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Research Article

Fault Detection for Wireless Networked Control Systems with Stochastic Switching Topology and Time Delay

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This paper deals with the fault detection problem for a class of discrete-time wireless networked control systems described by switching topology with uncertainties and disturbances. System states of each individual node are affected not only by its own measurements, but also by other nodes' measurements according to a certain network topology. As the topology of system can be switched in a stochastic way, we aim to design H_∞ fault detection observers for nodes in the dynamic time-delay systems. By using the Lyapunov method and stochastic analysis techniques, sufficient conditions are acquired to guarantee the existence of the filters satisfying the H_∞ performance constraint, and observer gains are derived by solving linear matrix inequalities. Finally, an illustrated example is provided to verify the effectiveness of the theoretical results.

1. Introduction

Dynamics analysis for wireless networked control systems (WiNCS) has recently been a hot research issue that has been attracting much attention from scholars [1–4], and fault detection for WiNCS has got fruitful result in both theoretical researches and practical cations [5–8]. Compared with the traditional point to point control systems or the wired networked control systems, using WiNCS can not only avoid a lot of wired interconnections, but also meet some needs of special occasions. Besides, WiNCS can serve as natural models for many practical systems such as power grid networks, cooperate networks, neural networks, and environmental monitoring systems [9–13]. Inspired by the *BA* scale-free model proposed by *Barabási* and *Albert* in 1999, complex networks have become a focus of research and have attracted increasing attention in various fields of science and engineering [14–17]. From a rich body of literature, stochastic systems associated with the complex networks played an important role in network dynamics, and system failure usually occurred when topology switched. In this case, research on fault detection for WiNCS with stochastic switching topology is essential.

To the best of our knowledge, switching topology in sensor networks is a hot topic, and great effort has been devoted to dealing with this problem when designing observers for state estimation or fault detection [18–23]. In [24], synchronization problem for complex networks with switching topology was studied. For both fixed and arbitrary switching topology, synchronization criteria were established and stability condition and switching law design method for time-varying switched systems were also presented. In [25], state estimation problem for discrete-time stochastic system with missing measurement was studied. Authors supposed that there was no centralized processor to collect all the information from the sensors, so nodes should estimate its own states according to certain topology, and sufficient conditions were proposed to make sure that the augmented system was asymptotical stable. In [26], stability problem of interconnected multiagent system was investigated. Agents in system were connected via a certain connection rule; two algebraic sufficient conditions were derived under the circumstance that the topology was uncontrollable. Reference [27] investigated the stability analysis problem on neural networks with Markovian jumping parameters. Both of Lyapunov-Krasovskii stability theory and Itô differential rule were established to deal with global asymptotic stability and global exponential stability. Sufficient conditions were acquired based on linear matrix inequality to make the system both stochastically globally exponentially stable and stochastically globally asymptotically stable, respectively. Reference [28] designed a decentralized guaranteed cost dynamic feedback controller to achieve the synchronization of the network, whose topology was randomly changing.

Information flow between sensor nodes is time consuming, which leads to transmission delay in WiNCS; many scholars focused on this problem because the time delay is a common issue in many distributed systems [29–32]. Reference [33] designed a sliding mode observer for a class of uncertain nonlinear neutral delay system. Both the reachable motion and the sliding motion were investigated and a sufficient condition of asymptotic stability was proposed in terms of linear matrix inequality for the closed-loop system. Reference [34] focused on analyzing discrete-time Takagi-Sugeno (T-S) fuzzy systems with time-varying delays, and a delay partitioning method was used to analyze the scaled small gain of the model. Reference [35] studied the problem of uncertain nonlinear singular time-delay systems, and a switching surface function was designed by utilizing singular matrix.

Besides all what is mentioned above, another important factor which arouses unstably in WiNCS is external disturbance. Since H_∞ filtering does not need the accurate statistics of disturbances and ensures an estimation error less than a given disturbance attenuation level, many scholars were devoted to the research of H_∞ filtering; see, for example, [36–38] and the references therein. Reference [36] proposed a novel concept of bounded H_∞ synchronization, which captured the transient behavior of the time-varying complex networks over a finite horizon. Reference [38] investigated the robust filtering problem for time-varying Markovian jump systems with randomly occurring nonlinearities and saturation; a robust filter was designed such that the disturbance attenuation level was guaranteed.

Motivated by the previous researches stated above, our target is focused on the fault detection problem for WiNCS described by discrete-time systems with switching topology and uncertainties. The main contributions of this paper can be summarized as follows. (1) The stochastic switching topology of WiNCS is introduced to describe the binary switch between two kinds of topologies governed by a Bernoulli-distributed white noise sequence. (2) H_∞ observers are designed to ensure an estimation error less than a given disturbance attenuation level. (3) Distributed fault detection observers are designed for each individual node according to the given topologies.

The rest of paper is organized as follows. In Section 2, the fault detection problem of WiNCS is formulated. In Section 3, we present sufficient conditions to make the filtering error system exponentially stable in the mean square, which also satisfies H_∞ constraints. Furthermore, the gains of observers are also designed through LMI. A numerical example is given in Section 4 to show the effectiveness of proposed method. Finally, we give our conclusions in Section 5.

Notations. The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n\times m$ real matrices; the superscript "T" stands for matrix transposition; I is the matrix of appropriate dimension; $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of matrix; the notation $X > 0$ (resp., $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$, means that the matrix X is real symmetric positive definite (respective positive definite). $\mathbb{E}\{\cdot\}$ stands for the expectation operator. $\dim\{\cdot\}$ is the dimension of a matrix. What is more, we use $(*)$ to represent the entries implied by symmetry. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

2. Problem Formulation

Consider a type of WiNCS, whose topology can be switched at a random instant. In this case, the state of each individual node is affected not only by itself, but also by the connection relationship with other nodes. In this paper, we suppose that the system structure can only be switched between two topologies. The dynamic networks with stochastic switching topology can be described by

$$\begin{aligned} x_i(k+1) = {} & (A+\Delta A)\,x_i(k) + Bx_i(k-\tau_k) \\ & + \alpha\sum_{j=1}^{N} D_{ij}^{\alpha} x_j(k) + (1-\alpha)\sum_{j=1}^{N} D_{ij}^{1-\alpha} x_j(k) \\ & + E_f f(k) + E_v v(k), \\ y_i(k) = {} & Cx_i(k), \\ z_i(k) = {} & Lx_i(k), \\ x_i(j) = {} & \varphi_i(j), \quad j = -\tau, -\tau+1, \ldots, 0; \ \ i = 1, 2, \ldots, N, \end{aligned} \tag{1}$$

where $x_i(k) = (x_{i1}(k), x_{i2}(k), \ldots, x_{in}(k))^T \in \mathbb{R}^n$ is the system state vector of the ith node, $y_i(k)$ is the measured output vector of the ith node, $z_i(k)$ is the controlled output vector of the ith node, $v(k)$ is the disturbance, and $f(k)$ is a fault. A, B, C, L, E_f, and E_v are known constant matrices with appropriate dimensions, ΔA is the system uncertainty arising from uncertain factors, and $D^{\alpha} = [D_{ij}^{\alpha}]_{n\times n}$ and $D^{1-\alpha} = [D_{ij}^{1-\alpha}]_{n\times n}$ are two coupled configuration matrices standing for different topologies which can be switched to each other. D_{ij}^{α} is defined as follows: if there is a connection from node i to node j $(i \neq j)$, then $D_{ij}^{\alpha} = 1$; otherwise $D_{ij}^{\alpha} = 0$ $(i \neq j)$, and the diagonal elements of the matrix are defined as $D_{ii}^{\alpha} = -\sum_{j=1, j\neq i}^{N} D_{ij}^{\alpha} = -\sum_{j=1, j\neq i}^{N} D_{ji}^{\alpha}$, and $D_{ij}^{1-\alpha}$ has the same notation as D_{ij}^{α} does. α is a Bernoulli-distributed white noise sequence with

$$\begin{aligned} &\operatorname{Prob}\{\alpha = 1\} = \mathbb{E}\{\alpha\} = \overline{\alpha}, \\ &\operatorname{Prob}\{\alpha = 0\} = 1 - \mathbb{E}\{\alpha\} = 1 - \overline{\alpha}, \end{aligned} \tag{2}$$

where $\overline{\alpha} \in [0, 1]$ is a known constant.

For the system shown in (1), we make the following assumption throughout the paper.

Assumption 1. The perturbation parameter of the system satisfies

$$\Delta A = MD(k)H, \tag{3}$$

where M and N are, respectively, known constant matrices, $D(k)$ is a time-varying delay uncertain matrix, yet Lebesgue measurable, and $D^T(k)D(k) \leq I$.

Assumption 2. The function τ_k describes the transmission delay which satisfies

$$0 \leq \tau_k \leq \tau. \tag{4}$$

Remark 3. Sensor nodes in WiNCS are usually in dynamic motion. When two nodes are within communication range, the linkage between them can be established; otherwise, their linkage may be broken off. The relative distance between nodes arouses in the topology switches. For the purpose of simplicity, we suppose that the system only switches between two topologies, D^{α} and $D^{1-\alpha}$, and binary switches for a certain node occur according to a given probability distribution.

We construct the following state observer for node i:

$$\begin{aligned} \widehat{x}_i(k+1) &= A\widehat{x}_i(k) + \sum_{j=1}^{N} k_{ij}\left[y_j(k) - \widehat{y}_j(k)\right], \\ \widehat{y}_i(k) &= C\widehat{x}_i(k), \\ \widehat{z}_i(k) &= L\widehat{x}_i(k), \end{aligned} \tag{5}$$

where $\widehat{x}_i(k) \in \mathbb{R}^n$ is the estimation value of $x_i(k)$, $\widehat{y}_i(k)$ is the estimation value of $y_i(k)$, $\widehat{z}_i(k)$ is the estimation value of $z_i(k)$, and $k_{ij} \in \mathbb{R}^{n\times n}$ is the gain of observer to be designed.

Define the state error $e_{x_i}(k)$, measured output error $e_{y_i}(k)$, and the controlled output error $\widetilde{z}_i(k)$ of the system

$$\begin{aligned} e_{x_i}(k) &= x_i(k) - \widehat{x}_i(k), \\ e_{y_i}(k) &= y_i(k) - \widehat{y}_i(k), \\ \widetilde{z}_i(k) &= z_i(k) - \widehat{z}_i(k). \end{aligned} \tag{6}$$

If system (1) has no fault, the residual is close to zero, and we set up residual evaluation function J and fault threshold J_{th} as follows:

$$\begin{aligned} J &= \left\{\sum_{k=1}^{N} e_{y_i}^T(k) e_{y_i}(k)\right\}^{1/2} \\ J_{\text{th}} &= \sup_{f(k)=0} J. \end{aligned} \tag{7}$$

So the system fault can be detected by comparing J and J_{th} as follows:

$$\begin{aligned} &J \leq J_{\text{th}} \quad \text{No fault happens,} \\ &J > J_{\text{th}} \quad \text{Fault happens.} \end{aligned} \tag{8}$$

By utilizing the Kronecker product, the error system can be obtained from (1) and (5) as follows:

$$\begin{aligned} e(k+1) = {} & \left(\widetilde{A} - K\widetilde{C}\right)e(k) + \Delta\widetilde{A}x(k) + \widetilde{B}x(k-\tau_k) \\ & + (\alpha - \overline{\alpha})\left(\widetilde{D}_1 - \widetilde{D}_2\right)x(k) + \overline{\alpha}\widetilde{D}_1 x(k) \\ & + (1-\overline{\alpha})\widetilde{D}_2 x(k) + \widetilde{E}_f f(k) + \widetilde{E}_v v(k), \\ \widetilde{z}(k) = {} & \widetilde{L}e(k), \end{aligned} \tag{9}$$

where

$$\begin{aligned} & x(k) = \left[x_1^T(k), x_2^T(k), \ldots, x_N^T(k)\right]^T, \\ & \widehat{x}(k) = \left[\widehat{x}_1^T(k), \widehat{x}_2^T(k), \ldots, \widehat{x}_N^T(k)\right]^T, \\ & y(k) = \left[y_1^T(k), y_2^T(k), \ldots, y_N^T(k)\right]^T, \\ & \widehat{y}(k) = \left[\widehat{y}_1^T(k), \widehat{y}_2^T(k), \ldots, \widehat{y}_N^T(k)\right]^T, \\ & z(k) = \left[z_1^T(k), z_2^T(k), \ldots, z_N^T(k)\right]^T, \\ & \widehat{z}(k) = \left[\widehat{z}_1^T(k), \widehat{z}_2^T(k), \ldots, \widehat{z}_N^T(k)\right]^T, \\ & \widetilde{A} = I_N \otimes A, \qquad \widetilde{B} = I_N \otimes B, \\ & \widetilde{C} = I_N \otimes C, \qquad \Delta\widetilde{A} = I_N \otimes \Delta A, \\ & \widetilde{D}_1 = D^{\alpha} \otimes I_{\dim(A)}, \qquad \widetilde{D}_2 = D^{1-\alpha} \otimes I_{\dim(A)}, \\ & K = (k_{ij})_{n\times n}, \qquad \widetilde{E}_f = I_N \otimes E_f, \\ & \widetilde{E}_v = I_N \otimes E_v, \qquad \widetilde{L} = I_N \otimes L, \\ & \widetilde{z}(k) = z(k) - \widehat{z}(k). \end{aligned} \tag{10}$$

By introducing an augmented vector $\eta(k) = \left[x^T(k)\ \ e^T(k)\right]^T$, we have the following augmented system:

$$\begin{aligned}\eta(k+1) &= \mathscr{A}\eta(k) + \Delta\mathscr{A}\eta(k) + \mathscr{B}\eta(k-\tau_k) \\ &\quad + (\alpha - \overline{\alpha})\mathscr{D}\eta(k) + \overline{\alpha}\mathscr{D}_1\eta(k) \\ &\quad + (1-\overline{\alpha})\mathscr{D}_2\eta(k) + \mathscr{E}_f f(k) + \mathscr{E}_v v(k), \\ \widetilde{z}(k) &= \mathscr{L}\eta(k),\end{aligned} \quad (11)$$

where

$$\begin{gathered}\mathscr{A} = \begin{bmatrix}\widetilde{A} & 0 \\ 0 & \widetilde{A} - K\widetilde{C}\end{bmatrix}, \qquad \Delta\mathscr{A} = \begin{bmatrix}\Delta\widetilde{A} & 0 \\ \Delta\widetilde{A} & 0\end{bmatrix}, \\ \mathscr{B} = \begin{bmatrix}\widetilde{B} & 0 \\ \widetilde{B} & 0\end{bmatrix}, \qquad \mathscr{D} = \begin{bmatrix}\widetilde{D}_1 - \widetilde{D}_2 & 0 \\ \widetilde{D}_1 - \widetilde{D}_2 & 0\end{bmatrix}, \\ \mathscr{D}_1 = \begin{bmatrix}\widetilde{D}_1 & 0 \\ \widetilde{D}_1 & 0\end{bmatrix}, \qquad \mathscr{D}_2 = \begin{bmatrix}\widetilde{D}_2 & 0 \\ \widetilde{D}_2 & 0\end{bmatrix}, \\ \mathscr{E}_f = \begin{bmatrix}\widetilde{E}_f \\ \widetilde{E}_f\end{bmatrix}, \qquad \mathscr{E}_v = \begin{bmatrix}\widetilde{E}_v \\ \widetilde{E}_v\end{bmatrix}, \\ \mathscr{L} = \begin{bmatrix}0 & \widetilde{L}\end{bmatrix}.\end{gathered} \quad (12)$$

Definition 4 (see [39]). Filtering error system (11) is said to be exponentially stable in the mean square for any initial conditions when $f(k) = 0$ and $v(k) = 0$, if there exist constants $\delta > 0$ and $0 < \kappa < 1$ such that the following inequality holds:

$$\mathbb{E}\left\{\|\eta(k)\|^2\right\} \le \delta_\kappa^k \sup_{-\tau \le i \le 0} \mathbb{E}\left\{\|\eta(i)\|^2\right\}, \quad \forall k \ge 0. \quad (13)$$

In this paper, we are going to design the fault detection observers for a class of WiNCS with randomly switching topology such that filtering error system (11) satisfies the following requirements simultaneously.

(C1) Filtering error system (11) with $f(k) = 0$, $v(k) = 0$ is exponentially stable in the mean square.

(C2) For any $f(k) = 0$, $v(k) \neq 0$ under the zero initial condition, the filtering error satisfies

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\frac{1}{N}\|\widetilde{z}(k)\|^2\right\} \le \gamma^2 \sum_{k=0}^{\infty}\mathbb{E}\left\{\|v(k)\|^2\right\}, \quad (14)$$

where γ is a given scalar.

Besides, some useful and important lemmas that will be used in deriving out results will be introduced below.

Lemma 5 (Schur complement [40]). *Given a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22}\end{bmatrix}$, where S_{11} is $r \times r$ dimensional, the following three conditions are equivalent:*

(1) $S < 0$;

(2) $S_{11} < 0$ *and* $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0$ *and* $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 6 (see [25]). *For any $x, y \in R^n$, and $\mu > 0$, the following inequality holds:*

$$2x^T y \le \mu x^T x + \frac{1}{\mu} y^T y. \quad (15)$$

Lemma 7 (see [41]). *Let $Y = Y^T$, M, N, and $D(t)$ be real matrix of proper dimensions and $D^T(t)D(t) \le I$; then inequality $Y + MDN + (MDN)^T < 0$ holds if there exists a constant ε, which makes the following inequality hold:*

$$Y + \varepsilon NN^T + \varepsilon^{-1} M^T M < 0 \quad (16)$$

or equivalently

$$\begin{bmatrix} Y & M & \varepsilon N^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I\end{bmatrix} < 0. \quad (17)$$

3. Main Results

In this section, by constructing a proper Lyapunov-Krasovskii functional combined with linear matrix inequalities, we are going to propose sufficient conditions such that filtering error system (11) is asymptotically stable in the mean square.

Theorem 8. *Consider system* (1) *and suppose that observer gain K is given. Filtering error system* (11) *is said to be asymptotically stable in the mean square, if there exist positive definite matrix $P = \operatorname{diag}\{P_1, P_2\}$ and $Q > 0$ with proper dimensions satisfying the following inequality:*

$$\Pi = \begin{bmatrix}\Pi_{11} & 0 \\ 0 & \Pi_{22}\end{bmatrix} < 0, \quad (18)$$

where

$$\begin{aligned}\Pi_{11} &= 4\mathscr{A}^T P\mathscr{A} + 4\Delta\mathscr{A}^T P\Delta\mathscr{A} + \overline{\alpha}(1-\overline{\alpha})\mathscr{D}^T P\mathscr{D} \\ &\quad + 4\overline{\alpha}\mathscr{D}_1^T P\mathscr{D}_1 + (4 - 4\overline{\alpha})\mathscr{D}_2^T P\mathscr{D}_2 - P + (1+\tau)Q, \\ \Pi_{22} &= 4\mathscr{B}^T P\mathscr{B} - Q.\end{aligned} \quad (19)$$

Proof. For the stability analysis of system (11), we set $f(k) = 0$, $v(k) = 0$, and system (11) can be rewritten as

$$\begin{aligned}\eta(k+1) &= \mathscr{A}\eta(k) + \Delta\mathscr{A}\eta(k) + \mathscr{B}\eta(k-\tau_k) \\ &\quad + (\alpha - \overline{\alpha})\mathscr{D}\eta(k) + \overline{\alpha}\mathscr{D}_1\eta(k) \\ &\quad + (1-\overline{\alpha})\mathscr{D}_2\eta(k).\end{aligned} \quad (20)$$

Then, choose the following Lyapunov-Krasovskii functional:

$$V(k) = V_1(k) + V_2(k) + V_3(k), \quad (21)$$

where

$$\begin{aligned}V_1(k) &= \eta^T(k) P\eta(k),\\ V_2(k) &= \sum_{i=k-\tau_k}^{k-1} \eta^T(i) Q\eta(i),\\ V_3(k) &= \sum_{j=1-\tau}^{0} \sum_{i=k+j}^{k-1} \eta^T(i) Q\eta(i).\end{aligned} \tag{22}$$

By calculating the difference of $V(k)$ along system (20), we have

$$\begin{aligned}&\mathbb{E}\{\Delta V_1\}\\ &\quad= \mathbb{E}\left\{\eta^T(k+1) P\eta(k+1) - \eta^T(k) P\eta(k)\right\}\\ &\quad= \eta^T(k)\mathscr{A}^T P\mathscr{A}\eta(k) + 2\eta^T(k)\mathscr{A}^T P\Delta\mathscr{A}\eta(k)\\ &\qquad+ 2\eta^T(k)\mathscr{A}^T P\mathscr{B}\eta(k-\tau_k) + 2\overline{\alpha}\eta^T(k)\mathscr{A}^T P\mathscr{D}_1\eta(k)\\ &\qquad+ 2(1-\overline{\alpha})\eta^T(k)\mathscr{A}^T P\mathscr{D}_2\eta(k)\\ &\qquad+ \eta^T(k)\Delta\mathscr{A}^T P\Delta\mathscr{A}\eta(k)\\ &\qquad+ 2\eta^T(k)\Delta\mathscr{A}^T P\mathscr{B}\eta(k-\tau_k)\\ &\qquad+ 2\overline{\alpha}\eta^T(k)\Delta\mathscr{A}^T P\mathscr{D}_1\eta(k)\\ &\qquad+ 2(1-\overline{\alpha})\eta^T(k)\Delta\mathscr{A}^T P\mathscr{D}_2\eta(k)\\ &\qquad+ \eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{B}\eta(k-\tau_k)\\ &\qquad+ 2\overline{\alpha}\eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{D}_1\eta(k)\\ &\qquad+ 2(1-\overline{\alpha})\eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{D}_2\eta(k)\\ &\qquad+ \overline{\alpha}(1-\overline{\alpha})\eta^T(k)\mathscr{D}^T P\mathscr{D}\eta(k)\\ &\qquad+ \overline{\alpha}^2\eta^T(k)\mathscr{D}_1^T P\mathscr{D}_1\eta(k)\\ &\qquad+ 2\overline{\alpha}(1-\overline{\alpha})\eta^T(k)\mathscr{D}_1^T P\mathscr{D}_2\eta(k)\\ &\qquad+ (1-\overline{\alpha})^2\eta^T(k)\mathscr{D}_2^T P\mathscr{D}_2\eta(k) - \eta^T(k) P\eta(k).\end{aligned} \tag{23}$$

In terms of Lemma 6, we have

$$\begin{aligned}&2\eta^T(k)\mathscr{A}^T P\Delta\mathscr{A}\eta(k)\\ &\quad\le \eta^T(k)\mathscr{A}^T P\mathscr{A}\eta(k)\\ &\qquad+ \eta^T(k)\Delta\mathscr{A}^T P\Delta\mathscr{A}\eta(k),\\ &2\eta^T(k)\mathscr{A}^T P\mathscr{B}\eta(k-\tau_k)\\ &\quad\le \eta^T(k)\mathscr{A}^T P\mathscr{A}\eta(k)\\ &\qquad+ \eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{B}\eta(k-\tau_k),\\ &2\overline{\alpha}\eta^T(k)\mathscr{A}^T P\mathscr{D}_1\eta(k)\\ &\quad\le \overline{\alpha}\eta^T(k)\mathscr{A}^T P\mathscr{A}\eta(k)\\ &\qquad+ \overline{\alpha}\eta^T(k)\mathscr{D}_1^T P\mathscr{D}_1\eta(k),\\ &2(1-\overline{\alpha})\eta^T(k)\mathscr{A}^T P\mathscr{D}_2\eta(k)\\ &\quad\le (1-\overline{\alpha})\eta^T(k)\mathscr{A}^T P\mathscr{A}\eta(k)\\ &\qquad+ (1-\overline{\alpha})\eta^T(k)\mathscr{D}_2^T P\mathscr{D}_2\eta(k),\\ &2\eta^T(k)\Delta\mathscr{A}^T P\mathscr{B}\eta(k-\tau_k)\\ &\quad\le \eta^T(k)\Delta\mathscr{A}^T P\Delta\mathscr{A}\eta(k)\\ &\qquad+ \eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{B}\eta(k-\tau_k),\\ &2\overline{\alpha}\eta^T(k)\Delta\mathscr{A}^T P\mathscr{D}_1\eta(k)\\ &\quad\le \overline{\alpha}\eta^T(k)\Delta\mathscr{A}^T P\Delta\mathscr{A}\eta(k)\\ &\qquad+ \overline{\alpha}\eta^T(k)\mathscr{D}_1^T P\mathscr{D}_1\eta(k),\\ &2(1-\overline{\alpha})\eta^T(k)\Delta\mathscr{A}^T P\mathscr{D}_2\eta(k)\\ &\quad\le (1-\overline{\alpha})\eta^T(k)\Delta\mathscr{A}^T P\Delta\mathscr{A}\eta(k)\\ &\qquad+ (1-\overline{\alpha})\eta^T(k)\mathscr{D}_2^T P\mathscr{D}_2\eta(k),\\ &2\overline{\alpha}\eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{D}_1\eta(k)\\ &\quad\le \overline{\alpha}\eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{B}\eta(k-\tau_k)\\ &\qquad+ \overline{\alpha}\eta^T(k)\mathscr{D}_1^T P\mathscr{D}_1\eta(k),\\ &2(1-\overline{\alpha})\eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{D}_2\eta(k)\\ &\quad\le (1-\overline{\alpha})\eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{B}\eta(k-\tau_k)\\ &\qquad+ (1-\overline{\alpha})\eta^T(k)\mathscr{D}_2^T P\mathscr{D}_2\eta(k),\\ &2\overline{\alpha}(1-\overline{\alpha})\eta^T(k)\mathscr{D}_1^T P\mathscr{D}_2\eta(k)\\ &\quad\le \overline{\alpha}(1-\overline{\alpha})\eta^T(k)\mathscr{D}_1^T P\mathscr{D}_1\eta(k)\\ &\qquad+ \overline{\alpha}(1-\overline{\alpha})\eta^T(k)\mathscr{D}_2^T P\mathscr{D}_2\eta(k).\end{aligned} \tag{24}$$

Next, we have derived that

$$\begin{aligned}\mathbb{E}\{\Delta V_2\} &= \mathbb{E}\{V_2(k+1) - V_2(k)\}\\ &= \sum_{i=k+1-\tau_{k+1}}^{k} \eta^T(i) Q\eta(i) - \sum_{i=k-\tau_k}^{k-1} \eta^T(i) Q\eta(i)\\ &= \eta^T(k) Q\eta(k) - \eta^T(k-\tau_k) Q\eta(k-\tau_k)\\ &\quad+ \sum_{i=k+1-\tau_{k+1}}^{k-1} \eta^T(i) Q\eta(i) - \sum_{i=k+1-\tau_k}^{k-1} \eta^T(i) Q\eta(i)\end{aligned}$$

$$\leq \eta^T(k) Q\eta(k) - \eta^T(k-\tau_k) Q\eta(k-\tau_k) + \sum_{i=k+1-\tau_k}^{k} \eta^T(i) Q\eta(i),$$

$$\begin{aligned} \mathbb{E}\{\Delta V_3\} &= \mathbb{E}\{V_3(k+1) - V_3(k)\} \\ &= \sum_{j=1-\tau}^{0} \sum_{i=k+1+j}^{k} \eta^T(i) Q\eta(i) - \sum_{j=1-\tau}^{0} \sum_{i=k+j}^{k-1} \eta^T(i) Q\eta(i) \\ &= \sum_{j=1-\tau}^{0} \left\{\eta^T(k) Q\eta(k) - \eta^T(k+j) Q\eta(k+j)\right\} \\ &= \tau\eta^T(k) Q\eta(k) - \sum_{i=k+1-\tau}^{k} \eta^T(i) Q\eta(i). \end{aligned} \tag{25}$$

Substituting (23)–(25) into (21), we have

$$\mathbb{E}\{\Delta V\} = \mathbb{E}\{\Delta V_1 + \Delta V_2 + \Delta V_3\} = \delta^T(k) \Pi\delta(k), \tag{26}$$

where

$$\delta(k) = \left[\eta^T(k) \;\; \eta^T(k-\tau_k)\right]^T. \tag{27}$$

According to Theorem 8, we have $\Pi < 0$. For all the $\delta(k) \neq 0$, $\mathbb{E}\{\Delta V\} < 0$, and there is a sufficiently small scalar $\varepsilon_0 > 0$ such that

$$\Pi + \varepsilon_0 \operatorname{diag}\{I, 0\} < 0. \tag{28}$$

Therefore, we can conclude from (26) and (28) that

$$\mathbb{E}\{\Delta V\} \leq -\varepsilon_0 \mathbb{E}\left\{\|\eta(k)\|^2\right\}. \tag{29}$$

According to (21), we obtain that

$$\mathbb{E}\{V(k)\} \leq \lambda_1 \mathbb{E}\left\{\|\eta(k)\|^2\right\} + \lambda_2 \sum_{i=k-\tau}^{k-1} \mathbb{E}\left\{\|\eta(i)\|^2\right\}, \tag{30}$$

where $\lambda_1 = \lambda_{\max}(P)$ and $\lambda_2 = (\tau+1)\lambda_{\max}(Q)$.

For any scalar $\sigma > 1$, taking (21) into consideration, we have

$$\begin{aligned} &\sigma^{k+1}\mathbb{E}\{V(k+1)\} - \sigma^k \mathbb{E}\{V(k)\} \\ &\quad = \sigma^{k+1}\mathbb{E}\{\Delta V\} + \sigma^k(\sigma-1)\mathbb{E}\{V(k)\} \\ &\quad \leq \epsilon_1(\sigma)\sigma^k \mathbb{E}\left\{\|\eta(k)\|^2\right\} \\ &\qquad + \epsilon_2(\sigma) \sum_{i=k-\tau}^{k-1} \sigma^k \mathbb{E}\left\{\|\eta(i)\|^2\right\}, \end{aligned} \tag{31}$$

where $\epsilon_1(\sigma) = (\sigma-1)\lambda_1 - \sigma\varepsilon_0$ and $\epsilon_2(\sigma) = (\sigma-1)\lambda_2$.

Besides, for integer $M \geq \tau + 1$, summing up both sides of (31) from 0 to $M-1$, we have

$$\begin{aligned} &\sigma^M \mathbb{E}\{V(k+1)\} - \mathbb{E}\{V(0)\} \\ &\quad \leq \epsilon_1(\sigma) \sum_{k=0}^{M-1} \sigma^k \mathbb{E}\left\{\|\eta(k)\|^2\right\} \\ &\qquad + \epsilon_2(\sigma) \sum_{k=0}^{M-1} \sum_{i=k-\tau}^{k-1} \sigma^k \mathbb{E}\left\{\|\eta(i)\|^2\right\}. \end{aligned} \tag{32}$$

For $\tau > 1$,

$$\begin{aligned} &\sum_{k=0}^{M-1} \sum_{i=k-\tau}^{k-1} \sigma^k \mathbb{E}\left\{\|\eta(i)\|^2\right\} \\ &\quad \leq \left(\sum_{i=-\tau}^{-1} \sum_{k=0}^{i+\tau} + \sum_{i=0}^{M-1-\tau} \sum_{k=i+1}^{i+\tau} + \sum_{i=M-1-\tau}^{M-1} \sum_{k=i+1}^{M-1}\right) \\ &\qquad \times \sigma^k \mathbb{E}\left\{\|\eta(i)\|^2\right\} \\ &\quad \leq \tau \sum_{i=-\tau}^{-1} \sigma^{i+\tau} \mathbb{E}\left\{\|\eta(i)\|^2\right\} + \tau \sum_{i=0}^{M-1-\tau} \sigma^{i+\tau} \mathbb{E}\left\{\|\eta(i)\|^2\right\} \\ &\qquad + \tau \sum_{i=M-1-\tau}^{M-1} \sigma^{i+\tau} \mathbb{E}\left\{\|\eta(i)\|^2\right\} \\ &\quad \leq \tau\sigma^\tau \max_{-\tau\leq i\leq 0} \mathbb{E}\left\{\|\eta(i)\|^2\right\} + \tau\sigma^\tau \sum_{i=0}^{M-1} \sigma^i \mathbb{E}\left\{\|\eta(i)\|^2\right\}. \end{aligned} \tag{33}$$

Then from (32) and (33), we have

$$\begin{aligned} &\sigma^k \mathbb{E}\{V(k)\} \\ &\quad \leq \mathbb{E}\{V(0)\} + (\epsilon_1(\sigma) + \bar{\epsilon}_2(\sigma)) \sum_{i=0}^{k-1} \sigma^i \mathbb{E}\left\{\|\eta(i)\|^2\right\} \\ &\qquad + \bar{\epsilon}_2(\sigma) \sum_{-\tau\leq i\leq 0} \mathbb{E}\left\{\|\eta(i)\|^2\right\}, \end{aligned} \tag{34}$$

where

$$\bar{\epsilon}_2(\sigma) = \tau\sigma^\tau(\sigma-1)\lambda_2. \tag{35}$$

We set $\lambda_0 = \lambda_{\min}(P)$ and $\lambda = \max\{\lambda_1, \lambda_2\}$; it is easy to follow that

$$\mathbb{E}\{V(k)\} \geq \lambda_0 \mathbb{E}\left\{\|\eta(k)\|^2\right\}. \tag{36}$$

Besides, we can conclude from (30) that

$$\mathbb{E}\{V(0)\} \leq \lambda \max_{-\tau\leq i\leq 0} \mathbb{E}\left\{\|\eta(i)\|^2\right\}. \tag{37}$$

It can be verified that there exists $\sigma_0 > 1$ that

$$\epsilon_1(\sigma_0) + \bar{\epsilon}_2(\sigma_0) = 0. \tag{38}$$

So it is clear to see from (34) to (38) that

$$\mathbb{E}\left\{\|\eta(k)\|^2\right\} \leq \left(\frac{1}{\sigma_0}\right)^k \frac{\lambda + \bar{\epsilon}_2(\sigma_0)}{\lambda_0} \max_{-\tau\leq i\leq 0} \mathbb{E}\left\{\|\eta(i)\|^2\right\}. \quad (39)$$

So augmented system (11) is exponentially mean-square stable according to Definition 4 when $f(k) = 0$ and $v(k) = 0$, and the proof of Theorem 8 is complete. □

In addition, we are going to analyze the H_∞ performance of filtering error system (11).

Theorem 9. *For the given disturbance attenuation level $\gamma > 0$ and observer gain K, filtering error system (11) is said to be asymptotically stable in the mean square and satisfies H_∞ constraints in (14) with $f(k) = 0$, $v(k) \neq 0$, if there exist positive definite matrix $P = \operatorname{diag}\{P_1, P_2\}$, $Q > 0$ with proper dimensions, and $\varepsilon > 0$ satisfying the following inequality:*

$$\Gamma = \begin{bmatrix} \Gamma_{11} & 0 & \Gamma_{13} \\ * & \Pi_{22} & \Gamma_{23} \\ * & * & \Gamma_{33} - \gamma^2 I \end{bmatrix} < 0, \quad (40)$$

where

$$\begin{aligned} \Gamma_{11} &= \Pi_{11} + \Delta\mathscr{A}^T P \Delta\mathscr{A} + \frac{1}{N}\mathscr{L}^T\mathscr{L}, \\ \Gamma_{13} &= \mathscr{A}^T P \mathscr{E}_v + \bar{\alpha}\mathscr{D}_1^T P \mathscr{E}_v + (1-\bar{\alpha})\mathscr{D}_2^T P \mathscr{E}_v, \\ \Gamma_{23} &= \mathscr{B}^T P \mathscr{E}_v, \\ \Gamma_{33} &= 2\mathscr{E}_v^T P \mathscr{E}_v. \end{aligned} \quad (41)$$

Π_{11} and Π_{22} are defined in Theorem 8.

Proof. According to Theorem 8, filtering error system (11) is asymptotically stable in the mean square with $f(k) = 0$, $v(k) = 0$. By constructing the same Lyapunov-Krasovskii functional as in Theorem 8 and setting $f(k) = 0$, we have

$$\begin{aligned} \mathbb{E}\{\Delta V_k\} \leq \mathbb{E}\Big\{&\delta^T(k)\Pi\delta(k) + 2\eta^T(k)\mathscr{A}^T P\mathscr{E}_v v(k) \\ &+ 2\eta^T(k)\Delta\mathscr{A}^T P\mathscr{E}_v v(k) \\ &+ 2\eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{E}_v v(k) \\ &+ 2\bar{\alpha}\eta^T(k)\mathscr{D}_1^T P\mathscr{E}_v v(k) \\ &+ 2(1-\bar{\alpha})\eta^T(k)\mathscr{D}_2^T P\mathscr{E}_v v(k) \\ &+ v^T(k)\mathscr{E}_v^T P\mathscr{E}_v v(k)\Big\}, \end{aligned} \quad (42)$$

where $\delta(k)$ and Π are previously defined.

It follows from Lemma 6 that

$$\begin{aligned} 2\eta^T(k)\Delta\mathscr{A}^T P\mathscr{E}_v v(k) &\leq \eta^T(k)\Delta\mathscr{A}^T P\Delta\mathscr{A}\eta(k) \\ &\quad + v(k)^T\mathscr{E}_v^T P\mathscr{E}_v v(k). \end{aligned} \quad (43)$$

Substituting (43) into (42), we have

$$\begin{aligned} \mathbb{E}\{\Delta V_k\} \leq \mathbb{E}\Big\{&\delta^T(k)\Pi\delta(k) + 2\eta^T(k)\mathscr{A}^T P\mathscr{E}_v v(k) \\ &+ \eta^T(k)\Delta\mathscr{A}^T P\Delta\mathscr{A}\eta(k) \\ &+ 2\eta^T(k-\tau_k)\mathscr{B}^T P\mathscr{E}_v v(k) \\ &+ 2\bar{\alpha}\eta^T(k)\mathscr{D}_1^T P\mathscr{E}_v v(k) \\ &+ 2(1-\bar{\alpha})\eta^T(k)\mathscr{D}_2^T P\mathscr{E}_v v(k) \\ &+ 2v^T(k)\mathscr{E}_v^T P\mathscr{E}_v v(k)\Big\}. \end{aligned} \quad (44)$$

By setting $\tilde{\delta}(k) = \left[\delta^T(k) \;\; v^T(k)\right]^T$, (44) can be written as

$$\begin{aligned} &\mathbb{E}\{\Delta V(k)\} \\ &\quad \leq \mathbb{E}\left\{\tilde{\delta}^T(k)\begin{bmatrix} \Pi_{11} + \Delta\mathscr{A}^T P\Delta\mathscr{A} & 0 & \Gamma_{13} \\ * & \Pi_{22} & \Gamma_{23} \\ * & * & \Gamma_{33} \end{bmatrix}\tilde{\delta}(k)\right\}, \end{aligned} \quad (45)$$

where $\Gamma_{13} = \mathscr{A}^T P\mathscr{E}_v + \bar{\alpha}\mathscr{D}_1^T P\mathscr{E}_v + (1-\bar{\alpha})\mathscr{D}_2^T P\mathscr{E}_v$, $\Gamma_{23} = \mathscr{B}^T P\mathscr{E}_v$, and $\Gamma_{33} = 2\mathscr{E}_v^T P\mathscr{E}_v$.

In order to deal with the H_∞ performance of (11), we introduce the following index:

$$J(n) = \mathbb{E}\sum_{k=0}^{n}\left\{\frac{1}{N}\tilde{z}^T(k)\tilde{z}(k) - \gamma^2 v^T(k)v(k)\right\}, \quad (46)$$

where n is a nonnegative integer.

When the system is under zero initial condition, we have

$$\begin{aligned} J(n) &= \mathbb{E}\sum_{k=0}^{n}\left\{\frac{1}{N}\tilde{z}^T(k)\tilde{z}(k) - \gamma^2 v^T(k)v(k) + \Delta V(k)\right\} \\ &\quad - \mathbb{E}\{V(n+1)\} \\ &\leq \mathbb{E}\sum_{k=0}^{n}\left\{\frac{1}{N}\tilde{z}^T(k)\tilde{z}(k) - \gamma^2 v^T(k)v(k) + \Delta V(k)\right\} \\ &\leq \mathbb{E}\sum_{k=0}^{n}\left\{\tilde{\delta}^T(k)\Gamma\tilde{\delta}(k)\right\}. \end{aligned} \quad (47)$$

According to Theorem 9, we have $J(n) \leq 0$. Furthermore, letting $n \to \infty$, we have

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{\frac{1}{N}\|\tilde{z}(k)\|^2\right\} \leq \gamma^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\|v(k)\|^2\right\}, \quad (48)$$

so the proof of Theorem 9 is complete. □

Next, sufficient condition is proposed for designing H_∞ filter for WiNCS as shown in (1).

Theorem 10. *For the given disturbance attenuation level $\gamma > 0$, filtering error system (11) is said to be asymptotically stable in the mean square and satisfies H_∞ constraints in (14) with $f(k) = 0$, $v(k) \neq 0$, if there exist positive definite matrix $P = \text{diag}\{P_1, P_2\}$, $Q > 0$, a general matrix X with proper dimensions, and $\varepsilon > 0$ satisfying the following inequality:*

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & 0 & \Upsilon_{13} & \Upsilon_{14} & 0 \\ * & -Q & \Gamma_{23} & \Upsilon_{24} & 0 \\ * & * & \Gamma_{33} - \gamma^2 I & 0 & 0 \\ * & * & * & \Upsilon_{44} & \Upsilon_{45} \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{49}$$

where

$$\begin{aligned} \Upsilon_{11} &= (1+\tau) Q - P + \frac{1}{N}\mathscr{L}^T\mathscr{L} + \varepsilon \overline{N}^T \overline{N}, \\ \Upsilon_{13} &= \Omega \mathscr{E}_v + \alpha \mathscr{D}_1^T P \mathscr{E}_v + (1-\overline{\alpha}) \mathscr{D}_2^T P \mathscr{E}_v, \\ \Upsilon_{14} &= \begin{bmatrix} 2\Omega & 0 & \alpha_1 \mathscr{D}^T P & \alpha_2 \mathscr{D}_1^T P & \alpha_3 \mathscr{D}_2^T P & 0 \end{bmatrix}, \\ \Upsilon_{24} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2\mathscr{B}^T P \end{bmatrix}, \\ \Upsilon_{44} &= \text{diag}\{-P, -P, -P, -P, -P, -P\}, \\ \Upsilon_{45}^T &= \begin{bmatrix} 0 & \widetilde{M}^T \overline{P}^T & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \alpha_1 &= \sqrt{\overline{\alpha}(1-\overline{\alpha})}, \qquad \alpha_2 = 2\sqrt{\overline{\alpha}}, \qquad \alpha_3 = 2\sqrt{1-\overline{\alpha}}, \\ \widetilde{N} &= I_N \otimes N, \qquad \widetilde{M} = I_N \otimes M, \\ \overline{P} &= \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \qquad \overline{N} = \begin{bmatrix} \widetilde{N} & 0 \end{bmatrix}, \\ \Omega &= \begin{bmatrix} \widetilde{A}^T P_1 & 0 \\ 0 & \widetilde{A}^T P_2 - \widetilde{C}^T X \end{bmatrix}. \end{aligned} \tag{50}$$

Γ_{23} and Γ_{33} are defined in (41). So the gain of fault detection observer is

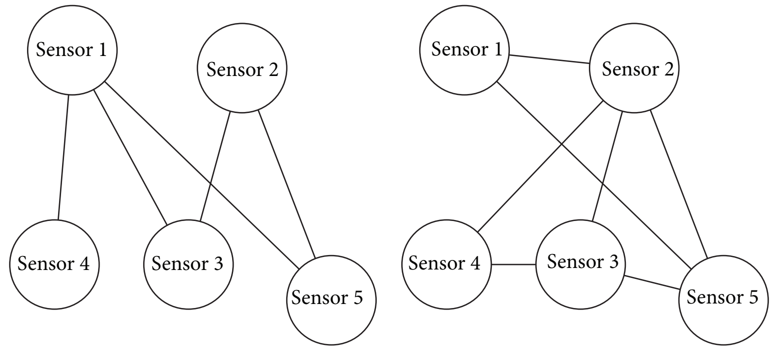
$$K = P_2^{-1} X^T. \tag{51}$$

Proof. According to Lemma 5, inequality (40) can be rewritten into the following:

$$\begin{bmatrix} (1+\tau)Q - P + \frac{1}{N}\mathscr{L}^T\mathscr{L} & 0 & \Gamma_{13} & 2\mathscr{A}^T & \frac{5}{2}\Delta\mathscr{A}^T & \sqrt{\overline{\alpha}(1-\overline{\alpha})}\mathscr{D}^T & 2\sqrt{\overline{\alpha}}\mathscr{D}_1^T & 2\sqrt{1-\overline{\alpha}}\mathscr{D}_2^T & 0 \\ * & -Q & \Gamma_{23} & 0 & 0 & 0 & 0 & 0 & 2\mathscr{B}^T \\ * & * & \Gamma_{33} - \gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -P^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -P^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & -P^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & -P^{-1} & 0 \\ * & * & * & * & * & * & * & * & -P^{-1} \end{bmatrix} < 0. \tag{52}$$

Multiplying $\text{diag}\{I, I, I, P, P, P, P, P, P\}$ on both sides of the above matrix inequality, we have

$$\begin{bmatrix} (1+\tau)Q - P + \frac{1}{N}\mathscr{L}^T\mathscr{L} & 0 & \Gamma_{13} & 2\mathscr{A}^T P & \frac{5}{2}\Delta\mathscr{A}^T P & \sqrt{\overline{\alpha}(1-\overline{\alpha})}\mathscr{D}^T P & 2\sqrt{\overline{\alpha}}\mathscr{D}_1^T P & 2\sqrt{1-\overline{\alpha}}\mathscr{D}_2^T P & 0 \\ * & -Q & \Gamma_{23} & 0 & 0 & 0 & 0 & 0 & 2\mathscr{B}^T P \\ * & * & \Gamma_{33} - \gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -P & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & * & -P & 0 & 0 \\ * & * & * & * & * & * & * & -P & 0 \\ * & * & * & * & * & * & * & * & -P \end{bmatrix} < 0. \tag{53}$$



(a) (b)

Figure 1: Two topology structures of WiNCS.

By the use of Lemma 7, inequality (53) can be rewritten into

$$\begin{bmatrix} (1+\tau)Q - P + \frac{1}{N}\mathscr{L}^T\mathscr{L} + \varepsilon\overline{N}^T\overline{N} & 0 & \Gamma_{13} & 2\mathscr{A}^T P & 0 & \sqrt{\overline{\alpha}(1-\overline{\alpha})}\mathscr{D}^T P & 2\sqrt{\overline{\alpha}}\mathscr{D}_1^T P & 2\sqrt{1-\overline{\alpha}}\mathscr{D}_2^T P & 0 & 0 \\ * & -Q & \Gamma_{23} & 0 & 0 & 0 & 0 & 0 & 2\mathscr{B}^T P & 0 \\ * & * & \Gamma_{33} - \gamma^2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -P & 0 & 0 & 0 & 0 & \overline{P}\widetilde{M} \\ * & * & * & * & * & -P & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -P & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -P & 0 & 0 \\ * & * & * & * & * & * & * & * & -P & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0. \tag{54}$$

We set $K^T P_2 = X$, so $K = P_2^{-1} X^T$. Substituting it into (54), we can get the result easily, and the proof of Theorem 10 is complete. □

4. Numerical Simulations

In this section, a simulation result is presented to show the effectiveness of the proposed method. Consider system (1) with

$$A = \begin{bmatrix} 0.1910 & 0.1854 \\ 0.1742 & 0.1701 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.1075 & 0.0046 \\ -0.0088 & 0.0288 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.8899 & 0.3414 \\ 0.4046 & 0.2373 \end{bmatrix}, \qquad E_f = \begin{bmatrix} 0.1252 \\ 0.0880 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.0463 & 0.0927 \\ 0 & 0.1066 \end{bmatrix},$$

$$D(k) = \begin{bmatrix} 0.8\sin(0.7k) & 0 \\ 0 & 0.8\sin(0.7k) \end{bmatrix},$$

$$H = \begin{bmatrix} 0.3337 & 0.2410 \\ 0.1947 & -0.3245 \end{bmatrix},$$

$$E_v = \begin{bmatrix} 0.2017 \\ 0.0512 \end{bmatrix}, \qquad L = \begin{bmatrix} 0.92 & 0.41 \end{bmatrix},$$

$$v(k) = 3e^{-0.9k}\sin(0.2k). \tag{55}$$

Suppose that there are five nodes in WiNCS with interconnection topology as shown in Figure 1, and the coupled configuration matrices are

$$D^{\alpha} = \begin{bmatrix} -3 & 0 & 1 & 1 & 1 \\ 0 & -2 & 1 & 0 & 1 \\ 1 & 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & -2 \end{bmatrix},$$

$$D^{1-\alpha} = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 0 & 1 & -3 & 1 & 1 \\ 0 & 1 & 1 & -2 & 0 \\ 1 & 1 & 1 & 0 & -3 \end{bmatrix} \tag{56}$$

with probability $\overline{\alpha} = 0.3$ and disturbance attenuation level $\gamma = 0.88$.

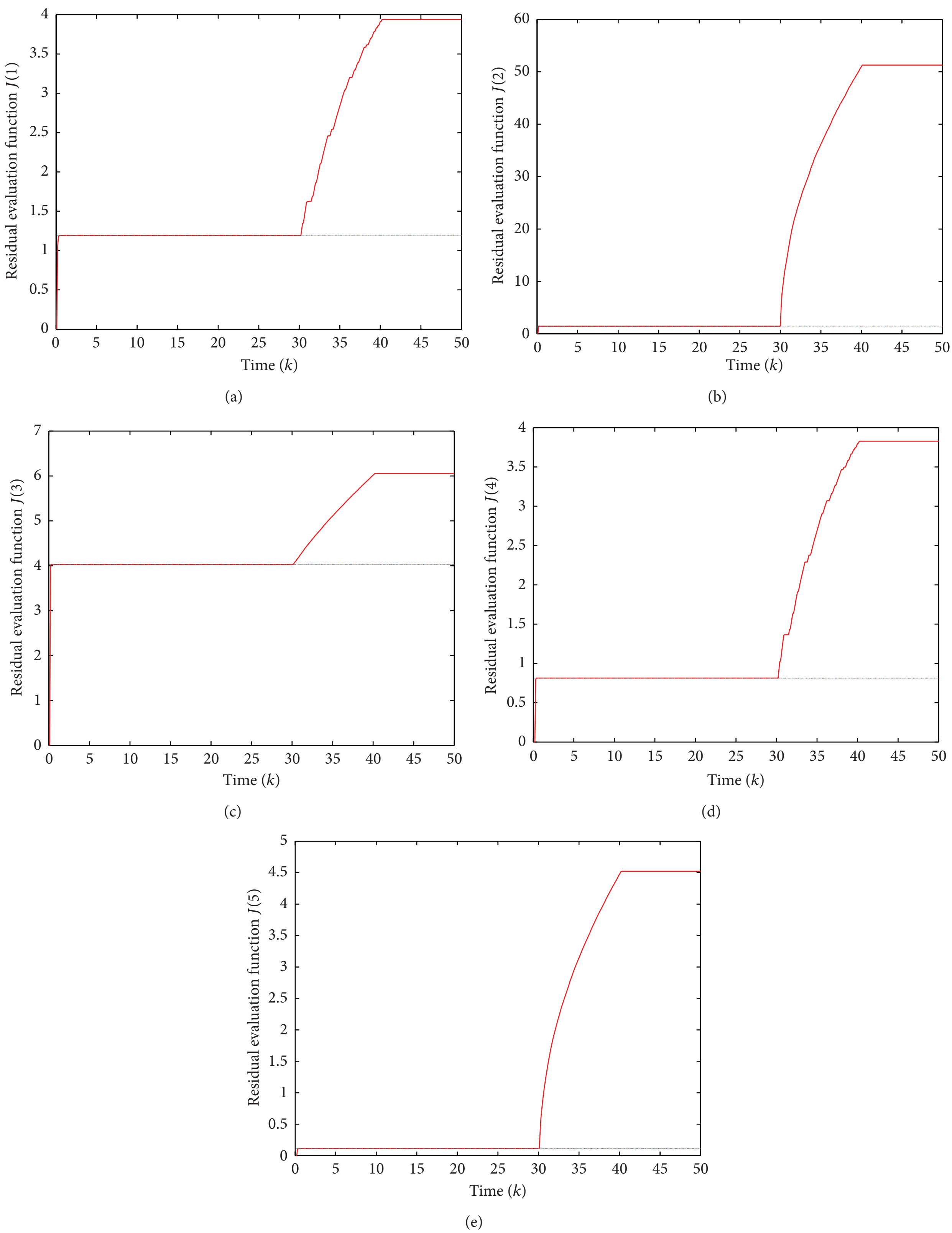


Figure 2: System residual evaluation function.

Table 1: Gain matrices.

k_{ij}	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
$i=1$	$\begin{bmatrix}-0.8360 & 2.2935\\ -0.4851 & 1.4932\end{bmatrix}$	$\begin{bmatrix}0.0671 & -0.1090\\ 0.0169 & -0.0272\end{bmatrix}$	$\begin{bmatrix}-0.0226 & 0.0699\\ -0.0057 & 0.0176\end{bmatrix}$	$\begin{bmatrix}0.0468 & -0.0874\\ 0.0118 & -0.0221\end{bmatrix}$	$\begin{bmatrix}0.1513 & -0.2661\\ 0.0384 & -0.0675\end{bmatrix}$
$i=2$	$\begin{bmatrix}0.0590 & -0.0896\\ 0.0149 & -0.0225\end{bmatrix}$	$\begin{bmatrix}-1.0733 & 2.7564\\ -0.5455 & 1.6103\end{bmatrix}$	$\begin{bmatrix}0.1716 & -0.3058\\ 0.0431 & -0.0766\end{bmatrix}$	$\begin{bmatrix}0.0868 & -0.1814\\ 0.0231 & -0.0478\end{bmatrix}$	$\begin{bmatrix}0.1625 & -0.2788\\ 0.0407 & -0.0696\end{bmatrix}$
$i=3$	$\begin{bmatrix}-0.0165 & 0.0549\\ -0.0042 & 0.0140\end{bmatrix}$	$\begin{bmatrix}0.1671 & -0.3039\\ 0.0425 & -0.0772\end{bmatrix}$	$\begin{bmatrix}-0.9403 & 2.4938\\ -0.5116 & 1.5439\end{bmatrix}$	$\begin{bmatrix}0.0765 & -0.1160\\ 0.0190 & -0.0287\end{bmatrix}$	$\begin{bmatrix}0.1198 & -0.2280\\ 0.0305 & -0.0581\end{bmatrix}$
$i=4$	$\begin{bmatrix}0.0413 & -0.0747\\ 0.0106 & -0.0191\end{bmatrix}$	$\begin{bmatrix}0.0842 & -0.1494\\ 0.0213 & -0.0376\end{bmatrix}$	$\begin{bmatrix}0.0786 & -0.1313\\ 0.0200 & -0.0334\end{bmatrix}$	$\begin{bmatrix}-0.7223 & 2.0971\\ -0.4566 & 1.4440\end{bmatrix}$	$\begin{bmatrix}-0.0752 & 0.1590\\ -0.0190 & 0.0401\end{bmatrix}$
$i=5$	$\begin{bmatrix}0.1587 & -0.2833\\ 0.0402 & -0.0717\end{bmatrix}$	$\begin{bmatrix}0.1616 & -0.2933\\ 0.0411 & -0.0744\end{bmatrix}$	$\begin{bmatrix}0.1194 & -0.2258\\ 0.0304 & -0.0575\end{bmatrix}$	$\begin{bmatrix}-0.0812 & 0.1885\\ -0.0210 & 0.0485\end{bmatrix}$	$\begin{bmatrix}-0.9518 & 2.5147\\ -0.5144 & 1.5490\end{bmatrix}$

The initial states of each sensor node are

$$x_1(0)=\begin{bmatrix}3.2\\-3.5\end{bmatrix},\quad x_2(0)=\begin{bmatrix}2.2\\-1.6\end{bmatrix},\quad x_3(0)=\begin{bmatrix}2.72\\3.25\end{bmatrix},\quad x_4(0)=\begin{bmatrix}-3.6\\-1.36\end{bmatrix},\quad x_5(0)=\begin{bmatrix}3.122\\-1.45\end{bmatrix}. \tag{57}$$

Parameters can be acquired based on the proposed theorems, they are omitted here for brevity concern, and observer gain matrices are listed in Table 1.

We make fault detection for the system shown in (1), and we assume that fault only occurs in node 2 at time instant $k=30$, system fault can be delivered to other nodes by their interconnections, and simulation results are shown in Figure 2, where red line and dotted line represent evaluation function J and threshold value J_{th}, respectively. From the results we can see that J rises quickly when fault happens, and threshold values are designed as $J_{\text{th}1}=1.1939$, $J_{\text{th}2}=1.4556$, $J_{\text{th}3}=4.0299$, $J_{\text{th}4}=0.8139$, and $J_{\text{th}5}=0.1127$. Figure 3 indicates the stochastic switching for two topologies associated with this example.

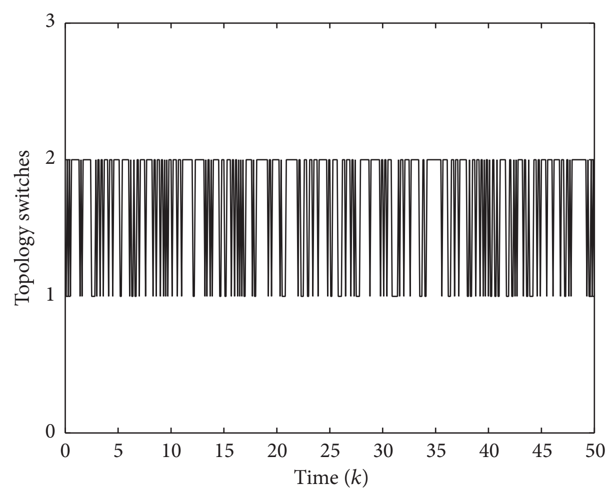


Figure 3: Topology switches.

In WiNCS, states of node are affected not only by itself, but also by other nodes' measurement according to the topology, so node's failure can be transmitted to other nodes via signal channel. Intuitively, a node with more connection means more importance in the system, and failure can be spread to entire topology in a short time, so detecting failure in time is quite important, which will affect the stability of the system.

5. Conclusion

In this paper, we have considered the fault detection problem for a class of discrete-time wireless networked control systems, which has stochastic switching topology, combined with uncertainty and disturbance. The states of each node in WiNCS are affected not only by itself, but also by other nodes' measurements according to a certain topology. We get sufficient conditions based on Lyapunov stability theory to guarantee the existence of the filters satisfying the H_∞ performance constraint, and the gains of observers are also acquired by solving linear matrix inequalities. However, there are only five nodes in the simulation and fault detection for WiNCS composed of large number of nodes is still a difficult problem, which is our future research task.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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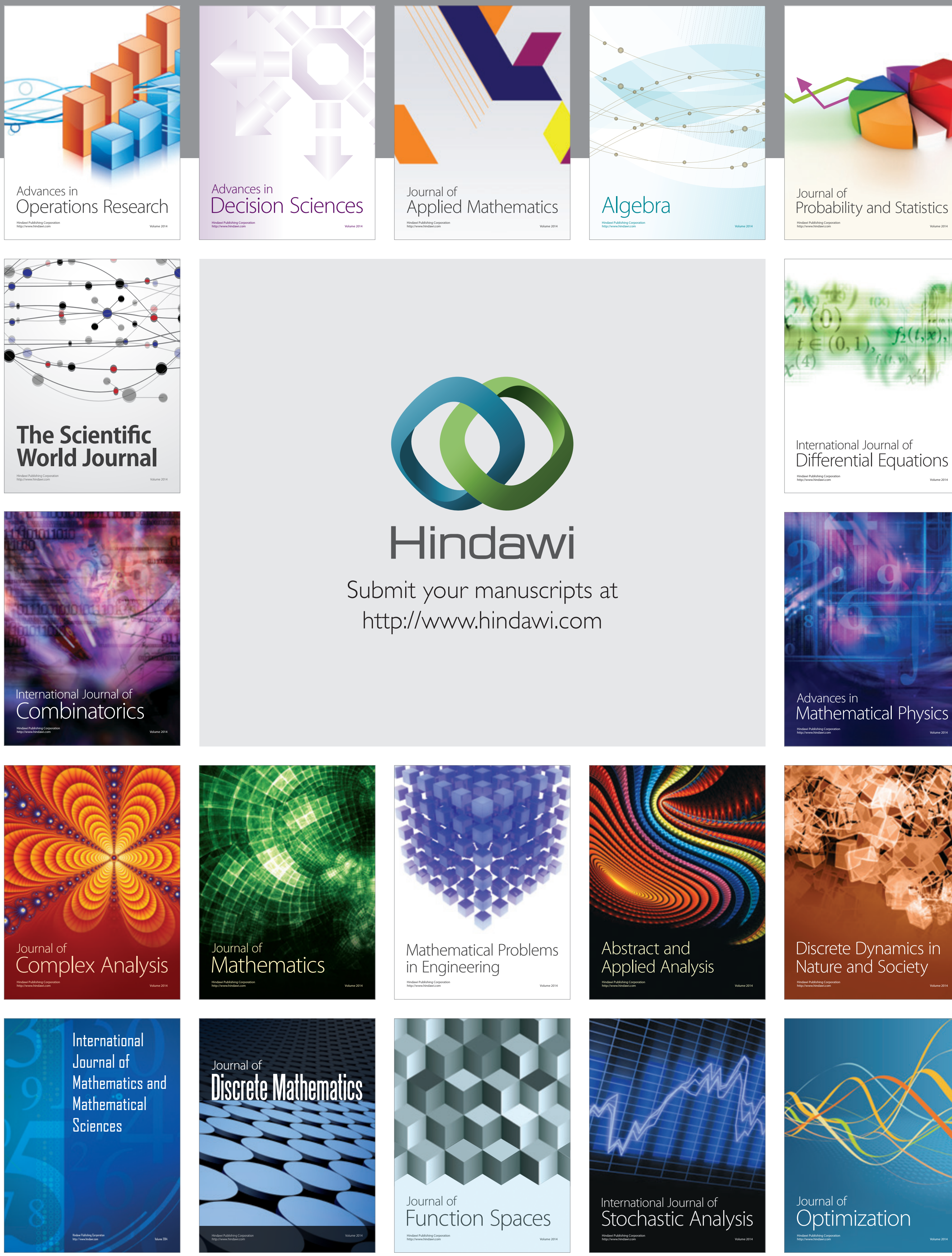
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