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Modeling and Performance Evaluation of Multistage Serial Manufacturing Systems with Rework Loops and Product Polymorphism

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Abstract

This paper studies multistage serial manufacturing systems with the integrated consideration of machine failures, process defects, multiple rework loops, etc. In particular, multiple rework loops and product polymorphism lead to a more complex conversion of internal material flows, and therefore it's difficult to model and analyse such manufacturing systems. A modular modeling method based on Generalized Stochastic Petri Nets (GSPN) is presented to characterize the material flows, it is capable of representing the processing differences resulting from product polymorphism comparing with traditional Markov model or Queuing network model. By analysing the model, the processing ratio of each workstation is inferred. Using 2M1B (two-machine and one-buffer) Markov cell model as the building blocks, which is obtained based on the GSPN models for their isomorphism, an overlapping decomposition method is then developed for evaluating the performance of the multistage serial systems with rework loops. Numerical experiments and a case study of a powertrain assembly line illustrate the efficiency of the proposed method.

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Keywords: Manufacturing system; Rework loops; Product polymorphism

1. Introduction

This paper studies multistage serial manufacturing systems with rework loops and product polymorphism. In many manufacturing systems, some defective products are unexpectedly produced due to process variations or other factors. Usually rework is done instead of scrapping these products for economic reasons as seen in semiconductor, glass, steel, food industries, etc. [1]. The products in the systems can be often divided into four states, the qualified, the defective, the rework, and the scrapped. To manage, operate and improve the performance of such systems, modeling and performance evaluation are necessary and important.

A number of numerical analysis methods have been proposed for stochastic manufacturing systems with unreliable machines, multiple failure modes, preventive maintenance, etc. Queueing network models, Markov models and decomposition methods have been widely used as a faster and more viable

alternative to simulation in the analysis of the systems [2]. When considering quality issues, the material flows in the systems, including rework flow and scrap flow, complicate modeling and make it more challenging to study the systems [3]. Connors et al. [4] proposed an open queueing network model for analysis of semiconductor manufacturing facilities, where the wafer lot sizes are affected by rework and scrap. Kang et al. [5] analyzed a parallel machine with rework. A dispatching algorithm is given to evaluate total tardiness, maximum lateness and mean flow-time, etc. Ju et al. [6], Lin et al. [7] and Biller et al. [8] studied multistage manufacturing systems with a single rework loop, where defective products are mostly assumed to be randomly generated with Bernoulli-type quality failure. Being constructed as a stochastic-flow network, systems are divided into several general processing paths and one rework path using decomposition technique. In addition, Liu et al. [9] proposed an approximation method of transforming an M-machine re-entrant line into a 2M-machine

serial line. However, models for evaluating multistage manufacturing systems with generally complex Markovian machines are not available. Some researchers have extended approximate decomposition methods, such as using two machines and one finite intermediate buffer (2M1B) [10] or three machines and one buffer (3M1B) [11] as building blocks. Considering multiple rework loops, Cao et al. [12] developed a new 3M1B model in addition to the 2M1B models. In the decomposition approach, both 2M1B and 3M1B models are used.

In practice, the production systems are more complex. For instance, there are several rework loops in power train assembly lines. Due to the construction of the system, the defective and scrapped must be sent out of the line only through the rework entrance as well as the rework being sent back on line. Before assembly processes, each workstation will read the RFID information of the product and identify the state of product. While the product is the qualified from upstream or the one need to be reworked here, the process will start. Otherwise, the

product will be immediately sent out of the workstation if it's the defective, or the scrapped from upstream, or the rework that will not be reworked here but downstream. However, the material flows information is not characterized in these models above. Thus these models are not extensible to multistage manufacturing system with multiple rework loops and product polymorphism.

In this paper, an optimal solution to model and evaluate the performance of multistage serial manufacturing systems with rework loops and productive polymorphism is proposed. Unlike the previous models proposed in the literature, a model based on Generalized Stochastic Petri Nets (GSPN) is presented to characterize the complex state transition resulting from the conversion of internal material flows. Based on analyzing the process differences in the model affected by multiple rework loops and products polymorphism and using 2M1B model as the building blocks, a decomposition approach is developed for evaluating the performance of the systems.

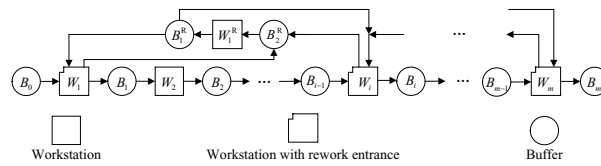


Fig. 1. The manufacturing system with rework loops

2. Problem Formulation and System Modeling

2.1. Problem Formulation

The manufacturing system studied in this paper is configured in serial layout. The main line consists of m on-line workstations, $m-1$ on-line buffers of finite capacity and r rework entrances that form $r-1$ rework loops. The rework entrances are located on the several workstations. Between two rework entrances $R_j, j = 1, 2, \dots, r-1$ and R_{j+1} adjacent, there are $m_j \geq 3$ on-line workstations and $m_j - 1$ on-line buffers. With other off-line buffers B_j^R and off-line pre-rework stations W_j^R , all of them above construct a standard rework loop L_j . Fig. 1 shows the system with rework loops.

Due to the construction of the system, it is assumed that the defective and scrapped from the workstations between W_{j-1}^R and W_j^R which are the workstations with rework entrances, must be sent out of the line through the entrance R_j , while the rework after pre-rework processing must be sent back through R_{j-1} . In addition, the defective and scrapped from W_i^R , must be sent out from the current workstation through R_i while the rework corresponding must be sent back through R_i .

For convenience, the following notations are used throughout the paper:

- μ_i = The processing rate of workstation $W_i, i = 1, 2, \dots, m$;
- λ_i, r_i = The failure and repair rate of $W_i, i = 1, 2, \dots, m$;
- α_i = The state of workstation $W_i, \alpha_i \in \{0(\text{down}), 1(\text{up})\}$ (see assumption 2), $i = 1, 2, \dots, m$;
- θ_i = The qualified rate of $W_i, i = 1, 2, \dots, m$;
- PR_i = The production rate of $W_i, i = 1, 2, \dots, m$;
- k_i = The capacity of on-line buffer $B_i, i = 1, 2, \dots, m-1$;
- x = The inventory of the system.

We make the following assumptions regarding the system:

- 1) The workstation won't process the defective or the scrapped from the upstream, or the reworked to the downstream. In addition, the process time of the pre-rework stations and the transport time are negligible.
- 2) All workstations are unreliable and subjected to operation-dependent failures. Therefore, each unreliable workstation has two states: up and down.
- 3) In this model, the capacity of off-line buffers B_j^R is infinite. the upstream workstation is never starved and the downstream workstation is never blocked.
- 4) The system is based on First Come First Serve (FCFS). While the products arrive simultaneously, the product from the upstream on-line buffer is a higher priority than the one from the off-line buffer. This considers to avoid the deadlock which may occur in the system.
- 5) If the products have been reworked failed, they are marked as the scrapped and sent through the entrance specified to the scrapped area. Each defective product has only one time to be reworked.

2.2. Modeling for Systems

In the system, multiple rework loops and products polymorphism lead to a more complex conversion of internal material flow. Fig. 2a and Fig. 2b show the material flow conversion in the general workstation and the workstation with a rework entrance respectively.

Based on Fig. 2, GSPN model blocks of workstations are built as shown in Fig. 3a and Fig. 3b respectively. The main meanings of places and transitions are shown in Table 1 and Table 2.

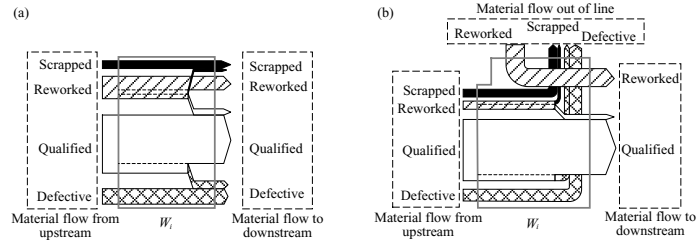


Fig. 2. The material flow conversion in workstations

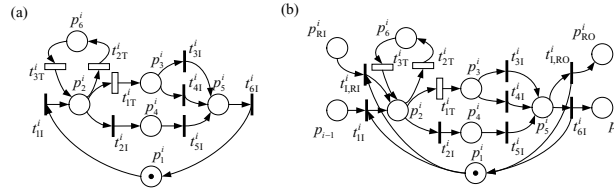


Fig. 3. GSPN blocks of the workstations

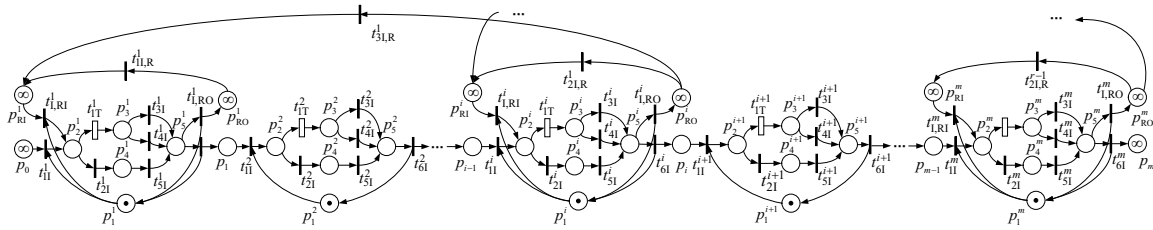


Fig. 4. GSPN of the system

Table 1. Main meanings of places of Fig. 3a and Fig. 3b model.

Place	Meaning of place
p_1^i	The workstation is ready.
p_2^i	Load the product and identified its state.
p_3^i	Product is being processed.
p_4^i	The product has not been processed.
p_5^i	Ready to unload the product.
p_6^i	Break down and stop.
p_i	Buffer B_i .
p_{ri}^j	Buffer for products send into rework entrance.
p_{ri}^j	Buffer for products out from rework entrance.

Table 2. Main meanings of transitions of Fig. 3a and Fig. 3b model.

Transition	Meaning of transition	Category
t_{11}^i	Load the product and read the information.	Immediate
t_{21}^i	Process does not start.	Immediate
t_{31}^i	The product is qualified.	Immediate
t_{41}^i	The product is defective.	Immediate
t_{51}^i	The product passes.	Immediate
t_{61}^i	Unload the product to the downstream.	Immediate
t_{1T}^i	Process start.	Timed
t_{2T}^i	Break down.	Timed
t_{3T}^i	Repair.	Timed

Combining the GSPN blocks of workstations and the layout of the system, the GSPN of the system is as shown in Fig. 4. To simplify the model view, the failure and repair transitions of each workstation are not shown in Fig. 4, but still need to be considered when calculating.

3. Decomposition of Systems

In this section, we first deduce the processing ratio based on analyzing the GSPN model. Then a decomposition method is developed for the system with multiple rework loops and product polymorphism.

3.1. Analysis of the Workstation

Based on material flow conservation, for a rework loop $L_j, j = 1, 2, \dots, r-1$, the trigger probability $\varphi_{i,j}, i = 1, 2, \dots, m_j$ of t_{1T}^i , which means the processing rate in the practice, is directly related to the qualified rates $\theta_{i,j}, i = 1, 2, \dots, m_j$. By conversation of material flows, $\varphi_{i,j}$ is shown as Eqs. (1).

As Fig. 2b shows, while the workstation W_i is the one with the rework entrance, the outgoing material flows consist of 1) the qualified, 2) the defective and 3) the scrapped flow from the current rework loop L_j , and 4) the rework flow from the next rework loop L_{j+1} . But in fact, the material flow into buffer B_i

consists of 1) the qualified flow from rework loop L_j , 4) the rework flow from rework loop L_{j+1} . As the processing time of 2) the defective and 3) the scrapped flow is assumed to be 0, the actual trigger probability $\phi'_{i,j}$ for the downstream eventually is shown as Eqs. (3).

$$\phi_{i,j} = \begin{cases} \frac{(2-\theta_{i,j})}{1+(1-\theta_{i,j})+\sum_{l=2}^{m_j-1}\left\{\left[\prod_{k=1}^{l-1}\theta_{k,j}(2-\theta_{k,j})\right]\cdot(1-\theta_{l,j})\right\}}, & i=1 \\ \frac{\left[\prod_{k=2}^i(2-\theta_{k,j})\right]\cdot\left[\prod_{k=1}^{i-1}\theta_{k,j}\right]\cdot\frac{1}{\theta_{i,j}}}{(2-\theta_{2,j})+\sum_{l=3}^{m_j-1}\left\{\left[\prod_{k=2}^{l-1}(2-\theta_{k,j})\theta_{k,j}\right]\cdot(1-\theta_{l,j})\right\}}, & i=2,\dots,m_j-1 \\ \frac{\left[\prod_{k=2}^{m_j}(2-\theta_{k,j})\right]\cdot\left[\prod_{k=1}^{m_j-1}\theta_{k,j}\right]\cdot\frac{1}{\theta_{i,j}}}{(2-\theta_{2,j})+\sum_{l=3}^{m_j}\left\{\left[\prod_{k=2}^{l-1}(2-\theta_{k,j})\theta_{k,j}\right]\cdot(1-\theta_{l,j})\right\}+o(\varphi_{m_j})}, & i=m_j \end{cases} \quad (1)$$

Where $o(\varphi_{m_j})$ is the compensation parameter of $\varphi_{m_j,j}$ for considering the rework flow from the next rework loop L_{j+1} .

$$o(\varphi_{m_j}) = \begin{cases} 0, & j = r-1 \\ \frac{\sum_{l=2}^{m_{j+1}-1}\left\{\left[\prod_{k=1}^{l-1}(2-\theta_{k,j+1})\theta_{k,j+1}\right]\cdot(1-\theta_{l,j+1})\right\}\cdot\left(\prod_{k=2}^{m_j}\theta_{k,j}\right)}{(2-\theta_{i,j+1})\cdot 2-\theta_{i,j}}, & \text{other} \end{cases} \quad (2)$$

$$\phi'_{i,j} = \begin{cases} \frac{\varphi_{i,j}\cdot\left\{1+\sum_{l=2}^{m_j-1}\left[\prod_{k=1}^{l-1}\theta_{k,j}(2-\theta_{k,j})\right]\cdot(1-\theta_{l,j})\right\}}{1+(1-\theta_{i,j})+\sum_{l=2}^{m_j-1}\left\{\left[\prod_{k=1}^{l-1}\theta_{k,j}(2-\theta_{k,j})\right]\cdot(1-\theta_{l,j})\right\}}, & i=1 \\ \varphi_{i,j}, & i=2,\dots,m_j-1 \\ \frac{\varphi_{i,j}\cdot\left(\prod_{k=1}^{m_j}\theta_{k,j}\right)}{(2-\theta_{2,j})+\sum_{l=3}^{m_j}\left\{\left[\prod_{k=2}^{l-1}(2-\theta_{k,j})\theta_{k,j}\right]\cdot(1-\theta_{l,j})\right\}+o(\varphi_{m_j})}, & i=m_j \end{cases} \quad (3)$$

Finally, the actual process rate $\mu'_{i,j}$ of $W_{i,j}$ in rework loop L_j is as follows:

$$\mu'_{i,j} = \phi_{i,j} \cdot \mu_{i,j} \quad (4)$$

3.2. Decomposition Method for Systems

(1) Approach for 2M1B Subsystem

Let (x, α_u, α_d) denote the state of the subsystem, and $P(x, \alpha_u, \alpha_d)$ denote the steady-state probability while the subsystem is in the state (x, α_u, α_d) . The balance equations for the Markov for the subsystem are presented as:

$$\mathbf{P}(x, \alpha_u, \alpha_d) \mathbf{M}_x = \mathbf{P}(x-1, \alpha_u, \alpha_d) \boldsymbol{\mu}_{u,x-1} + \mathbf{P}(x+1, \alpha_u, \alpha_d) \boldsymbol{\mu}_{d,x+1} \quad (5)$$

As \mathbf{M}_{x-1} , $\boldsymbol{\mu}_{u,x-1}$, $\boldsymbol{\mu}_{d,x+1}$ are the submatrices of the state transition matrix \mathbf{A} :

$$\mathbf{A} = \begin{bmatrix} \mathbf{M}_0 & \boldsymbol{\mu}_{u,0} & & & \\ \boldsymbol{\mu}_{d,1} & \mathbf{M}_1 & & & \\ & & \ddots & & \\ & & & \boldsymbol{\mu}_{u,k_i+1} & \mathbf{M}_{k_i+1} & \boldsymbol{\mu}_{d,k_i+1} \\ & & & & \boldsymbol{\mu}_{d,k_i+2} & \mathbf{M}_{k_i+2} \end{bmatrix} \quad (6)$$

By solving the matrix equation, the steady-state probabilities of all the states can be obtained. Subsequently, the following performance measurements can be obtained:

$$PR_u = \mu'_u \cdot \left[\sum_{\alpha_2} \sum_{x=0}^{k+2} P(x, 1, \alpha_2) \right] \quad (7)$$

$$PR_d = \mu'_d \cdot \left[\sum_{\alpha_1} \sum_{x=1}^{k+2} P(x, \alpha_1, 1) \right] \quad (8)$$

$$\bar{x} = \sum_{\alpha_1} \sum_{\alpha_2} \sum_{x=2}^{k+2} (x-2) \cdot P(x, \alpha_1, \alpha_2) \quad (9)$$

(2) Decomposition Method for Systems

The decomposition method is present as follows:

Step.1 Initialize $\mu_i^u = \mu'_i$, $\lambda_i^u = \lambda_i$, $r_i^u = r_i$, $i = 1, 2, \dots, (m-1)$;

$\mu_i^d = \mu'_{i+1}$, $\lambda_i^d = \lambda'_{i+1}$, $r_i^d = r'_{i+1}$, $i = 1, 2, \dots, (m-1)$.

Step.2 Calculate $P_i(k_i+2, 1, 0)$, PR_i^u , $P_i(0, 0, 1)$ and PR_i^d .

Step.3 For $i = 2 : m-1$, update λ_i^u , r_i^u , e_i^u , μ_i^u using Eqs. (10), (11), (12), (13) respectively and calculate $P_i(0, 0, 1)$, PR_i^d in turn.

$$\lambda_i^u = \lambda_i + \frac{P_i(i-1; 0, 0, 1) \cdot r_{i-1}^u \cdot \mu_i^u}{PR_{i-1}^d} \quad (10)$$

$$r_i^u = \left(1 - \frac{P_i(i-1; 0, 0, 1) \cdot r_{i-1}^u \cdot \mu_i^u}{\lambda_i^u \cdot PR_{i-1}^d} \right) \cdot r_i + \frac{P_i(i-1; 0, 0, 1) \cdot r_{i-1}^u \cdot \mu_i^u}{\lambda_i^u \cdot W_{i-1}^d} \cdot r_{i-1}^u \quad (11)$$

$$e_i^u = \frac{r_i^u}{\lambda_i^u + r_i^u} \quad (12)$$

$$\mu_i^u = \frac{1}{e_i^u} \cdot \frac{1}{\frac{1}{e_i \cdot \mu_i} + \frac{1}{PR_{i-1}^d} - \frac{1}{e_{i-1}^d \cdot \mu_{i-1}^d}} \quad (13)$$

Step.4 For $i = m-2 : 1$, update λ_i^d , r_i^d , e_i^d , μ_i^d using Eqs. (14), (15), (16), (17) respectively and calculate $P_i(k_i+2, 1, 0)$, PR_i^u in turn.

$$\lambda_i^d = \lambda_{i+1} + \frac{P_i(i; k_i+2, 1, 0) \cdot r_{i+1}^d \cdot \mu_i^d}{PR_{i+1}^u} \quad (14)$$

$$r_i^d = \left[1 - \frac{P_i(i; k_i+2, 1, 0) \cdot r_{i+1}^d \cdot \mu_i^d}{\lambda_i^d \cdot PR_{i+1}^u} \right] \cdot r_{i+1} + \frac{P_i(i; k_i+2, 1, 0) \cdot r_{i+1}^d \cdot \mu_i^d}{\lambda_i^d \cdot PR_{i+1}^u} \cdot r_{i+1}^d \quad (15)$$

$$e_i^d = \frac{r_i^d}{\lambda_i^d + r_i^d} \quad (16)$$

$$\mu_i^d = \frac{1}{e_i^d} \cdot \frac{1}{\frac{1}{e_{i+1} \cdot \mu_{i+1}} + \frac{1}{PR_{i+1}^u} - \frac{1}{e_{i+1}^u \cdot \mu_{i+1}^u}} \quad (17)$$

Step.5 Loop Step.3 and Step.4 until:

$$\Delta_i = |PR_{i-1}^d - PR_i^u|, i = 2, \dots, (m-1) \quad (18)$$

$$\max(\Delta_i) \leq \varepsilon \quad (19)$$

Where ε is a very small positive number.

Using the decomposition algorithm above, the steady-state availability of each on-line workstation $p_\Delta(i)$ and the average inventory of each on-line buffer k_i can be obtained. The total production rate of the qualified PR_S can be calculated as following:

$$PR_S = PR_{m-1}^d \cdot \varphi_m' \cdot (2-\theta_m) \cdot \theta_m \quad (20)$$

Table 3. Experiment parameters and comparison of results from the analytical and simulation.

Case	m	Workstation with Rework Entrance	r	k_i	$PR_S^{Analytical}$ /min ⁻¹	$PR_S^{Simulation}$ /min ⁻¹	Error/%	CPU Time for the Analytical Approach/s	CPU Time for the Simulation/s
1	7	W_1, W_4, W_7	2	2	38.9245	38.7897	0.35	0.131	72
2	7	W_1, W_4, W_7	2	5	46.9714	45.8159	2.52	0.200	89
3	7	W_1, W_4, W_7	2	10	52.0718	50.6376	2.83	0.277	107
4	7	W_1, W_4, W_7	2	20	55.4617	54.1880	2.35	0.508	135
5	10	W_1, W_4, W_7, W_{10}	3	2	37.7621	37.6469	0.31	0.281	118
6	10	W_1, W_4, W_7, W_{10}	3	5	46.1255	44.9072	2.71	0.315	139
7	10	W_1, W_4, W_7, W_{10}	3	10	51.4470	49.9542	2.99	0.499	159
8	10	W_1, W_4, W_7, W_{10}	3	20	55.1212	53.7916	2.47	0.815	208
9	16	$W_1, W_4, W_7, W_{10}, W_{13}, W_{16}$	5	2	36.5174	36.5740	-0.15	0.468	195
10	16	$W_1, W_4, W_7, W_{10}, W_{13}, W_{16}$	5	5	45.1356	44.0888	2.37	0.819	249
11	16	$W_1, W_4, W_7, W_{10}, W_{13}, W_{16}$	5	10	50.7076	49.4036	2.64	1.233	331
12	16	$W_1, W_4, W_7, W_{10}, W_{13}, W_{16}$	5	20	54.6473	53.4310	2.28	2.018	390

Table 4. Parameters of the Study Case.

Workstation	μ_i /min ⁻¹	λ_i /min ⁻¹	r_i /min ⁻¹	θ_i /%	Workstation	μ_i /min ⁻¹	λ_i /min ⁻¹	r_i /min ⁻¹	θ_i /%
W_1	60/48	1/48371	1/63	99.98	W_9	60/38	1/58422	1/63	99.75
W_2	60/73.3	1/78431	1/45	99.98	W_{10}	60/74	1/78431	1/45	99.98
W_3	60/52	1/78431	1/45	99.98	W_{11}	60/36	1/58422	1/63	99.75
W_4	60/38	1/78431	1/45	99.98	W_{12}	60/48	1/60154	1/61	99.98
W_5	60/36	1/58422	1/63	99.75	W_{13}	60/73	1/78431	1/45	99.98
W_6	60/73	1/78431	1/45	99.98	W_{14}	60/53	1/57719	1/75	99.98
W_7	60/66	1/78431	1/45	99.98	W_{15}	60/65	1/78431	1/45	99.75
W_8	60/62	1/78431	1/45	99.98	W_{16}	60/70	1/57719	1/75	99.75

4. Result and Discussion

4.1. Model Validation

In this section, the accuracy of the approach proposed is investigated numerically. Corresponding simulation models are built by using Plant Simulation Version 12.0. The main parameters of each case are shown in Table 3. The other parameters are as follows:

- Processing rate for each on-line workstation: 60 min⁻¹;
- Failure rate for each on-line workstation: 1/50000 min⁻¹;
- Repair rate for each on-line workstation: 1/50 min⁻¹;
- Qualified rate for each on-line workstation: 99%.

Considering the pseudo-random number which will enable result of the simulation less accurate, according to the GPSN models above, the module of the workstation in the simulation models is rebuilt as Fig. 5 shows.

A personal computer with Intel Core i5 CPU (2.6 GHz) and 12 GB RAM was used to perform the numerical experiments for the analytical approach and simulation. Each simulation was run for 300 days with a warm-up period of 1000 parts time.

In each case, we compare the analytical results with the simulation results. The errors in production rate are calculated using Eqs. (21) and listed in Table 3.

$$error = \frac{PR_S^{Analytical} - PR_S^{Simulation}}{PR_S^{Simulation}} \times 100\% \tag{21}$$

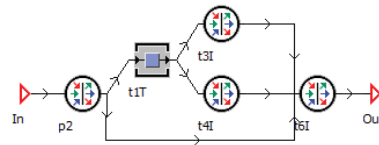


Fig. 5. Construction of workstation model in simulation

As Table 3 shows, most cases result in an error within 3%, which demonstrates that the approach is quite effective and efficient.

4.2. Case Study

In this section, the method proposed has been applied in a planning scheme of a powertrain assembly line to analyze the system performance and to provide guidance for continuous improvement processes. As Fig. 6 shows, main line of the system consists of 16 workstations. To simplify the model view, the off-line buffers and pre-rework stations are not shown in Fig. 6. The main parameters of the line are shown in Table 4. Otherwise, the capacity of each on-line buffer is 2. The production program requires the line has the production capability of 0.75 min⁻¹ in the model of PUSH.

Using the decomposition algorithm above, we evaluate the performance of the system. Table 5 shows the processing rate ϕ_i and the steady-state availability $p_A(i)$ of each workstation. Table 6 shows the average inventory k_i of each buffer.

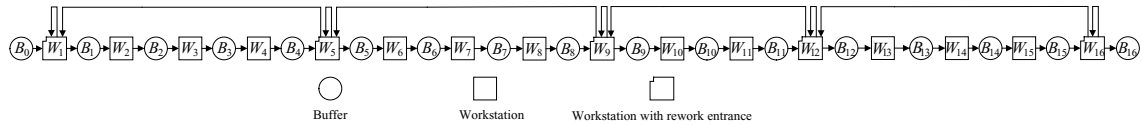


Fig. 6. Layout of the powertrain assembly line

Table 5. The processing ratio φ_i and the steady-state availability $p_A(i)$.

Workstation	φ_i	$p_A(i)$	Workstation	φ_i	$p_A(i)$
W_1	0.9994	0.4857	W_9	0.9967	0.4857
W_2	0.9996	0.7410	W_{10}	0.9975	0.7410
W_3	0.9996	0.5250	W_{11}	0.9998	0.5250
W_4	0.9996	0.6906	W_{12}	0.9944	0.6906
W_5	0.9988	0.3594	W_{13}	0.9973	0.3594
W_6	0.9996	0.7363	W_{14}	0.9973	0.7363
W_7	0.9996	0.6650	W_{15}	0.9996	0.6650
W_8	0.9996	0.6241	W_{16}	0.9971	0.6241

Table 6. The average inventory \bar{k}_i of each buffer.

Buffer	\bar{k}_i	Buffer	\bar{k}_i
B_1	1.2911	B_9	0.6181
B_2	0.6624	B_{10}	0.1992
B_3	0.8246	B_{11}	0.5639
B_4	0.5502	B_{12}	0.8422
B_5	1.0853	B_{13}	0.4410
B_6	0.5867	B_{14}	0.5664
B_7	0.3952	B_{15}	0.3993
B_8	0.2401		

Finally, the estimated qualified product production rate of the system is 0.5995 min^{-1} . The result can not reach the production rate requirement of the production program, which is obtained by the empirical methods. The future work will be directed to the improvements of the system by optimizing the buffer allocation and the rework entrance allocation.

5. Conclusions

This paper proposed an approach to model and evaluate the performance of multistage serial manufacturing systems with rework loops and productive polymorphism. Not only consider machine failures, starvation and blockage, but also pay attention to process defects, multiple rework loops and product polymorphism. To characterize the complex state transition resulting from the conversion of internal material flows, a model based on GSPN is presented. After analyzing the process differences resulting from rework loops and products polymorphism, A decomposition approach using 2M1B model as the building blocks is then developed. The accuracy of the approach is justified by numerical examples. In addition, this approach has been applied to a powertrain assembly line to evaluate the performance for the continuous improvements.

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