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From real to virtual globe: new technologies for digital cartographic representation

Summary
This work starts from the observation that today cartographic globes, present in almost every museum and art collection, are deprived of part of their value. They are actually considered as art objects, meant to decorate the halls of historical buildings and now relegated in museums, without recognizing the cartographic value that they deserve. From this consideration we want to investigate the different aspects of these cartographic spheres. We intend to improve the knowledge of their cartographic value under the double aspect of the material support and the depicted (or applied) cartography.

Introduction

Studying the globes, the significance of the disciplines that study the shape and representation of the Earth becomes apparent from the methodical point of view as well as for the application of new digital technologies. Indeed, in some sections of this work it will become clear that, as a consequence of the necessary approximation and scale differences, the representation and study systems recall those used for the Earth. Some aspects and problems typical of Geodesy will also be highlighted, such as those related to coordinate systems and reference surfaces, even if applied to cartographic globes. Furthermore, the globes represent the exemplification of the problem known since ancient times of unrolling a double curvature surface on a plan. Other than studying their shape, it will also be important to recognize their cartographic content.

We intend to improve the knowledge of their cartographic value under the double aspect of the material support and the depicted (or applied) cartography. As for geometry, we intend to verify if the wooden support was built according to an ideal spherical shape or if some adjustments were planned to resemble the actual shape, even if it is clearly impossible to consider them as physical models of the geoid. The central focus becomes therefore the transition from the sphere as an art object to the globe, intended as cartography to all intents and purposes. All these analyses allow defining a virtual sphere made of two different aspects: the geometry of the wooden support and the cartography extracted directly from the globe. Tests and analyses have been conducted on different globes, both originals (dating back to the end of XVII century) and current reproductions. In particular, the printed globes of Coronelli have been studied under the cartographic, projective and semantic aspects, as they are easily accessible at the Correr Museum and the Biblioteca Nazionale Marciana of Venice, but mainly because they represent the most common and documented cartographic globes, of which we have whole examples and the cartographic materials used in their assembly. The method tested on Coronelli Globe is going to be applied on the cartographic globe of Sanudo.

The research work has been grouped in different steps:
- survey of the geometrical shape of the wooden sphere;
- survey of the cartographic surface from the globe;
- projective content analysis.

The survey of the geometrical shape

The first part of the work consists in the survey of the sphere, typically a wooden structure covered with fabric, stucco and other materials, representing the physical support for the cartographic surface. The knowledge

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of the three-dimensional shape is undoubtedly research topic of high interest and, consequently, it is studied in different fields, from industry (in which reverse engineering emerges) to cultural heritage. First of all, the knowledge of the shape is necessary to understand if there was a project held under the generic term of sphere. In fact, it will be interesting to understand if the globe has a precisely round shape or not, and possibly to study, by representing deformations, the differences from the ideal shape, trying to attribute it to the project or to the effects of usage and time. Among current analyses, many aim at evaluating, processing and developing dimensional survey techniques and 3D rebuilding of objects and surfaces, with special attention to the metrological aspects of the problem, by using current improvements in computer science, optoelectronics and optical techniques to study Cultural Heritage. The problems about surveying the shape of a wooden sphere require an accurate selection of the measuring method, in order to address the problems due to the working environment and the object under study itself. The selection of the survey method should also consider the expected result itself, from a qualitative and quantitative point of view. The point cloud, which seems to be the survey goal, can be obtained in different ways and have different characteristics. It is important to establish the number of points (in terms of accuracy on the object), but most of all the precision in terms of XYZ coordinates of each point, in order to obtain an accurate estimate of the final sphere.

Different methods and instruments can be used for these analyses and each one of them allows achieving different results. From our tests the most suitable instruments are the scanner 3d (or range camera) based on the triangulation principle. Generally, these instruments consist in a known light source and a sensor, closely linked together. The light source, of different kinds depending on the instrument used, emits a laser stripe recorded by a sensor, for example a digital camera. With this method source centre, projection centre on the sensor and laser dot projected on the object form a triangle, with some known elements (distance between light source, sensor and the angle of the laser emitter) and others that can be calculated (position of the reflected point).
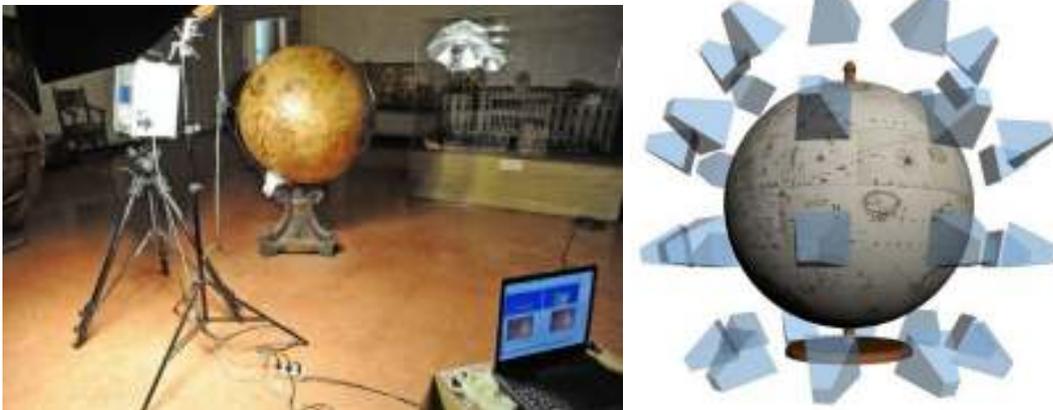

Figure 1. Acquisition set at Museo Correr and acquisition plan of 3d range camera

Triangulation acquisition systems have basically two advantages. In these active measurements systems the instrument acquires the studied surface automatically, provided that there is enough light, thus reducing acquisition time and the operating procedures required. The other advantage is the measurement quality. The resolution (i.e. the point density on the surface) and precision level are strictly related to the instrument used; we can easily reach resolutions of 1/10 mm and the instrumental uncertainty down to 25ηm. These values directly affect the resulting model and therefore the description of the three-dimensional shape recorded. Moreover range cameras do not require neither the application of positioning targets on the surveyed object, nor the materialization of a reference system and allow the simultaneous acquisition of geometric and radiometric information. The acquisition phases are not significantly different from other applications, for which an extended literature is available.

In the case of the cartographic globe of Coronelli, at Correr Museum, we used the Vivid 910 laser scanner, by Minolta. The choice was made by experimental tests, which showed a better operating mode compared to other instruments. The difference is due to the particular finishing of the globe's surface, which implies a high reflectivity of the sphere. In fact, the varnish is highly reflective of the light emitted by the projectors of pattern instruments, but did not create any problem with the thin light beam emitted by Vivid 910.

The globe was acquired using a Wide lens at a distance of ca. 70 cm from the globe's surface. Even though it would have been possible to increase this distance to obtain a larger acquisition area, we chose a 70 cm operating distance to guarantee the object resolution required to read the semantic content of the cartography.
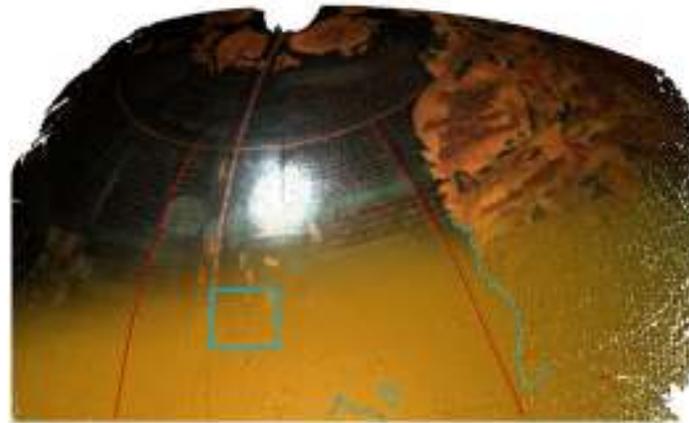

Figure 2. Surface obtained triangulating the point cloud. Physical discontinuities are evident (red) as well as those caused by abrupt luminosity transitions on the globe surface (blue). Behaim Globe.

The bigger sphere and the use of the Wide lens at a distance of 70 cm led to an increase in the number of acquisition belts. We acquired 6 longitudinal stripes as well and the two Polar icecaps for a total of 62 scans.

In the step of data processing by observing the triangulated mesh of a single cloud, we will notice that the surface is not smooth, but has features, i.e. discontinuities caused by various factors. In particular, scanners detected some discontinuities in overlapping areas of two gores, and there is actually a variation compared to the general curvature. However, all scans highlight other discontinuities that are not actually visible on the object.

After accurate observation, it appears clear that these discontinuities correspond to cartographic zones where the transition from bright to dark elements is abrupt. In this example, it results clear that the coast line is a three-dimensional element, because of the transition from bright land to dark water. In the middle of the ocean, we can also notice islands in relief and, in the figure, the corrugation effect is clearly due to the presence of bright writings on a dark background. This finding allows on one hand an approximate reading of the cartographic surface starting from the scan, even before we start using the textures acquired. However, discontinuities, both physical and real, are significant, because they characterize univocally part of the globe, i.e. the scanned spherical cap, that would otherwise have been difficult to record.

After this observation, globe data processing keeps on with the known pipeline which schedules the registration of all range maps in a single reference system. Between different strategies of registration, for globes we used the cloud on cloud registration method by the ICP (Iterative Closest Point) algorithm. This approach has been preferred thanks to the presence of discontinuities on the acquired surface and the RGB value of the range map.

At the end of data processing the final result is a 3d model of the globe with high accuracy. From the model we can extract horizontal and vertical profiles to verify the dimensions of the wooden sphere. However, the most interesting aspect is the possibility of studying the shape of the globe and its possible deformations. This analysis can be conducted starting from the deformations of the model generated by the laser scanner compared to the best interpolating sphere. The first step consists in determining the best fitting sphere, achievable

with reverse engineering software as Rapidform, Geomagic and others, which can provide information on the sphere and also show the globe deformations, using a radiometric scale.
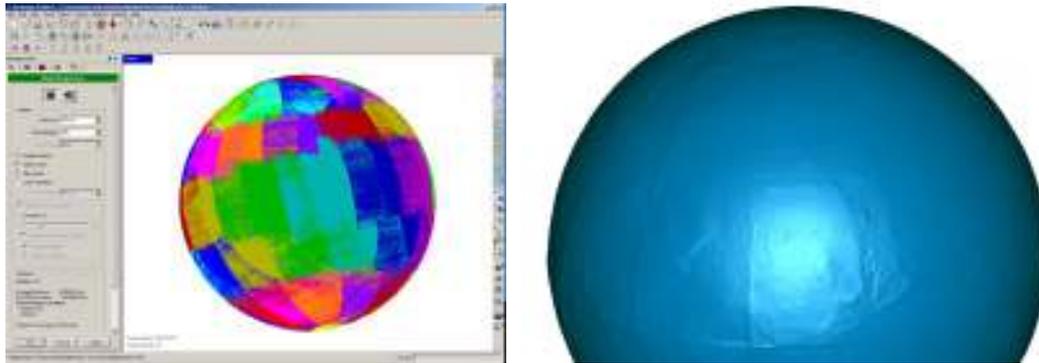

Figure 3. Registration of range maps in a single Reference System. Each range map is represented in a different colour.
Figure 4. Final model of Coronelli globe, shaded representation, with some evident physical features such as meridians, Equator…

For the same kind of analysis software was developed, on purpose, that calculates the interpolating sphere as well as producing a plane representation of deformations present on the sphere. The choice of writing an *ad hoc* program was due to the necessity of verifying the results achievable using the most common commercial software. Moreover, by writing this and the following software, we obtained an application specific to cartographic globes, capable of highlighting significant aspects of the sphere under study such as center, radius, curvature, deformations, residuals, etc.

In the software "Sphere parameters" to calculate the interpolating sphere we used the Cartesian equation of the spherical surface:

$$x^2+y^2+z^2+ax+by+cz+d=0$$

where a, b, c, d are real numbers, so that $a^2+b^2+c^2-4d >0$ because it is already linear compared to the unknown elements of the problem, such as a, b, c, d.

Given a geometric reference system, the sphere solution comes from:

$$x_c = -\frac{a}{2}$$

$$y_c = -\frac{b}{2}$$

$$x_c = -\frac{c}{2}$$

$$R_0 = \sqrt{a^2 + b^2 + c^2 - 4d}$$

The software calculates the least squares solution to obtain an estimate of the best interpolating sphere.

Now we can determine the globe's deformation compared to the ideal sphere, according to the classical cartographic problem of representing the equipotential surface of a geoid compared to the reference surface of the ellipsoid. In this case, the reference surface is further simplified and corresponds to the interpolating sphere previously calculated.
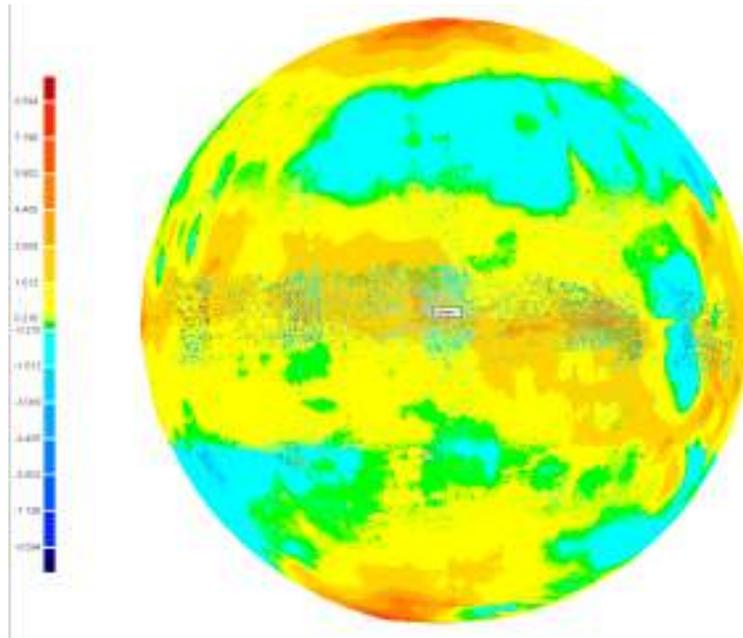
Figure 5. Representation of model's deformations compared to the best fitting sphere: the globe of Coronelli. maximum deformations [mm] are located in the polar zone and near the equator.

Specifically, in the software "Points", written for this application, deformation is expressed by means of an m x n matrix, with m = longitude e n = latitude, where the value in each single cell derives from the difference between the ray of the interpolating sphere and the ray calculated for that point. To obtain the matrix we choose, in order to simplify calculations, to perform a translation so that the sphere's centre has coordinates (0,0,0). Subsequently, each point of the model has been transformed from Cartesian to spherical coordinates using the formulas:

$$\lambda = arctg\left(\frac{y}{x}\right)$$

$$\varphi = arctg\left(\frac{z}{\sqrt{x^2 + y^2}}\right)$$

$$R = \sqrt{x^2 + by^2 + z^2}$$

Then we generate the matrix with values λ and φ as row and column indexes, while we enter in the cell the difference between the radius Rxyz (distance between the centre of the best fit sphere and the point P of coordinates x, y, z) and Rsph radius of the best fit sphere. The matrix is finally interpolated in Surfer with Kriging's algorithm and then represented according to an equirectangular projection. Therefore, we obtain false colour representations, or contour line representations, that we can superimpose on the globe's cartography to distinguish which parts of the cartography show the biggest deformation, due to the modifications in the wooden support.

The representation of the deformation obtained with this method provides information about the position of the main deformations regarding both the module (difference between the calculated radius and R0) and the direction of the deformation, outwards or toward the centre of the sphere itself. However, the three-dimensional visualization of this effect is limited by the unfolding on the plane of the calculated deformations. To allow a better understanding of the deformations, we wrote another application which generates a three-dimensional representation of the deformed sphere, following the example of the classical representations of the geoid. In this case, the coordinates of each point of the surveyed surface represent the starting point. By applying the formulas previously seen, each point has been transformed into spherical coordinates λ and φ and

the radius Rxyz where radius is the distance between the centre of the best fit sphere and the point P of coordinates x,y,z.
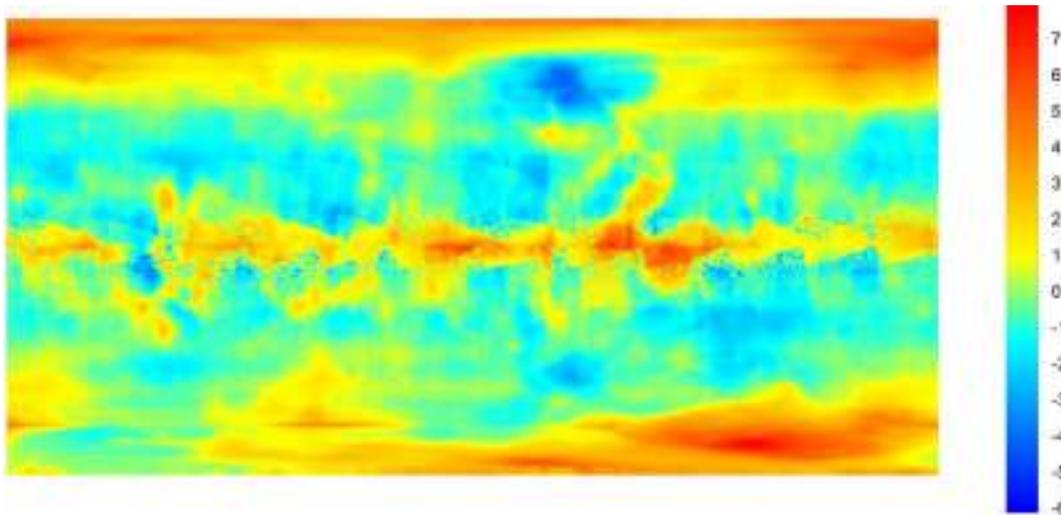

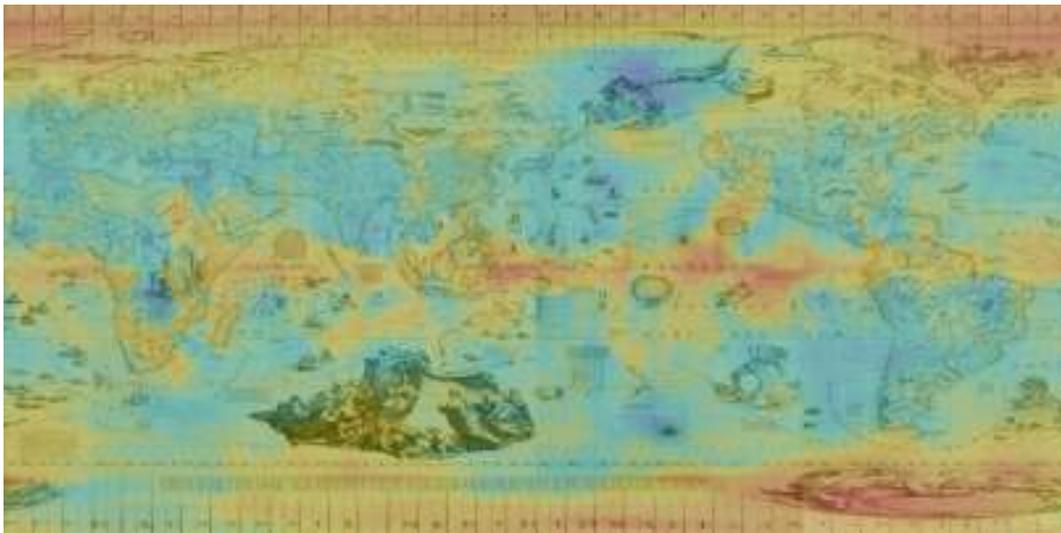Figure 6. Deformations of the Coronelli globe. Representation by false colour in a equirectangular projection.

Figure 7. Overlapping cartography with the map of deformation.

Then we calculate the difference between R_{xyz} and the radius R_{sph} of the best fit sphere to found Δr.

$$R_{xyz} - R_{sph} = \Delta r$$

Then the value Δr is amplified for a scale factor S, so:

$$R_{potato} = R_{sph} + \Delta r_{new}$$

so each point P is represented by $\lambda\ \varphi\ \Delta r_{new}$ and the it is transformed in $P_{xyz\ potato}$.
The new point cloud, realized by the software, can be imported in Geomagic or Rapidform to have the possibility of visualize and exploration. In this way, we obtain the equivalent of the classical cartographic representations, where the Z value is enhanced by a known factor. In cartography these are the so called two-and-a-half-dimensional representations, while the deformed model obtained is definitely three-dimensional. It can be viewed in the 3D space and we can use all spatial navigation operations typical of three-dimensional models.
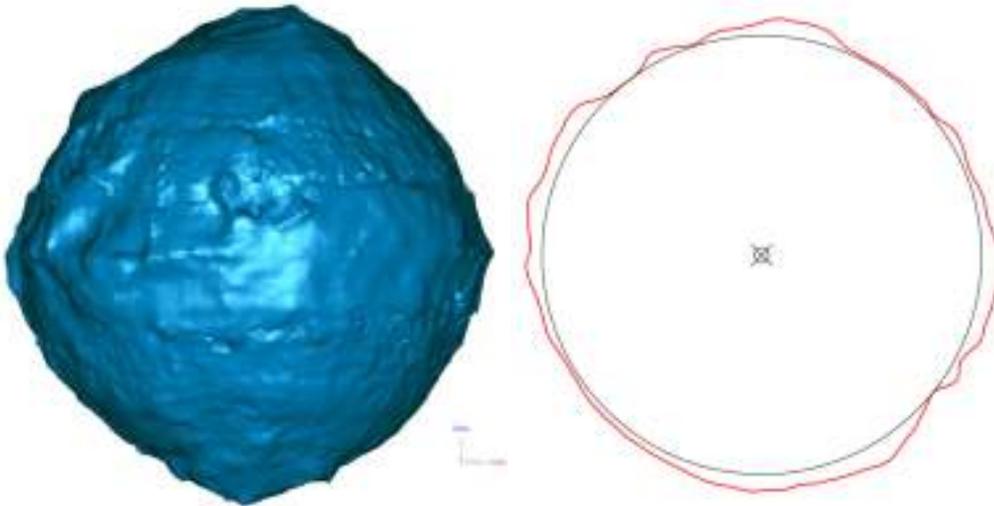
Figure 8. 3D representation of the globe with the exaggeration of deformation (exaggeration factor 10x)
Figure 9. Slice in plane XY from the model with the exaggeration of deformation (exaggeration factor 10x), top view. The black circle is referred to the best fit sphere of original surveyed data, the red line represents the slice of the exaggerated.

Moreover it's possible to attribute a colour to each point function of the real RGB value of the point (from the cartographic surface) or of the dimension of Δrnew. There is also the possibility to obtain some slices to represent the punctual value of deformation. This operation cannot be done on the real sphere because deformations have a little value and it is not possible to apply the exaggeration only to the value of deformation. Instead it is possible to apply the scale factor to spherical coordinates because we can amplify only the radius value.

The wooden sphere of Coronelli Globe, as we saw in the previous representations, shows many deformations with a maximum difference from the best fitting sphere of 7 mm. The deformations of this sphere can be divided into 2 groups. In the first group we found the ones sited in specific part of the sphere such as the Poles and the Equator, while in the other group the position is random. For the second kind of deformation, we can say that they are connected with incidents of the globe and with the attempt of restoration. Instead near Poles and Equator, the reasons of deformation are structural and they are strictly connected with the building methods of the wooden support.

The survey of the cartographic surface

Once we determined the shape of the wooden sphere, the next step is the survey of the cartographic surface. Our goal is to obtain an exhaustive knowledge of the cartographic globe, by assigning a cartographic content to the wooden support. The first step of the procedure, as shown in the flowchart, is photographic acquisition. This operation, apparently free of issues, encountered many difficulties due to the specific characteristics of the object. First, obstacles are due to the shape of the spherical object and to the operating mechanism. Often the equatorial areas and the poles are difficult to acquire as a consequence of the presence of the wooden support structure as well as the axis around which the globe rotates. Moreover, the poor conservation status may prevent the globe's rotation, while keeping the camera steady on the tripod. We have to resort to inconvenient shooting positions, made more awkward by the long acquisition time required in low light conditions.

Another problem encountered during photographic acquisition was related to the reflective surface of the globe itself. The surface is often in bad conditions and the covering varnish acts as a mirror, causing annoying reflections in the photographs, which make them useless for the map extraction process. Moreover, the reflective surface makes the use of photographic lighting equipment more complex due to glares in the picture so it is necessary to use more sophisticated acquiring system (polarized light: lamps and filter).
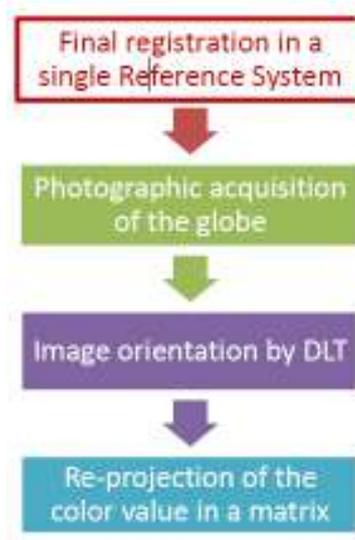
Figure 10. Photographic acquisition of the globe with colorimetric scale.
Figure 11. Photographic set for the globe.

The Coronelli globe to survey has been chosen to minimize all problems due to the external wooden structure, because the central axis of the globe is fixed to a metal stand.

For the photographic acquisition we used a Nikon D700 with 35 mm lens and a Rollei 6008 with a digital back of 39 Megapixel and with 80 mm lens. To reduce the glares, the images have been acquired with polarized filters on the lens and on the lamps. The format of the pictures is jpg and raw, to allow their radiometric processing for the white balance correction and to obtain homogeneous results.

Proceeding with the proposal process, the next step is the orientation of film frames. The goal of this phase is to position film frames where they were the picture was taken. We used the software NM3-digit based on D.L.T. (Direct Linear Transformation) realized by the Photogrammetry laboratory of CIRCE IUAV some years ago. This method presents the great advantage of solving the internal and external orientation, as seen above, of images acquired with non-metric digital cameras. It is however necessary to follow some indications related to the number of control points acquired and to their placement. In fact, for the algorithm to generate an accurate convergence, it is necessary to acquire more three-dimensional points in order to estimate the parameters (figure 12). We should also remember that the algorithm is sensible to the spatial distribution of the points, so it is essential that the points are correctly distributed around the object and that they lie on different planes. When applying the D.L.T. to the survey of the globes' surface, we want to identify the coordinates of the camera center, needed in the following phases of the analysis. To orient the film frames, the control points have been acquired directly from the range map sets oriented in a single reference system, by recognizing the point on the image through the RGB colour values of each single point. It is important to choose those points which allow to have a strongly 3d distribution.

Figure 12. Image to be oriented, with indication of the control points used and the DLT parameters calculated using NM3-digit software

In the last phase of this process, we move from the spherical surface to a known projection, considering the well known cartographic problem of the projection of a double curve surface. Like for the Earth's surface, the link between a point on the surface and one on the map is not immediate, but it is solved by means of a reference surface of known and simplified geometry. When we consider the Earth, the surface used is the rotational ellipsoid, while when we consider cartographic globes the geometrical shape used is the sphere. This approximation is introduced to bypass the deformation of the wooden sphere during the re-projection of the cartographic sphere. The suggested method aims to obtain a digital picture positioned in a given cartographic projection. By writing the software, we obtained a custom product that can be used on the cartographic globes. At the same time, we implemented and tested an algorithm specific for the sphere. Combined with the application described above, we created a software package useful to the examination of the globes under study. For this application we wrote custom software to associate the spherical coordinates, describing the sphere's points, to the pixels of the oriented digital image. The algorithm creates an empty image (matrix), where its coordinates (φ, λ), are the spherical coordinates of the model. As shown in the scheme, for a better understanding the software can be subdivided in smaller modules. In the first module, we create the reference surface. By using the classic sphere formula and determining the angular step in latitude and longitude, it is possible to reconstruct a sphere of known center and radius, according to the formulas that allow transforming spherical coordinates into Cartesian coordinates:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R\cos\phi \begin{bmatrix} \cos\lambda \\ \sin\lambda \\ \tan\phi \end{bmatrix}$$

where φ = vertical angle (Latitude) and λ = horizontal angle (Longitude). In the second module, we calculate the correspondence between each point of the reference system and the pixels of the oriented image. The equations are those used by the D.L.T. algorithm, which, once given the parameters calculated in the previous phase and object coordinates of the point, allow the identification of the corresponding pixel.

$$xi = \frac{b_1 x + b_2 y + b_4 z + b_4}{b_9 x + b_{10} y + b_{11} z + 1} \qquad yi = \frac{b_5 x + b_6 y + b_7 z + b_8}{b_9 x + b_{10} y + b_{11} z + 1}$$

Once we identified the pixel, it is possible to extract its RGB value. However, as shown in the diagram, each pixel of the image can correspond to two distinct spatial points, because the sphere is a round shaped object. To avoid this effect, we searched for an empirical method to place the color extracted from the pixel onto the precise corresponding point. The solution consists in introducing a point P between C and O, defining the distance CP. The constraint imposed is the distance CP, which has to be always less or equal to the distance OC, where OC is the segment that links the camera center to the sphere center.
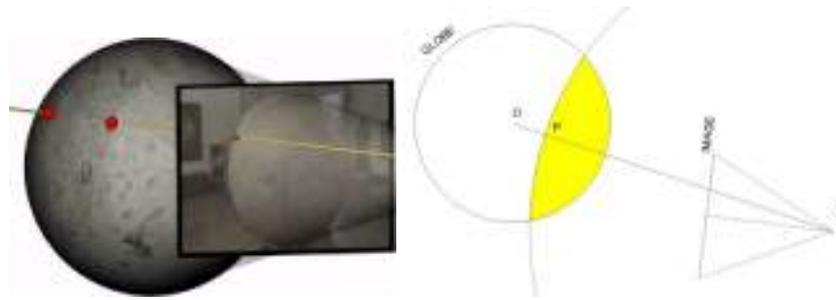
Figure: 13 Schematic representation of the point correspondence problem. For each image point there may be up to two matching points on the sphere.

Figure: 14 Functioning scheme of the empirical constraint imposed by software. The yellow area corresponds to the area selected using the CP value.

By applying this constraint, the surface implied in the picture corresponds to a spherical cap at the most. First, the software calculates the CP value and, subsequently, it allows modifying it. In fact, to avoid the stretching effect of pixels corresponding to the sphere's outermost areas (where CP is closer to CO), we can decrease the CP value with this software. Finally, in the third module, we build the final matrix. On a grid characterized by the angular step set in the first part of the procedure, we insert the RGB color values extracted from the pixels corresponding to the surface points. If during the computation there is no correspondence between an image point and a globe point, the corresponding cell in the matrix is filled with a constant value (black). This situation happens regularly in the areas of the image where we do not see the globe, but the surrounding environment. The resulting matrix can be imported in Matlab and therefore reprojected, choosing each time the cartographic projection most suitable for our representation requirements. At the same time, the software creates an image in a known projection, the equirectangular one, where the longitudinal degree is equal to the latitudinal degree for each point. Of course, the rows and columns size of the matrix corresponds to the dimensions of the digital image and is determined by the angular step chosen. For example, if we choose a angular step equal to 0. 1°, the matrix and the finale image will have the following sizes: 3600 pixel (360°/0.1°) in width-longitude and 1800 (180°/0.1°) in height-latitude. To obtain the final cartography, the last phase is image composition. If the images are realized with the same angular step, there is no necessity of mosaicking because they have the same dimension, and so the same reference system. Simply it's necessary cut images and t realize a radiometric correction.
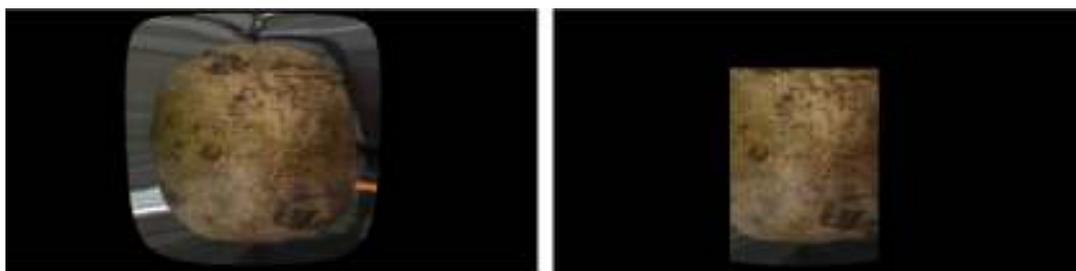

Figure 15: Single re-projected image, without any correction.

Figure 16: Single re-projected image with limit of computing.

The images show some step of the workflow. First of all a preview of the result (figure 15) where it's evident the deformation of the spherical shape projected in equirectangular projection. The second step is a selection of the area to compute. We can select the area defining a window by λ φ coordinate.

Even if we choose to compute a little part of the initial image, the dimensions of the final one don't change because they are set by the angular step. In this way the pixel is always referenced in the same reference system of the image which corresponds to the geographical one. The other images represent different step of the composition in only one image.

Figure 17: fusion of different re-projected images.

Figure 18: Detail of a reprojected photo: angular step 0.05 degree in longitude and latitude.

The final map shows only some problems in the polar are where meridians are not exactly parallel, but convergent. We have also to notice that the equirectangular projection is not suitable to represent the polar area. The final map obtained from the globe represents a very interest document because it contains all cartographic contents - semantic, projective, metric... - , but there are also many information connected with the story of the globe as a material object. This document becomes really important when we don't have any other source for the study of globes (such as printed gores like Coronelli book of Gores). The high resolution (figure

18) we can reach in the re-projection step allows us to analyze not only the main feature, but also the details, writings and other themes. Moreover the availability of a flat map of the spherical surface allows many other studies and comparison with different maps. At the same time, this kind of map is a document of history for the globe with all the defects and incidents. We have to pay attention to the deterioration of the surface which involves a degradation of the final map. Some examples are referred to the effect of light on the ink pigment but also the effects of woodworms which destroy the wooden support.

Analysis of the projective content

The study of the projective content of the historic cartography, which forms the subject of this chapter, can be analyzed from several points of view and it is strictly related to the historical period when the object was made. In the second half of the XVII century, Coronelli did not use an encoded and known projective system in his work. For this reason, the analysis of the projective content has been split in two aspects. First, we referred to his writings, both theoretical and practical, and to the documentation he left, and then we analyzed the gores forming the globe. Coronelli's "Cosmographic Epitome", published in 1693 and dedicated to the "Academy of the Argonauts", the first geographic institution in the world, contains a summary of: astronomy and celestial cartography in the first book; Earth and geography in the second book; and in the third book, "the Description of the different globes, that have been built so far" "and the practice" of the globe manufacture. We can also refer to the "Book of the Globes", on display at the Marciana National Library of Venice and edited in 1706. The Venetian monk, considering the popularity of his printed globes, published all the gores necessary to cover the wooden spheres in a collection. In this way, it was possible to ship to the client not the whole cartographic sphere, but simply the cartography to apply to the sphere and the instructions to assemble it. In the book, the globe with a 3.5 foot diameter (about 108 cm) is divided into 48 sheets for the gores, plus two sheets for the Polar icecaps. Each gore, 30° longitude and 140° latitude (-70° to +70°), is divided into 4 parts along the equatorial and tropical lines.

According to Coronelli's writings, the geometric construction does not take into account cartographic projections, as they are understood nowadays, but follows a geometric scheme. According to Coronelli's writings, the geometric construction does not take into account cartographic projections, as they are understood nowadays, but follows a geometric scheme. The mapmaker mentions the use of a modified geometric construction expounded by Henricus Glareanus in 1527 in the XIX chapter entitled "De inducenda papiro in globo" of his "D. Enrici Glareani poetae 16aureate de Geographia liber unus". The method consists of drawing a line equal to two and a half times the equator, dividing the line in 25 parts and using the 12 central parts to build the gores. The problems in building the gores relate to the identification of the external lines, representing the outermost semi-meridians of the gore. Both Glareanus and Coronelli solve the problem by choosing the shape of the circular arc and positioning the center on the prolongation of the smallest diameters. In particular, the circumference used has a radius equal to 10 gores, corresponding to 300° in longitude.

According to this geometric construction, as observed by Fiorini in 1894, it is clear that the lenticular (almond) shape does not correspond to the gore. In practice, to obtain an adequate coverage of the globe, the gore's paper sheet was dampened with glue and stretched to cover the semi-meridian, therefore producing further deformations in the cartography. If, as seen above, it is not possible to identify a known projection in the author's writings, at least it is possible to understand the projective genesis of the map. According to Tobler's [1966] approach, the problem is the investigation of a projective system that was probably in place, but is not one of the systems we know. A possible solution is the comparison between the map under study and other known reference models, according to the work presented by Boutoura [2006] and Balletti [2000] for portolan maps. For globes in particular, the comparison takes place between the transforms of the gore drawn by Coronelli and those of the cartographic projection chosen. In the specific case of Coronelli's gores, the procedure is simplified by the possibility to use the geographic grid of the map, instead of its content, to perform the comparisons. The comparison between the gore sector and the reference grid is carried out by calculating the differences between the observed and calculated coordinates of matching points.

The search for similarity consists in applying a global geometric transformation to the gore sector and the projected grid, testing in sequence the grid's "adaptation" (best fitting) to the map under projective study.
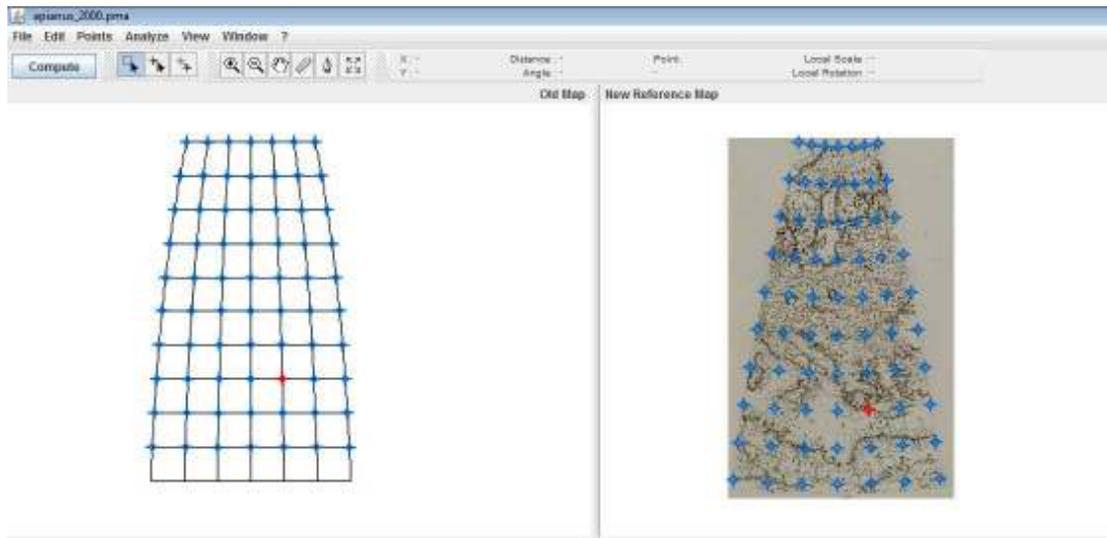

Figure 19. Screenshot of MapAnalyst software. We notice the historical gore (left) and the known projection (right) with the linked grid points.

The best fitting is assessed based on the residual values calculated after the deformations induced by the applied model. Depending on the results, we can define the projective system that better matches the historical map. To avoid systematic errors, it is necessary to choose the reticulate projection models carefully and then to identify a sufficient number of points with a homogeneous distribution. The analysis of the cartographic grid is carried out by generating a chart, which allows a clear view of the results obtained. We can also avoid any problems related to the choice of the reference systems. The same analysis, conducted analytically, of comparing the coordinates of different cartographic projections, implies the knowledge of the source of the reference system. The comparison has been performed using MapAnalyst software, which allows us to compare a modern map to an historical map. This software allows us to link the homologue points of the two maps and to perform a global geometric transformation in order to calculate the best fitting surface. The geometric transformation we used is Helmert transformation (roto-translation with change of scale), which is conformal and does not deform the map. We can view the differences between the two maps in several modes, but the one that better represents the similarity between them is the one used by Tobler [1986], which shows the deformation vectors of the gore grid compared to the known system. Obviously, bigger deformations correspond to vectors with a greater module.

The cartographic projections selected for the comparison with Coronelli's gore are:

- Transverse Apianus projection;
- Mollweide projection;
- Cassini projection;
- Transverse Mercator projection.

The first two are pseudocylindrical (or "false cylindrical"), while the third and the forth are cylindrical projections. The analyzed gore corresponds to an area of 30° in longitude and about 50° in latitude (from 20° N to 70°N), centered on its central meridian. The reference grids have been developed with Matlab software, in order to generate for each projection a gore with the same geographic extension of Coronelli's gore. Once all comparisons have been performed, it appears clear that the cylindrical transverse projections are the most similar to the projective system used by the author.
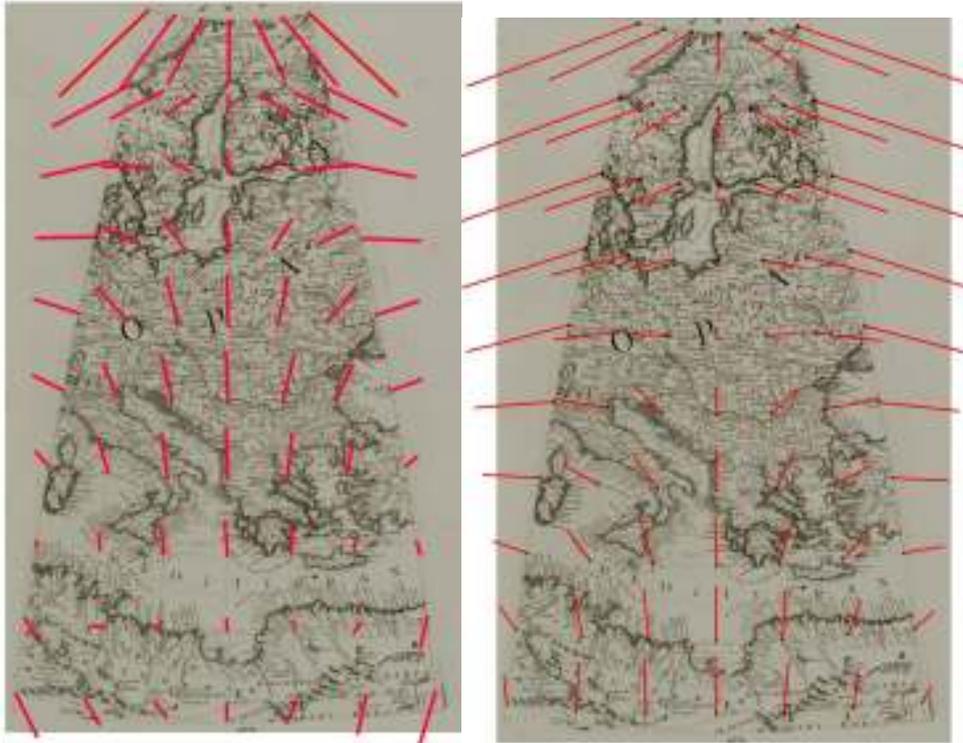
Figure 20_a: Comparison with transverse Apianus projection. Vectors are drawn 3x
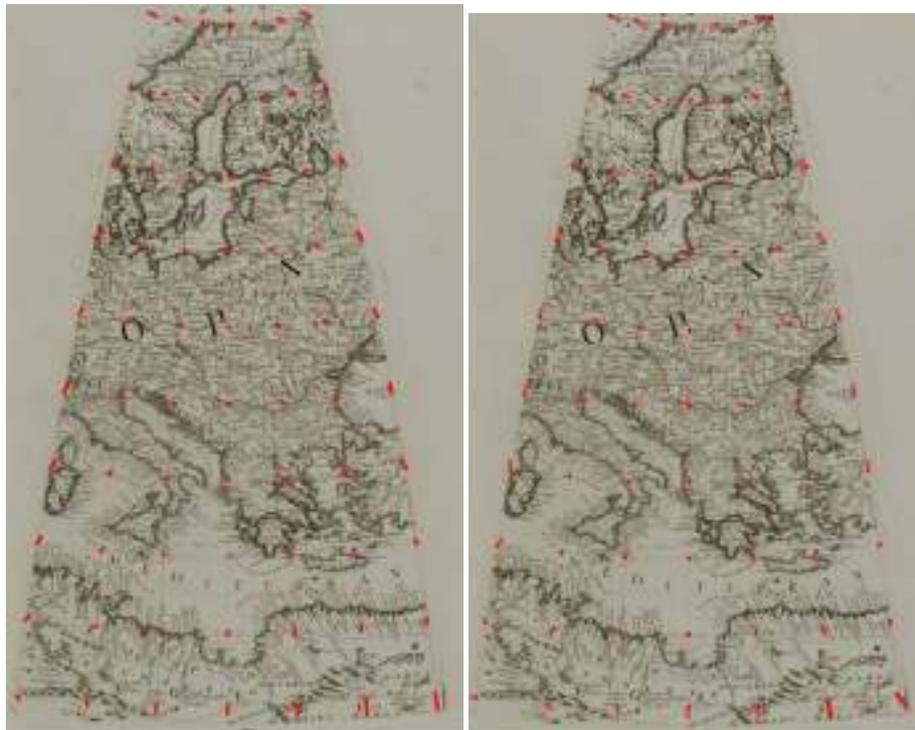Figure 20_b: Comparison with Mollweide projection. Vectors are drawn 3x

Figure 20_c: Comparison with Cassini projection. Vectors are drawn 3x.
Figure 20_d: Comparison with Transverse Mercartor projection. Vectors are drawn 3x.
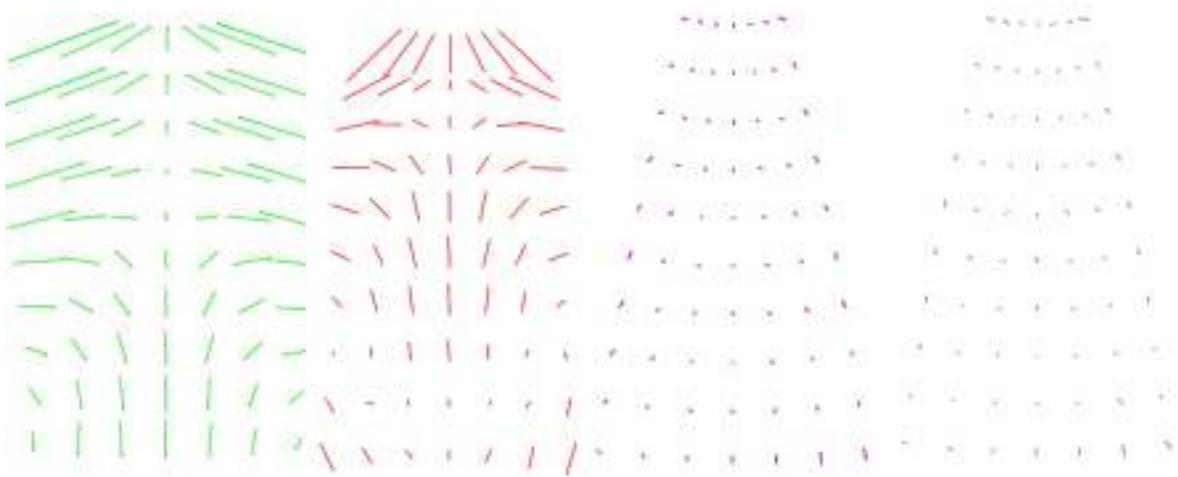
Figure 21. Synoptic comparison of the displacement vectors of the gore representations analyzed. From the left: Apianus, Mollweide, Cassini, Transverse Mercator.

Even if it is not possible to fully define the projection characteristics, considering the translation vectors from the historic map to the reference grid, we can state that Coronelli chose to represent the gores by using a map that was very similar to a Transverse Mercator projection. After defining the most correspondent projective system of Coronelli gores it is possible a new comparison between the original sheet of Coronelli, the re-projected images from cartographic surface of globe and present situation.

In the image 22_a is represented the sheet 50 of the Book of Globe of Vincenzo Coronelli. Particularly it is part of a gore with a dimension of 30° in longitude and 23°27 in latitude, corresponding with the Tropic.

The following image of gore, 22_b, is obtained using the matrix produced by the software in the re-projection step of an image. The gore is represented in the projection which best approximates the original gores as showed in the previous paragraphs (transverse Mercator). The third representation is an actual gore in the same projection. The comparison shows, first of all, the difference between the source of globe (printed gores) and the real globe. It is evident that the surface of real globe is damaged and this aspect does not allow an easy reading of the map. It is also interesting to compare the different coordinates of the same area between the first and second gore with the third one as it can be studied by a digital analysis. But this comparison is more significant because it represents like a closure in the circle of survey of the sphere and study of map. We started from gores as sources of the globe. At the end we arrived to the same gores in a digital form by another way: the survey of the wooden sphere, the acquisition of surface and the re-projection of images.

Conclusions

This work aims to present cartographic globes not only as an art object, but also to give them their role as a cartographic instrument. In the different steps of the research the only common aspect is the digital approach to data, both 3d coordinates, projective and semantic content.

This common denominator allows keeping on the analysis not only on Coronelli Globe, but also on other different spheres in order to realize a standard procedure for their survey. The digital approach, moreover, lead to a better access to historical globe, not only in the material shape, but also as a virtual copy, to spread on the net and to be available to scholars, cartography and map experts, and to the curiosity of the general public. The next step to promote the knowledge of the globes will be their online publication, to allow wider access, free from physic constraints. We could define this opportunity as sort of a virtual "globe-library", which would allow accessing a database containing the cartographies of the globes mapped on a spherical surface for 3D navigation.

Figure 22_b: Gore obtained in Matlab from the matrix of image re-projected. Transverse Mercator projection.

Figure 22_b: Gore obtained in Matlab from the matrix of image re-projected. Transverse Mercator projection.

Figure 22_c: Gore of actual world map. Transverse Mercator projection.

Bibliography

Adami A., Guerra F, 2008. Coronelli's Virtual Globe, *e-Perimetron,* Vol. 3, No. 4, 243-250.

Adami A., Fregonese L., Guerra F., Livieratos E., Tsioukas V., 2007. "Digital representations and analysis of deformations induced in map supporting materials". In Proceedings of CIPA conference 2007, Athens.

Balletti C., 2000. "Metodi analitici e quantitativi per lo studio del contenuto geometrico delle carte storiche"*, Tesi di dottorato in Scienze geodetiche e topografiche"*, XIII ciclo, Politecnico di Milano, Milano.;

Balletti C., Guerra F., 1998. "NM3 digit: the DLT algorithm in a digital photogrammetrical software for the survey teaching". The International Archives of Photogrammetry and Remote Sensing, Volume XXXII, Part 6W4, ISPRS WG VI/3 International Cooperation and Technology Transfer, Perugia.

Beraldin J.P., Guidi G., 2004. *Acquisizione 3D e modellazione poligonale*. Polidesign: Milano.

Bitelli, G., 2002. "Moderne tecniche e strumentazioni per il rilievo dei Beni Culturali". In Proceedings of 6° Conferenza Nazionale ASITA, Vol.1, IX-XXIV, Perugia;

Boutoura C. 2006. Assigning map projections to portolan maps, *e-Perimetron,* Vol. 1, No. 1, 40-50.

Boutoura C., Livieratos E., 2006. Some fundamentals for the study of the geometry of early maps by comparative methods*, e-Perimetron*, Vol. 1, No. 1, 60-70.

Fiorini M., 1881. *Le Projezioni delle Carte Geograftche,* Zanichelli: Bologna.

Fotou A., Livieratos E., Lombardini G:, Paraschakis I., 1991. Dome representation using photogrammetrically derived data and best fitting techniques. *ISPRS J.Photogramm. remote Sensing*, 46.

Guerra F., 1999. "Trasformazioni geometriche locali: trasformazione versus corrispondenza", Tesi di dottorato in Scienze geodetiche e topografiche"*,* XII ciclo, tutor prof. C. Monti, Politecnico di Milano, Milano.

Hruby F., Plank I., Riedl A., 2006. Cartographic heritage as shared experience in virtual space: A digital representation of the earth globe of Gerard Mercator (1541)*, e-Perimetron*, Vol.1, No. 2, 88-98.

Hruby F., Riedl A., Tomberger H., 2008. Virtual representations of antique globes - new ways of touching the untouchable*, International Journal of Digital Earth*, Vol. 1, Issue 1.

Jenny B., 2006. MapAnalyst - A digital tool for the analysis of the planimetric accuracy of historic maps*. e-Perimetron,* Vol. 1, No. 3, 239-245.

Livieratos E., 2008. On the cartography of Rigas Charta*, e-Perimetron*, Vol. 3, No. 3, 120-145.

Milanesi M., Schmidt R., 2007. *Sfere del Cielo. Sfere della Terra.* Electa: Milano.

Riedl A., 2005. "Digital Globes - from Virtual to Real"*,* In: Proceedings, 22. ICA Cartographic Conference, Coruna, Spain;

Snyder J.P., 1997. *Flattening the Earth: two thousand years of map projections.* University of Chicago Press: Chicago.

Tobler W.R., 1964. *Computation of the correspondence of the geographical patterns. 1964 Papers,* Regional Science Association, 15 (1965).

Tobler W. R., 1966. Medieval distortions: The projections of ancient maps, *Annals of the AAG*, Vol. 56, n. 2.

Tobler W.R. 1986. Measuring the Similarity of Map Projections, *The American Cartographer*, Vol. 13, 2.