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Probabilistic framework for multiaxial LCF assessment under material variability

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Abstract: The influence of material variability upon the multiaxial LCF assessment of engineering components is missing for a comprehensive description. In this paper, a probabilistic framework is established for multiaxial LCF assessment of notched components by using the Chaboche plasticity model and Fatemi-Socie criterion. Simulations from experimental results of two steels reveal that the scatter in fatigue lives can be well described by quantifying the variability of four material parameters $\{\sigma_f', \varepsilon_f', b, c\}$. A procedure for choosing the safety factor for fatigue design has been derived by using first order approximation.

Keywords: Multiaxial fatigue, elasto-plasticity, FE analysis, life prediction, uncertainty

Nomenclature

b	Fatigue strength exponent	$\sigma_{n,max}$	Maximum normal stress
b_γ	Shear fatigue strength exponent	σ_y	Yield strength
c	Fatigue ductility exponent	σ_f'	Fatigue strength coefficient
c_γ	Shear fatigue ductility exponent	τ_f'	Shear fatigue strength coefficient
k	Normal stress sensitivity coefficient	ε_f'	Fatigue ductility coefficient
n'	Cyclic strain hardening exponent	γ_f'	Shear fatigue ductility coefficient
C_i, γ_i	Kinematic hardening constants	γ_F	Safety factor
E	Elastic modulus	v	Poisson's ratio
G	Shear modulus	v_p	Plastic Poisson's ratio
K'	Cyclic strength coefficient	$\varepsilon_{ea}, \varepsilon_{pa}$	Elastic and plastic strain amplitude
$\boldsymbol{\alpha}$	Back stress tensor	σ_a, ε_a	Stress and total strain amplitude
$\mathbf{D}^e$	Elastic stiffness matrix	$\Delta\gamma_{max}$	Maximum shear strain range
$\boldsymbol{\sigma}, \boldsymbol{\varepsilon}$	Stress and strain tensor	$\gamma_{a,eq}$	Equivalent shear strain amplitude
γ_R	Strain resistance at $\hat{N}$	N_f	Number of cycles to failure
$\hat{\gamma}$	Design local strain	N_{fp}	Predicted number of cycles to failure
$\hat{N}$	Target design life		

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1 Introduction

Existence of geometrical discontinuities like notches in fatigue critical structures and components often gives rise to localized plastic strain, which may lead to crack initiation under cyclic loadings. Failure of these structures and components generally exhibits a stochastic behavior due to the variations in material properties, microstructures, geometries and in-service loads. Experimental results and process observations often appeared random in nature. Fatigue assessment of these components involves quantifying information concerning multiple variations, and fusions the information into a form suitable for fatigue design and subsequent life prediction. Since 1950s, dealing with this *spread* or *scatter* in fatigue lives played a vital role to ensure that the probability of structural failure is acceptably low. Traditional descriptions of this 'fatigue variability' are usually characterized by a distribution of lifetime response, one way to ascertain its main descriptors (mean and variance) is based on a sufficient number of tests which is time consuming and costly to generate; the other one is to conduct numerical simulations to analyze cyclic response and fatigue behavior by interfacing with probabilistic approaches and Finite Element (FE) solvers [1][2]. Since structural integrity of engineering components are directly affected by physical variability, statistical uncertainty, model uncertainty and errors [3]–[5], quantifying and controlling these uncertainties are essential to enhance the competitiveness in designing fatigue critical products such as turbine engines and railway axles [6]–[9]. Moreover, the development and application of probabilistic approaches with Physics-of-Failure (PoF)-based methods is imperative for valid structural integrity assessment of engineering components under complex loadings, i.e. multiaxial loadings in a low cycle fatigue (LCF) regime [10].

From the engineering design point of view, experience/expert judgment-based safety factors are generally introduced by a designer to balance the effects of multiple sources of uncertainty, which tends to reach an acceptable level of likelihood of failure. However, the direct use of scatter factors for representing fatigue variability often result in conservative or even over-conservative results. A conservative safety factor can lead to heavy structures and unnecessary maintenance costs. In particular, empirical or semi-probabilistic approaches are suggested in the standards/codes, which provide the designer a series of partial safety factors accompanying with limit state design codes to achieve the target reliability [11]. Among them, Eurocode 3 [12] introduces S-N diagrams under 5% failure probability P_f for different welded details, and suggests a safety factor $\gamma_F = 1.35$ onto the S-N curve of the welded detail for duality assessment of safety critical components. Similarly, the FKM Guideline [13] prescribes S-N diagrams with $P_f = 2.5\%$ and $\gamma_F = 1.5$ for the steel and aluminum components. However, these prescriptions cannot precisely elaborate the relationship between the load dispersion and the chosen safety factor, and cannot be used for fatigue design under a given target P_f. According to the nuclear design code [14], design curves can be fitted from experimental data through lowering by a scatter factor-of-two on stress or a scatter factor-of-twenty on life. These factors are not safety margins but rather adjustments for the uncertainties and scatter resulting from the fatigue data of laboratory tests to those of actual components [15]. For aero engine components, life curve corresponding to $P_f = 10^{-3}$ (i.e. $\mu - 3\sigma$) obtained from experiments is referred for life assessment, which ignored the load variability and some other safety factors are needed for achieving the target reliability in LCF design [16]. For helicopter components, the so-called "TOS/$\mu - 3\sigma$" methodology was introduced by using a working S-N curve, which defines the 'fatigue strength' of the material by three standard deviations below the mean and an 'extreme load spectrum' by three standard deviations above the mean. However, a simple application of these concepts for multiaxial fatigue criteria hasn't yet been explored.

Until now, continuous efforts are being made in an attempt to reduce the over-conservatism in LCF design. Among them, probabilistic approaches provide a rational way for fatigue design under uncertainty. Systematically dealing with the uncertainties in fatigue significantly affects the performance in defining morphology and material of engineering components, and also the robustness of fatigue design. These combined effects might lead to a large scatter of structural performance. Thus, appropriate treatment of uncertainty in fatigue design has become a significant topic with widespread interest [17]–[24]. Prior work on statistical or probabilistic aspects of fatigue includes modeling of the variability in material properties (e.g., elastic modulus, fracture toughness, yield strength) [6], [17], [25]–[27], equivalent initial flaw size (EIFS) [21], [28]–[30], microstructures as well as defects [31]–[35], stress-life data [36]–[39], and under multiaxial conditions [40]–[45]. Generally, two aspects need to be addressed for probabilistic fatigue design: a valid PoF-based fatigue model and a probabilistic framework for treating both the random material variables and the uncertainty on model parameters in the fatigue model [46], which has been reviewed in detail recently by Pineau et al. [47].

In order to quantify LCF life variability, most of available researches [5], [17], [37], [38], [48]–[50] focused primarily on obtaining the lifetime distribution from variation under given loading conditions as well as, the variation in the crack formation and crack growth rates.

Among them, Correia et al. [49] explored the generalization of various fatigue damage parameters to obtain the Weibull percentile curves, which provide new perspectives for utilizing the probabilistic approach to estimate fatigue life of structural components. Naderi et al. [26] investigated the effects of finite element types and sizes, initial flaws and material property variations on the fatigue life scatter of metals, and predicted the plastically induced fatigue damage for four types of metals. Pessard et al. [51] developed a probabilistic multiaxial modelling framework to describe the Kitagawa–Takahashi diagram by combining two fatigue damage mechanisms, which is based on the weakest link hypothesis. Recent research in [38] demonstrated that current safe-life design methods [12]–[14], [52], [53] introduced a series of partial safety factors to empirically or semi-empirically consider the variability in applied stress and material behavior. Sandberg et al. [22] pointed out that the fatigue failure probability should be used in design by modeling all uncertainties that influence the problem, i.e. the uncertainty stems from material properties and the variation in loading of components. These types of analyses indicated that combination of numerical simulation and cyclic plasticity modeling can yield more accurate results than traditional ones. However, current statistical approaches developed to account for fatigue behavior and life scatter ignored certain aspects of material variability and load fluctuations. Thus, a complete and systematic framework for probabilistic modeling of material variability and load fluctuations for fatigue design is expected.

Because of material variability, mechanical response in a component level is going to be stochastic as well. From the point of view of constitutive modeling, deterministic models generally represent a description of the average macroscopic behavior by the regression of test data. Few theoretical basis is available to infer further mechanical response variability of the material. Zhao et al. [54] performed a statistical investigation on the cyclic stress-strain response of a nuclear material. They introduced a probability-based Ramberg-Osgood form to model the random response of the nuclear material [55]. Different from the analytic approximation approach based on Neuber rule and Ramberg-Osgood equation applied in [38], this paper follows the combination of FE analysis and experiments dealing with stochastic nature of material response numerically by using Latin hypercube sampling technique.

The traditional deterministic approach for multiaxial LCF design has been the dominating method in the industry, however, current certified methods are essentially phenomenological, and based on design codes/standards. Such methods are empirical and conservative and cannot adapt easily to a change in materials or process, and often be over-conservative in stress concentration areas. Moreover, few studies have explored the chosen safety factor from a probabilistic design set-up prospective of material variability and load dispersion. According to this, the purpose of this study is to quantify the effects of material variability and load fluctuations on notched fatigue behavior and life scatter of materials in a multiaxial LCF regime, and suggest the scatter in fatigue lives accurately rather than conservatively presented, then provide an indication for choosing or defining valid safety factors in multiaxial LCF design.

The rest of the paper is organized as follows. Section 2 elaborates the material variability in uniaxial and multiaxial LCF. In addition, the material and model parameters are identified by using stochastic modeling and curve fitting techniques. In Section 3, a numerical framework for probabilistic multiaxial LCF design and life prediction is proposed by interfacing FE with Latin hypercube samplings. Section 4 compares the results of simulations, first order approximations (FOA) with the experimental data of 950X and 9CrMo steels, and discussions together with designs under different safety factors and load fluctuations are given. Finally, Section 5 concludes the remarks of this paper.

2 Uncertainty in material response

2.1 Material variability in uniaxial LCF

Looking to the history of researches relate to fatigue, most of them focused on developing or using deterministic methods to improve resistance of materials/structures against fatigue. However, the fatigue cracking (both of crack initiation and propagation) process is inherently random due to material variability, microstructural irregularities and uncontrolled test conditions. In addition, various sources of uncertainty arising from a simplified representation of actual physical process (primarily using semi-empirical or empirical models) and/or sparse information on manufacturing, material properties, and loading profiles contribute to stochastic behavior or scatter of failure mechanism modeling and analysis [4][23].

In fatigue design of safety critical components in mechanical systems, various sources of uncertainty including variations in material property, loading and geometry, which can be broadly divided into two categories: aleatory and epistemic [1]. The first one drives from the inherent variation resulting from the physical process, and cannot be reduced but need to be better quantified; the latter one derives where knowledge or information is lacking in fatigue cracking analysis, and can be reduced after the acquirement of new information and/or better use of the data and/or more accurate modeling methods. The cyclic stress-strain response in a structure/component provides the necessary relation for cyclic plasticity modeling and analysis. Note from the cyclic stress-strain relationships that cyclic loading significantly affects the deformation behavior of the material. However, its scatter is seldom considered in traditional fatigue design even in relative reliability analysis, which depends on inherent behavior of the material and whether such behavior can be altered by prior loading or after periods of service aging.

Variations of the stress amplitudes with cyclic response and fatigue life fraction for 950X steel under uniaxial fatigue in our previous work [38] are shown in Fig. 1. It's worth noting that the cyclic stress-strain responses for present material exhibited a significant scatter. In another words, a given load may lead to the random cyclic strain response, which might result from the difference of microstructural growth processes ahead of the main crack tips.

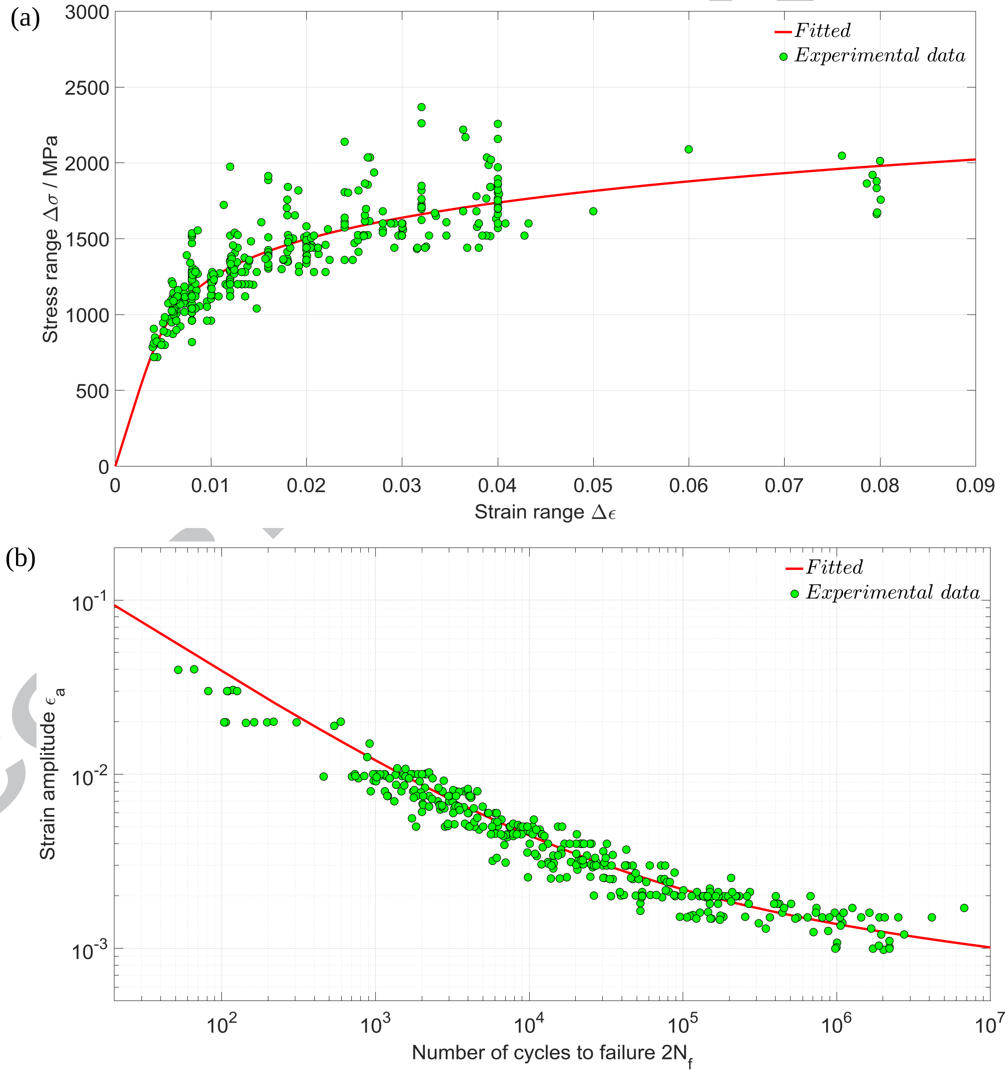


Fig. 1 Variations of (a) cyclic response and (b) stain-life for 950X steel

Through simulating the cyclic stress-strain response based on fatigue testing of smooth specimens, the so-called strain-life approach that includes both of the elastic and plastic components of deformation has been commonly used for fatigue failure analysis, which relates the local elastic-plastic behavior of the material under fatigue loadings. Based on this, the relationship between total strain amplitude ε_a and fatigue life N_f in reversals can be given by the Coffin-Manson equation,

$$\varepsilon_a = \varepsilon_{ea} + \varepsilon_{pa} = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \tag{1}$$

where σ_f' and b denote the fatigue strength coefficient and exponent; ε_f' and c represent the fatigue ductility coefficient and exponent, respectively.

When using Eq. (1) to model fatigue behavior of a material, its cyclic stress-strain response can be generally described by the Ramberg-Osgood equation as

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'} = f_{RO}(\sigma) \tag{2}$$

where ε and σ are the local strain and stress at a given location; E is the elastic modulus; K' and n' denote the cyclic strength coefficient and cyclic strain hardening exponent, respectively.

Assuming that elastic and plastic components in the Coffin-Manson equation (1) correlate well with that of Ramberg-Osgood equation (2) leads the following equations [56]

$$\frac{\Delta\sigma}{2} = \sigma_f'\left(2N_f\right)^b = K'\left(\varepsilon_f'\right)^{n'}\left(2N_f\right)^{cn'} \Rightarrow \begin{cases} \sigma_f' = K'\left(\varepsilon_f'\right)^{n'} & (3) \\ n' = \dfrac{b}{c} & (4) \end{cases}$$

Using Eq. (1) to Eq. (4), cyclic response and fatigue behavior of materials can be well described, where only four of the six material parameters $\{\sigma_f', \varepsilon_f', b, c, K', n'\}$ can be independently fitted from actual measurements, as reviewed in [38], [57], [58]. In engineering practice, a traditional strain-life curve that only represents median response of the material was found to be insufficiently robust, thus a probabilistic one that incorporates material variability is necessary. In addition, the variability of material response in Eq. (1) and Eq. (2) can be included by means of variability of these six parameters.

The key to assess the reliability of fatigue critical components is to realistically quantify the input variables and their variations and potential correlations. In particular, those six parameters can be fitted and/or derived from the cyclic stress-strain relationship and strain-life curve according to Eq. (1) and Eq. (2) for each batch of experimental data. Recently, several methods have been developed for assessing the Coffin-Manson parameters [57][58]. From a statistical point of view, as indicated by Meggiolaro et al. [57][58], accurate estimations can be derived by treating the exponents b and c as constant values, and σ_f' as a linear function of ultimate strength σ_u.

In the present work, a statistical analysis of material properties for 16 different batches of test data of 950X steel [33] and 12 different batches of 9CrMo steel at 550℃ [63], including the Coffin-Manson parameters $\{\sigma_f', \varepsilon_f', b, c\}$, Ramberg-Osgood parameters $\{K', n'\}$, elastic modulus E, and yield strength σ_y, is performed. The correlation analysis has shown that: (1) the correlations between $log(\sigma_f')$ and b as well as $log(\varepsilon_f')$ and c are so strong that a uniform scatter along $logN$ both for the elastic and plastic part of Eq. (1) can be obtained as taken in [38][39]; (2) other correlations are poor or very weak; (3) Compared to the other properties, variability in elastic modulus E is small and generally ignored. These findings enable us to treat the parameters b, c and E as constants, and treat $log(\sigma_f')$ and $log(\varepsilon_f')$ as independent Gaussian random variables, while the Ramberg-Osgood parameters K' and n' for the ith batch can be simply calculated using Eq. (3) and Eq. (4). The consequences of these assumptions will be discussed in Section 2.2.3.

For uniaxial LCF analysis of the two steels, the variability of material response both in the Ramberg-Osgood and Coffin-Manson diagrams can be well described by considering the variability of the four parameters $\{\sigma_f', \varepsilon_f', b, c\}$, their mean values have been calculated as listed in Table 1. In particular, the Ramberg-Osgood parameters $\{K', n'\}$ and yield strength σ_y are intermediate variables, and σ_y is obtained by the 0.05% offset rule ($\varepsilon_{pa} = 0.05\%$) as

$$\sigma_y = K'(\varepsilon_{pa})^{n'} \tag{5}$$

Table 1 Material parameters of the two steels

Parameter	950X	9CrMo at 550℃
ε_f'	$\mathrm{Lognormal}(-0.3065, 0.12597^2)$	$\mathrm{Lognormal}(0.2755, 0.10645^2)$
σ_f'/MPa	$\mathrm{Lognormal}(2.9298, 0.045119^2)$	$\mathrm{Lognormal}(2.5984, 0.035078^2)$
b	-0.0921	-0.0498
c	-0.5679	-0.7917
E/MPa	200892	179477

2.2 Material variability in multiaxial LCF

2.2.1 Cyclic plasticity model

Valid modeling and description of cyclic plasticity provides the indispensable relationship that reveals the elasto-plastic behavior of materials under cyclic loadings [59]. In this study, the Chaboche nonlinear kinematic hardening model [60] is introduced to model the plastic behavior of the two steels under study, which can simulate the ratchetting and shakedown effects during the FE analysis. The rate-independent material behavior can be modeled by using an additive rule, which is composed of elastic and nonlinear inelastic analyses based on the von Mises yield surface. Under isothermal cyclic loading conditions, the main equations are

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p \tag{6}$$

$$\boldsymbol{\sigma} = \mathbf{D}^e : \boldsymbol{\varepsilon}^e \tag{7}$$

$$d\boldsymbol{\varepsilon}^p = d\lambda \frac{\partial f}{\partial \boldsymbol{\sigma}} \tag{8}$$

For the initially isotropic homogenous material, the von Mises condition is used as

$$f = \sqrt{\tfrac{3}{2}(\mathbf{s} - \boldsymbol{\alpha}) : (\mathbf{s} - \boldsymbol{\alpha})} - \Upsilon = 0 \tag{9}$$

where $\mathbf{D}^e$ is the elastic stiffness matrix; $\mathbf{s}$ is the deviatoric part of stress tensor $\boldsymbol{\sigma}$; $\boldsymbol{\alpha}$ is the deviatoric part of back stress tensor $\mathbf{a}$; Υ is the size of yield surface.

The plastic multiplier $d\lambda$ in Eq. (8) corresponds to the accumulative plastic strain increment

$$dp = \sqrt{\tfrac{2}{3} d\boldsymbol{\varepsilon}^p : d\boldsymbol{\varepsilon}^p} \tag{10}$$

In order to accurately describe the nonlinearity of stress-strain loops, a significant improvement on the Armstrong-Frederick model [61] was given by Chaboche and Rousselier [60] by dividing the deviatoric part of back stress tensor $\boldsymbol{\alpha}$ into M independent parts as

$$\boldsymbol{\alpha} = \sum_{i=1}^{M} \boldsymbol{\alpha}^{(i)} \tag{11}$$

whereas evolution of each kinematic part is expressed according to [61]

$$d\boldsymbol{\alpha}^{(i)} = \tfrac{2}{3} C_i d\boldsymbol{\varepsilon}^p - \gamma_i \boldsymbol{\alpha}^{(i)} dp \tag{12}$$

where C_i and γ_i are experimentally fitted material constants in the rate-independent scheme; the suitable choice of the last back stress part γ_M leads to an accurate model for uniaxial ratcheting behavior.

In order to describe the variability in cyclic stress-strain response accurately for a notched component, a procedure to determine the constitutive coefficients is outlined here for using the Chaboche model with three evolution parts ($M = 3$), which provides sufficient variations to calibrate the nonlinear behavior of materials. More information on the determination of Chaboche model parameters can be referred to [59]. According to the usage of Armstrong-Frederick evolution law, the stress range $\Delta\sigma$ can be expressed by a function of plastic strain range $\Delta\varepsilon_p$ as

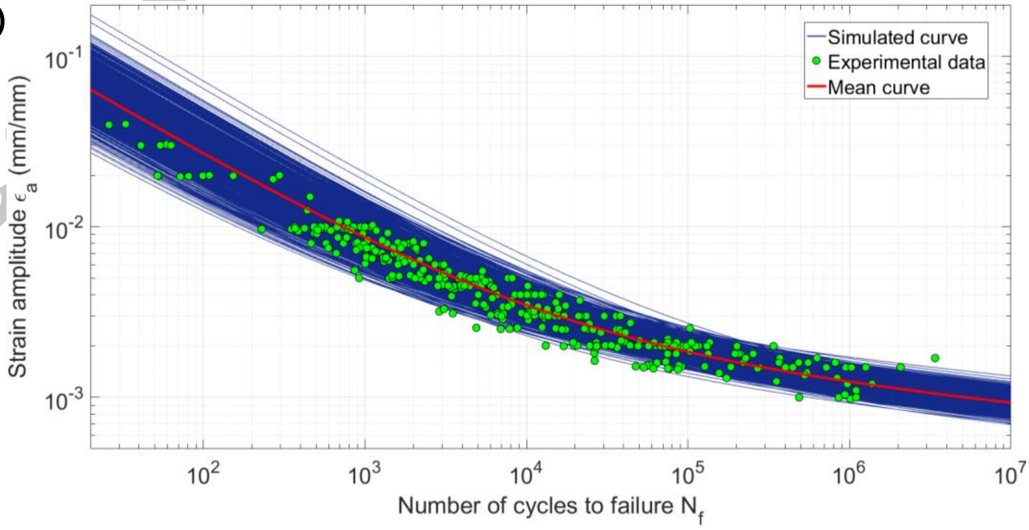
$$\frac{\Delta\sigma}{2} = \sigma_y + \sum_{i=1}^{M} \frac{C_i}{\gamma_i} \tanh\left(\gamma_i \frac{\Delta\varepsilon_p}{2}\right) \tag{13}$$

where the parameters C_i and γ_i $(i = 1, 2, \cdots, M)$ and the initial yield strength σ_y can be determined from uniaxial tests [62][63] using Eq. (5) and Eq. (13).

Generally, two steps fitting procedure are introduced to calibrate the Chaboche model parameters: the first one is to apply the Ramberg-Osgood equation to model the cyclic stress-strain curves of the material, which can produce basic smoothing and extrapolation; then conduct the curve fitting of a typical rate-independent form of the Chaboche model with 3 kinematic back stresses, as shown in Eq. (13), according to [59]. Specifically, the constitutive parameters can be determined using some numerical optimization scheme through fitting the Chaboche model to the Ramberg-Osgood extrapolated cyclic stress-strain curves, which minimizes the differences between experimental and predicted cyclic stress-strain curves.

In this study, the uncertainties associated with material variability and the analytical procedures employed (such as strain-life method, i.e. the Coffin-Manson equation) are quantified in cyclic stress-strain data and the derived constitutive equation parameters (such as Ramberg-Osgood equation parameters). Based on the stochastic material properties in Table 1, an example plot of simulated strain-life curves, stochastic cyclic stress-strain response and Chaboche model parameters C_1~C_3 in Eq. (13) of 950X steel is given for $s = 100MPa$ with $CV_{\hat{S}} = 0.05$ as shown in Fig. 2. The abovementioned optimization-based identification procedure can be conducted by means of an inverse method in a C++ environment/MATLAB® platform, or directly by using an embedded curve fitting tool of ANSYS® 17.0.

(a)



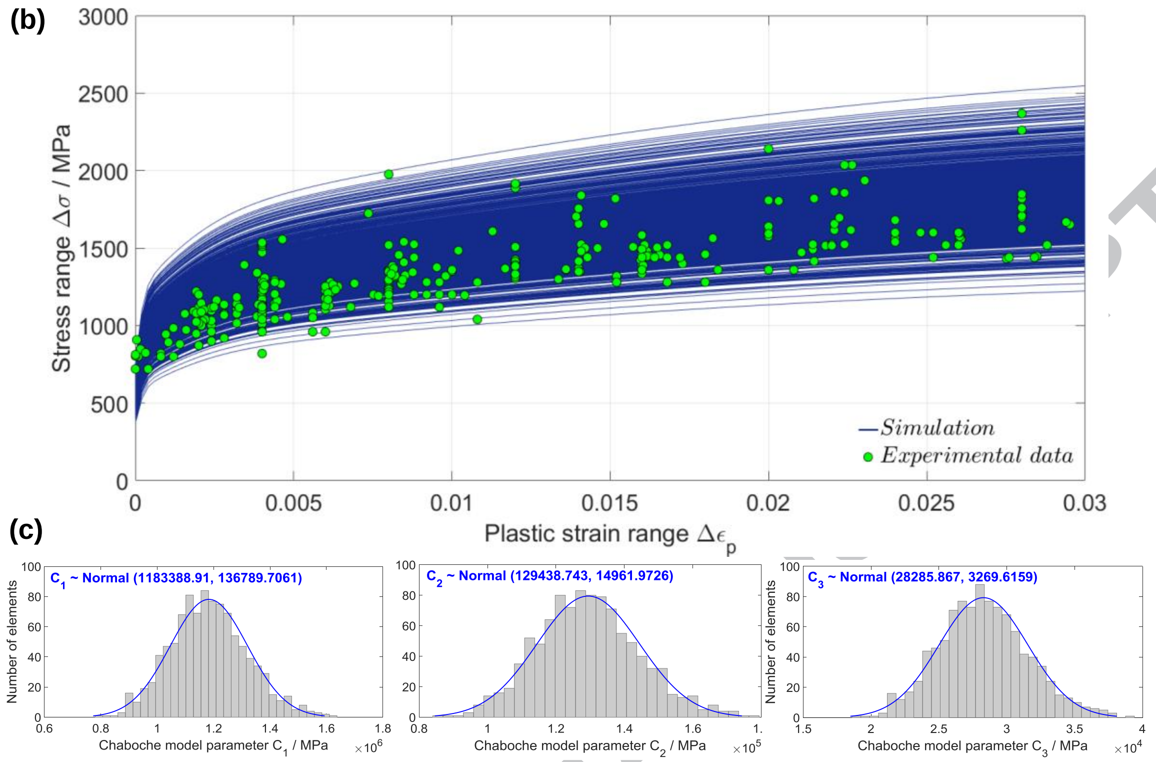


Fig. 2 Simulated cyclic response using Chaboche model (case $M = 3$): (a) simulated strain-life curves, (b) simulated cyclic stress-strain response with experimental data and (c) histograms of the three parameters C_1, C_2 and C_3

2.2.2 Fatemi-Socie damage criterion

Notch root analysis for evaluating driving forces during crack formation and estimating fatigue life has been critical in multiaxial fatigue design of notched components [64]–[66]. Generally, the scatter on fatigue predictions depends on the proper methods used for fatigue failure mechanism and damage modeling, the identification of load spectrums, and the characterization of elastic-plastic deformation behavior. Moreover, the uncertainties associated with material variability in notched fatigue analysis enable the statistical assessment of fatigue damage indicator in a multiaxial LCF regime. Due to the complexity of real stress/strain states in engineering components, a multiaxial stress/strain state can be reduced to an equivalent uniaxial one by using critical plane approaches, and fatigue failure occurs once the cumulative fatigue damage on the material plane reached the damage threshold [67]. Among them, a well-known Fatemi-Socie multiaxial fatigue criterion [68] is used for estimating fatigue life and locating fracture plane positions, although other multiaxial criteria can also be applied [69].

As a critical plane criterion for shear-dominated crack formation and initial stages of crack growth, the Fatemi-Socie criterion is presented based on the equivalent shear strain amplitude $\gamma_{a,eq}$, a modified cyclic shear strain by the normal stress, which includes the crack closure effect as

$$\gamma_{a,eq} = \frac{\Delta\gamma_{max}}{2}\left(1 + k\frac{\sigma_{n,max}}{\sigma_y}\right) = \frac{\tau_f'}{G}\left(2N_f\right)^{b_\gamma} + \gamma_f'\left(2N_f\right)^{c_\gamma} = f_{FS}\left(N_f\right) \tag{14}$$

where k is the normal stress sensitivity coefficient that describes the influence of normal stress on fatigue life, which is a material and life dependent parameter and generally fitted from uniaxial to torsion fatigue tests; $\Delta\gamma_{max}$ and $\sigma_{n,max}$ are the maximum shear strain and the maximum normal stress on the critical plane, respectively.

Model in Eq. (14) characterizes the cracking behavior in tension and torsion LCF, and also considers the effects of mean stress and non-proportional hardening. For situations where the local dominating stresses and strains are elastic, the McDiarmid criterion [70] yields the same estimations as the Fatemi-Socie criterion. Thus, the Fatemi-Socie criterion is introduced for the notch root analysis case in the present work due to its universality.

Similar to the Coffin-Manson equation, the right-hand side of Eq. (14) models macroscopic elastic-plastic behavior at the notch root and correlates the number of cycles to crack initiation N_f, and shear strain-life properties can be estimated by [44]

$$\begin{cases} \tau_f' \approx \frac{\sigma_f'}{\sqrt{3}} \\ \gamma_f' \approx \sqrt{3}\varepsilon_f' \\ b_\gamma \approx b \\ c_\gamma \approx c \end{cases} \tag{15}$$

where τ_f' and b_γ are the shear fatigue strength coefficient and exponent, respectively; γ_f' and c_γ are the shear fatigue ductility coefficient and exponent; G is the shear modulus, $G = E/2(1+v)$, v is the Poisson's ratio.

For the normal stress sensitivity coefficient k, it can be estimated by $k \approx \sigma_y/\sigma_f'$ under limited test data conditions [71]. In reality, the Fatemi-Socie parameters can be approximately obtained from Coffin-Manson parameters under uniaxial loadings by Eq. (15), or fitted from different batches of experimental data under multiaxial loadings. In general, the coefficient k can be identified based on tension-compression and/or torsion tests by

$$k = \left[\frac{\frac{\tau_f'}{G}\left(2N_f\right)^{b_\gamma} + \gamma_f'\left(2N_f\right)^{c_\gamma}}{(1+v)\frac{\sigma_f'}{E}\left(2N_f\right)^{b} + (1+v_p)\varepsilon_f'\left(2N_f\right)^{c}} - 1\right]\frac{2*\sigma_y}{\sigma_f'\left(2N_f\right)^{b}} \tag{16}$$

where v_p is the plastic Poisson's ratio, and $v_p = 0.5$.

2.2.3 Simulation of material behavior

In order to characterize the material variability on the cyclic stress-strain and strain-life responses of the two steels under multiaxial fatigue, the five Fatemi-Socie parameters $\{\tau_f', \gamma_f', b_\gamma, c_\gamma, k\}$ have been calculated for different batches of the two steels. In details, the experimental $\gamma_{a,eq} \sim N_f$ curve is fitted from uniaxial fatigue tests as shown in Fig. 3. Note from Eq. (16) that k is a function of σ_f' and ε_f', and can be estimated by the transforming the Fatemi-Socie life curves into $k - N_f$ curves during life in the relevant life range (in the example $[1E4,1E5]$). Thus, for multiaxial fatigue analysis of notched components, the variability of the material behavior both in the Chaboche mode and Fatemi-Socie damage criterion can be well modeled by using the four parameters $\{\sigma_f', \varepsilon_f', b, c\}$ (reported in Table 1). Using Eq. (5), Eq. (13) and Eq. (16), the six parameters, including the Ramberg-Osgood parameters $\{K', n'\}$, yield strength σ_y, Fatemi-Socie parameter k and Chaboche parameters $\{C_1, C_2, C_3\}$ are intermediate variables during the simulations.

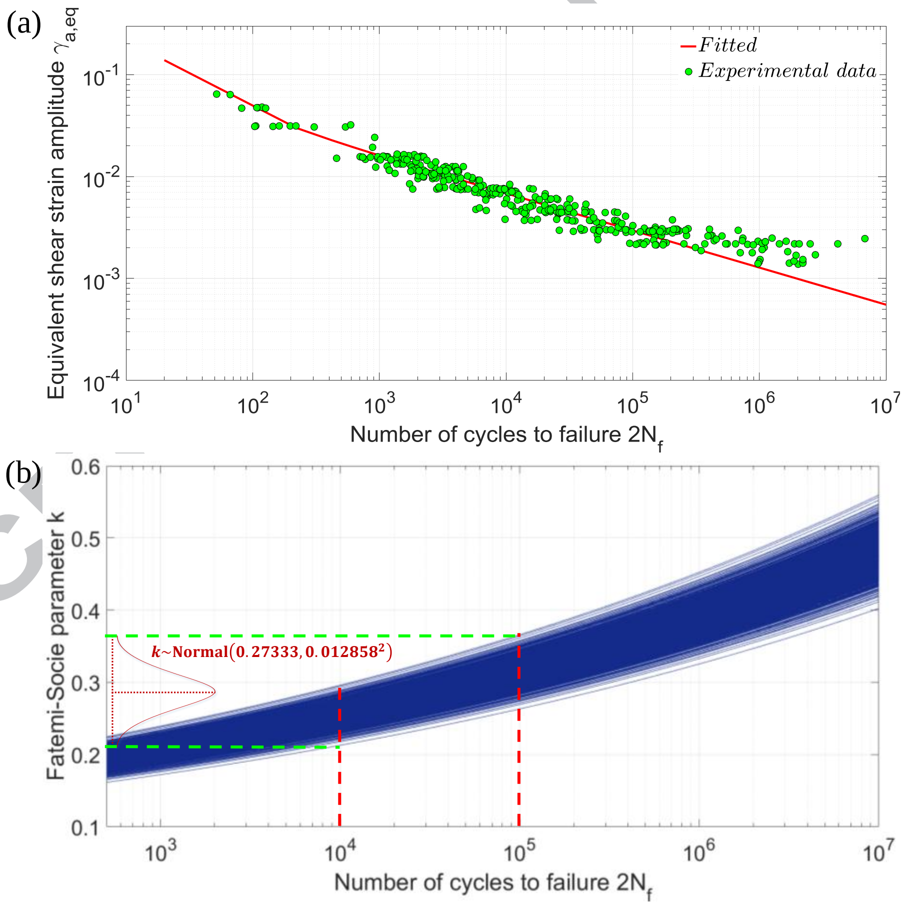


Fig. 3 Lognormal format of (a) Fatemi-Socie criterion and (b) k-N relationship of 950X at $s = 100MPa$ with $CV_{\hat{s}} = 0.05$

In this study, 16 batches of uniaxial fatigue test datasets of 950X steel and 12 batches of 9CrMo steel at 550℃ are taken from the literature [39] for variability analysis in material response under multiaxial fatigue loadings. Using Eqs. (3)-(4), through random extraction of parameters $\log(\sigma_f')_i$ and $\log(\varepsilon_f')_i$ (reported in Table 1) using 10^3 Latin hypercube samples, the local material response of the notched component can be simulated by calculating K_i', and as well as the yield strength σ_y using Eq. (5) and the Fatemi-Socie parameter k using Eq. (16).

Based on the stochastic material properties, a correlation analysis among the simulated 8 parameters was conducted as shown in Fig. 4. The relevant conclusions that can be drawn are: *i*) K' has a strong correlation with ε_f' and σ_f' according to Eq. (3); *ii*) k shows a correlation with the other parameters according to Eq. (16); *iii*) the shape of the cyclic curve fixed as $n' = b/c$ makes the parameters C_1, C_2 and C_3 perfectly correlated (so they are not independent) and that C_1 is simply a function of K'. The last conclusion is very important since it demonstrates how the Chaboche plasticity model can be made consistent with a reasonable yet simple description of material variability. Thus, a simple probabilistic analysis of mechanical response of the notched component can be conducted under different load dispersions.

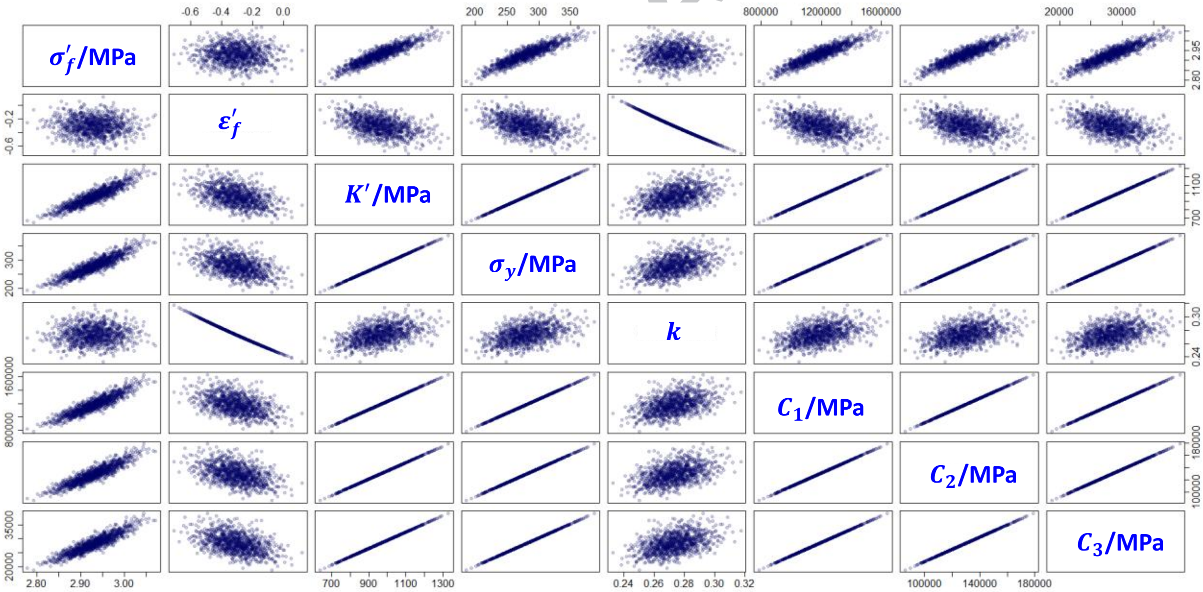


Fig. 4 Correlation analysis among simulated 8 parameters

3 Probabilistic framework for multiaxial LCF assessment

Elastic-plastic behavior of fatigue critical components is generally modeled by using material properties parameters based on calibrated tests in a deterministic way. However, this mechanical behavior is inherently stochastic. Deterministic simulations have so far been almost exclusively conducted using safety factors to consider the material variability and model uncertainty that were neglected.

From the viewpoint of practical applications, the probabilistic analysis process should be as simple as possible while maintaining reasonable accuracy for calculating the failure probability of fatigue critical components. In this study, a procedure for probabilistic multiaxial LCF design and life prediction is developed that interfaced commercially available probabilistic software, including simultaneous solutions of the constitutive relation using ANSYS® and the life calculation using MATLAB®. During the probabilistic FE analysis of the notched component, material properties and load fluctuations are the random inputs, where the variability of the applied load $\hat{S}$ is modeled by Normal distributions with different Coefficient of Variations, namely $CV_{\hat{S}} = (0.5, 0.10, 0.15)$, according to its load spectrum analysis [38]. Specifically, the Latin hypercube sampling procedure is introduced to update the specific material characteristics for each simulation, as shown in the flow chart of Fig. 5. The Latin hypercube sampling technique gives comparable results to the Monte Carlo sampling, but with fewer samples. It was implemented using MATLAB®.

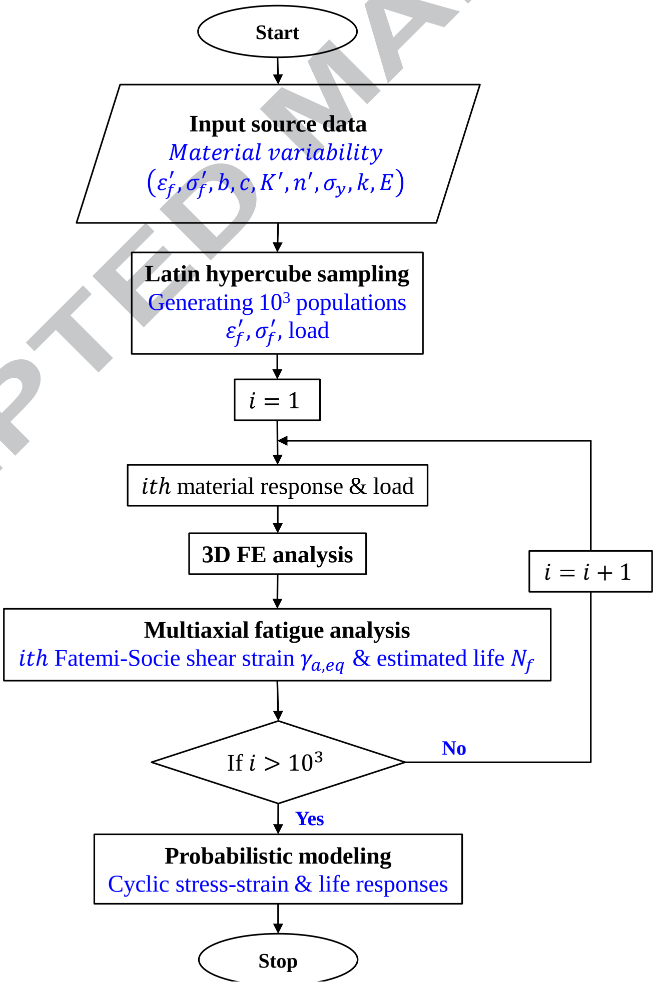


Fig. 5 Flow chart for Latin hypercube sampling-based simulations

Procedure 1 outlines the detail algorithm for probabilistic multiaxial LCF design and analysis of notched components:

Procedure 1 Probabilistic multiaxial LCF design of notched components	
Inputs:	
Structural model of the notched component; distributions of material properties (σ_f', ε_f'); load and its fluctuations ($CV_{\hat{S}}$); cyclic plasticity model; failure criterion under multiaxial LCF	
	Material and model parameters identification
1:	Correlation analysis of material property parameters, clarify and generate the independent random variables;#
2:	Calculate the cyclic response for the *i*th batch, and quantify the scatter of cyclic response by using Latin hypercube samplings and solving Eqs. (3) - (4);#
3:	Curve fitting of Chaboche and Fatemi-Socie model parameters under material variability;#
	Probabilistic multiaxial LCF design
4:	3D nonlinear FE analysis of the notched component, and output the stress-strain states of the element with the maximum Fatemi-Socie equivalent shear strain amplitude;*****, #
5:	Determination of the critical plane by using the Fatemi-Socie criterion;#
6:	Calculate the number of cycles to fatigue failure using Eq. (14);#
7:	Conduct 10^3 Latin hypercube samples to quantify the influence of material variability and load fluctuation on the cyclic stress-strain and strain-life behavior of the notched component;#
8:	Output the distributions of Fatemi-Socie equivalent shear strain amplitude and life;#
9:	Quantification of cyclic stress-strain response, life scatter and safety factors in multiaxial LCF design;#
10:	Estimation of failure probability P_f using first order approximation (FOA) format;#
11:	Calculation of the design point $(\hat{N}, \hat{\gamma})$ for achieving the target reliability;#
Output: Robust design point $(\hat{N}, \hat{\gamma})$ with a target reliability under multiaxial LCF	

Note: ***** and # represent the use of ANSYS and MATLAB, respectively.

The abovementioned procedure is conducted through coupling of ANSYS® and MATLAB® calculations. The block diagram in Fig. 6 illustrates the process on probabilistic multiaxial LCF design of notched fatigue behavior and life scatter of fatigue critical components.

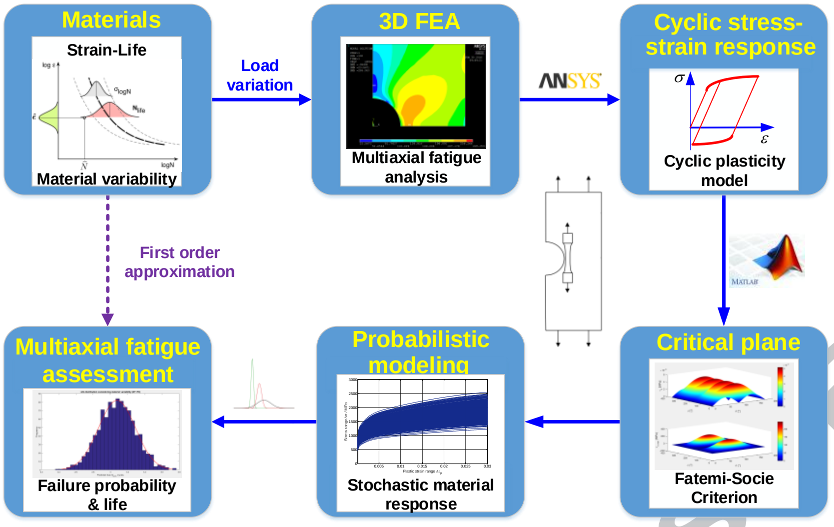


Fig. 6 Schematic interpretation of numerical procedure for multiaxial LCF assessment

According to the probabilistic simulations conducted in Fig. 6, the randomness of fatigue resistance of the material and actual loading process is quantified for probabilistic description of fatigue variability in structural integrity assessment and design. In addition, distributions of predicted fatigue life after a period of random loadings can be obtained. Through this probabilistic analysis of multiaxial LCF design set-up, an improved control over safety will be gained and this will lead us to robust safe-life design for achieving the target reliability.

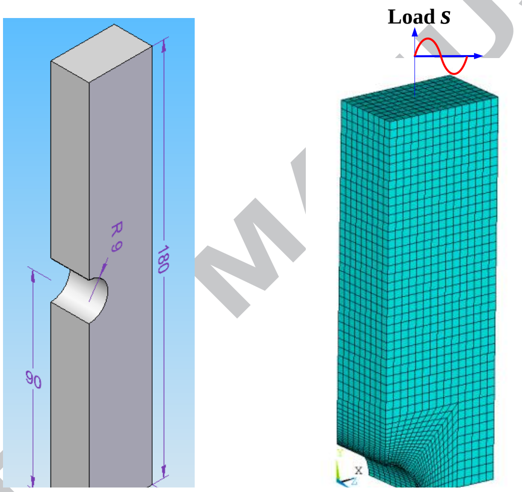
4 Model application to a notched component

In order to verify the previous framework for multiaxial LCF assessment of notched fatigue behavior and life scatter in ANSYS®, 16 batches of strain controlled fatigue test datasets of 950X steel and 12 batches of 9CrMo steel at 550℃ are taken from the literature [39][72] for model validation and application.

4.1 3D Finite element modeling

To apply the strain-life approach, a precise determination of the strain at the location of interest is required, usually the notch root in components. Using the constitutive model and Fatemi-Socie damage criterion outlined in Section 2, a computational model of single semicircular edge-notched plate is developed for cyclic stress-strain analysis, fatigue design and life assessment under cyclic loadings (see Fig. 7a). Specifically, nonlinear 3D FE calculations are conducted for the two steels using the Chaboche non-linear kinematic hardening model embedded in ANSYS® 17.0. Due to the symmetric condition, a one-half model of the single semicircular edge-notched plate is built and meshed using Solid 185 elements with a minimum size of 0.16 mm at the notch root chosen after a convergence analysis (see Fig. 7b).

The stochastic material properties used in the 3D FE model, including the variability in material properties as shown in Table 1, are assigned through coupling of Latin hypercube sampling and FE analysis. Remote applied loads were imposed via pressure boundary condition on the upper surfaces, the lower surface in Fig. 7b is a plane of symmetry. The elastic and elastic-plastic constants are extracted from a stable hysteresis loop under uniaxial fatigue tests according to Section 2.2.1. Moreover, the amplitude of imposed sinusoidal fully reversed loads ($R = -1$) is repeated and three loading cycles in each simulation are loaded on the specimen in the Y-axial direction in all the simulations.



(a) Schematic dimension (mm) (b) 3D FE model

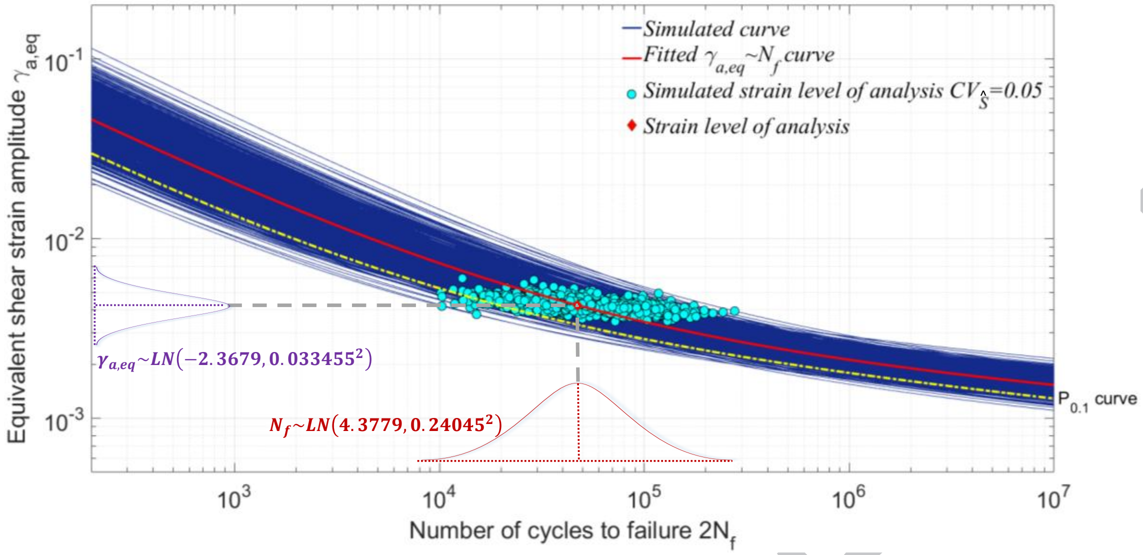
Fig. 7 Single semicircular edge-notched plate

4.2 Results

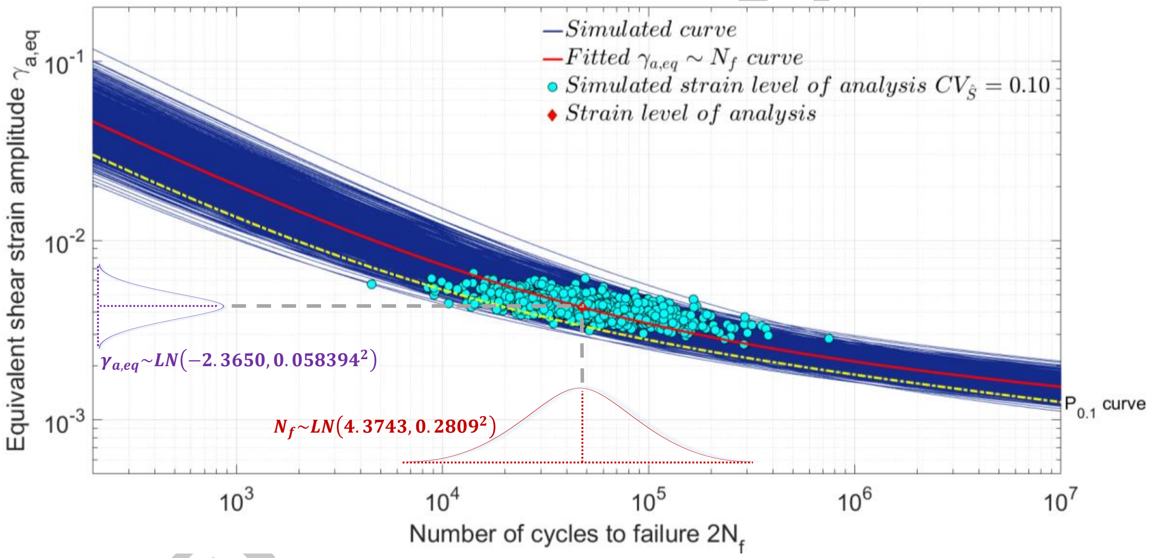
A notched component made of the two steels as shown in Fig. 7b is adopted for multiaxial LCF assessment and life scatter analysis under fully reversed cyclic loadings. Simulations on different stress levels have been conducted to validate the proposed procedure for each material in different regions of the strain-life diagram. Different strain levels $\hat{\gamma}$ correspond to different $\hat{s}$ levels, which are listed in Table 2.

Table 2 Life at different strain levels $\hat{\gamma}$

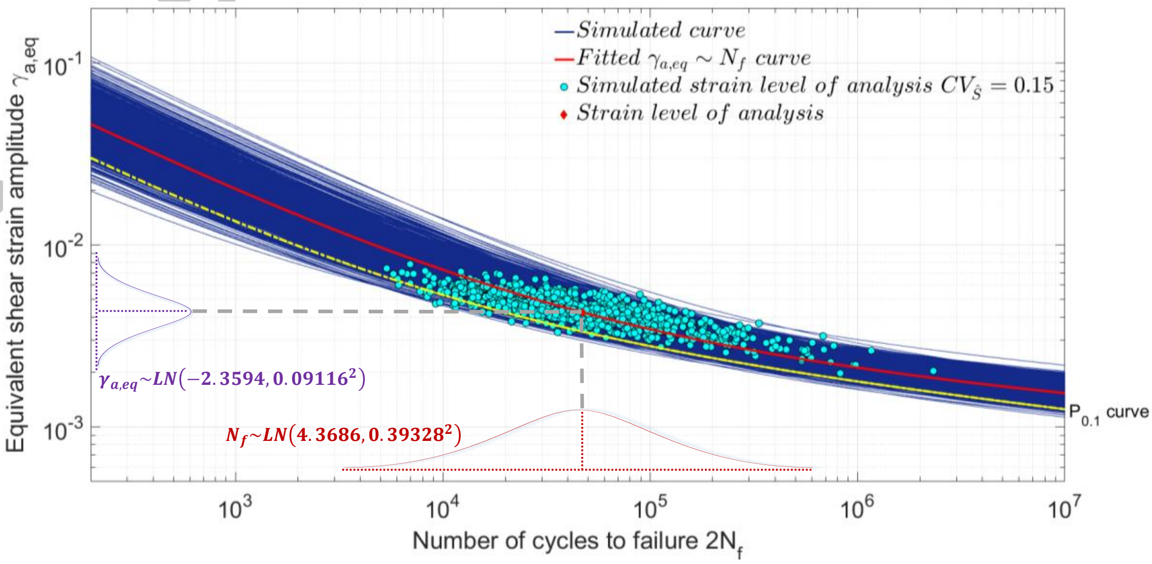
	Strain levels			
$\hat{s}$ (MPa)	90	100	150	950X steel
$\hat{\gamma}$ (mm/mm)	$3.703E-3$	$4.25E-3$	$7.815E-3$	
$\overline{N}$ (cycles)	38580	23870	4211	
$\hat{s}$ (MPa)	65	80	90	9CrMo steel
$\hat{\gamma}$ (mm/mm)	$3.339E-3$	$5.185E-3$	$1.053E-2$	
$\overline{N}$ (cycles)	10961	4657	972	



(a) $CV_{\hat{S}} = 0.05$



(b) $CV_{\hat{S}} = 0.10$



(c) $CV_{\hat{S}} = 0.15$

Fig. 8 Simulations for edge-notched plate made of 950X steel at $s = 100MPa$ with $CV_{\hat{S}} = 0.05 \sim 0.15$

Through following the framework in Section 3, Fig. 8 show the histogram and regression line of cycles to failure and $\gamma_{a,eq} - N$ curves of the simulations of an edge-notched plate made of 950X steel. According to the numerical results shown in Fig. 8, the lognormal approximation yields accurate estimations for the dispersions of both model predictions and Fatemi-Socie shear strain responses, i.e. $N_{fp} \sim LN(4.3779,\, 0.24045^2)$ and $\gamma_{a,eq} \sim LN(-2.3679, 0.033455^2)$ at $s = 100MPa$ with $CV_{\hat{S}} = 0.05$ in Fig. 8a. In addition, Fig. 8 presents the results of 10^3 Latin hypercube samples of the equivalent shear strain-life response, note that large randomness can be quantified by considering the stochastic material properties and load variations. Similarly, the abovementioned calculations have been carried out for the edge-notched plate made of 9CrMo steel at 550℃ under three different simulated $\hat{s}$ levels with $CV_{\hat{S}} = 0.10$. As it can be seen, the simulations have proven the effects of stochastic material properties on cyclic stress-strain and strain-life response.

4.3 Discussions with first order approximation (FOA)

An alternative to traditional deterministic multiaxial LCF design is probabilistic multiaxial LCF design (see Fig. 9). For the current investigation, typical variability of loads and resistance are identified and explicitly modeled to avoid unnecessary conservatism. Then, a robust design under material variability with a target failure probability P_f can be obtained.

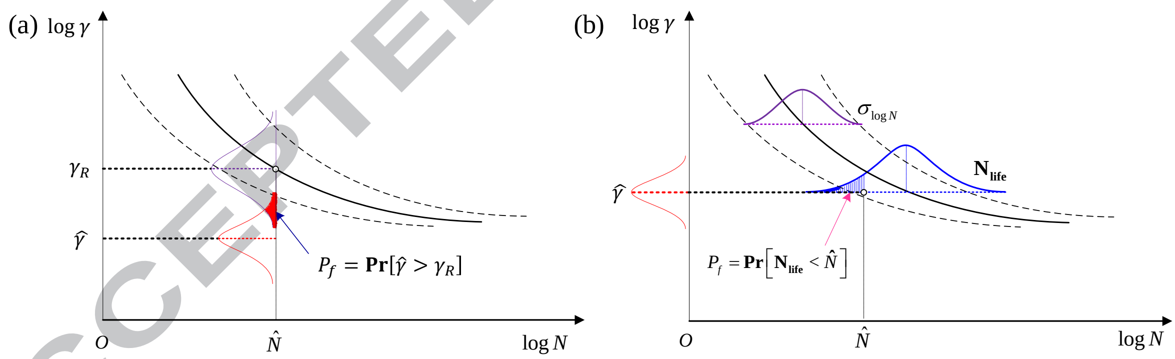


Fig. 9 Probabilistic formats for multiaxial LCF design set-up: (a) Stress-strength and (b) Load-life interferences

In order to estimate the failure probability of a notched component, the experimental $\gamma_{a,eq} \sim N_f$ curve is fitted from uniaxial fatigue tests by using Eq. (14), as shown in Fig. 3a. Similar to the uniaxial stress-strain response, a relationship between the loading stress S and Fatemi-Socie equivalent shear strain $\gamma_{a,eq}$, namely $S = R_{FS}(\gamma_{a,eq})$, can be derived through FE analysis. Following the first order approximation (FOA), $\log\gamma_{a,eq}$ can be viewed as a Gaussian distribution with parameters

$$\begin{cases} \mu_{log\gamma_{a,eq}} = logR_{FS}^{-1}(\hat{S}) \\ \sigma_{log\gamma_{a,eq}} = \left|\dfrac{\partial log\gamma_{a,eq}}{\partial logS}\right|_{log\gamma_{a,eq}} \cdot \sigma_S \end{cases} \tag{17}$$

Using Eq. (14), the prospective number of cycles to failure can be calculated as

$$N_{f,i} = f_{FS}^{-1}|_i(\gamma_{a,eqi}) \tag{18}$$

where the prospective distribution of N_{life} can be obtained by 10^3 Latin hypercube samples using Eq. (17).

As shown in Fig. 9a, the fatigue failure probability P_f can be readily calculated according to the stress-strength interference theory for the Gaussian load $\log\hat{\varepsilon}$ and strength $\log\gamma_R$

$$P_f = P_r[\hat{\gamma} > \gamma_R] = \Phi\left(-\frac{\mu_{\log\gamma_R} - \mu_{\log\hat{\gamma}}}{\sqrt{\sigma_{\log\gamma_R}^2 + \sigma_{\log\hat{\gamma}}^2}}\right) \tag{19}$$

where $\log\gamma_R$ can be estimated by a Gaussian distribution for a design life $\hat{N}$ using first order approximation

$$\begin{cases} \mu_{log\gamma_R} = log f_{FS}(\hat{N}) \\ \sigma_{log\gamma_R} = \left|\frac{\partial log\gamma}{\partial logN}\right|_{log\hat{N}} \cdot \sigma_{logN} \end{cases} \tag{20}$$

Similarly, a load-life interference model as shown in Fig. 9b can be derived for calculating the failure probability P_f as

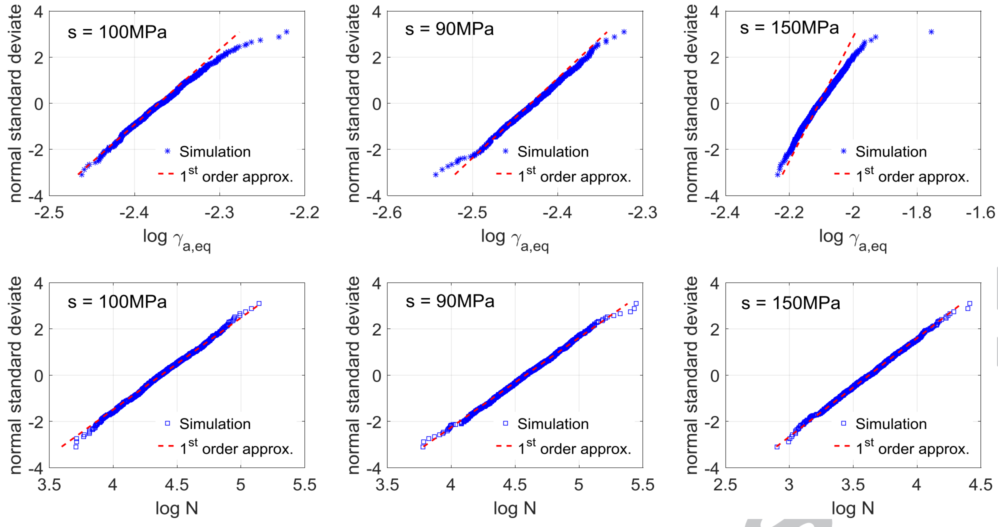
$$P_f = P_r\left[N_{life} < \hat{N}\right] = \Phi\left(\frac{\hat{N} - \mu_{\log N_{life}}}{\sigma_{\log N_{life}}}\right) \tag{21}$$

and

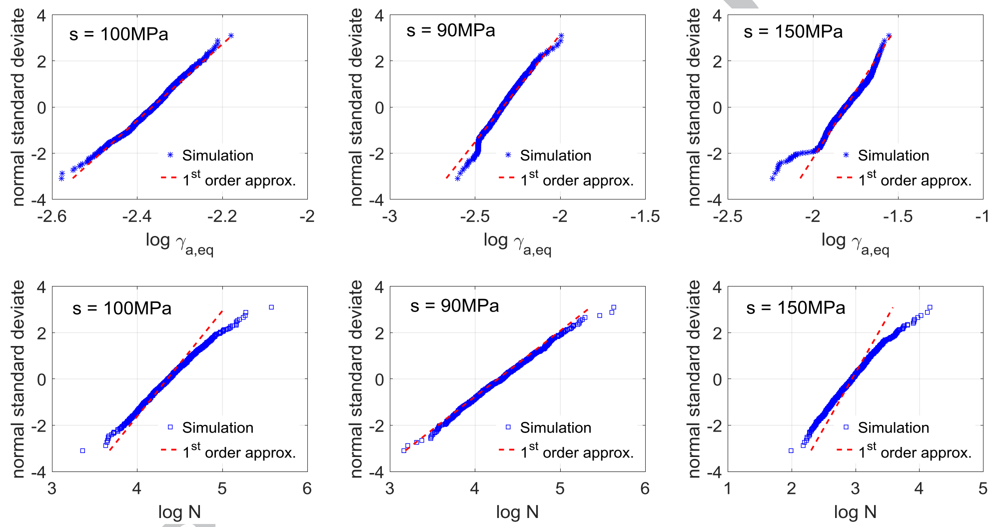
$$\begin{cases} \mu_{logN_f} = f_{FS}^{-1}\left(exp\left(\mu_{log\hat{\gamma}_{a,eq}}\right)\right) \\ \sigma_{logN_f} = \sqrt{\left(\frac{\sigma_{log\hat{\gamma}_{a,eq}}}{b}\right)^2 + \sigma_{logN}^2} \end{cases} \tag{22}$$

where f_{FS}^{-1} denotes the inverse of Eq. (14), $\hat{\gamma}_{a,eq}$ is a kind of loading stress amplitude $\hat{S}$ in the Fatemi-Socie criterion, σ_{logN} denotes the constant standard deviation of log-life which can be obtained from statistical tests [73], and $b = \left|\frac{\partial log\gamma_{a,eq}}{\partial logN}\right|_{\mu_{log\hat{\gamma}_{a,eq}}}$.

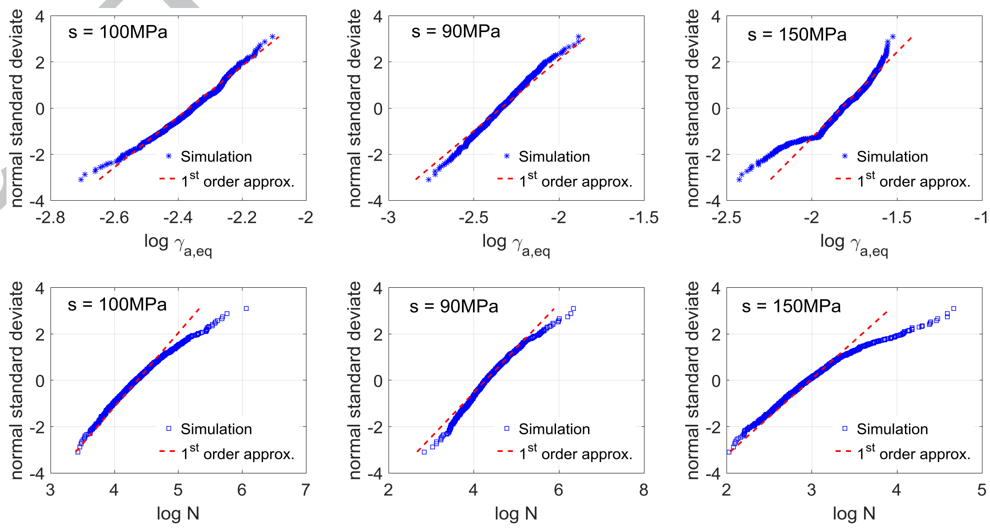
Simulated material response $\gamma_{a,eqi}$ and fatigue life $N_{f,i}$ for the two steels are compared with the log-normal distributions obtained from the FOA using Eq. (19) and Eq. (21). Example plots of this comparison of simulations and FOAs for 950X steel with $CV_{\hat{S}} = 0.05\sim0.15$ are presented as shown in Fig. 10, respectively. Note that FOA gives a good approximation for the lower tail of the N_f samples, while the description of $\gamma_{a,eq}$ is less precise.



(a) $CV_{\hat{s}} = 0.05$



(b) $CV_{\hat{s}} = 0.10$



(c) $CV_{\hat{s}} = 0.15$

Fig. 10 Simulated $log\gamma_{a,eq}$ and $logN$ for 950X steel with (a-b-c) $CV_{\hat{s}} = 0.05\sim0.15$

The conclusion is that, failure probability P_f can be calculated using the load-life interference equation (21) for different design load levels by the FOA. Such calculations are shown in Fig. 11 for two notched components under different load variations. It's worth noting that the FOA method yields reasonable estimates of failure probability.

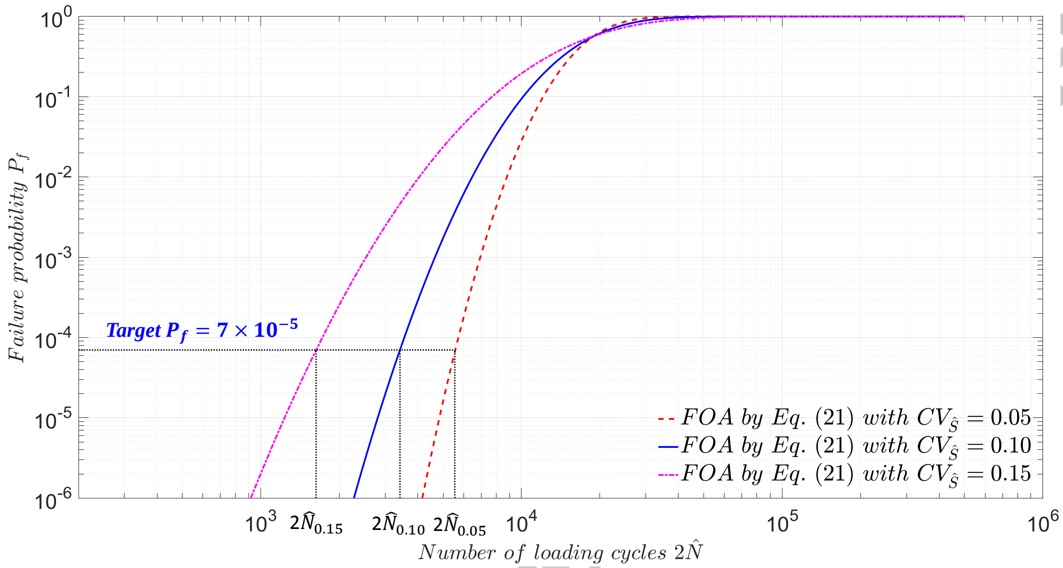
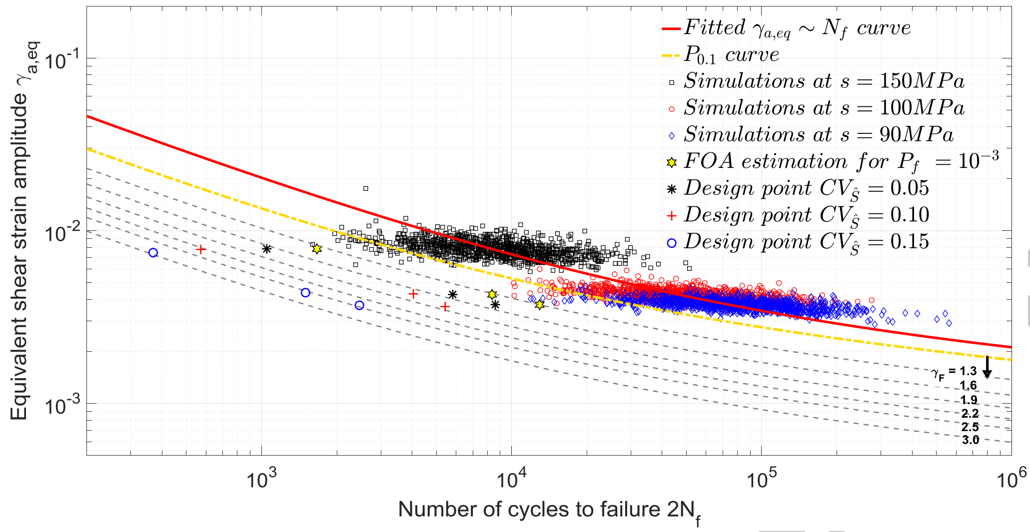
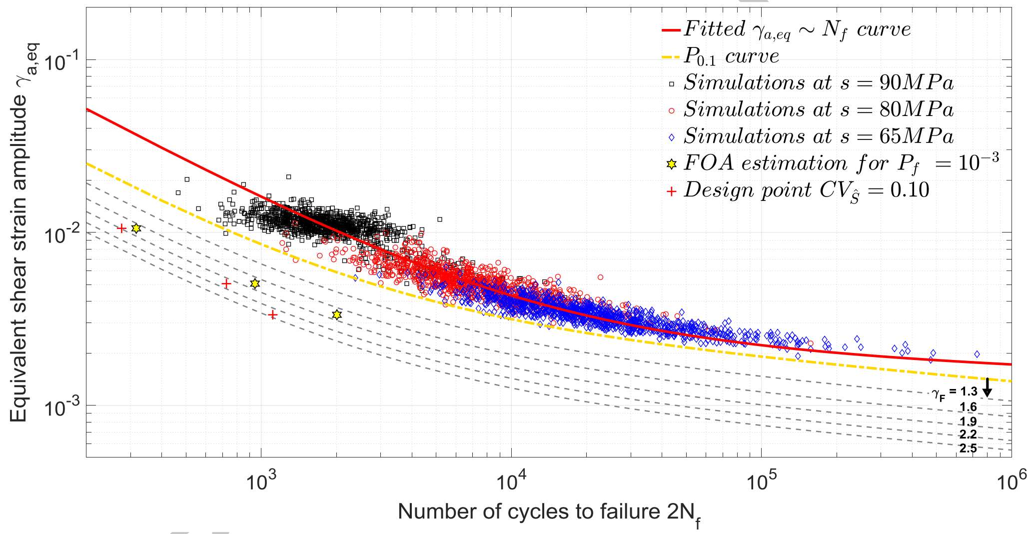


Fig. 11 Estimation of failure probabilities at $s = 100MPa$ for 950X steel with $CV_{\hat{S}} = 0.05 \sim 0.15$

From a safe-life design prospective, it is then possible to find the design point under a given fatigue failure probability P_f or safety factor γ_F. Using Eq. (23), for the two steels to reach a given target failure probability $P_f = 7 \times 10^{-5}$, design points with $\beta = 3.8$ under different safety factors have been calculated at different strain levels with different $CV_{\hat{S}}$, which are reported in Fig. 11 and Fig. 12.



(a)



(b)

Fig. 12 Definitions of the design points and the safety factors γ_F with target $P_f = 7 \times 10^{-5}$ ($\beta = 3.8$) for (a) 950X steel and (b) 9CrMo steel at $T = 550$℃

As it can be seen, the design points are well below the $\mu - 3\sigma$ curve and their positions can be simply estimated by applying a positive safety factor γ_F to be applied to the $\mu - 3\sigma$ curve [32]. In details, this corresponds to $\gamma_F = 1.3$~1.6 for $CV_{\hat{S}} = 0.05$, $\gamma_F = 1.6$~2.2 for $CV_{\hat{S}} = 0.10$ and $\gamma_F = 2.5$~3.0 for $CV_{\hat{S}} = 0.15$. It is worth mentioning that these γ_F values are slightly higher than the ones proposed in [32].

Similar calculations were carried out for the notched component, adopting the cyclic properties of a 9CrMo steel at 550℃, calculations are shown in Fig. 12b, the typical safety factors are $\gamma_F = 1.9$~2.5 for $CV_{\hat{S}} = 0.10$. In particular, due to the higher contribution of load variations to the material response scatter, it should be pointed out that higher safety factors should be chosen for the lower design lifetimes during the design phase, respect to the ones obtained from the Neuber rule-based approximation [38]. Thus, with the known cyclic stress-strain response and strain-life parameters, the design points with different target reliability or P_f can be derived with FOA and the safety factor γ_F.

5 Conclusions

In this study, probabilistic LCF design and life scatter of notched components made of two steels under multiaxial loading is investigated by combining stochastic analysis of material variability, load fluctuations and cyclic constitutive parameters. The conclusions are drawn as follows:

(1) A numerical framework based on 3D finite element modeling is proposed for multiaxial LCF assessment of notched components, which quantifies the stochastic response results from random non-uniformity of material properties and/or inherent variability of constitutive behavior of materials.

(2) Consideration of probabilistic modeling of plastically induced stress-strain response yields more accurate results for fatigue design and analysis, where the Chaboche constitutive model was introduced to model the elastoplastic stress-strain states at the notch root and the Fatemi-Socie damage criterion for analyzing notched fatigue behavior in FEA.

(3) For both of uniaxial and multiaxial fatigue, the scatter in fatigue lives or the variability of material response can be well described by quantifying the variability of the four material parameters $\{\sigma_f', \varepsilon_f', b, c\}$. Moreover, the choosing of an appropriate safety factor depends also on the scatter of applied loads. The normal and lognormal distributions are introduced to account for the material variability and load variations during the simulations.

(4) A relationship between the safety factor for choosing design curve and strain-life variability has been derived for obtaining the design point by using first order approximation based on the load-life and/or stress-strength interferences, which elaborated the safety factor accurately in safe-life design of fatigue critical components.

Comparisons of the numerical and experimental results of notched components made of the two steels reveal the validity and accuracy of the proposed framework.

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References

[1] A. Chernatynskiy, S. R. Phillpot, and R. LeSar, "Uncertainty quantification in multiscale simulation of materials: A prospective," *Annu. Rev. Mater. Res.*, vol. 43, no. 1, pp. 157–182, 2013.

[2] G. Stefanou, "The stochastic finite element method: Past, present and future," *Comput. Methods Appl. Mech. Eng.*, vol. 198, no. 9–12, pp. 1031–1051, 2009.

[3] S. P. Zhu, H. Z. Huang, V. Ontiveros, L. P. He, and M. Modarres, "Probabilistic low cycle fatigue life prediction using an energy-based damage parameter and accounting for model uncertainty," *Int. J. Damage Mech.*, vol. 21, no. 8, pp. 1128–1153, 2012.

[4] S. P. Zhu, H. Z. Huang, W. Peng, H. K. Wang, and S. Mahadevan, "Probabilistic Physics of Failure-based framework for fatigue life prediction of aircraft gas turbine discs under uncertainty," *Reliab. Eng. Syst. Saf.*, vol. 146, pp. 1–12, 2016.

[5] D. Hu, R. Wang, and Z. Tao, "Probabilistic design for turbine disk at high temperature," *Aircr. Eng. Aerosp. Technol.*, vol. 83, no. 4, pp. 199–207, 2011.

[6] J. H. Kim, T. Chau-Dinh, G. Zi, W. W. Lee, and J. S. Kong, "Probabilistic fatigue integrity assessment in multiple crack growth analysis associated with equivalent initial flaw and material variability," *Eng. Fract. Mech.*, vol. 156, pp. 182–196, 2016.

[7] S. P. Zhu, H. Z. Huang, Y. F. Li, Y. Liu, and Y. J. Yang, "Probabilistic modeling of damage accumulation for time-dependent fatigue reliability analysis of railway axle steels," *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit*, vol. 229, no. 1, pp. 23–33, 2015.

[8] S.P. Zhu, Y.J. Yang, H.Z. Huang, Z. Lv, and H.K. Wang, "A unified criterion for fatigue-creep life prediction of high temperature components," *Proc. Inst. Mech. Eng. Part G J. Aerosp. Eng.*, vol. 231, no. 4, pp. 677–688, 2017.

[9] S. C. Wu, S. Q. Zhang, Z. W. Xu, G. Z. Kang, and L. X. Cai, "Cyclic plastic strain based damage tolerance for railway axles in China," *Int. J. Fatigue*, vol. 93, pp. 64–70, 2016.

[10] M. M. Calvente, S. Blasón, A. F. Canteli, and A. de J. J. Correia, "A probabilistic approach for multiaxial fatigue criteria," *Frat. ed Integrità Strutt.*, vol. 39, pp. 160–165, 2017.

[11] L. Gallegos Mayorga, S. Sire, J. A. F. O. Correia, A. M. P. De Jesus, C. Rebelo, A. Fernández-Canteli, M. Ragueneau, and B. Plu, "Statistical evaluation of fatigue strength of double shear riveted connections and crack growth rates of materials from old bridges," *Eng. Fract. Mech.*, vol. in press, p. 10.1016/j.engfracmech.2017.05.039, 2017.

[12] ISO EN 1993, *Eurocode 3: design of steel structures*. 2005.

[13] FKM Guideline, *Analytical strength assessment of components in mechanical engineering.* 2012.

[14] ASME BPVDC III-Div. 1., *ASME boiler & pressure vessel design code – III Div. 1 – rules for construction of nuclear facility components*. 2005.

[15] W. J. O'Donnell, "Code design and evaluation for cyclic loading – sections III and VIII," *Rao K,*

Ed. Companion Guid. to ASME Boil. Press. Vessel code, vol. 2, 3rd ed., ASME, New York;, 2009.

[16] Y. Tong, "Literature review on aircraft structural risk and reliability analysis," *Tech Rep DSTR-1306. DSTO Aeronaut. Marit. Res. Lab.*, 2001.

[17] W. A. Grell and P. J. Laz, "Probabilistic fatigue life prediction using AFGROW and accounting for material variability," *Int. J. Fatigue*, vol. 32, no. 7, pp. 1042–1049, 2010.

[18] R. McClung, M. Enright, H. Millwater, G. Leverant, and S. Hudak, "A software framework for probabilistic fatigue life assessment of gas turbine engine rotors," *J. ASTM Int.*, vol. 1, no. 8, pp. 1–16, 2004.

[19] M. P. Enright, S. J. Hudak, R. C. McClung, and H. R. Millwater, "Application of probabilistic fracture mechanics to prognosis of aircraft engine components," *AIAA J.*, vol. 44, no. 2, pp. 311–316, 2006.

[20] S. Sankararaman, Y. Ling, and S. Mahadevan, "Uncertainty quantification and model validation of fatigue crack growth prediction," *Eng. Fract. Mech.*, vol. 78, no. 7, pp. 1487–1504, 2011.

[21] S. Sankararaman, Y. Ling, C. Shantz, and S. Mahadevan, "Inference of equivalent initial flaw size under multiple sources of uncertainty," *Int. J. Fatigue*, vol. 33, no. 2, pp. 75–89, 2011.

[22] D. Sandberg, R. Mansour, and M. Olsson, "Fatigue probability assessment including aleatory and epistemic uncertainty with application to gas turbine compressor blades," *Int. J. Fatigue*, vol. 95, pp. 132–142, 2017.

[23] J. M. Larsen, S. K. Jha, C. J. Szczepanski, M. J. Caton, R. John, A. H. Rosenberger, D. J. Buchanan, P. J. Golden, and J. R. Jira, "Reducing uncertainty in fatigue life limits of turbine engine alloys," *Int. J. Fatigue*, vol. 57, pp. 103–112, 2013.

[24] S. K. Jha, J. M. Larsen, and A. H. Rosenberger, "Towards a physics-based description of fatigue variability behavior in probabilistic life-prediction," *Eng. Fract. Mech.*, vol. 76, no. 5, pp. 681–694, 2009.

[25] P. J. Laz, B. A. Craig, and B. M. Hillberry, "A probabilistic total fatigue life model incorporating material inhomogeneities, stress level and fracture mechanics," *Int. J. Fatigue*, vol. 23, pp. 119–127, 2001.

[26] M. Naderi, S. H. Hoseini, and M. M. Khonsari, "Probabilistic simulation of fatigue damage and life scatter of metallic components," *Int. J. Plast.*, vol. 43, pp. 101–115, 2013.

[27] B. Jeremic and K. Sett, "On probabilistic yielding of materials," *Commun. Numer. Methods Eng.*, vol. 25, no. 3, pp. 291–300, 2009.

[28] X. Guan, J. Zhang, S. Zhou, E. M. Rasselkorde, and W. Abbasi, "Probabilistic modeling and sizing of embedded flaws in ultrasonic non-destructive inspections for fatigue damage prognostics and structural integrity assessment," *NDT E Int.*, vol. 61, pp. 1–9, 2014.

[29] Y. Liu and S. Mahadevan, "Probabilistic fatigue life prediction using an equivalent initial flaw size distribution," *Int. J. Fatigue*, vol. 31, no. 3, pp. 476–487, 2009.

[30] J. A. F. O. Correia, S. Blasón, A. M. P. De Jesus, A. F. Canteli, P. M. G. P. Moreira, and P. J.

Tavares, "Fatigue life prediction based on an equivalent initial flaw size approach and a new normalized fatigue crack growth model," *Eng. Fail. Anal.*, vol. 69, pp. 15–28, 2016.

[31] M. D. Sangid, H. J. Maier, and H. Sehitoglu, "An energy-based microstructure model to account for fatigue scatter in polycrystals," *J. Mech. Phys. Solids*, vol. 59, no. 3, pp. 595–609, 2011.

[32] D. L. McDowell, "Simulation-based strategies for microstructure-sensitive fatigue modeling," *Mater. Sci. Eng. A*, vol. 468–470, no. SPEC. ISS., pp. 4–14, 2007.

[33] K. S. Chan, M. P. Enright, J. Moody, and S. H. K. Fitch, "A microstructure-based time-dependent crack growth model for life and reliability prediction of turbopropulsion systems," *Metall. Mater. Trans. A Phys. Metall. Mater. Sci.*, vol. 45, no. 1, pp. 287–301, 2014.

[34] K. Wallin, "Statistical aspects of fatigue life and endurance limit," *Fatigue Fract. Eng. Mater. Struct.*, vol. 33, no. 6, pp. 333–344, 2010.

[35] D. Hu, Q. Ma, L. Shang, Y. Gao, and R. Wang, "Creep-fatigue behavior of turbine disc of superalloy GH720Li at 650°C and probabilistic creep-fatigue modeling," *Mater. Sci. Eng. A*, vol. 670, pp. 17–25, 2016.

[36] E. Castillo, A. Fernández-Canteli, and A. S. Hadi, "On fitting a fatigue model to data," *Int. J. Fatigue*, vol. 21, no. 1, pp. 97–106, 1999.

[37] M. Guida and F. Penta, "A Bayesian analysis of fatigue data," *Struct. Saf.*, vol. 32, no. 1, pp. 64–76, 2010.

[38] S. Beretta, S. Foletti, E. Rusconi, A. Riva, and D. Socie, "A log-normal format for failure probability under LCF: Concept, validation and definition of design curve," *Int. J. Fatigue*, vol. 82, pp. 2–11, 2016.

[39] D. Socie and S. Downing, "Statistical Strain-Life Fatigue Analysis," *Sae Tech. Pap.*, no. 960566, 1996.

[40] Y. Liu, L. Liu, B. Stratman, and S. Mahadevan, "Multiaxial fatigue reliability analysis of railroad wheels," *Reliab. Eng. Syst. Saf.*, vol. 93, no. 3, pp. 456–467, 2008.

[41] C. Doudard, F. Hild, and S. Calloch, "A probabilistic model for multiaxial high cycle fatigue," *Fatigue Fract. Eng. Mater. Struct.*, vol. 30, no. 2, pp. 107–114, 2007.

[42] F. Morel and L. Flacelière, "Data scatter in multiaxial fatigue: From the infinite to the finite fatigue life regime," *Int. J. Fatigue*, vol. 27, no. 9, pp. 1089–1101, 2005.

[43] A. Carpinteri, A. Spagnoli, S. Vantadori, and C. Bagni, "Structural integrity assessment of metallic components under multiaxial fatigue: The C-S criterion and its evolution," *Fatigue Fract. Eng. Mater. Struct.*, vol. 36, no. 9, pp. 870–883, 2013.

[44] L. Susmel, G. Meneghetti, and B. Atzori, "A simple and efficient reformulation of the classical Manson-Coffin curve to predict lifetime under multiaxial fatigue loading-Part II: notches," *J. Eng. Mater. Technol. Trans. ASME*, vol. 131, no. 2, p. 21009, 2009.

[45] A. Banvillet, T. Łagoda, E. Macha, A. Niesłony, T. Palin-Luc, and J. F. Vittori, "Fatigue life under non-Gaussian random loading from various models," *Int. J. Fatigue*, vol. 26, no. 4, pp.

349–363, 2004.

[46] S. P. Zhu, H. Z. Huang, R. Smith, V. Ontiveros, L. P. He, and M. Modarres, "Bayesian framework for probabilistic low cycle fatigue life prediction and uncertainty modeling of aircraft turbine disk alloys," *Probabilistic Eng. Mech.*, vol. 34, pp. 114–122, 2013.

[47] A. Pineau, D. L. McDowell, E. P. Busso, and S. D. Antolovich, "Failure of metals II: Fatigue," *Acta Mater.*, vol. 107, pp. 484–507, 2016.

[48] D. Hu, R. Wang, J. Fan, and X. Shen, "Probabilistic damage tolerance analysis on turbine disk through experimental data," *Eng. Fract. Mech.*, vol. 87, pp. 73–82, 2012.

[49] J. Correia, N. Apetre, A. Arcari, A. De Jesus, M. Muñiz-calvente, R. Calçada, F. Berto, and A. Fernández-canteli, "Generalized probabilistic model allowing for various fatigue damage variables," *Int. J. Fatigue*, vol. 100, pp. 187–194, 2017.

[50] A. M. P. De Jesus, H. Pinto, A. Fernández-canteli, E. Castillo, and J. A. F. O. Correia, "Fatigue assessment of a riveted shear splice based on a probabilistic model," *Int. J. Fatigue*, vol. 32, no. 2, pp. 453–462, 2010.

[51] E. Pessard, D. Bellett, F. Morel, and I. Koutiri, "A mechanistic approach to the Kitagawa-Takahashi diagram using a multiaxial probabilistic framework," *Eng. Fract. Mech.*, vol. 109, pp. 89–104, 2013.

[52] BS 7910, "Guide on methods for assessing the acceptability of flaws in metallic structures," in *British Standards*, 2005.

[53] O. Chopra and W. Shack, *Review of the margins for ASME code fatigue design curve – effects of surface roughness and material variability. Tech Rep NUREG/ CR6815, ANL-02/39. U.S. Nuclear Regulatory Commission*. 2003.

[54] Y. X. Zhao, Q. Gao, and J. N. Wang, "Random cyclic stress-strain responses of a stainless steel pipe-weld metal I - a statistical investigation," *Nucl. Eng. Des.*, vol. 199, no. 3, pp. 303–314, 2000.

[55] Y. X. Zhao, Q. Gao, and J. N. Wang, "Random cyclic stress-strain responses of a stainless steel pipe-weld metal II - a modeling," *Nucl. Eng. Des.*, vol. 199, no. 3, pp. 315–326, 2000.

[56] N. E. Dowling, *Mechanical behavior of materials-engineering methods for deformation, fracture, and fatigue*, 2nd Editio. NJ: Prentice Hall, 1998.

[57] M. A. Meggiolaro and J. T. P. Castro, "Statistical evaluation of strain-life fatigue crack initiation predictions," *Int. J. Fatigue*, vol. 26, no. 5, pp. 463–476, 2004.

[58] V. T. Troshchenko and L. A. Khamaza, "Strain-life curves of steels and methods for determining the curve parameters. Part 1 conventional methods," *Strength Mater.*, vol. 42, no. 6, pp. 647–659, 2010.

[59] J. L. Chaboche, "A review of some plasticity and viscoplasticity constitutive theories," *Int. J. Plast.*, vol. 24, no. 10, pp. 1642–1693, 2008.

[60] J. L. Chaboche and G. Rousselier, "On the plastic and viscoplastic constitutive equations, Part I

and II," *J. Press. Vessel Technol.*, vol. 105, no. 2, pp. 153–164, 1983.

[61] C. O. Frederick and P. J. Armstrong, "A mathematical representation of the multiaxial Bauschinger effect," *Mater. High Temp.*, vol. 24, no. 1, pp. 1–26, 2007.

[62] G. B. Broggiato, F. Campana, and L. Cortese, "The Chaboche nonlinear kinematic hardening model: Calibration methodology and validation," *Meccanica*, vol. 43, no. 2, pp. 115–124, 2008.

[63] S. Imaoka, "Chaboche Nonlinear Kinematic Hardening Model," *Memo Number STI0805A, ANSYS Release 12.0.1*, 2008.

[64] S. Zhu, Q. Lei, H. Huang, Y. Yang, and W. Peng, "Mean stress effect correction in strain energy-based fatigue life prediction of metals," *Int. J. Damage Mech.*, in press, doi: 10.1177/1056789516651920, 2016.

[65] F. Ding, M. Feng, and Y. Jiang, "Modeling of fatigue crack growth from a notch," *Int. J. Plast.*, vol. 23, no. 7, pp. 1167–1188, 2007.

[66] S. P. Zhu, Q. Lei, and Q. Y. Wang, "Mean stress and ratcheting corrections in fatigue life prediction of metals," *Fatigue Fract. Eng. Mater. Struct.*, in press, doi: 10.1111/ffe.12569, 2017.

[67] Z. Y. Yu, S. P. Zhu, Q. Liu, and Y. Liu, "A new energy-critical plane damage parameter for multiaxial fatigue life prediction of turbine blades," *Materials*, vol. 10, no. 5, pp. 513, 2017.

[68] A. Fatemi and D. F. Socie, "A critical plane approach to multiaxial fatigue damage including out-of-phase loading," *Fatigue Fract. Eng. Mater. Struct.*, vol. 11, no. 3, pp. 149–165, 1988.

[69] A. Karolczuk and E. Macha, "A review of critical plane orientations in multiaxial fatigue failure criteria of metallic materials," *Int. J. Fract.*, vol. 134, no. 3–4, pp. 267–304, 2005.

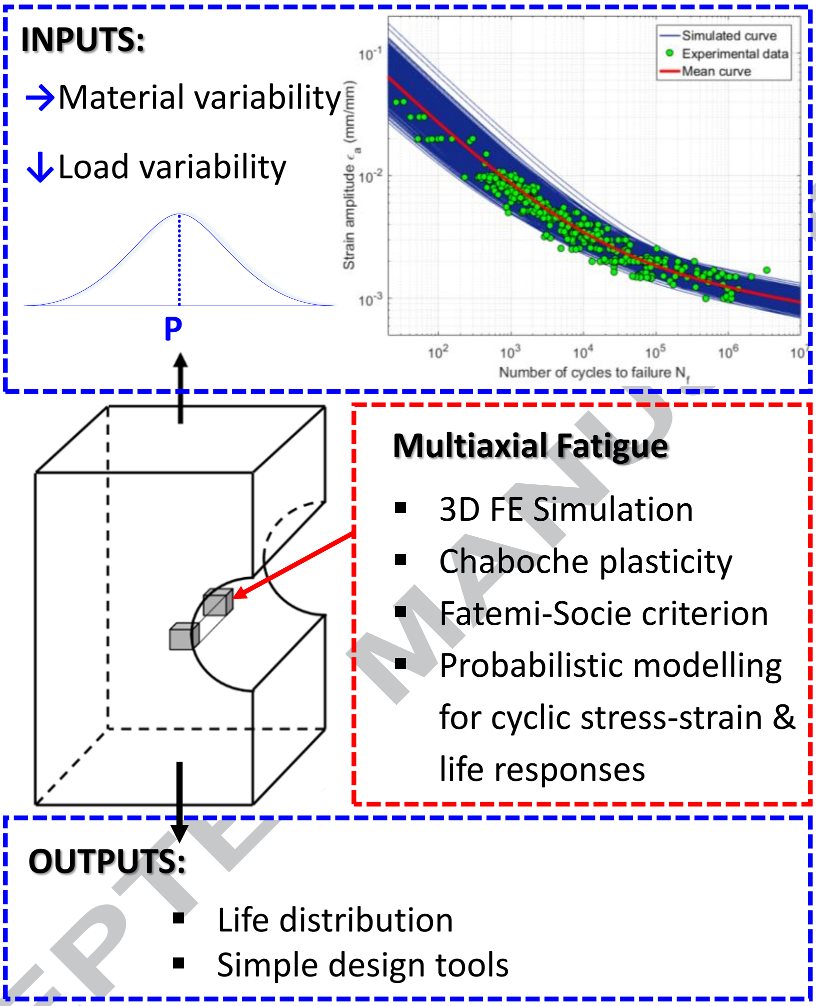
[70] D. L. McDiarmid, "A shear stress based critical-plane criterion of multiaxial fatigue failure for design and life prediction," *Fatigue Fract. Eng. Mater. Struct. Ltd*, vol. 17, no. 12, pp. 1475–1484, 1994.

[71] D. F. Socie and G. B. Marquis, *Multiaxial fatigue*. SAE Inc., 2000.

[72] G. Thomas and R. Varma, "Evaluation of low cycle fatigue test data in the BCR/VAMAS intercomparison programme. Tech Rep CD-NA-14105-EN-C. Commission of European Communities.," 1993.

[73] ASTM E739, "Standard practice for statistical analysis of linear or linearized stress-life (S-N) and strain-life (ε-N) fatigue data," *Am. Soc. Test. Mater.*, 2015.

Graphical Abstract:



Research Highlights:

(1) A probabilistic framework for multiaxial LCF assessment is proposed.

(2) Scatter in fatigue lives can be well described by material response variability $\{\sigma_f', \varepsilon_f', b, c\}$.

(3) Probabilistic plasticity induced stress-strain response yields more accurate results.

(4) A simple application of the safety factor for multiaxial fatigue criteria has been explored.