Diego Bellan[1,*], Gabrio Superti-Furga[2]

J. Electrical Systems 14-1 (2018): 188-198

Regular paper

Space-vector state-equation analysis of three-phase transients

Journal of Electrical Systems

This paper investigates the analysis of transients in three-phase systems by means of the Clarke transformation. Under the commonly accepted assumption of phase symmetry (i.e., three-phase basic elements with symmetrical parameters), the alpha and beta dynamic circuits are independent and characterized by the same circuit parameters. Thus, since the space vector is defined as the combination of alpha and beta variables, the state equation approach based on space vector variables results in an effective tool for three-phase transient analysis. In fact, the space vector approach presented in this work exploits the system symmetry providing state equations with reduced dynamic order. Moreover, it is shown that the space vector shape on the complex plane provides a concise and rich representation of the transients of the three phase variables. Indeed, despite the assumption of system symmetry, it is shown that the transient behavior of the three phase variables is not symmetrical. In particular, maximum over voltages and over currents can be easily detected from the space vector shape. Numerical examples are presented in order to show the effectiveness and adequacy of the general methodology presented in this work for the analysis of three-phase dynamic circuits

Keywords: Three-phase transient analysis, space vector, Clarke transformation, state equation.

Article history: Received 21 December 2017, Accepted 16 February 2018

1. Introduction

Circuit analysis of three-phase power systems under steady-state conditions is usually performed by resorting to the symmetrical component transformation (SCT). In fact, it is well known that, thanks to the assumption of phase symmetry, the use of SCT results in three uncoupled single-phase circuits. As far as transient analysis is concerned, however, such an approach is no longer effective since a direct time-domain analysis would be more suited [1]-[5]. To this aim, power system engineers usually perform transient analysis by resorting to numerical methods through commercial simulation tools like ElectroMagnetic Transients Program [5]. The numerical approach is attractive because it can handle even large and complex power networks. As a general principle, however, a numerical approach to a problem prevents theoretical and physical insight into the phenomenon under analysis. For this reason an analytical tool similar to SCT but working in the time domain would be very useful in order to provide analytical solutions to transients in three-phase circuits. To this aim, the Clarke transformation is a proper candidate since it works in the time domain and provides uncoupled modal circuits under the assumption of phase symmetry. Traditionally, the Clarke transformation and the related space vector is widespread in the dynamics and control of rotating electrical machines, and power electronics engineering. Its use in power system analysis, however, is still uncommon [4]-[11].

Indeed, three-phase power systems are conventionally analysed by resorting to single-phase equivalent circuits under sinusoidal steady state conditions by means of the standard

[*] Corresponding author: D. Bellan, Department of Electronics, Information and Bioengineering, Politecnico di Milano, piazza Leonardo da Vinci 32, 20133 Milan, Italy, E-mail: diego.bellan@polimi.it

[1] Department of Electronics, Information and Bioengineering, Politecnico di Milano, Milan, Italy

[2] formerly with the Department of Electrical Engineering, Politecnico di Milano, Milan, Italy

****** *This paper is an extended version of a paper presented in International Conference on Recent Advances in Electrical Systems, ICRAES'17, Tunisia, 2017*

phasor approach. However, the space vector approach is a more rigorous approach for a formal analysis of three-phase systems. In fact, it leads to the conventional standard phasors in the special case of sinusoidal steady state, whereas it is a suitable tool for proper analysis under distorted steady state and dynamic conditions. In particular, the main objective of this work is showing the powerful and complete features of the space vector when three-phase transients are considered.

In this paper the use of the Clarke transformation and space vectors for the analytical solution of transients in symmetrical three-phase circuits is presented. Several interesting properties and advantages of the space vector approach will be pointed out. More specifically, the paper is organized as follows. In Section 2 the space vector definition and its relationship with the SCT are briefly recalled. In Section 3 the space-vector state-equation approach is introduced for general Nth-order three-phase circuits. Some transient analyses of a specific second-order circuit are shown in Section 4, where the results are presented in terms of the shape of the space vector on the complex plane. Such concise and effective representation of three-phase transients will be explained and discussed. Concluding remarks are drawn in Section 5.

2. Space vector and zero component under transient and steady state conditions

Space vector is an effective tool for analyzing three-phase power systems in time domain under both transient and steady-state conditions. Definition of space vector is based on the Clarke transformation (in rational form) given by the following orthogonal matrix [4]-[10]:

$$\mathbf{T}_0 = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{1}$$

Let us consider three phase currents in time domain i_a, i_b, and i_c (similar derivations hold for phase voltages). The transformed currents according to the Clarke transformation are given by:

$$\mathbf{i}_T = \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \mathbf{T}_0 \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \mathbf{T}_0 \mathbf{i} \tag{2}$$

The current space vector is defined as a complex valued time-domain function with real and imaginary parts given by the α and β components, respectively:

$$\bar{i} = i_\alpha + j i_\beta \tag{3}$$

It can be readily proven that the straightforward formula to obtain the space vector from phase variables can be written as:

$$\bar{i} = \sqrt{\frac{2}{3}}(i_a + a i_b + a^2 i_c) \tag{4a}$$

where $a = \exp(j\,2\pi/3)$.

Similarly, the zero component is defined as:

$$i_o = \frac{i_a+i_b+i_c}{\sqrt{3}} \tag{4b}$$

The phase variables can be readily recovered from the space vector and from the zero component i_0 as

$$i_a(t) = \sqrt{\frac{2}{3}}\mathrm{Re}(\bar{\imath}(t)) + \frac{1}{\sqrt{3}} i_0(t) \tag{5a}$$

$$i_b(t) = \sqrt{\frac{2}{3}}\mathrm{Re}\left(a^2\bar{\imath}(t)\right) + \frac{1}{\sqrt{3}} i_0(t) \tag{5b}$$

$$i_c(t) = \sqrt{\frac{2}{3}}\mathrm{Re}\left(a\bar{\imath}(t)\right) + \frac{1}{\sqrt{3}} i_0(t) \tag{5c}$$

Notice that from (5), in case of null zero-sequence current i_0 (i.e., a pure three-phase system), the instantaneous phase currents $i_a(t)$, $i_b(t)$, $i_c(t)$ can be recovered by taking the orthogonal components of the space vector $\bar{\imath}(t)$ on three axes with geometrical phase displacement $2\pi/3$ each other (see Fig. 1).

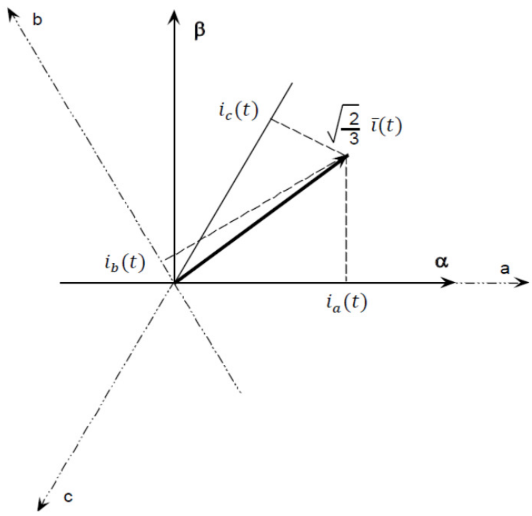


Figure 1. The phase currents i_a, i_b, and i_c can be recovered from the components of the space vector $\bar{\imath}$ on the axes *a*, *b*, and *c*.

Under steady-state sinusoidal conditions, the space vector can be put into relation with positive and negative sequence variables obtained from the symmetrical component transformation (SCT) operating on phasor variables as [11]-[12]:

$$\begin{bmatrix} I_p \\ I_n \\ I_0 \end{bmatrix} = \mathbf{S} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{6}$$

where **S** (in rational form) is the unitary matrix:

$$\mathbf{S} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \tag{7}$$

The relationship between the space vector (4a) and the sequence currents (6) can be written as [4], [10]:

$$\bar{\imath}(t) = I_p e^{j\omega t} + I_n^* e^{-j\omega t} \tag{8}$$

where asterisk denotes complex conjugate, and $\omega = 2\pi f$ is the angular frequency. Eq. (8) provides the fundamental relationship between space vector and conventional phasors under sinusoidal steady state. The space vector (8) follows an ellipse shape on the complex plane, with semi-major axis $R = |I_p| + |I_n|$, semi-minor axis $r = ||I_p| - |I_n||$, and inclination angle $\varphi = (\varphi_p - \varphi_n)/2$, where φ_p and φ_n are the angles of I_p and I_n, respectively [8]-[9]. Under ideal conditions the negative-sequence current I_n is equal to zero and therefore the space vector follows a circle shape with radius $|I_p|$ on the complex plane.

In sinusoidal steady-state, the zero component (4b) is a sinusoidal waveform since it is defined as a combination of sinusoids with the same frequency. Therefore, by using the conventional phasor representation of sinusoids we obtain:

$$i_0(t) = \sqrt{2}\mathrm{Re}\{I_0 e^{j\omega t}\} \tag{9}$$

where I_0 is the zero-sequence phasor.

3. Space vector analysis of three-phase transients

3.1. First-order circuits

Let us consider first a single-phase RL circuit branch with impressed total voltage $e(t)$ such that:

$$e(t) = Ri(t) + L\frac{d}{dt}i(t) \tag{10}$$

The solution of (10) can be written as:

$$i(t) = Ae^{-\frac{t}{\tau}} + i_s(t) \tag{11}$$

where $\tau = L/R$ is the time constant of the transient, $i_s(t)$ is the steady-state solution of (10) (depending on the forcing term $e(t)$), and $A = i(0) - i_s(0)$. The best approach to obtain the explicit solution of the form given in (11) is to obtain first the steady-state solution $i_s(t)$ through a simple phasor analysis (assuming sinusoidal impressed voltage $e(t)$), and second to evaluate in $t = 0$ such steady-state solution to obtain the constant A (assuming known initial condition $i(0)$).

The simple approach shortly outlined above can be readily applied to space vectors [4]. Let us consider a three-phase balanced star-connected RL branch with resistances R, selfinductances L_p, and mutual inductances L_m (see Fig. 2). A sinusoidal three-phase star-connected voltage source is applied to the RL branch at $t = 0$ by closing a three-phase switch. By applying the Clarke transformation, and by taking into account that for the space vectors the central point of star connections can be treated as short circuits [12], the space-vector differential equation describing the circuit is given by:

$$\bar{e}(t) = R\bar{\imath}(t) + L\frac{d}{dt}\bar{\imath}(t) \tag{12}$$

where $L = L_p - L_m$. Notice that (12) is a first-order equation, whereas the circuit in Fig. 2 is a second order circuit (two independent inductive meshes). However, due to system symmetry, the circuit would show one repeated (i.e., multiple) eigenvalue. The space vector approach is able to exploit this feature, leading to a state equation with order one instead of two.

Clearly, (12) is similar to (10). According to the procedure outlined above, in order to obtain the solution of (12) the steady-state solution should be calculated first. The forcing term $\bar{e}(t)$ is formed in general (see (8)), by the sum of two terms, i.e., the positive and negative (conjugated) sequence components rotating in positive and negative directions with angular frequency ω:

$$\bar{e}(t) = E_p e^{j\omega t} + E_n^* e^{-j\omega t} \tag{13}$$

Since the forcing term has the form given in (13), it is expected that also the steady-state solution can be written in the form:

$$\bar{\imath}_s(t) = I_1 e^{j\omega t} + I_2 e^{-j\omega t} \tag{14}$$

By substituting (13) and (14) into (12) we obtain:

$$I_1 = \frac{E_p}{R+j\omega L}, \qquad I_2 = \frac{E_n^*}{R-j\omega L} \tag{15}$$

According to (11) the complete solution of (12) is given by:

$$\bar{\imath}(t) = Ae^{-\frac{t}{\tau}} + \bar{\imath}_s(t) \tag{16}$$

where $A = \bar{\imath}(0) - \bar{\imath}_s(0)$. Notice that in this case (switches closing at $t = 0$) we have $\bar{\imath}(0) = 0$. Moreover, $\bar{\imath}_s(0) = I_1 + I_2$. Thus, from (14)-(16) we obtain the following expression for the complete solution:

$$\bar{\imath}(t) = \frac{E_p}{R+j\omega L}\left(e^{j\omega t} - e^{-\frac{t}{\tau}}\right) + \frac{E_n^*}{R-j\omega L}\left(e^{-j\omega t} - e^{-\frac{t}{\tau}}\right) \tag{17}$$

Notice that the dynamics of the current appears as superposition of the transients related to positive and negative sequences. The same approach is suitable to other circuit topologies and to first-order RC circuits.

Finally, it should be observed that in case of a four-wire system, the transient involving the zero components must be also evaluated to obtain a complete description of the transient. A conventional approach can be readily used for zero component transients.

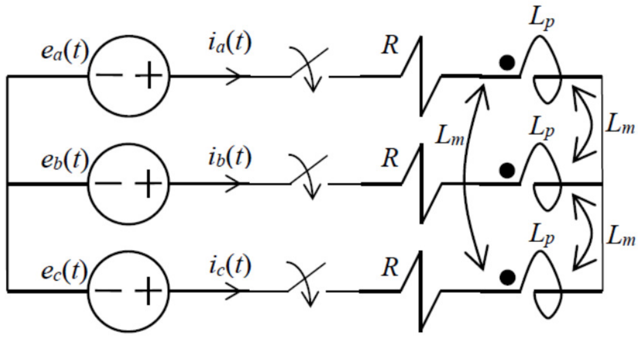


Figure 2. Three-phase RL circuit described by (12). The switches close at $t = 0$.

3.2. N^{th}-order circuits

The approach outlined above for a first-order circuit can be generalized to the case of an arbitrary number of three-phase dynamic components by resorting to the conventional state-equation approach.

The general methodology can be described as follows. The basic assumption of the proposed approach is the phase symmetry of the three-phase system. It means that for each three-phase basic element (i.e., resistive, inductive, or capacitive element) the self-parameters take the same value, and the mutual parameters (if any) take the same value. This is the common assumption underlying all the mathematical methods for three-phase system analysis. Thus, the parameter matrix (R, L, or C) for each three-phase basic element can be written in the following form:

$$\mathbf{P} = \begin{bmatrix} p & p_m & p_m \\ p_m & p & p_m \\ p_m & p_m & p \end{bmatrix} \tag{18}$$

It should be stressed that the symmetry assumption is related only to the element parameters, whereas no assumption is made on the sources (i.e., arbitrary three-phase sources can be assumed).

By applying the Clarke transformation matrix (1) to all the voltage/current variables of the three-phase system, each three-phase basic element (18) is transformed as:

$$\mathbf{P}_T = \mathbf{T}_0 \mathbf{P} \mathbf{T}_0^{-1} = \begin{bmatrix} p - p_m & 0 & 0 \\ 0 & p - p_m & 0 \\ 0 & 0 & p + 2p_m \end{bmatrix} = \begin{bmatrix} p_\alpha & 0 & 0 \\ 0 & p_\beta & 0 \\ 0 & 0 & p_0 \end{bmatrix} \tag{19}$$

Therefore, since $\mathbf{P}_T$ is a diagonal matrix, according to (2) the Clarke transformation results in decoupled differential equations for the α, β, and 0 variables. Notice that (19) is formally the same as the SCT in the phasor domain.

Notice that the star connections in the three-phase system result in short circuits for the α and β variables, whereas for the zero component the equivalent circuit depends on the connection of the star centre: open circuit for an isolated star centre, and an ideal transformer with ratio $\sqrt{3}$ in case the star centre is connected to a single phase network. Such single phase network can be reported on the three-phase side according to the conventional properties of ideal transformers. These results are similar to the SCT properties already derived and discussed in [12].

From the above assumptions and the outlined procedure we obtain three independent dynamic circuits:

$$\frac{d}{dt}\mathbf{x}_\alpha = \mathbf{F}\mathbf{x}_\alpha + \mathbf{B}\boldsymbol{u}_\alpha \tag{20a}$$

$$\frac{d}{dt}\mathbf{x}_\beta = \mathbf{F}\mathbf{x}_\beta + \mathbf{B}\boldsymbol{u}_\beta \tag{20b}$$

$$\frac{d}{dt}\mathbf{x}_0 = \mathbf{F}_0\mathbf{x}_0 + \mathbf{B}_0\boldsymbol{u}_0 \tag{20c}$$

where $\mathbf{x}_{\alpha,\beta,0}$ are the vectors of the state variables (in the Clarke domain), $\mathbf{u}_{\alpha,\beta,0}$ are the input vectors, **F** is the state matrix and **B** is the input matrix. A key point is that for α and β circuits we have the same matrices **F** and **B**. This is a consequence of (19) where $p_\alpha = p_\beta$ for all the three-phase basic elements. On the contrary, for the zero component circuit in general we obtain different matrices $\mathbf{F}_0$ and $\mathbf{B}_0$ because in this case we have different parameters p_0 and the contribution of possible single phase networks connected to the three-phase system.

As a fundamental result, the α and β circuits have the same eigenvalues. This is the reason explaining why the analysis of a three-phase system in terms of space vectors (i.e., by combining the α and β variables according to (3)) results in a dynamic system with reduced order *N* (e.g., first-order space-vector system for the second-order three-phase system in Fig. 2). Thus, in terms of space vectors we obtain:

$$\frac{d}{dt}\bar{\mathbf{x}} = \mathbf{F}\bar{\mathbf{x}} + \mathbf{B}\bar{\mathbf{u}} \tag{21}$$

where $\bar{\mathbf{x}}$ is the $N\times1$ state vector (i.e., the vector consisting of the *N* space vectors of independent inductor currents and capacitor voltages), **F** is the $N\times N$ state matrix, **B** is the $N\times M$ input matrix (where *M* is the number of space vector sources), and $\bar{\mathbf{u}}$ is the $M\times1$ vector of the input space vectors. Notice that (21) is the generalization of (12) to a N^{th}-order space-vector circuit.

Actually, the space vector exploits the system symmetry by combining the repeated eigenvalues resulting from such assumption. The total number of eigenvalues in the Clarke domain must be the same as in the original system. If the zero component is also present, the eigenvalues of the zero component circuit contribute to the total number of eigenvalues. Indeed, the Clarke transformation cannot change the number of state variables nor the number and values of the system eigenvalues. Thus, the proposed space-vector approach is capable of reducing the apparent dynamic order of the system by exploiting the system symmetry resulting in repeated eigenvalues. In case of a system with null zero components, the apparent space-vector dynamic order is half the actual dynamic order.

The steady-state solution to an input of the form $\bar{\mathbf{u}} = \mathbf{U}\exp(\gamma t)$ (where γ can denote either $+j\omega$ or $-j\omega$) can be readily obtained by substituting $\bar{\mathbf{u}}$ into (21) and searching a solution of the form $\bar{\mathbf{x}} = \mathbf{X}\exp(\gamma t)$:

$$\bar{\mathbf{x}}_s(t) = \mathbf{X}_s e^{\gamma t} = (\gamma \mathbf{1} - \mathbf{F})^{-1}\mathbf{B}\mathbf{U}e^{\gamma t} \tag{22}$$

The complete solution of (21) can be written as the sum of the general solution of the homogeneous form of (21) (i.e., assuming $\mathbf{B} = 0$) and the steady-state solution (22). Thus, for the *k*th space-vector state variable the complete solution can be written in the form:

$$\bar{x}_k(t) = \sum_{n=1}^{N} C_{kn} e^{\lambda_n t} + \bar{x}_{sk}(t) \tag{23}$$

where $\{\lambda_n\}_{n=1}^{N}$ is the set of the eigenvalues of the state matrix **F** (under the assumption of *N* distinct eigenvalues), and the set of constants $\{C_{kn}\}_{n=1}^{N}$ can be obtained by imposing initial values $\mathbf{x}_0 = \bar{\mathbf{x}}(0) - \bar{\mathbf{x}}_s(0)$ [13]:

$$\mathbf{C} = [\mathbf{x}_0 \quad \mathbf{F}\mathbf{x}_0 \quad \dots \quad \mathbf{F}^{N-1}\mathbf{x}_0] \begin{bmatrix} 1 & \lambda_1 & \lambda_1^2 & \cdots & \lambda_1^{N-1} \\ 1 & \lambda_2 & \lambda_2^2 & \cdots & \lambda_2^{N-1} \\ 1 & \lambda_3 & \lambda_3^2 & \cdots & \lambda_3^{N-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & \lambda_N & \lambda_N^2 & \cdots & \lambda_N^{N-1} \end{bmatrix}^{-1} \tag{24}$$

4. Numerical example

The general approach outlined in Subsection 3.2 was applied to the three-phase circuit shown in Fig. 3. The circuit presents two dynamic three-phase elements, i.e., three star-connected coupled inductors and three star-connected capacitors. The space-vector equivalent circuit is shown in Fig. 4 where it was taken into account that Clarke transformation results in short-circuit connections between star centers. The second-order space-vector circuit in Fig. 4 can be modeled through the state equation (21) where:

$$\bar{\mathbf{x}} = \begin{bmatrix} \bar{\imath} \\ \bar{v} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} -\frac{R_2}{L} & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{R_1 C} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{R_1 C} \end{bmatrix}, \quad \bar{\mathbf{u}} = \bar{e} \tag{25}$$

Notice again that the circuit in Fig. 3 when solved by means of conventional state-equation techniques would result in a fifth-order circuit. However, due to system symmetry, the circuit shows two repeated eigenvalues, resulting in the second-order model (25). The fifth eigenvalue is related to a capacitor in the zero component circuit, where, however, due to the open star connections, the state variable has no time evolution.

A 50Hz - voltage source was considered consisting of only the positive sequence component $E_p = 100\ V$. Thus, according to (13), the input in (22) consists only of the term corresponding to $\gamma = +j\omega$. Initial values of state variables are assumed zero. Passive components are given as $R_1 = 10\ \Omega, R_2 = 1\ \Omega$, $L_p = 30\ mH$, $L_m = 25\ mH$ (i.e., $L = L_p - L_m = 5\ mH$), and $C = 10\ mF$. The corresponding complex conjugated eigenvalues are given by $\lambda_{1,2} = -60 \pm j135.65\ s^{-1}$.

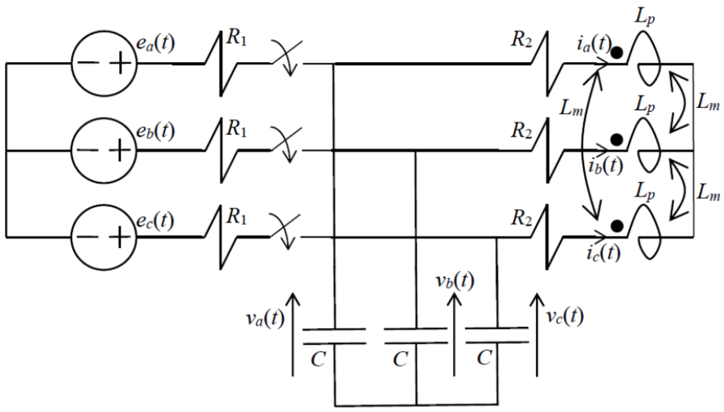


Figure 3. Three-phase RLC circuit. The switches close at $t = 0$.

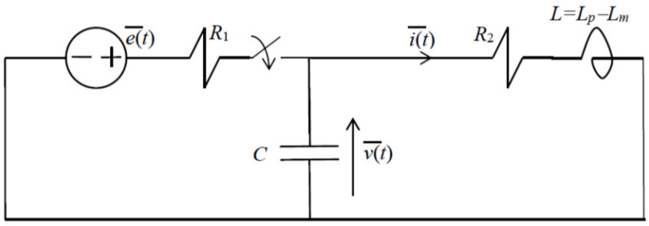


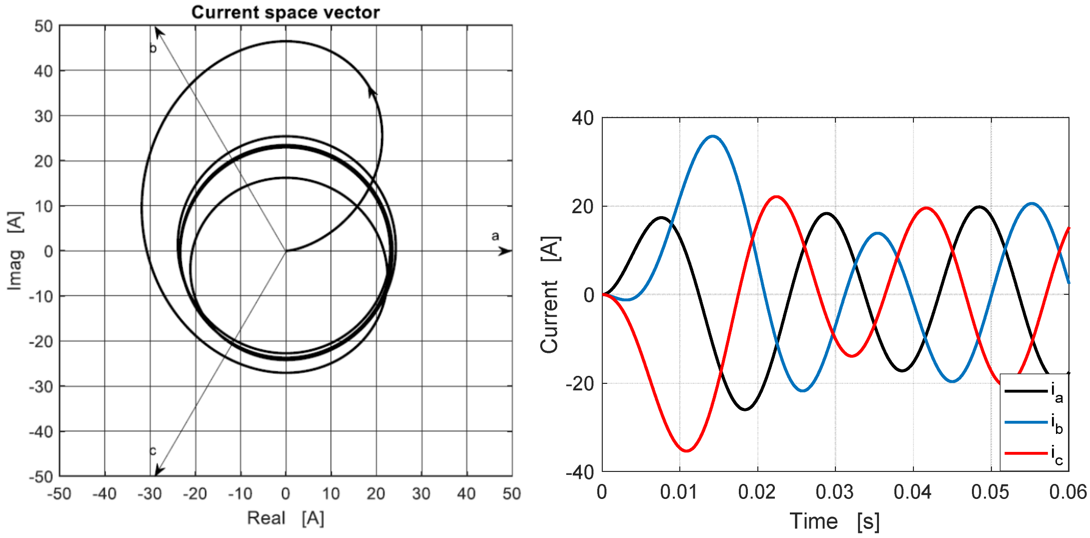
Figure 4. Space-vector equivalent circuit for the three-phase circuit shown in Fig. 3.

Fig. 5a shows the behavior on the complex plane of the current space vector $\overline{\imath}(t)$ provided by the complete solution (23). The steady-state shape is a counterclockwise circle since the negative sequence component is zero. Fig. 5b shows the time behavior of the three phase currents according to (5). Figs. 6a and 6b refer to the capacitor voltage $\overline{v}(t)$ and the related phase voltages, respectively.

Notice that the transient behavior of each phase variable can be readily related to the components of the space vector on the *a, b, c* axes in Figs. 5a and 6a. Therefore, the space vector approach is well suited to provide a rich and concise description of three-phase transients.

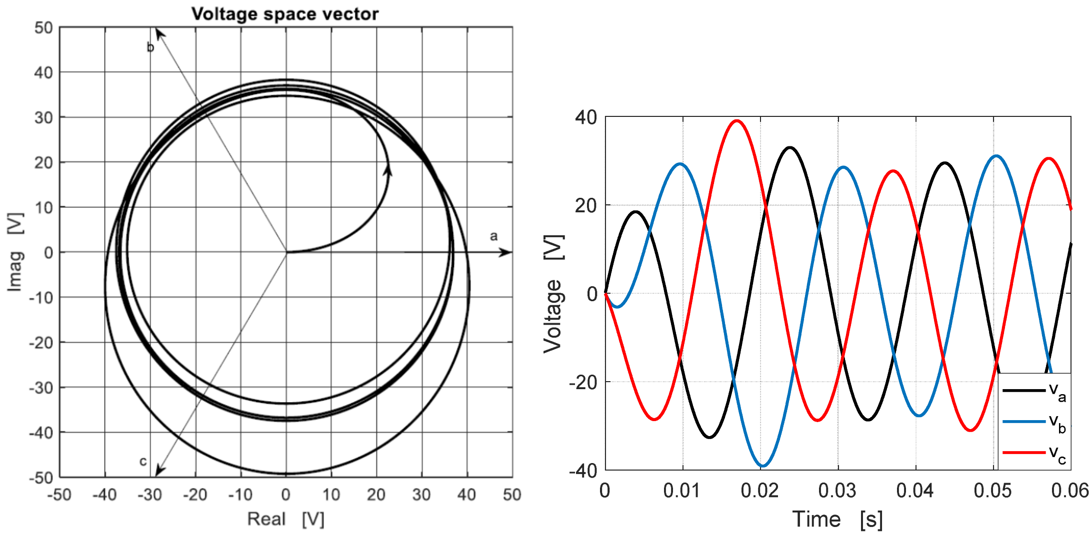
Moreover, a crucial point can be stressed from the transients shown in Figs. 5b and 6b, i.e., the three phase transients show asymmetrical time behavior. For example, the peaks of currents and voltages take different values for phases a, b and c. Therefore, despite the symmetry of the three-phase system and of the short circuit, the three phase variables show asymmetrical behavior. This point prevents the use of any single-phase equivalent circuit for a proper analysis of transients even in symmetrical systems.

A further key-point highlighting the suitability of the space-vector approach is related to the straightforward evaluation of the actual worst-case in terms of maximum overcurrent/overvoltage. In fact, such worst case is given by the maximum space-vector magnitude. Since the phase variables are given by the space-vector components on the *a, b, c* axes, the actual worst case can be observed on a phase variable only in case the maximum space-vector magnitude is reached exactly at one of the *a, b, c* axes. Otherwise, the peaks of the phase variables correspond to the components of the maximum space-vector magnitude, leading to an underestimate of the possible worst case. Notice that a change in the initial phase (usually unknown) of the forcing term would result in a simple angular displacement in the graphs in Figs. 5a and 6a. This feature is a further point showing the space-vector capability to provide a synthetic representation of three-phase transients.



(a) (b)

Figure 5. Space vector shape (a) and phase currents (b) in the three-phase inductor in Fig. 4



(a) (b)

Figure 6. Space vector shape (a) and phase voltages (b) across the three-phase capacitor in Fig. 4.

5. Conclusion

The well-known conventional state-equation approach for the analysis of dynamic circuits has been extended to space vector variables resulting from the Clarke transformation applied to three-phase systems. The only assumption was the phase symmetry of system parameters. Two main results have been obtained and discussed.

First, the space vector approach exploits the intrinsic system symmetry resulting in repeated eigenvalues. Thus, the apparent dynamic order of the system in terms of space vectors is lower than the order of the original three-phase system. This is an important result leading to a simplified and complete representation of three-phase dynamic systems. On the contrary, considering a three-phase dynamic system in terms of its natural variables would result in redundant information due to the symmetry assumption.

Second, the shape of the space vector on the complex plane provides concise and rich information about the transient of the corresponding three variables. This is a major point since, despite the system symmetry, the time behaviour of the three phase variables is not symmetrical. Indeed, overvoltages/overcurrents can be different for the three phases. The maximum magnitude of the space vector provides the estimate of the worst case for the peak variables. A change in the starting time of the transient corresponds to a simple rotation of the space vector shape on the complex plane.

Thus, the space vector approach has proven to be a powerful and effective tool for the analysis of symmetrical three-phase transients even if its use is not common in power system analysis.

References

[1] M. H. J. Bollen and I. Yu-Hua Gu, On the analysis of voltage and current transients in three-phase power systems, *IEEE Trans. on Power Delivery*, *22*(2), 1194-1201, 2007.

[2] N. R. Watson and A. Farzanehrafat, Three-phase transient state estimation algorithm for distribution systems, *IET Generation, Transmission & Distribution*, *8*(10), 1656-1666, 2014.

[3] M. Kumari, P. Thakur, and D. N. Badodkar, Transient analysis of three phase high power voltage source inverter with nonlinearities in hybrid electric vehicles, accepted for publication on *IEEE Trans. on Power Electronics*, DOI 10.1109/TPEL.2017.2712065.

[4] D. Bellan and G. Superti-Furga, Three-phase transient analysis based on the space vector approach, in Proc. of the *2017 IEEE Region 10 Conference (TENCON)*, 2161-2164, 2017.

[5] M. Owen, Transient analysis using component transforms, in Proc. of *2011 IEEE Power Engineering and Automation Conference*, 4-9, 2011.

[6] L. Cristaldi, A. Ferrero, and G. Superti-Furga, Current decomposition in asymmetrical, unbalanced three-phase systems under nonsinusoidal conditions, *IEEE Trans. on Instrum. Meas.*, *43*(1), 63-68, 1994.

[7] J. Aller, A. Bueno, and T. Paga, Power system analysis using space vector transformation, *IEEE Transactions on Power Systems*, *17*(4), 957-965, 2002.

[8] V. Ignatova, P. Granjon, and S. Bacha, Space vector method for voltage dips and swells analysis, *IEEE Trans. on Power Delivery*, *24*(4), 2054-2061, 2009.

[9] D. Bellan, On the statistics of noisy space vector in power quality analysis, *International Journal of Engineering and Technology*, *8*(5), 2177-2183, 2016.

[10] G. Superti-Furga, S. Barcellona, and E. Tironi, Space-vector approach in three-phase unbalance and distortion analysis, in Proc. of *17th International Conference on Harmonics and Quality of Power (ICHQP)*, 721-726, 2016.

[11] G. C. Paap, Symmetrical components in the time domain and their application to power network calculations, *IEEE Trans. on Power Systems*, *15*(2), 522-528, 2000.

[12] D. Bellan, S. A. Pignari, and G. Superti-Furga, Consistent circuit technique for zero-sequence currents evaluation in interconnected single/three-phase power networks, *Journal of Electrical Systems*, *12*(2), 230-238, 2016.

[13] J. A. Rosendo Macìas, A. Gomez Exposito, and A. Bachiller Soler, A comparison of techniques for state-space transient analysis of transmission lines, *IEEE Trans. Power Delivery*, *20*(2), 894-903, 2005.