frac{R_3^{-1}}{\tau_{m1}} + \frac{R_4^{-1}}{\tau_{m2}}, \quad \Pi_{2,9} = M_{11} + M_{21}, \quad \Pi_{2,11} = Z_{11} + Z_{21},\\
&\Pi_{3,9} = M_{12} + M_{22}, \quad \Pi_{4,9} = M_{12} + M_{22}, \quad \Pi_{5,10} = N_{11} + N_{21}, \quad \Pi_{5,11} = Z_{13} + Z_{23},\\
&\Pi_{6,10} = N_{12} + N_{22}, \quad \Pi_{7,10} = N_{12} + N_{22}.
\end{aligned}$$

On the other hand, since, ρ_i is unknown constant, we have $\dot{\hat{\rho}}_i(t) = \dot{\tilde{\rho}}_i(t)$. Now, if we choose the adaptive law as

$$\begin{aligned}
\dot{\hat{\rho}}_i(t) &= \underset{[\min_j\{\underline{\rho}_i^j\},\max_j\{\overline{\rho}_i^j\}]}{Proj}\{L_{1i}\}\\
&= \begin{cases} 0, & \text{if } \begin{array}{l}\hat{\rho} = \min\limits_j\{\underline{\rho}_i^j\} \text{ and } L_{1i} \le 0\\ \text{or } \hat{\rho} = \max\limits_j\{\overline{\rho}_i^j\} \text{ and } \{L_{1i}\} \ge 0;\end{array}\\ L_{1i}, & \text{otherwise}\end{cases}
\end{aligned}$$

where $L_{1i} = -l_ix^T(t)[PB^iK_b(\hat{\rho}) + PBK_{ai}]x(t-\tau_2(t))$, then $\psi^T(t)P\overline{\mathcal{B}}_{\tau_2}\psi(t-\tau_2(t)) + \left(\psi^T(t)P\overline{\mathcal{B}}_{\tau_2}\right.$

$\psi(t-\tau_2(t)))^T + 2\sum_{i=1}^{m}\tilde{\rho}_i\frac{\dot{\hat{\rho}}_i}{l_i} \geq 0$, then we have

$$\dot{V}(\psi(t)) \leq \zeta^T(t)[\Pi]_{11\times 11}\zeta(t)$$

To apply concurrent transformation, pre-and-post multiplying $[\Pi]_{11\times 11}$ by $diag\{X, X, X, X, X, X, X, I, X, X, X\}$ and choose $X = P^{-1}$, $\hat{Q}_i = XQ_iX$, $\hat{R}_i = XR_iX$, $\hat{M}_{jk} = XM_{jk}X$, $\hat{N}_{jk} = XZ_{jk}X$, $\hat{Z}_{jk} = XZ_{jk}X$, $(i = 1,2,3,4,\ \ j = 1,2,\ k = 1,2,3)$. By Lemma 2.3, if the LMIs (15)–(17) hold for any $\rho \in \mathcal{N}_\rho$ and $\hat{\rho}(t)$ satisfying Eq. (18) then $[\Pi]_{11\times 11} < 0$ for all $\rho \in \mathcal{S}$. Hence the LMIs (15)–(17) hold which implies that $\dot{V}(\psi(t)) < 0$ for any $\psi(t) \neq 0$ and for all $\rho \in \mathcal{S}$. Thus, the closed-loop system (12) is asymptotically stable and the controller robustly tracks the reference signal for all $\rho \in \mathcal{S}$.□

From the above theorem, we can easily deduce the traditional state feedback reliable controller. For this purpose, we consider the closed-loop system as follows:

$$\dot{\phi}(t) = \overline{\mathcal{A}}\phi(t) + \overline{\mathcal{B}}_{\tau_1}\phi(t-\tau_1(t)) + \hat{\mathcal{B}}_{\tau_2}\phi(t-\tau_2(t)), \tag{34}$$

where

$$\hat{\mathcal{B}}_{\tau_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (I-\rho)BK_0 & -(I-\rho)BK_0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \phi(t) = \left[\hat{x}^T(t)\ \ \Delta x^T(t)\ \ x_r^T(t)\ \ x_F^T(t)\right]^T.$$

In this consequence we can obtain the following corollary:

Corollary 3.2. *Consider the closed-loop system* (34). *Given constants* $0 \leq \tau_{1m} \leq \tau_{1M}$, $0 \leq \tau_{2m} \leq \tau_{2M}$ *and* ϵ_i $(i = 1,2,3,4)$, *the closed-loop system* (34) *is asymptotically stable if there exists symmetric matrix* $P > 0$, $Q_i > 0$, $R_i > 0$, $(i = 1,2,3,4)$ *and* Θ *with*

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{12}^T & \Theta_{22} \end{bmatrix},$$

where $\Theta_{11}, \Theta_{22} \in R^{mn\times mn}$ *such that the LMIs* (15)–(17) *hold with* Y_{ai} *and* Y_{bi} $(i = 1,2,\ldots,m)$ *be zero.*

Remark 3.3. It should be noted that in Theorem 3.1, the feedback controller and observer gains (K_0, K_{ai}, K_{bi} and L $(i = 1,2,\ldots,m)$) are calculated by off-line, while the estimation $\hat{\rho}$ is automatically updating the gains through online according to the adaptive law (18). Thus, due to the introduction of adaptive mechanism, the proposed reliable controller gain in $u^F(t)$ act as variable controller gain. The positive scalar l_i can be chosen to meet the low-energy control input requirement since a high-gain controller may be unstable for the NCS. It can be seen that if $K_{ai} = 0$ and $K_{bi} = 0$ $(i = 1,2,\ldots,m)$, then the adaptive reliable controller equivalently turns to the conventional one, which reveals that the proposed adaptive reliable controller is more general one.

Remark 3.4. Generally, it is difficult to measure the state of the networked control system at each instant, but the proposed adaptive reliable controller design contains the observer which estimates the state for the feedback loop. Moreover, by using EID technique the estimated disturbance is given into the control channel which helps to highly suppress the effect of the external disturbances which yields the proposed controller is robust one. The sufficient conditions for the existence of the controller (17) and (18) are based on the solvability of LMIs, which can be easily solved by using available numerical software.

4. Numerical examples

In this section, two numerical examples are presented to demonstrate the developed results and validate the effectiveness of the obtained adaptive reliable controller over the conventional controller.

Example 4.1. To demonstrate the proposed controller under more than one actuators with different fault modes, we consider this example with two actuators and two different fault modes. Consider the networked control system (12) with the following matrices:

$$A = \begin{bmatrix} -2 & 2 & 1 \\ -1 & 0 & -1 \\ 5 & 1 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \\ 3 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad C = [1 \ \ 0 \ \ 0].$$

Now, to track the state of the system to the reference output, we consider the adaptive reliable controller $u^F(t)$ with two actuators in the presence of normal mode and two possible fault modes. In order to do this, we take the parameters as $\tau_{1m} = 0.1$, $\tau_{1M} = 0.568$, $\tau_{2m} = 0.3$, $\tau_{2M} = 0.732$, $\epsilon_1 = 0.01$, $\epsilon_2 = 0.015$, $\epsilon_3 = 0.4$, $\epsilon_4 = 0.2$, $l_1 = 75$ and $l_2 = 75$. For simulation purpose, we consider the reference model by

$$\dot{x}_r(t) = \begin{bmatrix} 0 & 1 & 0 \\ -6 & -5 & 1 \\ 0 & 0 & -4 \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$
$$y_r(t) = [1 \ \ 0 \ \ 0] x_r(t).$$

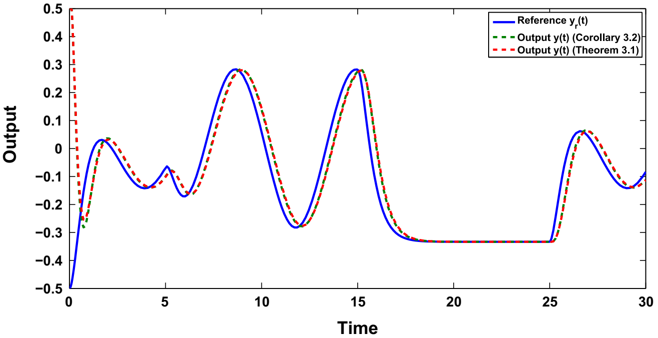

Fig. 1. Tracking performance of the EID based adaptive reliable controller (Theorem 3.1) and conventional reliable controller (Corollary 3.2) under normal mode.

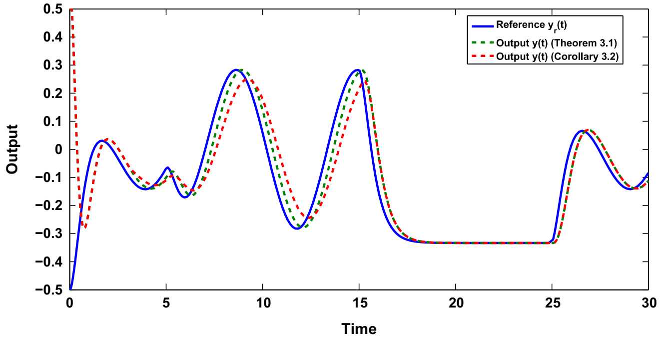

Fig. 2. Tracking performance of the EID based adaptive reliable controller (Theorem 3.1) and conventional reliable controller (Corollary 3.2) in Fault mode 1.

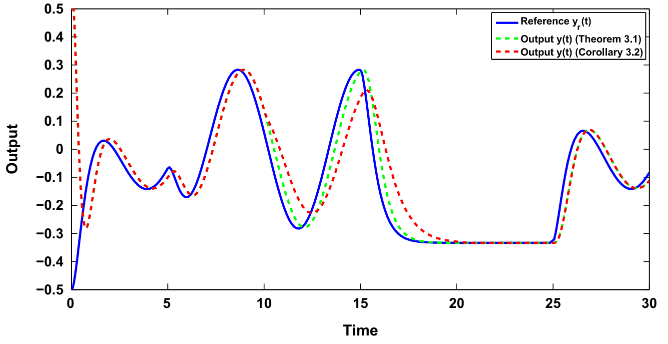

Fig. 3. Tracking performance of the EID based adaptive reliable controller (Theorem 3.1) and conventional reliable controller (Corollary 3.2) in Fault mode 2.

Furthermore, reference and disturbances input are taken as

$$r(t) = \begin{cases} 2\sin(t), & 5 \le t \le 15 \\ -2, & 15 < t \le 25 \\ \cos(t), & t > 25 \end{cases}$$

$$w(t) = \begin{cases} 5\sin(t) & 0 \le t \le 20 \\ 0 & \text{otherwise} \end{cases}$$

The main purpose is to obtain an EID based adaptive reliable controller by solving the LMIs in Theorem 3.1 and Corollary 3.2. Furthermore, to illustrate the applicability of the developed control technique, we consider the actuator faulty modes in both outage and normal cases which are given as follows:

Normal mode: Two actuators are normal, it means that $\rho_1 = \rho_2 = 0$.

Fault mode 1: The first actuator is outage and second actuator may be normal, that is,

$$\rho_1^1 = 1, \quad 0 \le \rho_2^1 \le 0.7$$

Fault mode 2: The second actuator is outage and first actuator may be normal, that is,

$$\rho_1^2 = 1, \quad 0 \le \rho_2^2 \le 0.85$$

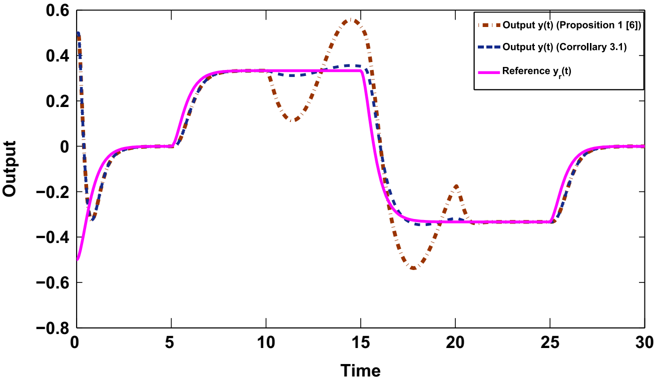

Fig. 4. Tracking performance of fault free system with EID based reliable controller. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

With the constant gain $u_F(t) = K_0(\hat{x}(t-\tau_2(t)) - x_r(t-\tau_2(t)))$ and the initial condition $x_0 = [0.5\ 0\ -1]^T$, $x_{r0} = [0\ 0\ 0]$, the tracking performance of the developed EID based adaptive reliable controller based on Theorem 3.1 and conventional reliable controller based on Corollary 3.2 when the actuator is normal is shown in Fig. 1. Further, based on Theorem 3.1 and Corollary 3.2, the tracking performances under EID based adaptive reliable and conventional reliable controllers for the aforementioned two faulty cases (i.e. fault mode 1 and fault mode 2) are shown in Figs. 2 and 3 respectively.

From the simulation result, it can be seen that when the actuator meets some failures, the conventional reliable controller is not able to perform well. In this situation, EID based adaptive reliable control proposed in Theorem 3.1 is compensate fault effect in on-line and improves the control performance. From Figs. 2 and 3, it is clear that the classical reliable controller could not estimate the fault in the actuator, but adaptive reliable controller estimates the fault and improves the tracking performance in better way. In summary, from Figs. 1–3, it is revealed that the traditional reliable controller could not be used to achieve a stable tracking performance for the system in a network environment. From the simulation result, it is concluded that a stable and satisfactory tracking effect can be achieved by the developed EID based adaptive reliable controller.

Example 4.2. In the second example, a moving mobile robot model whose dynamic equation is presented in [11] is considered in the context of NCSs to demonstrate the effectiveness and superiority of the proposed EID based adaptive reliable controller by performing output tracking over a network. The identification phase of the mobile robot model is given by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_1 w(t), \\ y(t) = Cx(t) \end{cases}$$

where $x(t) = [x_1(t)\ x_2(t)]$ with $x_1(t)$ and $x_2(t)$ are the displacement and velocity of the mobile robot, respectively; $u(t)$ and $w(t)$ are the control and disturbance input vectors, respectively. Now, we take the parameter values as provided in [11]

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -11.32 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 11.32 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [1\ \ 0].$$

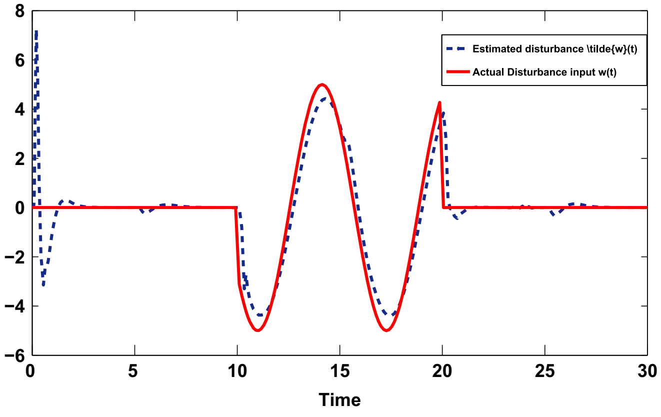

Fig. 5. Trajectories of the actual disturbance $w(t)$ and estimated disturbance $\tilde{w}(t)$ of fault free system with EID based approach.

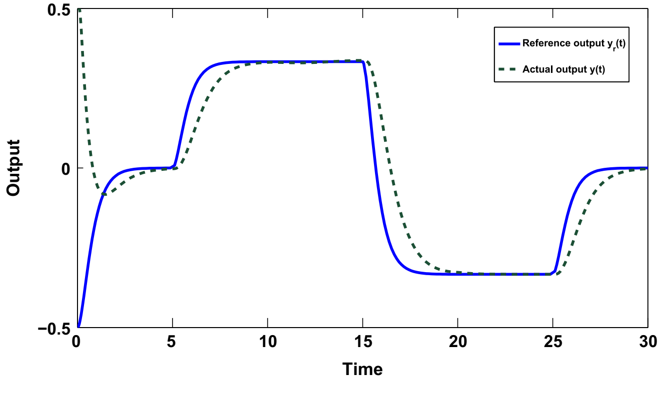

Fig. 6. Tracking performance of the closed-loop system in the presence of fault with EID based reliable control.

For the considered NCSs, the reference model and low pass filter are taken, respectively, as

$$\begin{cases} \dot{x}_r(t) = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \\ y_r(t) = [1 \ \ 0] x_r(t) \end{cases}$$

and

$$\begin{cases} \dot{x}_F(t) = -101 x_F(t) + 100 w_e(t) \\ y_F(t) = x_F(t) \end{cases}$$

In order to compare the performance of EID based reliable controller with the existing observer-based controller proposed in [11], we assume that actuator has no fault and choose the delay bounds as provided in [11], $\tau_{m1} = 20$ ms, $\tau_{M1} = 60$ ms, $\tau_{m2} = 40$ ms, $\tau_{M2} = 120$ ms, $\epsilon_1 = 0.02$, $\epsilon_2 = 0.01$, $\epsilon_3 = 0.2$, $\epsilon_4 = 0.8$. Also, for simulation purposes, we choose the disturbance input as $w(t) = 5\sin(t)$ and the initial condition as $x(0) = [0.5 \ 0]^T$, $\hat{x}(0) = [0 \ 0]^T$

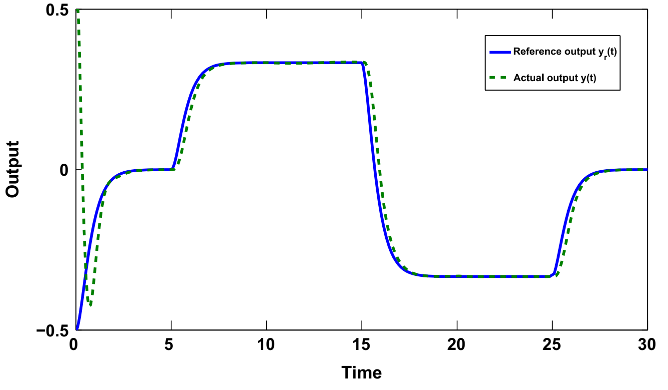

Fig. 7. Tracking performance of the closed-loop system in the presence of fault with EID based adaptive reliable control.

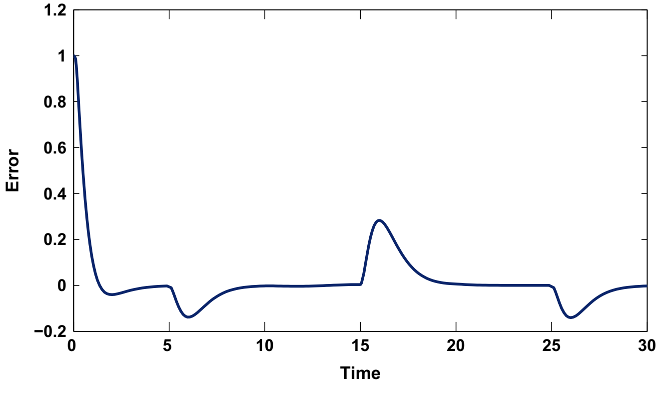

Fig. 8. Tracking error responses in the presence of fault under EID based reliable control.

and $x_r(0)=[-0.5\ 0]^T$. By solving the LMIs in Corollary 3.2 we obtain the feedback controller gain as $K_0=[-5.3241\ -0.8732]$ and observer gain as $L=[-5.89781\ 0.68721]^T$. Fig. 4 depicts the tracking performance of the proposed EID based reliable controller in Corollary 3.2 and the observer-based controller designed with the methodology proposed in [11]. In Fig. 4, the solid red lines represent the state response results with the proposed EID based reliable control law, while the dotted gray curves represent the state responses with the control law designed with the methodology presented in [11]. In particular, the tracking performance of the controlled output to reference output of the closed-loop system with EID based approach provided in Fig. 4. From Fig. 4, it is concluded that the EID based reliable controller effectively remove the external disturbances in an active manner without knowing any prior knowledge of the disturbance signal compare with the controller proposed in [11]. The result reveals that the proposed EID based reliable controller achieves satisfactory tracking performance in the network environment better than the controller proposed in [11] which ensures the effectiveness of the proposed EID based reliable controller. Moreover, Fig. 5 shows the trajectories of the actual disturbance $w(t)$ and estimated disturbance $\tilde{w}(t)$ based on the proposed EID based reliable controller. It is noted that from the above gain values that it is possible to obtain the controllers with less control effort with the proposed EID based reliable controller than the conventional controller in [11]. Thus, we can

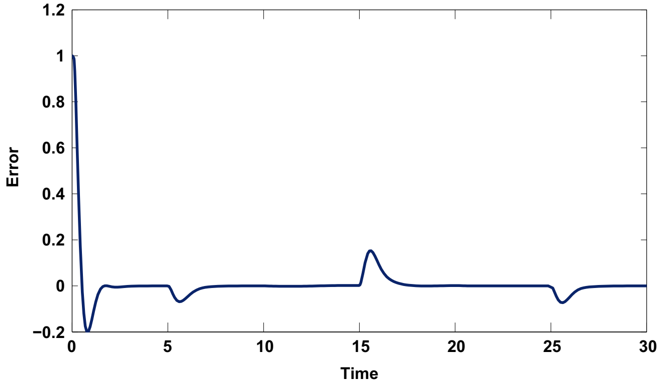

Fig. 9. Tracking error responses in the presence of fault under EID based adaptive reliable control.

conclude that EID based reliable control output tracking performance is effective one due to its less control gain and better rejection of the external disturbances.

Suppose there exist actuator faults in the control system, it will affect the system desired performances significantly. We assume that the actuator fault will be in the range of $0.2 \leq \rho \leq 0.9$. Further, by choosing the remaining parameters as mentioned above in the no actuator failure case and solving the LMIs in Eqs. (15), (17) of Theorem 3.1 and estimate the actuator fault ρ based on the adaptive fault estimator $\hat{\rho}$ in Eq. (18). Figs. 6 and 7 depict the EID based controller tracking performance of the NCSs in the presence of actuator fault without and with adaptive mechanism. From Figs. 6 and 7, it can be seen that adaptive controller yields better tracking performance against actuator fault. Figs. 8 and 9 show the tracking error responses in the presence of fault based on the EID based reliable controller without and with adaptive mechanism. It can be seen from Fig. 9 that despite the actuator failures, the states of the controlled plant still track well those of the tracked plant, which demonstrates the effectiveness of the proposed EID based adaptive reliable controller.

The result reveals that the satisfactory tracking performance is achieved by using the proposed EID based adaptive controller with no actuator fault case as well as the actuator fault occurred in the presence of external disturbances. In particular, all satisfactory performance such as stability and tracking performance can be achieved by using this simple control structure with better performance. From the simulation result, it is revealed that the proposed EID based adaptive reliable control is more superior and estimates both the external disturbance and the internal actuator faults on-line and rejects them effectively.

Remark 4.3. It should be pointed out that the proposed controller contains two safety layers in which EID helps to product the controller from the external disturbances and adaptive mechanism in the controller provides the automatic update against the actuator fault.

Remark 4.4. In [11], the authors designed the observer based H_∞ tracking controller in the presence of network-induced delays and packet dropouts in the controller-to-actuator channel. In this paper, EID based adaptive reliable control design is proposed in the same network environment. In order to show the advantage and efficiency of the proposed controller, a comparison is provided with the controller proposed in [11]. From the simulation results in Example 4.2, it is concluded that the proposed EID based reliable controller yields better tracking performance in the network environment with less control effort over the controller proposed in [11]. Thus, the proposed method is conservative.

5. Conclusion

In this paper, we have investigated an output tracking control problem for NCSs subject to actuator faults and external disturbances by using EID based adaptive reliable control technique. In particular, the proposed output feedback adaptive reliable controller has been designed to guarantee the asymptotic stability of resulting closed-loop NCSs. By choosing a proper Lyapunov functional, a new set of sufficient conditions have been derived in terms of linear matrix inequalities to obtain the required result. As a result, stability and robust tracking property can be achieved simultaneously by the developed EID based adaptive reliable controller. Simulation results have been provided to verify the benefits and effectiveness of the proposed control scheme. The result shows that the EID based adaptive reliable controller can reduce the effect of external disturbance significantly and consume less control effort to achieve the desired tracking performance.

Acknowledgment

The work of P. Selvaraj was supported by the NBHM/DAE under Grant no. 2/48(4)/2013/NBHM(R.P)/R&D ll/687.

References

[1] Y.-L. Wang, T.-B. Wang, Q.-L. Han, Fault detection filter design for data reconstruction-based continuous-time networked control systems, Inf. Sci. 328 (2016) 577–594.

[2] Q. Aibing, J. Bin, W. Chenglin, M. Zehui, Fault estimation and accommodation for networked control systems with nonuniform sampling periods, Int. J. Adapt. Control Signal Process. 29 (2015) 427–442.

[3] C. Hua, C. Yu, X. Guan, Neural network observer-based networked control for a class of nonlinear systems, Neurocomputing 133 (2015) 103–110.

[4] H. Zhang, Q. Hong, H. Yan, F. Yang, G. Guo, Event-based distributed H_∞ filtering networks of 2DOF quarter-car suspension systems, IEEE Trans. Ind. Inform. http://dx.doi.org/10.1109/TII.2016.2569566.

[5] L. Zou, Z. Wang, H. Gao, Observer-based H_∞ control of networked systems with stochastic communication protocol: the finite-horizon case, Automatica 63 (2016) 366–373.

[6] H. Yan, F. Qian, F. Yang, H. Shi, H_∞ filtering for nonlinear networked systems with randomly occurring distributed delays, missing measurements and sensor saturation, Inf. Sci. , http://dx.doi.org/10.1016/j.ins.2015.09.027.

[7] D. Zhang, Q.-L. Han, X. Jia, Network-based output tracking control for a class of T–S fuzzy systems that can not be stabilized by non-delayed output feedback controllers, IEEE Trans. Cybern. 45 (2015) 1511–1524.

[8] J.-Y. Zhai, B. Wang, S.-M. Fei, Tracking control for switched nonlinear systems with multiple time-varying delays, Nonlinear Anal.: Hybrid Syst. 17 (2015) 44–55.

[9] J. Li, J. Zhao, H_∞ output tracking control for discrete-time switched systems via output feedback, Int. J. Robust Nonlinear Control 25 (2015) 430–442.

[10] H. Zhang, H. Yan, T. Liu, Q. Chen, Fuzzy controller design for nonlinear impulsive fuzzy systems with time delay, IEEE Trans. Fuzzy Syst. 19 (2011) 844–856.

[11] D. Zhang, Q.L. Han, X.C. Jia, Observer-based H_∞ output tracking control for networked control systems, Int. J. Robust Nonlinear Control 24 (2014) 2741–2760.

[12] G.H. Yang, D. Ye, Reliable Control and Filtering of Linear Systems with Adaptive Mechanisms, CRC Press, Florida, US, 2011.

[13] Z. Zhang, W. Chen, Adaptive output feedback control of nonlinear systems with actuator failures, Inf. Sci. 179 (2009) 4249–4260.
[14] R.J. Wai, M.W. Chen, Y.K. Liu, Design of adaptive control and fuzzy neural network control for single-stage boost inverter, IEEE Trans. Ind. Electron. 62 (2015) 5434–5445.
[15] H. Wang, X. Liu, K. Liu, H.R. Karimi, Approximation-based adaptive fuzzy tracking control for a class of non strict-feedback stochastic nonlinear time-delay systems, IEEE Trans. Fuzzy Syst. 23 (2015) 1746–1760.
[16] H. Li, Y. Gao, P. Shi, H.K. Lam, Observer-based fault detection for nonlinear systems with sensor fault and limited communication capacity, IEEE Trans. Autom. Control , http://dx.doi.org/10.1109/TAC.2015.2503566.
[17] H. Li, H. Gao, P. Shi, X. Zhao, Fault-tolerant control of Markovian jump stochastic systems via the augmented sliding mode observer approach, Automatica 50 (2014) 1825–1834.
[18] H. Yan, F. Qian, H. Zhang, F. Yang, G. Ge., H_∞ fault detection for networked mechanical spring–mass systems with incomplete information, IEEE Trans. Ind. Electron. , http://dx.doi.org/10.1109/TIE.2016.2559454.
[19] H. Li, P. Shi, D. Yao, L. Wu, Observer-based adaptive sliding mode control of nonlinear Markovian jump systems, Automatica 64 (2016) 133–142.
[20] M. Lin, S. Zhang, H. Chen, W. Sheng, H_∞ output tracking control of discrete-time nonlinear systems via standard neural network models, IEEE Trans. Neural Netw. Learn. Syst. 25 (2014) 1928–1935.
[21] Q. Zhou, H. Li, P. Shi, Decentralized adaptive fuzzy tracking control for robot finger dynamics, IEEE Trans. Fuzzy Syst. 23 (2015) 501–510.
[22] H. Zhang, G. Feng, H. Yan, Q. Chen, Observer-based output feedback event-triggered control for consensus of multi-agent systems, IEEE Trans. Ind. Electron. 61 (2014) 4885–4894.
[23] H.K. Lam, H. Li, Output-feedback tracking control for polynomial fuzzy-model-based control systems, IEEE Trans. Ind. Electron. 60 (2013) 5830–5840.
[24] J. She, M. Fang, Y. Ohyama, H. Hashimoto, M. Wu, Improving disturbance-rejection performance based on an equivalent-input-disturbance approach, IEEE Trans. Ind. Electron. 55 (2008) 380–389.
[25] R.J. Liu, G.P. Liu, M. Wu, Z.Y. Nie, Disturbance rejection for time-delay systems based on the equivalent-input-disturbance approach, J. Frankl. Inst. 351 (2014) 3364–3377.
[26] R.J. Liu, G.P. Liu, M. Wu, F.C. Xiao, J. She, Robust disturbance rejection based on equivalent-input-disturbance approach, IET Control Theory Appl. 7 (2013) 1261–1268.
[27] A. Zhang, X. Lai, M. Wu, J. She, Global stabilization of under actuated spring-coupled three-link horizontal manipulator using position measurements only, Appl. Math. Modell. 39 (2015) 1917–1928.
[28] R.J. Liu, G.P. Liu, M. Wu, J. She, Z.Y. Nie, Robust disturbance rejection in modified repetitive control system, Syst. Control Lett. 70 (2014) 100–108.
[29] C. Qin, Z. Xiang, H.R. Karimi, Robust H_∞ reliable control for delta operator switched systems with time-varying delays under asynchronous switching, Trans. Inst. Meas. Control 37 (2015) 219–229.
[30] Y. Liu, Y. Niu, Y. Zou, H.R. Karimi, Adaptive sliding mode reliable control for switched systems with actuator degradation, IET Control Theory Appl. 9 (2015) 1197–1204.
[31] R. Sakthivel, S. Santra, K. Mathiyalagan, H. Su, Robust reliable control design for networked control system with sampling communication, Int. J. Control 88 (2015) 2510–2522.
[32] S. Selvi, R. Sakthivel, K. Mathiyalagan, A. Arunkumar, Reliable gain-scheduled control design for networked control systems, Complexity 21 (2015) 214–228.
[33] J. Hu, J. Liang, D. Chen, Reliable guaranteed-cost control for networked systems with randomly occurring actuator failures and fading performance output, Int. J. Gen. Syst. 44 (2015) 129–141.
[34] Y.-Y. Hou, Reliable stabilization of networked control systems with actuator faults, Nonlinear Dyn. 71 (2013) 447–455.
[35] W. Yang, S. Tong, Adaptive output feedback fault-tolerant control of switched fuzzy systems, Inf. Sci. 329 (2016) 478–490.
[36] F.-L. Qu, Z.-H. Guan, D.-X. He, M. Chi, Event-triggered control for networked control systems with quantization and packet losses, J. Frankl. Inst. 352 (2015) 974–986.
[37] M.S. Mahmoud, G.D. Khan, Robust mixed image control of networked control systems with random delays and partially known transition matrix, J. Frankl. Inst. 351 (2014) 5548–5564.
[38] H. Zhang, H. Yan, F. Yang, Q. Chen, Quantized control design for impulsive fuzzy networked systems, IEEE Trans. Fuzzy Syst. 19 (2011) 1153–1162.
[39] H. Li, C. Wu, S. Yin, H.K. Lam, Observer-based fuzzy control for nonlinear networked systems under unmeasurable premise variables, IEEE Trans. Fuzzy Syst. (2015), http://dx.doi.org/10.1109/TFUZZ.2015.2505331.
[40] J. Dong, G.-H. Yang, Reliable state feedback control of T–S fuzzy systems with sensor faults, IEEE Trans. Fuzzy Syst. 23 (2015) 421–433.

[41] Y. Liu, M. Li, An improved delay-dependent stability criterion of networked control systems, J. Frankl. Inst. 351 (2014) 1540–1552.

[42] J. Bai, R. Lu, H. Su, A. Xue, Modeling and $H\infty$ control of wireless networked control system with both delay and packetloss, J. Frankl. Inst. 352 (2015) 3915–3928.

[43] H. Yan, S. Yan, H. Zhang, H. Shi, L_2 control design of event-triggered networked control systems with quantizations, J. Frankl. Inst. 352 (2015) 332–345.

[44] Z. Xiang, S. Liu, M.S. Mahmoud, Robust H_∞ reliable control for uncertain switched neutral systems with distributed delays, IMA J. Math. Control Inf. 32 (2015) 1–19.

[45] Z.R. Xiang, S.L. Liu, Q.W. Chen, Tracking control for a class of switched non-linear systems with time-varying delay, Trans. Inst. Meas. Control 35 (2013) 398–406.

[46] S.L. Liu, Z.R. Xiang, Exponential L_1 output tracking control for positive switched linear systems with time-varying delays, Nonlinear Anal.: Hybrid Syst. 11 (2014) 118–128.

[47] S. Longo, G. Herrmann, P. Barber, Robust scheduling of sampled-data networked control systems, IEEE Trans. Control Syst. Technol. 20 (2012) 1613–1621.

[48] H. Wang, B. Zhou, C.C. Lim, R. Lu, A. Xue, H_∞ fault-tolerant control of networked control systems with actuator failures, IET Control Theory Appl. 8 (2014) 1127–1136.

[49] H. Yan, Z. Su, H. Zhang, F. Yang, Observer-based H_∞ control for discrete-time stochastic systems with quantisation and random communication delays, IET Control Theory Appl. 7 (2013) 372–379.

[50] K. Mathiyalagan, J.H. Park, R. Sakthivel, Robust reliable dissipative filtering for networked control systems with sensor failure, IET Signal Process. 8 (2014) 809–822.

[51] G. Yang, D. Ye, Reliable H_∞ control of linear systems with adaptive mechanism, IEEE Trans. Autom. Control 55 (2010) 242–247.

[52] R. Sakthivel, P. Selvaraj, K. Mathiyalagan, Ju H. Park, Robust fault-tolerant $H\infty$ control for offshore steel jacket platforms via sampled-data approach, J. Frankl. Inst. 352 (2015) 2259–2279.