

# 1 **A quantitative approach for risk assessment of a ship stuck in ice in Arctic waters**

2 Shanshan Fu <sup>1,2,3</sup>, Di Zhang <sup>2,3\*</sup>, Jakub Montewka <sup>4,5</sup>, Enrico Zio <sup>6,7</sup>, Xinping Yan <sup>2,3</sup>

3 1 College of Ocean Science and Engineering, Shanghai Maritime University, Shanghai, China

4 2 Intelligent Transport Systems Research Center, Wuhan University of Technology, Wuhan, China

5 3 National Engineering Research Center for Water Transport Safety, Wuhan, China

6 4 Gdynia Maritime University, Department of Transport and Logistics, Poland

7 5 Finnish Geospatial Research Institute, Masala, Finland

8 6 Chair on Systems Science and the Energetic Challenge (Foundation, EDF), Centrale-Supélec, Université Paris-Saclay, Paris, France

9 7 Department of Energy, Politecnico di Milano, Italy

10 Arctic waters have historically been regarded as harsh environments owing to their extreme weather conditions  
11 and remoteness from land. The advantages of shorter sea routes and hydrocarbon energy exploitation have recently  
12 led to increased marine activities in such harsh environments. To ensure safe operation within the area, the potential  
13 risks of ship accidents, need to be systematically analyzed, assessed and managed along with the associated  
14 uncertainties. The treatment of epistemic uncertainty in the likelihoods of adverse events due to lack of knowledge  
15 and information should also be considered. This paper presents a Frank copula-based fuzzy event tree analysis  
16 approach to assess the risks of major ship accidents in Arctic waters, taking uncertainty into consideration. The  
17 quantitative approach includes four steps, namely, accident scenario modeling by an event tree model, probability  
18 and dependence analysis of the associated intermediate events, risk assessment with respect to the consequent  
19 outcome events. A major ship accident in Arctic waters - ships stuck in ice, is chosen as a case to interpret the  
20 modeling process of the approach proposed. Crews and ships owners can use such approach to defining risk control  
21 options that enable optimal risk mitigation. Maritime management may also benefit from better risk assessment.

22 Key works: Arctic waters, ship accidents, accident scenario analysis, ship stuck in ice, fuzzy-event tree analysis,  
23 Frank copula

## 24 **List of abbreviations**

25	ET	Event Tree
26	IE	Intermediate Event
27	MSC	Maritime Safety Committee
28	NSR	Northern Sea Route
29	OE	Outcome Event
30	IMO	International Maritime Organization
31	PMCC	Product-Moment Correlation Coefficient
32	SEQ	Sequence number
33	TFN	Triangular fuzzy number

## 34 **1. Introduction**

35 Recently, Arctic waters have become more accessible for marine activities due to the increased melting of the  
36 Arctic sea ice (Ho, 2010; Verny and Grigentin, 2009; Parsons et al., 2011; ABS, 2014). On the other hand, the  
37 northern sea route (NSR) through the Arctic sea is attractive because it offers a shorter transit than the traditional

38 routes through Suez Canal or Panama Canal (Liu and Kronbak, 2010; Raza and Schøyen, 2014; Schøyen and Bråthen,  
39 2011). Moreover, the polar areas are attractive for exploitation of the hydrocarbon resources. These advantages  
40 explain why marine activities in Arctic waters were gradually increasing in recent years (NSR, 2016). Nevertheless,  
41 these waters still share only a small amount of international shipping transits and lack of appropriate response  
42 capacity in case of emergency. The reason is that Arctic waters have historically been regarded as harsh environments,  
43 including extended sea ice, severe operating conditions, unpredictable weather changes, poorly charted waters,  
44 remoteness of the polar areas for marine activities, and an overall high degree of uncertainty regarding navigational  
45 environment conditions (Meng et al., 2016). The increasing ship traffic and exploitation in this area, the safety of  
46 marine activities and operations in such harsh environments, thus, becomes of great interest (MSC, 2014). Hence,  
47 there is a need for risk analysis of major ship accidents in Arctic waters.

48 The analysis of the risk associated with ship operations in ice-covered waters has obtained much attention from  
49 academic and industry (Afenyo et al., 2016; Afenyo et al., 2016; Canada, 1998; MSC, 2014; Arctic Council, 2009;  
50 Fu et al., 2015; Fu et al., 2016; Fu et al., 2016; Goerlandt et al., 2016; Khan et al., 2014; Kotovirta et al., 2009; Kum  
51 and Sahin, 2015; Liu et al., 2016; Marken et al., 2015; Montewka et al., 2015; Sørstrand, 2012; Valdez Banda et al.,  
52 2016; Valdez Banda et al., 2015). The 2009 Arctic marine shipping assessment report (Arctic Council, 2009) focused  
53 on the future scenarios development and environmental considerations of Arctic shipping. The international code for  
54 ships operating in polar waters (Polar Code) was adopted by the International Maritime Organization (IMO) during  
55 its 94th Maritime Safety Committee meeting (MSC, 2014). The polar code highlighted a comprehensive list of  
56 hazards for marine operations in Arctic waters, but it scantily elaborated on the risk influencing factors (RIFs)  
57 involved in some individual operations, or on the appropriate modeling techniques to be used for formal safety  
58 assessment(MSC, 2013). Besides, a few event-oriented models were proposed for the risk analysis of major  
59 operations in ice-covered waters. Khan et al. (Khan et al., 2014) proposed a transportation risk analysis framework  
60 for collision accidents in Arctic waters by using a Bayesian network model. Kum and Sahin (Kum and Sahin, 2015)  
61 used a fuzzy fault tree method considering some causal risk factors in human and management aspects, concerning  
62 collision and grounding accidents in Arctic waters. Marken et al. (Marken et al., 2015) conducted a delay risk analysis  
63 of ship sailing in the NSR by using a traditional Bow-tie diagram, integrated by fault tree analysis and event tree  
64 (ET) analysis. Valdez Banda et al. (Valdez Banda et al., 2016; Valdez Banda et al., 2015) presented a risk  
65 management model for the Finnish-Swedish Winter Navigation System, by incorporating formal safety assessment  
66 and a Bayesian network model. Goerlandt et al. (Goerlandt et al., 2016) carried out an analysis of winter operations  
67 in the Northern Baltic Sea involving icebreakers and assisted ships, pointing to various relationships between the ice  
68 feature and operational characteristics. Afenyo et al. (Afenyo et al., 2016; Afenyo et al., 2016) presented a model of  
69 oil spill accidents in ice-covered waters. Montewka et al. (Montewka et al., 2015) and Fu et al. (Fu et al., 2016)  
70 presented Bayesian networks models for analyzing ship performance in dynamic ice and predicting the probability  
71 of ships getting stuck in ice in the Northern Baltic Sea and NSR, respectively. These publications focus on major  
72 accidents of ship operations in ice-covered waters, such as collision (Goerlandt et al., 2016; Khan et al., 2014; Valdez  
73 Banda et al., 2016; Valdez Banda et al., 2015), grounding (Kum and Sahin, 2015; Valdez Banda et al., 2016; Valdez  
74 Banda et al., 2015), ship delay (Marken et al., 2015), oil spill (Afenyo et al., 2016; Afenyo et al., 2016; Marken et  
75 al., 2015) and ship besetting/stuck in ice (Fu et al., 2016; Fu et al., 2016; Montewka et al., 2015). However, this is

76 still a limited amount of publications, compared with the studies of risk analysis of ship operations in open-water (Fu  
77 et al., 2016; Goerlandt and Montewka, 2015; Graziano et al., 2016; Hanninen et al., 2014; Li et al., 2012; Mazaheri  
78 et al., 2016; Mazaheri et al., 2015; Zhang et al., 2016; Zhang et al., 2013). Furthermore, very little research to date  
79 has focused on the risks of potential accident scenarios and undesirable consequences of ship operations in ice-  
80 covered waters (Kotovirta et al., 2009; Kubat et al., 2015).

81 The ET analysis is a distinct and graphically supported method used to develop a logical relationship between  
82 the events leading to an accident and estimated the level of risk associated (Ferdous et al., 2011; Huang, 2001; Zio,  
83 2007). In an ET model, the event that generates the accident is named an initiating event, and the follow-up ones are  
84 termed intermediate events (IEs) or safety barriers (AIChE, 2000; Ferdous et al., 2011). The ET analysis represents  
85 the progression of the dichotomous conditions (e.g. success/failure or yes/no) of the initiating event onto the  
86 subsequent IEs all the way to the outcome events (OEs) of the accident sequence (AIChE, 2000; Andrews, 2000). In  
87 general, the ET analysis is used under two basic assumptions. First, the probability of occurrence of the events is  
88 assumed to be precisely known; in practice, this is often difficult to obtain due to imperfect or incomplete information  
89 (Chang et al., 2015; SRA, 2015) that leads to epistemic uncertainty in the ET probability values. The treatment of  
90 this kind of epistemic uncertainty associated with the probability of occurrence of events in an ET model – parameter  
91 uncertainty, can be of great importance, particularly in situations where little data and information are available, like  
92 for ship accidents in Arctic waters. Furthermore, the dependence of collected IEs in the ET model is also uncertain  
93 (Ferson S., 2004; Janbu, 2009). The impacts of the two different types of epistemic uncertainties, namely, parameter  
94 uncertainty and dependence uncertainty, must, thoroughly, be considered in the risk assessment process (Ferdous et  
95 al., 2011).

96 The objective of this paper is to develop an original Frank-copula based fuzzy-ET approach for quantitative risk  
97 assessment of ship accidents in Arctic waters, by investigating the probabilities of potential accident scenarios of a  
98 certain ship accident. The primary feature of the quantitative approach proposed is that it enables us to describe,  
99 measure and propagate the effects of parameter and dependence uncertainties in the ET model. Fuzzy sets are used  
100 to describe the former uncertainty in the situation of scarce and limited datasets for IEs. For the latter uncertainty,  
101 The Frank-copula is used to describe the interdependence between dependent events and make a precise calculation  
102 for the probability of OEs in the ET model. A major ship accident in Arctic waters – ship stuck in ice, is chosen as a  
103 case to interpret the approach. For this, this study provides an insight into the combined effects of the probability of  
104 occurrence and potential consequences of the ship becoming stuck in ice, and it properly distinguishes between  
105 different accident scenarios. The approach can assist in determining risk control options that enable optimal risk  
106 mitigation.

107 The remainder of the paper is structured as follows. Section 2 proposes an ET model for the risk analysis of a  
108 ship stuck in ice in Arctic waters. Section 3 describes the methods for epistemic and dependence uncertainties  
109 modeling and propagation. The modeling process and the obtained results are described in section 4, and discussed  
110 in section 5. Section 6 concludes the research findings.

## 111 **2. Methods**

112 Quantitative risk assessment of ship accidents in Arctic waters is a challenging problem, due to the limited data

113 and information available. A quantitative method is proposed for analyzing accident risks in Arctic waters. The  
114 quantitative method can be used for estimating the risk of potential accident scenarios, with consideration of  
115 parameter and dependence uncertainties. The following sections describe the methodological framework adopted,  
116 along with the techniques of epistemic and dependence uncertainties modeling, and propagation.

### 117 2.1. Framework for quantitative risk assessment

118 The framework of the quantitative approach can be decomposed into four steps, as follows:

119 Step 1: **Accident scenarios modeling**. Analyze accident scenarios of a typical ship accident in Arctic waters by  
120 developing an ET model, including an initiating event, IEs and OEs, logically connected in the resulting accident  
121 sequences (Ferdous et al., 2011; Marken et al., 2015).

122 Step 2: **Probability analysis of the IEs**. Collect information and knowledge about the probability of occurrence  
123 of the IEs in the ET model proposed, from historical records, related literature and expert knowledge. Since  
124 information related to the initiating event and the IEs are uncertain for the ice-covered polar waters, an epistemic  
125 uncertainty modeling method- fuzzy sets (Zadeh, 1965) is used for collecting knowledge from the domain experts.

126 Step 3: **Dependence analysis of the IEs**. A possibilistic approach is incorporated into the fuzzy-ET analysis for  
127 generating sample data, to analyze the dependence relationships between the connected IEs in the ET model.  
128 Correlation analysis is used for calculating the Pearson product-moment correlation coefficient (PMCC) of the  
129 connected IEs.

130 Step 4: **Risk assessment**. According to the dependence based-fuzzy-ET analysis method, further, integrate Frank  
131 copula to conduct conjunction operations of the connected IEs in the ET model so as to calculate the probability of  
132 each accident sequence in the ET model. The probability of the OEs and the risk of the ship accident can also be  
133 calculated.

### 134 2.2. Fuzzy event tree analysis

135 Fuzzy sets are introduced by Zadeh (Zadeh, 1965) and have gained popularity in various fields, including  
136 reliability analysis and risk assessment (Sahin and Senol, 2015; Zio and Aven, 2013). The fuzzy set theory uses fuzzy  
137 numbers to capture the imprecision or vagueness in expert linguistic assessment. The membership function of a fuzzy  
138 number establishes a numerical relationship for uncertain values, ranging from 0 to 1. Triangular fuzzy numbers  
139 (TFNs) are flexible for uncertainty representation and propagation in the ET model (Baraldi and Zio, 2008; Ferdous  
140 et al., 2009; Ferdous et al., 2012).

141 Here, the TFN is a vector whose three elements range from 0 to 1 and are the lower bound, most likely, and  
142 upper bound values of the (uncertain) possibility/likelihood of occurrence for an event. In this paper, seven linguistic  
143 scales are used to describe expert knowledge relating to possibility values in the ET analysis. The linguistic terms  
144 and associated membership functions are reported in Table 1, taken from a study of transportation risk analysis in  
145 the Northern Sea Route (Marken et al., 2015).

146 **Table 1**

147 TFN of linguistic terms.

Linguistic terms	Membership function (TFNs)
Very low (VL)	(0, 0.025, 0.05)

Medium low (ML)	(0.045, 0.125, 0.2)
Low (L)	(0.15, 0.275, 0.4)
Medium (M)	(0.35, 0.5, 0.65)
High (H)	(0.6, 0.725, 0.85)
Medium high (MH)	(0.8, 0.875, 0.955)
Very high (VH)	(0.95, 0.975, 1)

148 The fuzzy set theory uses the fuzzy arithmetic operations based on the  $\alpha$ -cut formulation to manipulate fuzzy  
149 numbers (Slier & Buckley, 2005). Suppose  $\tilde{P}_A(p_{l_A}, p_{m_A}, p_{u_A})$  is a TFN representing the possibility of the occurrence  
150 of event A; its rules of multiplication and complement operations are calculated as follows:

$$151 \quad \lambda \times \tilde{P}_A = \lambda \times (p_{l_A}, p_{m_A}, p_{u_A}) = (\lambda \times p_{l_A}, \lambda p_{m_A}, \lambda p_{u_A}), \lambda > 0, \lambda \in R, \quad (1)$$

$$152 \quad \overline{\tilde{P}_A} = 1 - (p_{l_A}, p_{m_A}, p_{u_A}) = (1 - p_{l_A}, 1 - p_{m_A}, 1 - p_{u_A}), \quad (2)$$

153 where Eq. (1) is a multiplication operation between  $\tilde{P}_A$  and a crisp value, and  $\overline{\tilde{P}_A}$  is the possibility of the occurrence  
154 of the complement event of event A.

155 The  $\alpha$ -cut  $\tilde{P}_A^\alpha$  ( $\alpha \in [0,1]$ ) of the TFN represents a fuzzy interval with  $(1 - \alpha)\%$  degree of belief, which can be  
156 calculated as:

$$157 \quad \tilde{P}_A^\alpha = [p_{l_A} + \alpha \times (p_{m_A} - p_{l_A}), p_{u_A} - \alpha \times (p_{u_A} - p_{m_A})], \quad (3)$$

158 The possibility mean of a TFN is used to calculate the defuzzification value (Kurano, 2006), which can be  
159 computed as:

$$160 \quad P_{Adefuzzification} = [p_{l_A} + \alpha \times (p_{m_A} - p_{l_A})] + 2 * p_{m_A} + [p_{u_A} - \alpha \times (p_{u_A} - p_{m_A})]/4. \quad (4)$$

161 Suppose  $\tilde{P}_B(p_{l_B}, p_{m_B}, p_{u_B})$  is another TFN representing the possibility of the occurrence of event B. In case of  
162 event A and event B are independent, the possibility of their conjunction operation can be computed as the product  
163 of the possibilities of the two events, as follows:

$$164 \quad \tilde{P}(A \& B) = \tilde{P}_A \otimes \tilde{P}_B = (p_{l_A} \times p_{l_B}, p_{m_A} \times p_{m_B}, p_{u_A} \times p_{u_B}). \quad (5)$$

165 The probability of the OEs resulting from the accident scenarios in the ET model can be calculated by  
166 multiplying the conditional probabilities of the associated IEs along the corresponding sequence with branches on  
167 the ET, from the initiating event A to the OEs. Traditional fuzzy arithmetic operations assume that the connected  
168 events (or IEs) are independent (Ferdous et al., 2011), the formulations for the fuzzy-ET analysis can, then, be  
169 developed as follows:

$$170 \quad \tilde{P}_{SEQi,j}^\alpha = P_A \times \prod_{k=1}^n \tilde{P}_{IE_k}^\alpha, k = 1, 2, \dots, n, \quad (6)$$

171 where,  $k$  is the IEs index;  $n$  refers to the number of IEs in the ET model;  $P_A$  refers to the probability of occurrence of  
172 the initiating event A;  $\tilde{P}_{IE_k}^\alpha$  refers to the possibility of the  $k$ th IE in the  $\alpha$ -cut;  $\tilde{P}_{SEQi,j}^\alpha$  refers to the possibility of the

173 sequence number (SEQ)  $j$  for the  $i$ th OE in the  $\alpha$ -cut. Eq. (5) can also be seen as a multiplication operation between  
 174 the probability of an initiating event  $A$  and the possibility of the corresponding  $\tilde{P}_{IE_k}^\alpha$  in the SEQ  $j$  of the  $i$ th OE in the  
 175  $\alpha$ -cut for calculating  $\tilde{P}_{SEQ_{i,j}}^\alpha$ .

176 The total possibility of OEs in the event model can be calculated as:

$$177 \quad \tilde{P}_{OEs}^\alpha = \sum_{i=1}^N \tilde{P}_{OE_i}^\alpha = \tilde{P}_{SEQ_{i,1}}^\alpha \oplus \dots \oplus \tilde{P}_{SEQ_{i,j}}^\alpha, i = 1, 2, \dots, N, j = 1, 2, \dots, M, \quad (7)$$

178 where, according to the definition of the ET model in section 2 (as shown in Fig. 1),  $i$  is the index of the OEs;  $N$   
 179 refers to the number of possible OE scenarios;  $j$  is the SEQ index;  $M$  refers to the SEQ number in the ET model;  $\tilde{P}_{OE_i}^\alpha$   
 180 refers to the possibility of the  $i$ th OE in the  $\alpha$ -cut;  $\tilde{P}_{SEQ_{i,j}}^\alpha$  refers to the total possibility of OEs in the ET model in the  
 181  $\alpha$ -cut. Eq. (6) can also be seen as an addition operation between the corresponding possibility of the  $i$ th OE in the  $\alpha$ -  
 182 cut ( $\tilde{P}_{OE_i}^\alpha$ ) for calculating the total possibility  $\tilde{P}_{SEQ_{i,j}}^\alpha$ .

### 183 2.3. Dependence analysis

184 As mentioned in the previous section, the probability of occurrence of the events in the ET model is derived  
 185 from multi-expert judgment, with the majority of them being possibilistic regarding linguistic terms, as illustrated in  
 186 Table 1. A possibilistic approach integrating the Monte Carlo technique and fuzzy set theory (Baraldi and Zio, 2008;  
 187 Terje Aven, 2014), is then used to propagate the uncertainty of information using the following three main steps:

- 188 • Select possibility value  $\alpha$  and the corresponding cuts of the possibility distributions  $\tilde{P}_{IE_k}^\alpha$  as the interval of  
 189 possible values of the possibilistic variables  $IE_k$ .
- 190 • Compute the smallest and largest values of  $IE_k$ , denoted by  $p_{lIE_k}^\alpha$  and  $p_{uIE_k}^\alpha$  respectively, considering all  
 191 values of the possibilistic variables  $IE_k$  in the  $\alpha$ -cuts of their possibility distributions.
- 192 • Return to the first step and repeat for another  $\alpha$ -cut; after having repeated the above two steps for all the  $\alpha$ -  
 193 cuts of interest, the fuzzy random realization can then be obtained as the sample data from which to  
 194 calculate the correlation coefficient.

195 The PMCC is, then, used to estimate the correlation between two events, as follows (Freedman, 2010):

$$196 \quad r_{PMCC} = \frac{\sum_{i=1}^n (a_i - \bar{a})(b_i - \bar{b})}{\sqrt{\sum_{i=1}^n (a_i - \bar{a})^2} \sqrt{\sum_{i=1}^n (b_i - \bar{b})^2}}, \quad (8)$$

197 where  $a = P(A)$  and  $b = P(B)$ ,  $a_i$  and  $b_i$  are two variables with random possibility distribution calculated by a  
 198 possibilistic approach, and  $\bar{a}$  and  $\bar{b}$  are the means of the  $a_i$  and  $b_i$ , respectively.

199 The  $r_{PMCC}$  can describe the full range of dependencies ranging from -1 to 1, where 1 represents perfect  
 200 dependence, (0,1) represents positive dependence, 0 represents independence, (-1,0) represents negative  
 201 dependence and -1 represents opposite dependence.

202 According to the scale for categorizing the dependence between events proposed in Ferdous et al. (Ferdous et  
 203 al., 2011), the dependence relationship between events can be further decomposed into six types, as follows:

- 204 ● Perfect dependence. The value of the correlation coefficient between events is 1.000.
- 205 ● Very strong dependence. The value of the correlation coefficient between events is between 0.800 and
- 206 0.995.
- 207 ● Strong dependence. The value of the correlation coefficient between events is between 0.450 and 0.850.
- 208 ● Weak dependence. The value of the correlation coefficient between events is between 0.150 and 0.500.
- 209 ● Very weak dependence. The value of the correlation coefficient between events is between 0.005 and 0.200.
- 210 ● Perfect independence. The value of the correlation coefficient between events is 0.000.

211 A significance test can be conducted to justify the degree of belief in the dependence between the data, which is  
 212 calculated as:

$$213 \quad t^* = r \sqrt{\frac{n-2}{1-r^2}}. \quad (9)$$

214 For Eq. (8), if  $|t^*| > t_{\alpha/2}$ , this indicates that the correlation of the data  $a_i$  and  $b_i$  is significant at level  $\alpha$ ; for example,  
 215 if  $\alpha$  is set at 0.01 or 0.05, this would correspond to a degree of belief in the results of 99% or 95%, respectively.

#### 216 2.4. Frank copula-based conjunction operations

217 The fuzzy-ET analysis method above can be used to calculate the probability of each scenario and propagate  
 218 uncertainty in the ET model. However, the independent assumption underlying IEs simplifies the actual dependence  
 219 relationships between the connected IEs and, thus, adds dependence uncertainty. For this reason, an extended Frank  
 220 copula-based conjunction operation mathematic is proposed to express this dependence, as discussed in this section.

221 Frank copula (Frank, 1979) is a formulation for expressing the correlation of events, which is defined by

$$222 \quad C_{Frank}(a, b) = \log_s[1 + (s^a - 1)(s^b - 1)/(s - 1)], \quad (10)$$

$$223 \quad s = \tan(\pi(1 - r)/4), s \geq 0, \quad (11)$$

224 where a and b refer to the probability of event A and event B, respectively; and  $a = P(A)$  and  $b = P(B)$ , r is the  
 225 correlation coefficient between P(A) and P(B). Perfect dependence arises in the limit as s tends to 0; opposite  
 226 dependence arises when s goes to infinity, and independent corresponds to s equal to 1 (Ferson S., 2004).

227 In the Frank model of correlation between events, the probabilities of a conjunction of events A and B are given  
 228 by the formula:

$$229 \quad P_{Frank}(A\&B) = Frank(a, b, r) = \begin{cases} \min(a, b), & \text{if } r = 1 \\ ab, & \text{if } r = 0 \\ \max(1 + b - 1, 0), & \text{if } r = -1 \\ \log_s[1 + (s^a - 1)(s^b - 1)/(s - 1)], & \text{otherwise} \end{cases}. \quad (12)$$

230 This function is continuous; a special case arises when r is +1, 0 or -1.

231 Since the ET model uses fuzzy numbers for the uncertain event probabilities, the Frank copula must be extended  
 232 for using in the fuzzy-ET analysis. To do this, the monotonicity and bounds of Eq. (11) in each condition need to be

233 analyzed. According to Eq. (10), if  $r$  is defined on the interval  $(-1,0) \cup (0,1)$ , the value of  $s$  belongs to the interval  
 234  $(0,1) \cup (1, +\infty)$ .

235 Suppose  $f(a,b) = (s^a - 1)(s^b - 1)/(s - 1)$  in the condition  $r \in (-1,0) \cup (0,1)$ , the probability of the  
 236 conjunction of event A and event B can be worked out using the formula:

$$237 \quad F(s) = \log_s[1 + f(a,b)], a, b \in (0,1), s \in (0,1) \cup (1, +\infty). \quad (13)$$

238 According to the monotonicity of the logarithmic function, if the base of the logarithmic function belongs to interval  
 239  $(0,1)$  or  $(1, +\infty)$ , then the logarithmic function will be a decreasing or increasing function, respectively. Hence,  $F(s)$   
 240 is a decreasing function when  $s \in (0,1)$ , while  $F(s)$  is an increasing function when  $s \in (1, +\infty)$ .

241 On the other hand, the partial derivative of  $f(a,b)$  can be calculated as:

$$242 \quad df(a,b)/da = [(s^b - 1) * s^a * \ln s]/s - 1, a, b \in (0,1), s \in (0,1) \cup (1, +\infty). \quad (14)$$

243 In Eq. (14), if  $s$  is in interval  $(0,1)$ , then  $(s^b - 1) < 0$ ,  $s^a > 0$ ,  $\ln s < 0$ ,  $s - 1 < 0$  and the value of  $df/da$  will be  
 244 negative, namely  $df/da < 0$ ; if  $s$  is in interval  $(1, +\infty)$ , then  $(s^b - 1) > 0$ ,  $s^a > 0$ ,  $\ln s > 0$ ,  $s - 1 > 0$  and the  
 245 value of  $df/da$  will be positive, namely  $df/da > 0$ . Similarly, since  $a$  and  $b$  are symmetrical in the  $f(a,b)$ ,  $df/db$   
 246 is negative when  $s \in (0,1)$ , while  $df/db$  is positive when  $s \in (1, +\infty)$ .

247 It is clear that  $\text{Frank}(a,b,r)$  is a monotonic increasing function since both  $F(s)$  and  $f(a,b)$  are increasing  
 248 functions when  $s \in (0,1)$ , and decreasing functions when  $s \in (1, +\infty)$ . In Eq. (13), if the value of the correlation  
 249 coefficient  $r$  between events  $P(A)$  and  $P(B)$  is a constant, and the probabilities of the occurrence of events A and B  
 250 ( $a$  and  $b$ ) are intervals from fuzzy sets, the thresholds for the function can be calculated by using the lower and upper  
 251 bounds of the probabilities of occurrence  $(a_{min}, b_{min})$  and  $(a_{max}, b_{max})$ , respectively. By this way, the monotonic  
 252 increasing copula function  $\text{Frank}(a,b,r)$ , can be extended into fuzzy theory. If the input variables in the copula  
 253 function  $\text{Frank}(a,b,r)$  are TFNs, this function can be formulated as follows:

$$254 \quad \tilde{P}_{\text{Frank}}(A\&B) = \left( P_{\text{Frank}}(p_{l_A}, p_{l_B}), P_{\text{Frank}}(p_{m_A}, p_{m_B}), P_{\text{Frank}}(p_{u_A}, p_{u_B}) \right) \quad (15)$$

255 For two dependent IEs  $IE_i$  and  $\overline{IE}_j$ , the conjunction operation of these two events can then be calculated as:

$$256 \quad \tilde{P}_{\text{Frank}}(IE_i \& \overline{IE}_j) = \log_{s_{IE_i \& \overline{IE}_j}} \left[ 1 + \left( s_{IE_i \& \overline{IE}_j}^{p_{IE_i}^\alpha} - 1 \right) \left( s_{IE_i \& \overline{IE}_j}^{p_{\overline{IE}_j}^\alpha} - 1 \right) / (s_{IE_i \& \overline{IE}_j} - 1) \right], \quad (16)$$

$$257 \quad s_{IE_i \& \overline{IE}_j} = \tan(\pi(1 - r_{IE_i \& \overline{IE}_j})/4). \quad (17)$$

### 258 3. Risk model

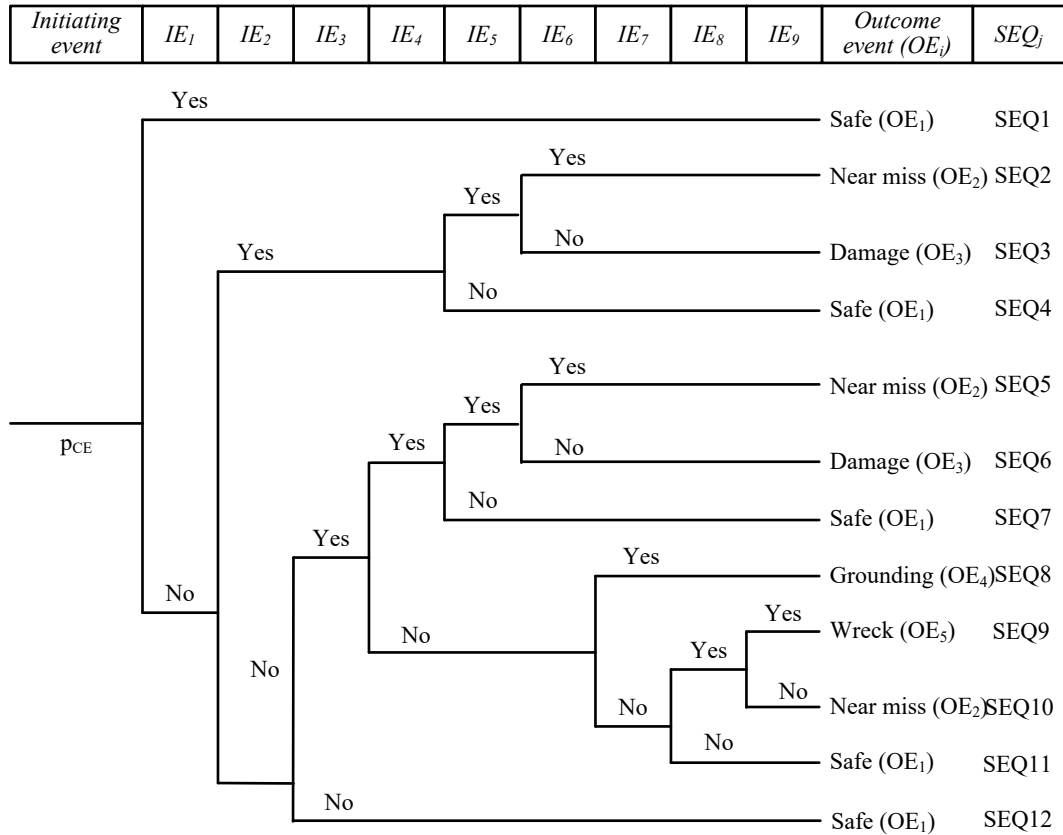
259 In this section, we present a risk model of a typical ship accident in Arctic waters – a ship stuck in ice. This is  
 260 chosen as a case to interpret the quantitative approach proposed in section 2.

#### 261 3.1. Accident scenarios modeling (step 1)

262 To model the ET of a ship stuck in ice, we consider the following technical and environmental factors: ice class



263 of the ship, navigation operations, ice conditions, rescue ability, channel depth, meteorological environment and the  
 264 ship's maintenance ability. The proposed ET model for the accident of a ship stuck in ice is depicted in Fig. 1. The  
 265 considered risk factors are associated with the technical and environmental aspects of shipping, and organizational  
 266 and human-related factors are beyond the scope of this paper and are not considered in the ET model.



- IE<sub>1</sub>: The stuck ship breaks the harsh sea ice by herself.
- IE<sub>2</sub>: There is nearby ship/icebreaker assistance.
- IE<sub>3</sub>: The stuck ship encounters fast moving ice.
- IE<sub>4</sub>: The ship is assisted by an icebreaker during a period of an uncontrolled drift.
- IE<sub>5</sub>: The ship collides with assisting ships (icebreakers) or objects.
- IE<sub>6</sub>: The damage extend of the ship is not significant.
- IE<sub>7</sub>: The depth of the fairway is less than the draught of the ship.
- IE<sub>8</sub>: The uncontrolled ship collides with ships or objects.
- IE<sub>9</sub>: The ship hull is breached upon collision with ships (icebreaking ships or icebreakers) or objects.

267  
 268 Fig. 1. An ET model for accident scenarios of a ship stuck in ice, including nine intermediate events (IEs), five outcome events (OEs)  
 269 and twelve accident sequences; upper branch corresponds to the occurrence of the event, lower branch to nonoccurrence. SEQ =  
 270 sequence number.

271 As shown in Fig. 1, the potential scenarios that can originate from the given initiating event - a ship stuck in ice  
 272 - are described by nine IEs and five OEs, logically structured in twelve possible accident sequences on the basis of  
 273 expert knowledge and related studies (Afenyo et al., 2016; Afenyo et al., 2016; Committee, 2014; Council, 2009; Fu  
 274 et al., 2016; Fu et al., 2016; Goerlandt et al., 2016; Khan et al., 2014; Kotovirta et al., 2009; Kubat et al., 2015; Kum  
 275 and Sahin, 2015; Marken et al., 2015; Montewka et al., 2015; Valdez Banda et al., 2016). The structure of the risk  
 276 model is developed as an ET with nine IEs, as follows:

- 277 • IE<sub>1</sub>: The stuck ship breaks the harsh sea ice by itself.

- 278 ● IE<sub>2</sub>: There is nearby ship/icebreaker assistance. There is available icebreakers or ice-going ships with a  
279 high ice class to assist the stuck ship in breaking up the harsh sea ice.
- 280 ● IE<sub>3</sub>: The stuck ship encounters fast moving ice. In some extreme weather conditions, ships may lose control  
281 and collide with ice floes or ice ridges. This event had happened and caused the wrecking of several ships  
282 in Arctic waters (Marchenko, 2011).
- 283 ● IE<sub>4</sub>: The ship is assisted by an icebreaker during a period of uncontrolled drift. Following the drift,  
284 icebreakers are available in some coastal areas and within the scope of research and rescue.
- 285 ● IE<sub>5</sub>: The ship collides with the assisting icebreaker. The forces of collision may lead to damage to the hull  
286 structure.
- 287 ● IE<sub>6</sub>: The damage extend of the ship is not significant.
- 288 ● IE<sub>7</sub>: The depth of the fairway is less than the draught of the ship. During an uncontrolled drift with the ice  
289 field, the ship may drift into shallow waters.
- 290 ● IE<sub>8</sub>: The uncontrolled ship collides with ships (icebreaking ships or icebreakers), an iceberg, or ice ridge  
291 ice.
- 292 ● IE<sub>9</sub>: The ship hull is breached in the collision accidents (Khan et al., 2014).

293 On the consequences of the ship accident following the development of the initiating event and possible IEs for  
294 the accident of a ship stuck in ice may lead to five major OEs, as follows:

- 295 ● OE<sub>1</sub>: Safe. The ship is released from the harsh sea ice and can continue its voyage.
- 296 ● OE<sub>2</sub>: Near miss. The ship sustains slight damages, which can be handled by the crew onboard. The ship  
297 can continue its voyage.
- 298 ● OE<sub>2</sub>: Damage. Damage occurs to the hull structure, which cannot be repaired by the crew on board. The  
299 ship is unable to continue and has to proceed to the nearest port, where the necessary repairs can be done.
- 300 ● OE<sub>4</sub>: Grounding. The uncontrolled ship runs aground and is unable to continue its voyage.
- 301 ● OE<sub>5</sub>: Wreck. The hull is breached, and the ship cannot continue its voyage.

302 As shown in Fig. 1, the ship will be safe (OE<sub>1</sub>) in SEQ 1, 4, 7 and 11; the ship will have a near miss (OE<sub>2</sub>) in  
303 SEQ 2, 5 and 10, suffering slight damages on board; the ship will be damaged (OE<sub>3</sub>) in SEQ 3 and 6, because of  
304 collision with objects; the ship will be grounded (OE<sub>4</sub>) in SEQ 8, if the water depth is deficient; the ship will wreck  
305 (OE<sub>5</sub>) in SEQ 9, owing to a significant breach in the hull structure after collision.

### 306 3.2. Probability analysis of the IEs (step 2)

307 The data and information used for probability analysis of the IEs in ET model are elicited from seven experts,  
308 including one captain, four professors, and two senior researchers. Their detail information is listed as follows:

- 309 ● Expert A: An associate professor engaged in risk management of ship operations in ice-covered waters since  
310 more than five years, from the National Engineering Research Center for Water Transport Safety and the  
311 Wuhan University of Technology.
- 312 ● Expert B: A professor engaged in navigation safety in polar waters since more than twenty years, from

- 313 School of Navigation of the Wuhan University of Technology.
- 314 ● Expert C: An associate professor engaged in safety management of ship accidents system since more than  
 315 five years, from the National Engineering Research Center for Water Transport Safety and the Wuhan  
 316 University of Technology.
- 317 ● Expert D: A specialist research scientist engaged in safety management and risk assessment of ship  
 318 operations in harsh environments, from the Finnish Geospatial Research Institute.
- 319 ● Expert E: An associate professor engaged in navigation safety in polar waters since more than a decade,  
 320 from Shanghai Maritime University, also engaged as a second officer on board.
- 321 ● Expert F: A senior researcher engaged in security management of ship operation in polar waters more than  
 322 fifteen years, from China Classification Society Certification Company Shanghai Branch.
- 323 ● Expert G: A senior captain with more than fifteen years' navigation experience in ice-covered waters, from  
 324 the Polar Research Institute of China.

325 The linguistic judgments of the seven experts are given in Table 2, concerning the linguistic terms of Table 1.  
 326 As shown in Table 2, experts' linguistic judgments as to the likelihood of various IEs are presented. Since these  
 327 experts have difference working experiences and stand for different stakeholders, their opinions for several IEs are  
 328 different. Some experts are negative for the undesired events. For example, the judgment of expert F for the IEs is  
 329 more cautious compared to the other experts. We will analyze these experts' judgments individually so as to make a  
 330 comprehensive analysis.

331 **Table 2**  
 332 Experts' linguistic judgments as to the likelihood of various IEs.

IE <sub>s</sub>	Expert A	Expert B	Expert C	Expert D	Expert E	Expert F	Expert G
IE <sub>1</sub>	ML	VL	L	M	L	L	M
IE <sub>2</sub>	H	VH	M	H	H	H	L
IE <sub>3</sub>	L	MH	MH	M	M	H	MH
IE <sub>4</sub>	VL	ML	L	H	M	ML	ML
IE <sub>5</sub>	ML	VL	L	VL	VL	M	ML
IE <sub>6</sub>	L	VL	VL	M-H	ML	L	VL
IE <sub>7</sub>	ML	VL	VL	ML	L	H	VL
IE <sub>8</sub>	H	ML	M	M	MH	M	ML
IE <sub>9</sub>	M	H	H	ML	L	MH	M

333 Use Eq. (3) to calculate the TFNs of the IEs from experts' judgment at 95% degree of belief ( $\alpha=0.05$ ). The TFNs  
 334 in the  $\alpha$ -cuts are listed in Table 3, which will be used in the subsequent dependence analysis, scenario analysis, and  
 335 risk assessment.

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341 **Table 3**

342 TFNs from experts' judgment for the IEs at 95% degree of belief ( $\alpha=0.05$ ).

$IE_s$	Expert A	Expert B	Expert C	Expert D
$IE_1$	(0.049,0.125,0.196)	(0.001,0.025,0.049)	(0.156,0.275,0.394)	(0.358,0.5,0.643)
$IE_2$	(0.606,0.725,0.844)	(0.951,0.975,0.999)	(0.358,0.5,0.643)	(0.606,0.725,0.844)
$IE_3$	(0.358,0.5,0.643)	(0.804,0.875,0.951)	(0.804,0.875,0.951)	(0.358,0.5,0.643)
$IE_4$	(0.001,0.025,0.049)	(0.049,0.125,0.196)	(0.156,0.275,0.394)	(0.606,0.725,0.844)
$IE_5$	(0.358,0.5,0.643)	(0.001,0.025,0.049)	(0.156,0.275,0.394)	(0.00,0.025,0.049)
$IE_6$	(0.156,0.275,0.394)	(0.001,0.025,0.049)	(0.001,0.025,0.049)	(0.179,0.2625,0.346)
$IE_7$	(0.049,0.125,0.196)	(0.001,0.025,0.049)	(0.001,0.025,0.049)	(0.049,0.125,0.196)
$IE_8$	(0.606,0.725,0.844)	(0.049,0.125,0.196)	(0.049,0.125,0.196)	(0.358,0.5,0.643)
$IE_9$	(0.358,0.5,0.643)	(0.606,0.725,0.844)	(0.606,0.725,0.844)	(0.049,0.125,0.196)
$IE_s$	Expert E	Expert F	Expert G	
$IE_1$	(0.156,0.275,0.394)	(0.156,0.275,0.394)	(0.358,0.5,0.643)	
$IE_2$	(0.606,0.725,0.844)	(0.606,0.725,0.844)	(0.156,0.275,0.394)	
$IE_3$	(0.358,0.5,0.643)	(0.606,0.725,0.844)	(0.804,0.875,0.951)	
$IE_4$	(0.358,0.5,0.643)	(0.049,0.125,0.196)	(0.049,0.125,0.196)	
$IE_5$	(0.001,0.025,0.049)	(0.358,0.5,0.643)	(0.358,0.5,0.643)	
$IE_6$	(0.049,0.125,0.196)	(0.156,0.275,0.394)	(0.001,0.025,0.049)	
$IE_7$	(0.156,0.275,0.394)	(0.606,0.725,0.844)	(0.001,0.025,0.049)	
$IE_8$	(0.804,0.875,0.951)	(0.358,0.5,0.643)	(0.049,0.125,0.196)	
$IE_9$	(0.156,0.275,0.394)	(0.804,0.875,0.951)	(0.358,0.5,0.643)	

343 *3.3. Dependence analysis of the IEs (step 3)*

344 To undertake this, we use the probability of occurrence of the IEs derived from the seven expert judgments to  
 345 calculate the probability of the corresponding complement events for Eq. (3). For each of the seven expert judgments,  
 346 the possibilistic approach is applied as follows:

347 (1) Set the possibility value  $\alpha=0$  and the corresponding cuts of the possibility distribution  $\tilde{P}_{IE_k}^{\alpha=0}$  as the intervals  
 348 of possible values of the possibilistic variables  $IE_k$  ( $k = 1, 2, \dots, n$ ). In this ET model of Fig. 1, the number of the  
 349 probabilities of the IEs  $n$  is 12.

350 (2) Compute the smallest ( $p_{l_{IE_k}}$ ) and largest ( $p_{u_{IE_k}}$ ) values in its  $\alpha$ -cut intervals  $[p_{l_{IE_k}}, p_{u_{IE_k}}]$  (calculated based  
 351 on Eq. (3) for 1000 Monte Carlo application sampled from a uniform distribution.

352 (3) Return to the first step and repeat for another  $\alpha$  value (here ten  $\alpha$ -cuts, for  $\alpha=0.1:0.1:1$ ).

353 Random realization can, then, be obtained by sampling the occurrence of each event 70,000 times. This is done  
 354 in order to calculate the correlation coefficients of the events. The results of the PMCC and associated significance  
 355 tests are shown in Table 4, as based on Eq. (8) and Eq. (9). The dependence relationships between connected IEs in

356 the ET model are also depicted in Table 4.

357

358

359 **Table 4**

360 Product-moment correlation coefficients ( $r_{PMCC}$ ) between the connected IEs in the ET model.

Events	$r_{PMCC}$	Dependence relationship	Events	$r_{PMCC}$	Dependence relationship
IE <sub>2</sub> IE <sub>3</sub>	-0.519**	Negative, strong dependence	$\overline{IE}_1$ IE <sub>2</sub>	0.658**	Positive, very strong dependence
			$\overline{IE}_2$	-0.658**	Negative, very strong dependence
IE <sub>2</sub> IE <sub>5</sub>	0.475**	Positive, weak-strong dependence	$\overline{IE}_2$ IE <sub>3</sub>	0.288**	Positive, weak dependence
			$\overline{IE}_3$	-0.288**	Negative, weak dependence
IE <sub>3</sub> IE <sub>4</sub>	-0.689**	Negative, strong dependence	$\overline{IE}_4$ IE <sub>7</sub>	0.066**	Positive, very weak dependence
			$\overline{IE}_7$	-0.066	Negative, very strong dependence
IE <sub>5</sub> IE <sub>6</sub>	0.203**	Positive, weak dependence	$\overline{IE}_7$ IE <sub>8</sub>	-0.424**	Negative, weak dependence
			$\overline{IE}_8$	0.424**	Positive, weak dependence
IE <sub>8</sub> IE <sub>9</sub>	-0.464**	Negative, weak-strong dependence			
			$\overline{IE}_9$	0.464**	Positive, weak-strong dependence

361 \*\*. Correlation is significant at the 0.01 level (2-tailed).

362 As shown in Table 4, the dependence relationships between the most of the connected IEs are significant since  
 363 the correlation coefficients ( $r_{PMCC}$ ) are more than 0.2, except the  $r_{PMCC}$  between  $\overline{IE}_4$  and  $IE_7/\overline{IE}_7$ . Hence, we use Eq.  
 364 (16) and Eq. (17) to calculate the conjunction operations for these couples of connected IEs. The dependence  
 365 relationships between  $\overline{IE}_4$  and  $IE_7/\overline{IE}_7$  are neglected in this paper, and a traditional formula – Eq. (5) are used to  
 366 calculate the conjunction operations for these two conditions.

## 367 4. Results of risk assessment

### 368 4.1. Probability of the accident scenarios

369 The probability of the initiating event for a ship stuck in ice is set to a crisp of 0.02, based on our earlier work  
 370 (Fu et al., 2016). Use the TFNs of the IEs listed in Table 3 to calculate the probability of the accident scenarios in the  
 371 ET model, according to Eq. (6), (16) and (17). Then, use Eq. (4) to calculate the defuzzification values for each  
 372 sequence. The sequence probability results for the twelve sequences, based on the seven experts' judgments, are  
 373 listed in Table 5.

374 **Table 5**

375 Resulting defuzzification numbers for each sequence from the seven experts' judgments in the ET model proposed for the accident of a  
 376 ship stuck in ice.

SEQ	Expert A	Expert B	Expert C	Expert D	Expert E	Expert F	Expert G
1	2.48E-03	5.00E-04	5.50E-03	1.00E-02	5.50E-03	5.50E-03	1.00E-02
2	2.06E-03	2.04E-05	7.86E-05	6.39E-05	5.00E-05	1.77E-03	6.48E-05
3	4.64E-03	4.73E-04	1.95E-03	1.40E-04	2.41E-04	3.92E-03	1.57E-03
4	7.23E-03	1.86E-02	6.38E-03	7.89E-03	1.10E-02	6.32E-03	2.21E-03
5	1.13E-05	4.94E-08	1.59E-05	7.06E-06	4.04E-06	5.51E-05	1.42E-05
6	2.22E-05	9.69E-07	3.35E-04	1.51E-05	1.83E-05	1.09E-04	2.99E-04

7	4.86E-05	5.08E-05	1.39E-03	1.00E-03	1.02E-03	2.41E-04	5.18E-04
8	1.13E-05	2.05E-06	5.84E-05	1.65E-04	3.49E-04	2.62E-04	2.71E-05
9	8.48E-04	3.70E-05	4.32E-04	3.34E-05	2.59E-04	3.90E-04	3.31E-04
10	1.13E-03	2.24E-05	2.54E-04	2.57E-04	8.23E-04	9.95E-05	4.51E-04
11	9.37E-04	3.96E-04	4.54E-03	3.55E-04	2.27E-04	5.59E-04	5.18E-03
12	2.44E-03	8.11E-05	9.55E-04	1.28E-03	1.96E-03	1.13E-03	9.55E-04

377 4.2. Probability of the outcome events

378 The probabilities of the OEs are calculated using Eq. (7), by multiplying the probability of the initiating event  
379 A with the total probabilities for the consequent scenarios:

380 
$$P_{OE_1} = P_{SEQ_1} + P_{SEQ_4} + P_{SEQ_7} + P_{SEQ_{11}} + P_{SEQ_{12}}, \quad (18)$$

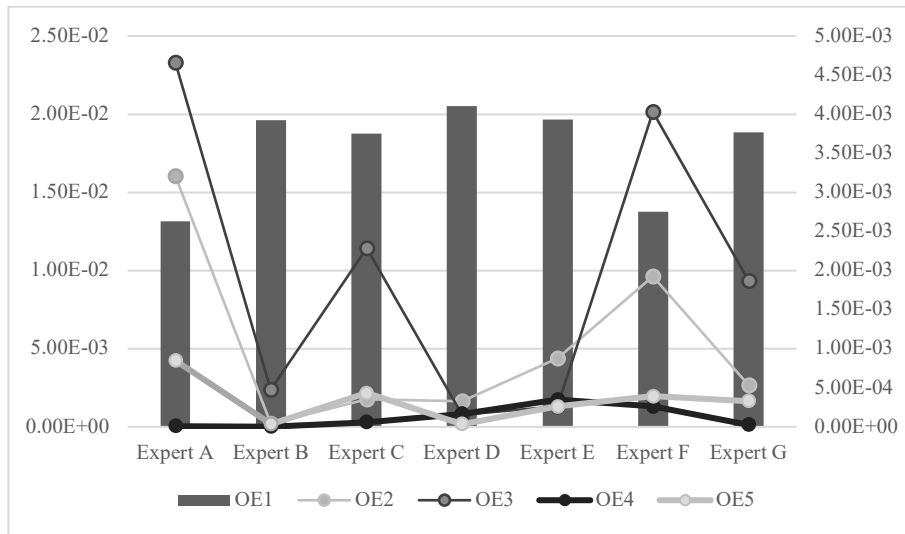
381 
$$P_{OE_2} = P_{SEQ_2} + P_{SEQ_5} + P_{SEQ_{10}}, \quad (19)$$

382 
$$P_{OE_3} = P_{SEQ_3} + P_{SEQ_6}, \quad (20)$$

383 
$$P_{OE_4} = P_{SEQ_8}, \quad (21)$$

384 
$$P_{OE_5} = P_{SEQ_9}. \quad (22)$$

385 According to Eq. (18) ~ (22), the resulting defuzzification numbers of the OEs provided by the seven experts'  
386 judgments are depicted in Fig. 2.



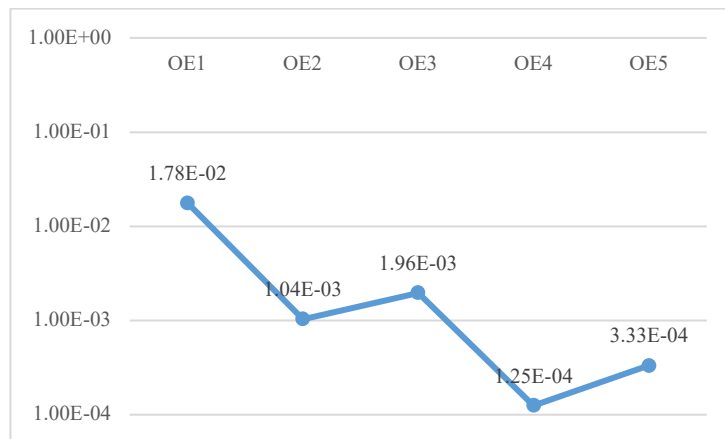
387  
388 Fig. 2. Resulting defuzzification numbers of the OEs according to seven experts' judgments in the ET model proposed for the accident  
389 of a ship stuck in ice.

390 As shown in Fig. 2, the grey bar refers to the probability of the safe scenario - OE<sub>1</sub> from the seven experts'  
391 judgment, using the left vertical axis as a coordinate. The four lines use the right vertical axis as a coordinate for  
392 representing the hazardous scenarios in the ET model. The fine grey line refers to the probability of the OE<sub>2</sub>, the fine  
393 black line refers to the probability of the OE<sub>3</sub>; the bold black line refers to the probability of the OE<sub>4</sub>; and the bold  
394 gray line refers to the probability of the OE<sub>5</sub>. The ranking of the resulting probability of the OEs from the seven  
395 experts' judgments can be obtained, as follows:

- 396 ● Expert A:  $OE_1 > OE_3 > OE_2 > OE_5 > OE_4$ ,
- 397 ● Expert B:  $OE_1 > OE_3 > OE_2 > OE_5 > OE_4$ ,
- 398 ● Expert C:  $OE_1 > OE_3 > OE_5 > OE_2 > OE_4$ ,
- 399 ● Expert D:  $OE_1 > OE_2 > OE_4 > OE_3 > OE_5$ ,
- 400 ● Expert E:  $OE_1 > OE_2 > OE_4 > OE_3 = OE_5$ ,
- 401 ● Expert F:  $OE_1 > OE_3 > OE_2 > OE_5 > OE_4$ ,
- 402 ● Expert G:  $OE_1 > OE_3 > OE_2 > OE_5 > OE_4$ .

403 According to the above rankings from the seven experts, it can be seen that  $OE_1$  and  $OE_4$  are seen both as the  
 404 most likely and most unlikely  $OE_s$  respectively for a ship stuck in ice;  $OE_3$  is the second most likely to occur;  $OE_2$  is  
 405 the third most likely; finally,  $OE_5$  is seen as the least likely to occur, by the majority of the experts' judgments.  $OE_1$   
 406 is considered by far the most likely OE by all seven experts, with probabilities approximately around  $10^{-2}$ ;  $OE_3$  is  
 407 considered the second likely OE by five experts' judgments, with probabilities ranging between  $2.59E-04$  and  $4.66E-$   
 408  $0$ ;  $OE_2$  is considered the third likely OE by four experts' judgments, with probabilities ranging between  $1.55E-04$   
 409 and  $1.87E-03$ ;  $OE_5$  is considered the fourth likely OE by four experts' judgments, with probabilities ranging between  
 410  $3.34E-05$  and  $8.48E-04$ ;  $OE_4$  is considered the most unlikely OE by five experts' judgments, with probabilities  
 411 ranging between  $2.05E-06$  and  $3.49E-04$ .

412 This ranking ( $OE_1 > OE_3 > OE_2 > OE_5 > OE_4$ ) is in accordance with the resulting average values of the probability  
 413 of the OEs provided by the seven experts. As shown in Fig. 3, it can be discerned that  $OE_1$  and  $OE_4$  are seen both as  
 414 the most likely and most unlikely  $OE_s$  for the accident of a ship stuck in ice, with probabilities  $1.78E-02$  and  $1.25E-$   
 415  $04$ , respectively;  $OE_3$  is the second most likely to occur with probability  $1.96E-03$ ;  $OE_2$  is the third most likely with  
 416 probability  $1.04E-03$ ; and finally,  $OE_5$  is seen as the least likely to occur with probability  $3.33E-04$ .



417  
 418 Fig. 3. The average values of the probability of the OEs in the ET model proposed for the accident of a ship stuck in ice.

419 The results obtained in the course of the presented study are compared with the similar studies conducted for  
 420 the Northern Baltic Sea, see for example (Valdez Banda et al., 2016; Valdez Banda et al., 2015). The results of the  
 421 probability of occurrence for the undesired events in this paper are between  $10^{-5}$  and  $10^{-3}$ , and these values are  
 422 comparable to those characterizing maritime transportation systems operating in ice-covered waters of the Northern  
 423 Baltic Sea. For example, the probability of occurrence of the damage event approximately equals to  $10^{-4}$ . However,

424 the resulting probability of occurrence for the grounding event in (Valdez Banda et al., 2015) is less than  $10^{-4}$ , which  
425 is lower than the corresponding result for the OE<sub>4</sub> with the value being 1.25E-04, as presented in Fig. 3. This  
426 difference is due to the diverse environment conditions considered.

427  
428

## 429 **5. Discussion**

430 Risk assessment of ship accidents in Arctic waters is a high concern issue in the marine industry with high  
431 uncertainty. The quantitative approach presented here shows a strong prediction power of the probability of ship  
432 stuck in ice in the current case study, in the absence of high-quality data in Arctic waters. The application of expert  
433 judgment in the proposed approach makes a supplement for such kind of data problem. However, it is difficult to  
434 judge the correction of the collected information for the disagreements among experts, which are influenced by  
435 complex factors, such as research background, status (stakeholder or not), personality (conservative or optimistic),  
436 working experience, etc. We intend to collect and analyze more expert judgments in the future.

437 The proposed ET model clearly depicts the IEs, scenarios and OEs in a figure for the initiating event of a ship  
438 stuck in ice in Arctic waters. If the conditions of some IEs are changed, we can see the new sequences through ET  
439 figure. Also, the risk of multiple scenarios of a ship stuck in ice can be calculated respectively, so that we can quickly  
440 find the worst scenario and make associated risk control options considering corresponding IEs.

441 Besides, both epistemic and dependence uncertainties in the quantitative risk assessment have been handled  
442 regarding the mathematical forms of fuzzy sets and Frank copula methods. The Frank copula based-ET analysis is a  
443 useful method that enables description and propagation the effects of uncertainties. Some other copula functions may  
444 be better than the Frank copula, but it is difficult to make a comparison the limited objective data. If high-quality data  
445 can be obtained, I would like to conduct further studies.

## 446 **6. Conclusions**

447 In this paper, a quantitative approach is proposed to analyze risks for ship accidents in Arctic waters. The  
448 occurrence probabilities of the initiating event and intermediate events are extracted from expert knowledge, and the  
449 probabilities of potential outcome events are computed. Fuzzy set theory and Frank copula are incorporated into the  
450 ET model to handle the parameter uncertainty in the probability values and the dependence uncertainty between  
451 dependent events, respectively. Additionally, a possibilistic approach integrating Monte Carlo simulation and fuzzy  
452 set theory is used to calculate the correlation relationship between events regarding linguistic terms. A typical event  
453 for ship operations in Arctic waters- ship stuck in ice is chosen as a case to interpret the approach proposed. The  
454 results show that the risk for ships stuck in ice in Arctic waters is comparable to that of maritime transportation  
455 systems operating in the Northern Baltic Sea. Through the case study analysis, the presented approach can be  
456 considered an appropriate approach for predicting the probability of the consequence of a ship getting stuck in ice.

457 The proposed risk approach enables to predict the risk for a ship stuck in ice in Arctic waters and also enables  
458 one to describe, measure and propagate the effects of uncertainties. Moreover, the approach provides an insight into  
459 the combined effects of the probability of occurrence and potential consequences of ship accidents. This study can



460 assist the management of accident prevention or a ship's crew in planning and conducting an actual sea passage  
461 through Arctic waters.

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463

464

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