Cost Allocation strategy for off grid system in rural area: a case study on irrigation for rural agricultural lands in India

Giorgio Bonamini[a], Fabio Riva[b]. Emanuela Colombo[c]

[a] *Department of Mathematics – Politecnico di Milano, Italy, giorgio.bonamini@polimi.it - CA*
[b] *Department of Energy – Politecnico di Milano, Italy, fabio.riva@polimi.it*
[c] *Department of Energy – Politecnico di Milano, Italy, emanuela.colombo@polimi.it*

Abstract

In rural areas of developing countries, irrigation systems may rely on water pumps powered by the national grid. Unfortunately, scheduled and unscheduled blackouts, or poor quality of the power supply often reduce the possibility of a secure electricity service for cultivation purposes. To increase the energy security, farmers could switch to the use of stand-alone power systems. Applying the concepts of cooperative game theory allows analysing different configurations for irrigation systems based on solar power (i.e. solar Photovoltaic (PV) pumps) and understanding if sharing such a system could be a socially acceptable and stable solution. In this work, we analyse the stability of the solution considering the effect of different electrical loads – due to the daily and monthly variability of irrigation patterns – on the choice of the final system configuration. We used the Shapley Value to allocate the costs of a solar generator that three Indian farmers could potentially share. We estimated different yearly load profiles for different irrigation patterns employing a stochastic bottom-up approach. The physical operation of the electric plants has been simulated by applying a MATLAB® code based on numerical methods, which consider accurate physical models for best describing each system component (i.e. PV array, battery bank, inverters, etc.) and an appropriate criterion (i.e. the objective function) to choose the best combination of components that addresses the load.

Keywords

Cooperative Game Theory, Cost Allocation, Energy-water Nexus, Rural Energy Systems.

1. Introduction

Access to energy is an always-controversial issue. People that lack access to energy have also limited ability to develop properly. Electricity is the form of energy that fosters economic development [1]. With the escalation of electricity connection – usually to the national grid – countries of all over the world have grown in both Gross Domestic Product (GDP) and Human Development Index (HDI)[2]. Despite their growth of GPD, some countries exhibit big disparities in terms of energy access: low per capita consumptions [1,3,4] and low quality of the electric supply [5].

High costs for grid extension – not balanced by local market – limit the progress of electrification in rural areas, exacerbating the problem of energy poverty for the people who live in those contexts [6]. Thanks to the growing consideration towards the target of universal access to energy, rural electrification and decentralised approaches are now gaining stronger momentum, at a global scale [7]. Off-grid small-scale electricity generation represents one of the most appropriate options, either as a first step in the electrification process or as a building-block for future grid development [4,8-11].

For what we said before, we decided to work on India as case study since it's facing a huge energy challenge in the last decades. According to the International Energy Agency, India's "energy demand has almost doubled since 2000, but energy consumption per capita is still only around one-third of the global average and around 240 million people have no access to electricity" [4]. Nevertheless, the power energy sector of the country is growing very fast, especially though a massive exploitation of renewable energy resources. In fact, Indian government considers solar power a key element in its expansion plans – solar power is defined as "the heart of India's push towards low-carbon energy sources" [4]. For this reason, off-grid systems are expected to assume an important role for unleashing universal access to energy, especially in rural areas of the country. According to the World Energy Outlook 2015 [12], the investment expected to the expansion of mini- and off-grid power generation capacity is around \$45 billion: for 240 million people without access in 2015, around 405% is expected to achieve access via off-grid systems and 35% via mini-grid systems.

Our work addresses one of the issues related to electricity access: sharing costs of an off-grid Photovoltaic (PV) system. A problem that arises when people want to reduce the investment costs, but finding a stable solution might be hard. In particular, we analysed the situation in which three Indian farmers have to decide whether to share a PV power plant or not for providing electricity to their own irrigation system. The authors solved the problem as a cost allocation game. We employed concepts of cooperative game theory – such as the *Shapley value* and the *Core* – to propose a stable allocation of the selected power system's total Net Present Cost (NPC) among the farmers. Our model requires detailed and fundamental inputs for defining a more realistic cost allocation that are the definition of yearly energy consumption profiles and the identification of the most robust PV power plant design.

2. Game theory for cost sharing methods: state of the art

Situations that face cost allocation in the power industry have been first analysed by Gately [13]. Other example have been exploited mainly in situation of transmission expansion planning [14]–[16]. Some studies, like [17]–[21], analysed also generation problems. What is important in rural energy access is not only to look for the cheapest solution for a group of people, but, also, how much each individual is going to pay the electricity. Gately approached the problem looking at the gains (or losses), while the most common approach is to look at the costs. In our work, we focused on the costs, because we assume that the beneficiaries will buy a new generator and they want to minimize their investment. In case they autonomously were to buy and share a new generator, the concept of subsidy-free sharing rule [20] is what we should think in this context. Game theory is one of the most common approaches to solve problems of cost allocation. Game theoretical concepts are often applied to answer questions like: which will be the system configuration, or which coalition structure will have the minimum cost for the considered beneficiaries? What is the fairest and most stable way to allocate cost?

Before the investment, the "pay-per-use" rule cannot be applied to share costs, so we propose the Shapley Value – as many already did [22]–[24]. Literature is full of examples where the Shapley value is used to allocate costs in electric networks problems [43,52,57–59]. In general, this approach is considered as one of the fairest way to assign costs. The main drawbacks of the Shapley value is that it takes extremely long time to compute when there are several hundreds of beneficiaries (or players, in game theory). The computational time is not only linked to the mere Shapley value calculation but to the cost function value for all possible coalitions.

3. Cost Allocation Game model

3.1. Cost allocation problems

Game theorists may transform a cost allocation problem into a cost game. Referring to [28], we have a pair $\langle N, c\rangle$ where $N \coloneqq \{1,2,\dots,n\}$ denotes the set of beneficiaries – also called players. The subsets of N are called coalitions, denoted with S. The cost function $c\colon 2^N \to \mathbb{R}$ assigns to any nonempty coalition S the minimal costs, $c(S)$, involved when the members of S cooperate. The cost of an empty coalition is null, $c(\emptyset) = 0$. Given a cost function, a cost allocation method defines a cost vector made of the costs that such method charges to each player of the cost game.

The players can group in m mutually exclusive and excluding coalitions S, forming a coalition structure, $\delta = \{S_1, S_2, \dots, S_m\}$[29]. δ is a coalition structure of N if it fulfils conditions (1).

$$\begin{aligned} & S_j \neq \varnothing; j = 1,2,\dots m \\ & S_i \bigcap S_j = \varnothing; \forall i \neq j \\ & \bigcup_{S_j \in \delta} S_j = N \end{aligned} \quad . \tag{1}$$

What is interesting to know in cost allocation problems is which coalition structure will derive from each cost game. If the cost function is subadditive[1], then the grand coalition will form ($\delta = \{N\}$).

3.2. Shapley Value

The Shapley value, introduced by Lloyd Shapley in 1952 [30], is "the main solution concept in cooperative game theory" [31]. In this context, the Shapley value, φ_i, defines the actual cost each beneficiary will face in a cost allocation problem. More precisely, it allocates an amount proportional to the cost each coalition pays when it has a specific player as a member [32].

The Shapley value is built upon four axioms: symmetry, efficiency, additivity, null player. Each axiom allows the Shapley value to be a fair solution. Formally, such value is the only one that satisfies all four axioms and is defined in (2):

$$\varphi_i = \sum_{S \subseteq N : i \in S} \frac{(s-1)!(n-s)!}{n!} \left[c(S) - c(S \setminus i) \right] . \tag{2}$$

Each φ_i correspond to the exact allocation to the i-th beneficiary of the value of the grand coalition, $c(N)$. This value takes into account the marginal contribution of each player in each coalition. Like Tijs and Driessen affirmed in [28], the Shapley method is efficient, anonymous and it possesses the dummy player property. What this method does not assure is a stable solution, meaning that with the Shapley value the rationality of the players may bring them to unleash some coalitions.

3.3. Stability analysis

To verify the stability of a cost allocation, we may use the concept of core (3) [28].

$$CORE(c) := \left\{ y \in \mathbb{R}^n ; \sum_{i \in N} y_i = c(N) \wedge \sum_{i \in S} y_i \leq c(S) \text{ for all } S \neq \varnothing \right\} . \tag{3}$$

In game theory, the core represents an extension of the property of individual rationality. If a cost allocation belongs to the core, the sum of the final cost assigned to the member of any coalition S should be smaller than the cost computed for the same coalition S.

4. Case study

The authors developed the cost allocation model to the case study of three Indian farmers that may need to share a PV power plant for irrigation purposes. In order to implement our model, we need information about the cost function relative to the system that will generate and transmit power to the beneficiaries. This information depends on the evaluation of three preliminary inputs: context analysis, prediction of electrical load curves and establishment of the most robust power plant design.

[1] $c(S \bigcup T) \leq c(S) + c(T)$ for all $S, T \in N$ and $S \bigcap T = \varnothing$

4.1. Context analysis

The village of Katgaon is located in India, precisely in the district of Osmanabad in the State of Maharashtra. The population amounts to about 7800 people and most of them rely on farming activities (about 70% of workers are farmers). The area is partially electrified with national grid and presents good availability of ground water. The supply of electricity from national grid is characterised by scheduled blackouts of continuous 16 hours that change weekly. In addition to this, unscheduled blackouts and low voltage situations make the national grid unreliable for farming activities, mainly for irrigation.

For this case study, we picked three farmers that owns only one electric pump and lack of a backup generator. Table 1 exhibits the detailed irrigation patterns of the farmers for three agricultural seasons: Rabi (from 1st October to 28th February), Summer (from 1st March to 31th June) and Kharif (from 1st June to 30th September).

Table 1. Seasonal irrigation patterns and pumping system information of the three farmers.

Farmer	Season	Crop-type	Daily window	Times per month	Pump power [kW (hp)]
1	Rabi	Sugarcane	7:00 – 9:00	6	2.23 (3)
		Jowar	10:00 – 12:00	12	
	Summer	-	-	-	
	Kharif	Sugarcane	7:00 – 9:00	6	
2	Rabi	Sugarcane	8:00 – 13:00	1	3.73 (5)
		Jowar	5:00 – 23:00	5	
		Onion	5:00 – 23:00	8	
		Corn	5:00 – 23:00	4	
		Wheat	5:00 – 23:00	4	
		Pigeon pea	5:00 – 23:00	1	
	Summer	-	-	-	
	Kharif	Sugarcane	8:00 – 13:00	1	
3	Rabi	Jowar	5-00 – 7:00	1	5.59 (7.5)
		Bitter	7:00 – 8:00	8	
		Chilly	9:00 – 10:00	8	
		Lady finger	10:00 – 11:00	8	
		Cluster beans	8:00 – 9:00	8	
		Drumstick	11:00 – 12:00	2	
	Summer	-	-	-	
	Kharif	-	-	-	

4.2. Prediction of electrical load curves

Users' electric consumptions (i.e. daily electrical load profiles) represent an essential input within the process of off-grid systems design. Nevertheless, from the scientific literature emerges that little attention has been devoted to introduce proper modelling or methods for their estimates in the design process of off-grid systems [33]. This is particularly true when dealing with off-grid *rural* electrification. In this framework, Mandelli et al. [33] filled the literature-gap by introducing a novel new mathematical procedure. They formalized this procedure in the software *LoadProGen* (Load Profile Generator), implemented in MATLAB®, based on a bottom-up approach and on a stochastic process which takes into consideration all the possible uncertainties involved by formulating different realistic load profiles according to the given the input data. For the details about the stochastic process and operation adopted by the model, we invite the reader to refer to [33], [34]. For the aim of this work, we computed 350 yearly load profiles with *LoadProGen* for the three farmers with both a stand-alone and four different shared solutions. At the end of this step, we obtained 7 matrixes, each one with 350 rows (i.e. the yearly load curves computed for each configuration) and 525600 columns containing the values of electric load to be satisfied each minute of the entire of the year.

4.3. Establishment of the most robust power plant design

From the analysis of the scientific literature, three different methods for sizing off-grid systems emerge[23][35]: intuitive, numerical and analytical. Each follows different mathematical and computational procedures, but commonly they express the quantitative results in terms of capacity to be installed (i.e. the size of panels and batteries) and often the present investment cost of the system. Numerical methods are preferred when more accurate results are required in order to optimize and design a power plant, despite they require intensive computational efforts and time calculation.

In this work, we employed the *Scenario Robust Design* (*SRD*) numerical method developed in [35][36], in order to achieve more accurate results to design and optimize a PV power plant for addressing the irrigation needs of the three Indian farmers. It compares different configurations of PV plants based on the installed power, the storage capacity, the NPC and the Levelized Cost of Electricity (LCoE), and employs the criterion of NPC minimization to choose the best combination of panels and batteries. We ran the program 7 times, according to the 7 different combinations of beneficiaries. Each time, the program receive the following inputs:

1. *Yearly electric curves.* For every year of the 20-years scenario, the program takes in input the always the same 525600x350 matrix developed by *LoadProGen.* The matrix is always the same every year since we did not suppose any growth of energy demand for irrigation along the 20 years.
2. *Incident solar irradiance on tilted surface.* We derived the such data by employing HOMER© software [37] to calculate the hourly values of the incident radiation on the tilted surface of the PV array over one year, based on the latitude and longitude of the village of Katgaon. We set the panels slope value at 26° and the azimuth value at 0°: local surveys confirmed these are the orientation values set by the Indian government for the installation of solar panels on the roof of a local national hospital.
3. *PV cell temperature.* Similarly to solar irradiance data, we derived the PV cell temperature from HOMER© software as well; it calculates the cell temperature in each time step over the year as function of the daily air temperature profile [38].
4. *Technical and economic inputs.* In Table 2, we report general inputs related to technical and economic parameters of the all components of the system.

Table 2. Technical and economic input of SRD program.

	Value	Unit
PV array		
Balance Of System: account for such factors as soiling of the panels, wiring losses, shading, snow cover, aging, and so on	0.85	-
Test Temperature	25	°C
Derating of panel's power due to temperature	0.004	1/°C
Battery		
Minimum allowed State Of Charge	0.4	-
Setting initial State Of Charge	1	-
Charge efficiency	0.85	-
Discharge efficiency	0.9	-
Number of charge/discharge battery cycle	2000	-
Maximum year before battery replacement	5	years

Ratio power / energy of the battery	0.5	-
Inverter		
Inverter efficiency	0.9	-
Economics		
PV panel cost	1000	€/kW
Inverter cost	500	€/kW
O&M cost for the overall plant	50	€/kW*year
Installation and BOS cost as % of investment cost of PV+B+Inv	0.2	-
Battery cost	140	€/kWh
Plant lifetime	20	year
Rate of interest	0.06	-

At the end of this process we obtained the most Scenario Robust Design of the PV power plant for each of the 7 combinations of farmers. Table 3 reports the main technical and economic features of the 7 optimum solutions.

Table 3. Main outputs of the SRD program for the most Scenario Robust Design of the PV plant for each of the 7 possible coalitions of beneficiaries.

	PV [kW]	**Batteries [kWh]**	**Distance [m]**	**NPC[2] [k€]**
Farmer 1	*3*	*7*	*0*	*10.370*
Farmer 2	*14*	*56*	*0*	*51.367*
Farmer 3	*3*	*18*	*0*	*18.150*
Farmer 1 , Farmer 2	*14*	*57*	*210*	*54.401*
Farmer 1 , Farmer 3	*5*	*19*	*316*	*24.923*
Farmer 2 , Farmer 3	*16*	*59*	*197*	*62.019*
Farmer 1 , Farmer 2 , Farmer 3	*17*	*56*	*407*	*65.170*

4.4. Application of the cost allocation game

With respect to the analysed context and based on the preliminary inputs described in the previous sections, we can formulate the cost allocation game for our case study, and to use the notation of game theory we have:

- N := {Farmer 1, Farmer 2, Farmer 3}
- c := computation of NPC

From Table 3 we derive the costs of each coalition, and so compute the cost vector computing the Shapley value for each farmer.

5. Results and discussion

Table 4. Net Present Cost corresponding to each coalition structure.

Coalition structure - δ		**Total Net Present Cost [k€]**
I	*{ Farmer 1 } , { Farmer 2 } , { Farmer 3 }*	*79.887*
IIa	*{ Farmer 1 , Farmer 2 } , { Farmer 3 }*	*76.290*
IIb	*{ Farmer 1 , Farmer 3 } , { Farmer 2 }*	*72.551*
IIc	*{ Farmer 1 } , { Farmer 2 , Farmer 3 }*	*72.389*
III	*{ Farmer 1 , Farmer 2 , Farmer 3 }*	*65.170*

[2] The Net Present Cost includes also transmission costs (2000 €/km)

Using the values in Table 3 is possible to verify the subadditivity of the cost function. We rearranged them in Table 4 to focus only on the total NPC for each coalition structure.

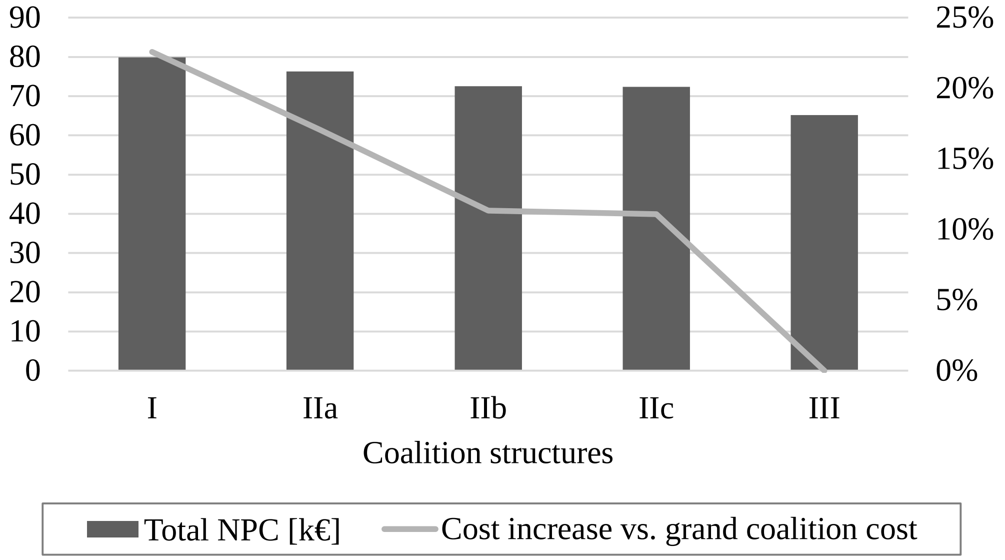

Fig. 1. Net Present Cost that each coalition structure must pay (in columns); percentage of cost increase, moving from the grand coalition to another coalition structure (line)

Figure 1 helps visualise the subadditivity of the cost function considered – both in absolute and relative terms. In this situation, we focus on assigning the Net Present Cost of the system correspondent to the grand coalition (NPC = 65.170 k€) to each beneficiary, using the Shapley value.

We computed the cost allocation for the three farmers that share an energy system using the Shapley value:

$$\varphi = [6.142 \text{ k€} ; 45.188 \text{ k€}; 13.841 \text{ k€}]. \quad (4)$$

Figure 2 shows the comparison between the costs of a shared system and the stand-alone solution. With Fig. 3, we want to emphasize how much money a farmer can save with the Shapley cost allocation.

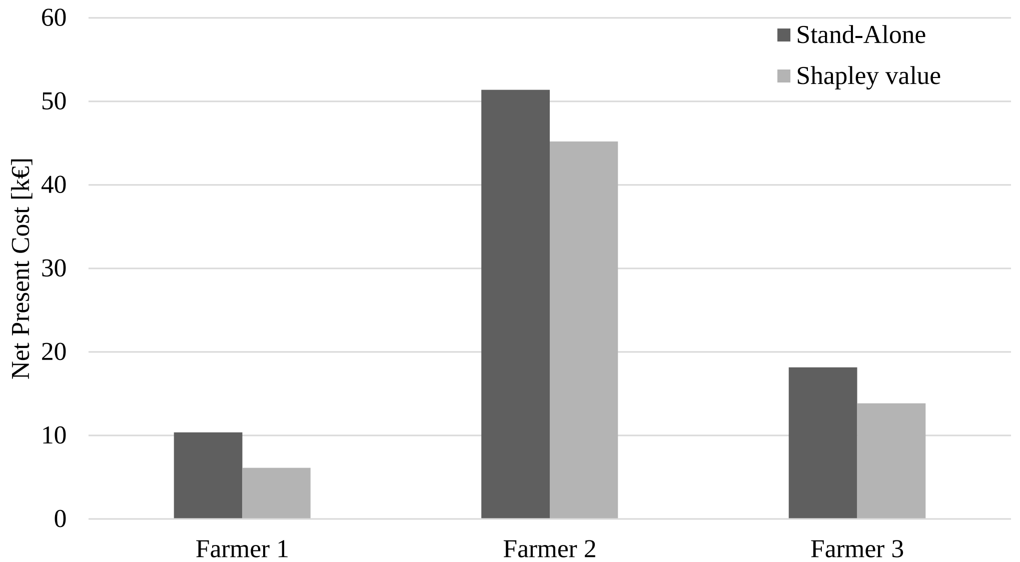

Fig. 2. Comparison between Shapley allocation and Stand-alone solution

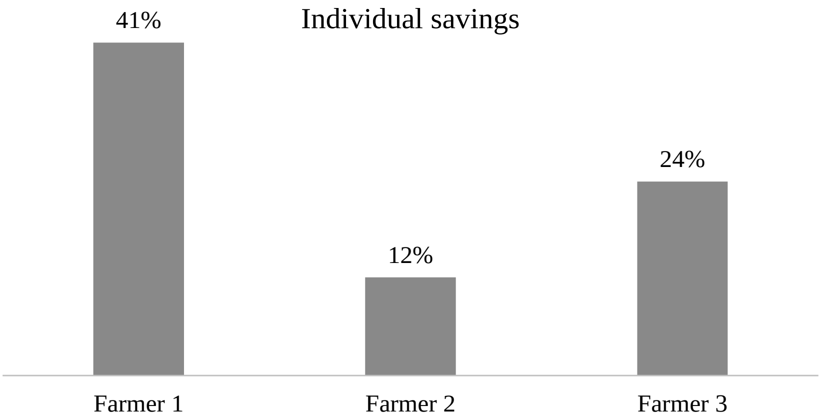

Fig. 3. Individual savings derived sharing an energy system instead buying a stand-alone system

From Fig. 3 it is evident that the farmers with small energy requirements save more sharing a system than the farmer with the highest load is the one pays more.

6. Conclusions

In this paper, we proposed a game theoretical approach with the aim of addressing the issue of cost assignment off-grid power systems. We adopted it in a case study in India, where three farmers have to decide whether to share a PV power plant or not for providing electricity to their irrigation system. We built our model evaluating three preliminary inputs: assessment of the local context, definition of the energy load curves and selection of the most economic PV plant configuration. We relied on concepts that belongs to cooperative game theory to define which coalition will form and how allocate the net present cost of the PV plant to the farmers. We used the Shapley value to allocate the NPC and verified its stability using the concept of core.

The results showed that a PV plant shared among all three farmers appears to be the cheapest solution and that Shapley allocation method allows the farmers to save at least 12% compared with the stand-alone solution.

In the end, we propose further improvements the Authors will consider in the next stage of their research work:

- The introduction of a model for predicting the growth in energy demand for irrigation in the 20-years scenario;
- Comparing different cost allocation methods for application of rural energy access.

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Nomenclature

N Set of players, (-)

c Cost function, (€)

S Coalition, (-)

m number of coalitions in a coalition structure, (-)

Greek symbols

δ Coalition structure

φ Shapley value

Subscript

i Player or beneficiary i-th

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