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A General Framework for MIMO Receivers with Low-Resolution Quantization

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***Abstract*—The capacity of a discrete-time, multi-input multi-output (MIMO) channel with output quantization is investigated for different receiver architectures. A general framework for low–resolution quantization is proposed in which the antenna outputs are processed by analog combiners and sign quantizers are used for analog–to–digital conversion. The configuration of the analog combiners is chosen as a function of the channel realization so that the transmission rate can be maximized over the set of available configurations. To exemplify the proposed approach, four analog receiver architectures are considered: (a) sign quantization of the antenna outputs, (b) single antenna selection, (c) multiple antenna selection, and (d) linear processing of the antenna outputs. In each scenario, capacity is investigated as a function of the transmit power, the number of transmit/receive antennas and sign quantizers. In particular, it is shown that architecture (a) is sufficient to approach the optimal high signal–to–noise ratio (SNR)performance for a MIMO receiver in which the number of receive antennas is larger than the number of sign quantizers. Numerical evaluations of the average performance are presented for the case in which the channel gains are i.i.d. Gaussian distributed.**

***Index Terms*—MIMO channel;Channel output quantization; Analog-to-digital conversion; One-bit quantization.**

I. Introduction

Low–resolution quantization is an important technology for massive MIMO and millimeter–wave communication systems as it enables low–power and low–complexity transceivers [1]. Although the performance of MIMO receivers with large antenna arrays and low–resolution quantizers has been widely investigated in the literature, a fundamental information theoretic understanding of such receiver architectures is currently not available. In this paper, we propose a unified framework to analyze and compare the performance of a MIMO system under various constraints on the analog–to–digital processing at the receiver. More specifically, we consider a $N_{\mathrm{r}} \times N_{\mathrm{t}}$ MIMO channel in which the receiver is comprised of N_{sq} sign quantizers. Each sign quantizer is connected to the antenna outputs via an analog combiner with limited processing capabilities and the configuration of the analog combiners can be chosen as a function of the channel realization. For this model formulation, we consider the problem of determining which configuration of the analog combiners yields the largest transmission rate.

Literature Review: Quantization in MIMO systems is a well–investigated topic in the literature; for the sake of brevity we focus here on results regarding sign quantization.[1] The authors in [2] are perhaps the first to point out that the capacity loss in MIMO channels due to coarse quantization is surprisingly small, although this observation is supported mostly through numerical evaluations. In [3], the authors derive fundamental properties of the capacity-achieving distribution for a single–input single–output (SISO) channel with output quantization. A lower bound on the capacity of sign-quantized MIMO channels with Gaussian inputs based on the Bussgang decomposition is derived in [4]. The high SNR asymptotics for complex MIMO channels with sign quantization are studied are [5].

Contributions: For the proposed problem formulation, we focus on four receiver architectures: (a) multiple antenna selection and sign quantization, (b) single antenna selection and multilevel quantization, (c) multiple antenna selection and multilevel quantization, and (d) linear combining and multilevel quantization. Architecture (c) is more general than both architectures (a) and (b), and (d) is the most general. For these architectures, we provide capacity bounds for the SIMO and MIMO case as a function of the channel realization, transmit power and number of sign quantizers. In particular, for the MIMO channel with linear combining and multilevel quantization, we derive an approximatively optimal transmission scheme as a variation of the classic water–filling power allocation scheme. This approximate solution shows that, if the number of antennas at the receiver is larger than the number of sign quantizers, then sign quantization is sufficient to approach the optimal high SNR performance. Numerical evaluations are provided for the case of i.i.d. Gaussian–distributed channel gains.

Paper Organization: Sec. II introduces the channel model. Sec. III reviews the available results in the literature. The main results are given in Sec. IV. Numerical evaluations are provided in Sec. V. Sec. VI concludes the paper.

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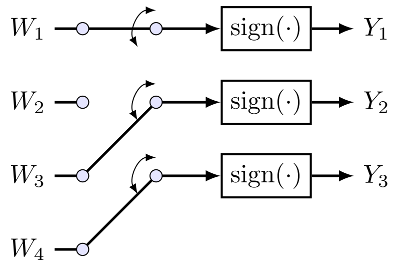
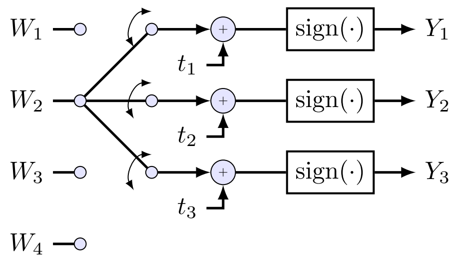
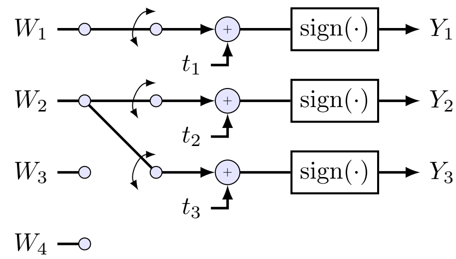
[1] In the literature, the term "one–bit quantization" most often refers to sign quantization of the antenna outputs. Here we prefer the term "sign quantization".



(a) Multiple antenna selection and sign quantization.



(b) Single antenna selection and multilevel quantization.



(c) Multiple antenna selection and multilevel quantization.

Fig. 1: Relevant receiver architectures in Sec. II for $N_{\rm r}=4$ and $N_{\rm sq}=3$.

Only sketches of the proofs are presented in the following; a complete version is provided online [6].

Notation: All logarithms are taken in base two. The binary entropy function is defined as $H_2(x) = -x\log x - (1-x)\log(1-x)$ and the tail probability of the standard normal distribution as $Q(x) = 1/\sqrt{2\pi}\int_x^{+\infty}\exp(-u^2/2)\mathrm{d}u$. The vector $\boldsymbol{\lambda}(M) = [\lambda_1 \ldots \lambda_{\mathsf{rank}(M)}]$ contains the eigenvalues of the matrix M. The identity matrix of size n is indicated as $\mathbf{I}_n$. The all–zero/all–one matrix of size $n \times m$ is denoted as $\mathbf{0}_{n\times m}$/$\mathbf{1}_{n\times m}$, respectively, and the set of all permutation matrices of size $m \times m$ as $\mathcal{P}^{\pi}_m$.

II. Channel Model

Consider a discrete-time, real-valued MIMO channel with $N_{\rm t}$ transmit and $N_{\rm r}$ receive antennas. At the $n^{\rm th}$ channel use, the antenna output vector $\mathbf{W}_n = [W_{1,n} \ldots W_{N_{\rm r},n}]^T$, is obtained from the channel input vector $\mathbf{X}_n = [X_{1,n} \ldots X_{N_{\rm t},n}]^T$ as

$$\mathbf{W}_n = \mathbf{H}\mathbf{X}_n + \mathbf{Z}_n, \quad n \in [1 \ldots N], \tag{1}$$

where $\mathbf{H}$ is a full rank matrix of size[2] $N_{\rm r} \times N_{\rm t}$ and $\mathbf{Z}_n$ is an $N_{\rm r}$-vector of i.i.d. Gaussian variables with zero mean and identity covariance matrix. For $N_{\rm t} = N_{\rm r} = 1$, let $\mathbf{H} = 1$ w.l.g. For $N_{\rm t} = 1$, $N_{\rm r} > 1$, let $\mathbf{H} = \mathbf{h}$ to improve clarity. The channel input vector is subject to the average power constraint $\sum_{n=1}^{N} \mathbb{E}[||\mathbf{X}_n||_2^2] \le NP$ where $||\cdot||_2$ is the 2-norm.

The antenna output vector is processed by $N_{\rm sq}$ sign quantizers, each receiving a linear combination of the antenna output vector plus a constant

$$\mathbf{Y}_n = \mathrm{sign}(\mathbf{V}\mathbf{W}_n + \mathbf{t}), \quad n \in [1 \ldots N], \tag{2}$$

where $\mathbf{V}$ is the analog combining matrix of size $N_{\rm sq} \times N_{\rm r}$, $\mathbf{t}$ is a threshold vector of length $N_{\rm sq}$ and $\mathrm{sign}(\mathbf{u})$ is the function producing the sign of each component of the vector $\mathbf{u}$ as plus or minus one, so that $\mathbf{Y}_n \in \{-1,+1\}^{N_{\rm sq}}$. For a given $\{\mathbf{V},\mathbf{t}\}$, the capacity of the model in (2) is

$$\mathcal{C}(\mathbf{V},\mathbf{t}) = \max_{P_{\mathbf{X}}(\mathbf{x}),\ \mathbb{E}[||\mathbf{X}||_2^2]\le P} I(\mathbf{X};\mathbf{Y}). \tag{3}$$

The analog processing capabilities at the receiver are modeled as a set of feasible values of $\{\mathbf{V},\mathbf{t}\}$, denoted as $\mathcal{F}$. Our goal is to maximize the capacity expression in (3) over $\mathcal{F}$, namely

$$\mathcal{C}(\mathcal{F}) = \max_{\{\mathbf{V},\mathbf{t}\}\in\mathcal{F}} \mathcal{C}(\mathbf{V},\mathbf{t}). \tag{4}$$

[2]This condition guarantees the existence of a right pseudo-inverse for $\mathbf{H}$ and holds with high probability in a richly scattering environment.

Relevant receiver architectures: For the problem formulation in (4), sign quantizers can be seen as a resource to be allocated optimally among a set of possible configurations $\mathcal{F}$. As such, it captures the trade-off between the quantization of few antennas with high precision versus the quantization of many antennas with low precision.

To exemplify the insights provided by our approach, we study four receiver architectures:

(a) Multiple antenna selection and sign quantization: Here $\mathcal{F}$ in (4) is selected as

$$\mathcal{F}_a = \left\{\mathbf{V} = \left[\mathbf{I}_{N_{\rm sq}}, \mathbf{0}_{N_{\rm sq}\times(N_{\rm r}-N_{\rm sq})}\right]K,\ K \in \mathcal{P}^{\pi}_{N_{\rm r}},\right.$$
$$\left.\mathbf{t} = \mathbf{0}_{N_{\rm sq}\times 1}\right\}, \tag{5}$$

that is, each sign quantizer is connected to one of the channel outputs.

Fig. 1a represents this model for $N_{\rm r} = 4$ and $N_{\rm sq} = 3$.

(b) Single antenna selection and multilevel quantization: In this configuration, a single antenna output is processed by all the quantizers:

$$\mathcal{F}_b = \left\{\mathbf{V} = \left[\mathbf{1}_{N_{\rm sq}\times 1}, \mathbf{0}_{N_{\rm sq}\times(N_{\rm r}-1)}\right]K,\ K \in \mathcal{P}^{\pi}_{N_r},\right.$$
$$\left.\mathbf{t} \in \mathbb{R}^{N_{\rm sq}}\right\}, \tag{6}$$

This model is presented in Fig. 1b.

(c) Multiple antenna selection and multilevel quantization: Each sign quantizer can select an antenna output and a voltage offset before performing quantization. This is obtained by choosing

$$\mathcal{F}_c = \left\{\mathbf{V} \text{ s.t. } V_{ij} \in \{0,1\},\ \sum_{j=1}^{N_{\rm r}} V_{ij} = 1,\ \mathbf{t} \in \mathbb{R}^{N_{\rm sq}}\right\}. \tag{7}$$

This architecture is shown Fig. 1c.

(d) Linear combining and multilevel quantization: Corresponds to

$$\mathcal{F}_d = \left\{\mathbf{V} \in \mathbb{R}^{N_{\rm sq}\times N_{\rm r}},\ \mathbf{t} \in \mathbb{R}^{N_{\rm sq}}\right\}. \tag{8}$$

Architecture (c) encompasses architectures (a) and (c); architecture (d) subsumes all other architectures. Note that an M–level multilevel quantizer is obtained utilizing $M-1$ sign quantizers: it follows that sign quantization produces the most information bits per sign quantizer and increasing the number of quantization levels increases the information bits only logarithmically.

III. Sign Quantization

The effect of quantization on the capacity of the MIMO channel has been investigated thoroughly in the literature. For conciseness, we review only the results on sign quantization of the channel outputs, corresponding to architecture (a) in Fig. 1a when $N_{\rm sq}=N_{\rm r}$.

The capacity of the SISO channel with sign quantization of the outputs is attained by antipodal signaling.

Lemma III.1. [3, Th. 2]: *The capacity of the SISO channel with sign quantization of the antenna output is*

$$\mathcal{C}_{\rm SISO}=1-H_2\left(Q\left(\sqrt{P}\right)\right). \quad (9)$$

The capacity of the MISO model with sign output quantization is obtained from the result in Lem. III.1 by transforming the MISO channel into a SISO channel through transmitter beamforming, thus yielding

$$\mathcal{C}_{\rm MISO}=1-H_2\left(Q\left(|\mathbf{h}|\sqrt{P}\right)\right). \quad (10)$$

For the SIMO and MIMO channel, capacity with sign quantization is known in the high-SNR regime. Here, capacity is obtained as the number of points in the received signal space that can be distinguished after sign quantization: this problem is closely related to classical problems in combinatorial geometry.

Lemma III.2. [5, Prop. 3]. *The capacity of the MIMO channel with sign quantization and $N_{\rm sq}=N_{\rm r}$ for which $\mathbf{H}$ satisfies a* general position *condition (see [5, Def. 1]), is bounded at high SNR as*

$$\log(K(N_{\rm sq},N_{\rm t}))\leq \mathcal{C}_{{\rm MIMO},a}^{{\rm SNR}\to\infty}\leq \log(K(N_{\rm sq},N_{\rm t})+1),$$

if $N_{\rm t}<N_{\rm sq}$, where

$$K(N_{\rm sq},N_{\rm t})=2\sum_{k=0}^{N_{\rm t}-1}\binom{N_{\rm sq}-1}{k}. \quad (11)$$

If $N_{\rm t}\geq N_{\rm sq}$, then $\mathcal{C}_{{\rm MIMO},a}^{{\rm SNR}\to\infty}=N_{\rm sq}$.

At finite SNR, upper and lower bounds on the capacity of the MIMO channel with sign quantization are known but are not tight in general.

Lemma III.3. [5, Sec. V.A]. *The capacity of a MIMO channel with output sign quantization with $N_{\rm t}\geq N_{\rm r}=N_{\rm sq}$ is bounded as*

$$N_R\left(1-H_2\left(Q\left(\sqrt{\frac{P}{{\rm trace}\,(\mathbf{K})}}\right)\right)\right) \leq \mathcal{C}_{MIMO,a}\leq N_R\left(1-H_2\left(Q\left(\sqrt{P\lambda_{\max}}\right)\right)\right), \quad (12)$$

where $\mathbf{K}=(\mathbf{H}\mathbf{H}^T)^{-1}$ and $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{K}$.

Note that, in the setting of [5], the authors consider the complex MIMO model while the results it Lem. III.2 and Lem. III.3 hold for a real channel.

IV. Main Results

We begin by considering the capacity of the SISO channel for the receiver architectures in Sec. II. Capacity for architecture (a) is provided in Lem. III.1 (as we have, necessarily, $N_{\rm sq}=1$). Architectures (b), (c) and (d) all correspond to the same model in which the channel output is quantized through an $(N_{\rm sq}+1)$-level quantizer.

Proposition 1. *The capacity of the SISO channel with multilevel output quantization, $N_{\rm sq}>1$, is upper–bounded as*

$$\mathcal{C}_{\rm SISO}\leq\frac{1}{2}\log\left(\min\left\{P+1,(N_{\rm sq}+1)^2\right\}\right), \quad (13)$$

*and capacity is to within 1 bits-per-channel-use (*bpcu*) from the upper bound in* (13)*.*

Proof: The proof substantially follows [7]. The upper bound (13) is the minimum between the capacity of the model without quantization constraints and the capacity of the channel without additive noise. For the achievability proof, the input is chosen as an equiprobable M-PAM signal for

$$M=\min\left\{\lfloor\sqrt{P}\rfloor,N_{\rm sq}+1\right\}, \quad (14)$$

in which the distance between the constellation points is chosen such that the power constraint is met with equality. At the receiver, the quantization thresholds are selected as the midpoints of the M-PAM constellation points. ■

In the remainder of the paper, we focus on approximate characterization of capacity for the SIMO and MIMO channel[3] in the spirit of Prop. 1, that is:

- **upper bounds** are obtained as the minimum among two expressions: the capacity for the channel without quantization constraint and the capacity of the channel without additive noise, and
- **lower bounds** rely on PAM modulation and uniform multilevel quantization, as the performance of these schemes can be more easily explicitly quantified.

1) SIMO case: For receiver architecture (a), capacity is at most 1 bpcu so that Lem. III.3 is sufficient to characterize capacity to within a small additive gap. For receiver architecture (b), capacity can be characterized through a rather straightforward extension of the result in Prop. 1.

Proposition 2. *The capacity of the SIMO channel with single antenna selection and multilevel quantization is upper–bounded as*

$$\mathcal{C}_{{\rm SIMO},b}\leq\frac{1}{2}\log\left(\min\left\{1+||\mathbf{h}||_\infty^2 P,(N_{\rm sq}+1)^2\right\}\right), \quad (15)$$

where $||\cdot||_\infty$ is the max*-norm; capacity is to within* 1 bpcu *from the outer bound in* (15)*.*

Note that (15) reduces to (13) when $N_{\rm r}=1$.

For architecture (c), sampling more antennas allows the receiver to collect more information on the input but reduces the number of samples that can be acquired from each antenna.

[3]Note that the MISO case follows from the SISO case as in (10) using transmitter beam-forming.

Proposition 3. *The capacity of the SIMO channel with multiple antenna selection and multilevel quantization for $P > \log(N_{\rm sq}) > 2$ and $h_i^2 > 1$ is bounded as*

$$\max_K \frac{1}{2}\log\left(\min\left\{1+||\mathbf{h}^{(K)}||_2^2 P, \left(\frac{N_{\rm sq}}{K}+1\right)^2\right\}\right) - 2 \quad (16a)$$

$$\leq \mathcal{C}_{{\rm SIMO},c} \leq \frac{1}{2}\log\left(1+||\mathbf{h}||_2^2 P, (N_{\rm sq}+1)^2\right), \quad (16b)$$

where $\mathbf{h}^{(K)}$ is the vector of the K largest channel gains in $\mathbf{k}$.

Note that the upper and lower bounds in (16a) and (16b) differ of at most 2 bpcu.

Proof: The upper bound in (16b) is derived similarly to Prop. 1. The inner bound in (16a) is obtained by considering the transmission scheme in which the channel input is the sum of a uniformly distributed M-PAM signal plus a dither. For the model in which the channel output undergoes infinite, uniform quantization, the dither has the effect of making the quantization noise independent of the channel input and of the additive Gaussian noise. By accounting for the rate loss incurred with finite, uniform quantization of the channel output under the conditions $P > \log(N_{\rm sq})$ and $h_i^2 P > 1$, we obtain the achievable rate in (16a). ■

Next, we consider the SIMO channel with architecture (d).

Proposition 4. *The capacity of the SIMO channel with linear combining and multilevel quantization is upper–bounded as*

$$\mathcal{C}_{{\rm SIMO},d} \leq \frac{1}{2}\log\left(\min\left\{1+|\mathbf{h}|_2^2 P, (N_{\rm sq}+1)^2\right)\right., \quad (17)$$

and the capacity is to within 1 bpcu *from the upper bound in* (17).

Proof: With this architecture, the maximal ratio combining at the receiver results in the equivalent SISO channel with channel gain $||\mathbf{h}||_2$. The result in Prop. 1 can then be used to obtain the approximate capacity. ■

The results in Prop. 2, Prop. 3 and Prop. 4 are related as follows: all the architectures attain the same high SNR performance, $\log(N_{\rm sq}+1)$, as in the high SNR regime the number of available quantizers determines the attainable rate, regardless of the architecture. Receiver architecture (b) performs close to architecture (c) when the number of receive antennas is sufficiently small or the channel gains vary widely. As for architectures (c) and (d): in architecture (d) the combining of the channel information occurs before quantization while, in architecture (c), the combining of the channel information takes place after quantization. Let us refer to the former scenario as *analog combining* and the latter scenario as *digital combining*. From Prop. 3 we gather precise conditions on the channel realizations under which digital combining performs sufficiently close to analog combining. The result in Prop. 3 can be further refined so that the performance gap from architecture (d) decreases with the number of available quantizers.

2) MIMO case: We begin by deriving an approximate characterization of capacity for architecture (b).

Proposition 5. *The capacity of the MIMO channel with single antenna selection and multilevel quantization is upper–bounded as*

$$\mathcal{C}_{{\rm MIMO},b} \leq \frac{1}{2}\log\left(\min\left\{1+|\mathbf{h}_{\max}^T|_2^2 P, (N_{\rm sq}+1)^2\right\}\right), \quad (18)$$

where $\mathbf{h}_{\max}^T$ is the row of $\mathbf{H}$ with the largest norm and the upper bound in (18) *can be attained to within* 2 bpcu.

For architecture (c), we are currently unable to provide matching inner and outer bounds. For architecture (d), the approximate capacity can be obtained as a variation of the classic water-filling solution. Through the classic singular value decomposition, the MIMO channel can be transformed in $K = \min\{N_t, N_r\}$ parallel sub-channel with independent additive noise and gains $\lambda(\mathbf{H})$. By assigning power P_i and $N_{\rm SQ,i}$ sign quantizers to the $i^{\rm th}$ sub-channel and coding as in Prop. 1, we attain the rate

$$\sum_{i=1}^{K} \frac{1}{2}\log\left(\min\left\{1+\lambda_i^2 P_i, (N_{SQ,i}+1)^2\right\}\right) - K, \quad (19)$$

where (19) can be maximized over $P_i \in \mathbb{R}^+$, $\sum_i P_i = P$, $N_{SQ,i} \in \mathbb{N}$, $\sum_i N_{SQ,i} = N_{\rm sq}$. By relaxing the integer constraint on $N_{SQ,i}$, standard convex optimization techniques yield the achievable region

$$\mathcal{C} \leq R^\star(\boldsymbol{\lambda}, P, N_{\rm sq}) = \begin{cases} \sum_{i=1}^{K} \frac{1}{2}\log(1+\lambda_i P_i) - K \\ \qquad \text{if } \sum_{i=1}^{K}\left(\sqrt{1+\lambda_i P_i}-1\right) \leq N_{\rm sq} \\ K\log\left(\frac{N_{\rm sq}}{K}+1\right) - K \\ \qquad \text{otherwise,} \end{cases} \quad (20)$$

where P_i are chosen as $P_i = (\mu - \lambda_i^{-2})^+$ and $\mu \in \mathbb{R}^+$ is the smallest value for which $\sum_i P_i = P$.

The achievable rate in (20) can be interpreted as follows. The optimal solution for the (relaxed) maximization in (19) has two regimes: either the rate is limited by the power constraint or by the quantization constraint. When the rate is limited by the power constraint, then classic water–filling solution determines the power allocation on each sub-channel while sign quantizers are allocated as in (14). When the rate is limited by the quantization constraint, then the optimal solution is the equal SNR and equal sign quantizer allocation in each sub-channel.

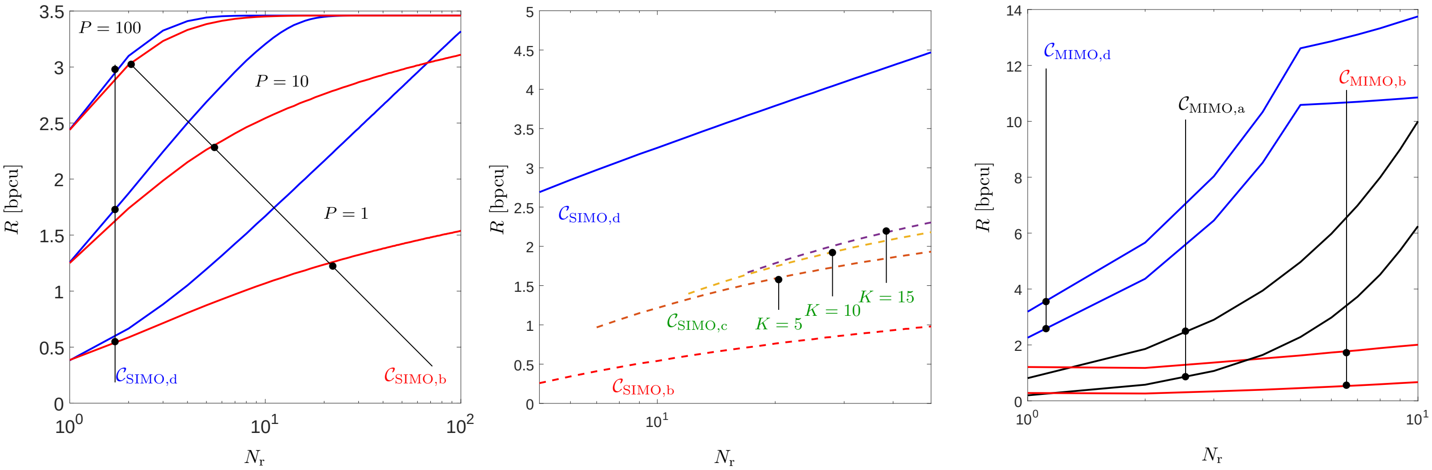
The approximate capacity for architecture (d) is obtained by deriving an outer bound matching the achievable rate in (20).

Proposition 6. *The capacity of a MIMO channel with linear combining and multilevel quantization is upper–bounded as*

$$\mathcal{C}_{{\rm MIMO},d} \leq R^\star(\boldsymbol{\lambda}, P, N_{\rm sq}) + 2K, \quad (21)$$

and capacity is to within a gap of $3K$ bpcu *from the upper bound in* (21) *for* $R^\star(\boldsymbol{\lambda}, P, N_{\rm sq})$ *and* K *in* (20).

The converse proof Prof. 6, as for the converse of Prop. 1, is obtained as the intersection of the capacity of the channel with-



(a) Prop. 2 vs. Prop. 4 for the SIMO channel with architectures (b) and (d) with $N_{SQ} = 10$, $N_{\mathrm{r}} \in [1 \ldots 10^2]$ and $P \in [1, 10, 100]$

(b) Prop. 2 vs. Prop. 3 vs. Prop. 4 for the SIMO channel with architectures (b), (c) and (d) with $N_{SQ} = 500$, $P = 10$ and $N_{\mathrm{r}} \in [1 \ldots 50]$.

(c) Lem. III.3 vs. Prop. 5 vs. Prop. 6 for the MIMO channel with architectures (a), (b) and (d) with $N_{\mathrm{sq}} = 10, N_{\mathrm{t}} = 5$, $N_{\mathrm{r}} \in [1 \ldots 10]$ and $P \in \{0.1, 1\}$.

Fig. 2: Average performance comparison.

out quantization constraint and the channel without additive noise. For the channel with additive noise, the combinatorial geometric approach in [5] is extended to architecture (d): each sign quantizer partitions the signal space along an hyperplane. Capacity is then obtained as the largest number of partitions of the received channel space that can be obtained with the available number of hyperplanes. Note that Prop. 6 does not provide a tight characterization of capacity as the gap grows with the rank of the channel matrix.

V. Numerical Evaluations

In this section, we evaluate the results in Sec. IV by considering the expected value of capacity $\mathcal{C}(\mathcal{F})$ in (4) when the channel gains H_{ij} are drawn from a Gaussian distribution with zero mean and unit variance. We begin by numerically evaluating the performance for the SIMO channel with single antenna and multilevel quantization selection in Prop. 2 and with linear combining in Prop. 4. Fig. 2a shows the upper bound expressions in (15) and (17) as a function of the number of receive antennas N_{r} and for a fixed transmit power P and number of sign quantizers N_{sq}. For $N_{\mathrm{r}} = 1$, the performance of the two architectures is the same as the SISO channel in Prop. 1 while, when N_{r} increases, the performance approaches $\log(N_{\mathrm{sq}}+1)$, albeit at a slower rate than for the single antenna selection case.

The performance of multiple antenna selection for the SIMO case is shown in Fig. 2b: in this figure, we plot the upper bound in Prop. 2 and the lower bounds in Prop. 3 and Prop. 4. From Fig. 2b we observe how increasing the number of antennas that are selected impacts the achievable rate, reducing the gap from the performance of the architecture with linear combining and multilevel quantization.

Upper bounds on the performance for the MIMO case are presented in Fig. 2c. Single antenna selection with multilevel quantization in Prop. 5 performs well when the number of receive antennas is small but its performance is surpassed by multi-antenna selection and sign quantization in Lem. III.3 as the number of receive antennas grows. This follows from the fact that the attainable rate with single antenna selection converges to $\log(N_{\mathrm{sq}}+1)$ as N_{r} grows while sign quantization converges to N_{sq}.

VI. Conclusion

A general approach to model MIMO channels with low–resolution output quantization is proposed. In this problem formulation, the antenna outputs undergo analog processing before being quantized using a fixed number of sign quantizers. Analog processing is embedded in the channel model description so that the transmission rate is maximized over the set of feasible analog processing operations. Through this formulation, it is then possible to optimize the transmission rate over the set of feasible analog processing operations while keeping the number of sign quantizers fixed.

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