Model predictive control-based non-linear fault tolerant control for air-breathing hypersonic vehicles

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1 Introduction

Air-breathing hypersonic vehicle (AHV) is a kind of vehicle which can fly at a speed of five times the speed of sound (Mach 5). With the utmost aim of feasible and affordable atmospheric flight, AHV has drove much attention in recent years [1]. The design of guidance and control systems for AHVs is a challenging task since the interactions between the airframe, the propulsion system and the structural dynamics are very strong, and AHVs are very sensitive to changes in flight condition and the aerodynamic characteristics [2–4].

Owing to the enormous complexity of the dynamics, only longitudinal models have been developed and used for control design. A lot of works have been done and several results are available in literature which consider control solutions for AHVs. A design strategy of a multi-input/multi-output adaptive sliding mode controller for the longitudinal dynamics of AHVs is reported in [5], in which the vehicle model is non-linear, multi-variable and unstable, and includes uncertain parameters. In [6], the authors consider the development of the control-oriented model and also provide an example of control design based on approximate feedback linearisation. In [7], the design problem of a non-linear robust adaptive controller for AHVs is discussed. In [8], the authors consider the longitudinal motion of a hypersonic aircraft containing inertial and aerodynamic uncertain parameters. By using stochastic robustness analysis approach, robust flight control systems with non-linear dynamic inversion structure are synthesised. In [9], the authors present an adaptive linear quadratic altitude and velocity tracking control algorithm for the longitudinal model of a generic air-breathing hypersonic flight vehicle. In [10], the authors deal with the adaptive model reference sliding output tracking control for AHVs. Although a lot of results have been obtained, the complex and challenging control problem for AHVs has not been fully investigated, especially when possible faults exist.

Owing to the requirements of precise control and the complexity of modern engineering systems, the reliability of the designed controller becomes very important. Unfortunately, similar to other airplanes and space vehicles, possible failures are unavoidable in AHVs. Hence, the stability of a system in which a fault occurs is very important, which motivates the research of fault tolerant control (FTC). The FTC of AHVs has been studied in recent years. In [11], a reference output tracking controller design method is proposed for AHVs with actuator delay and uncertainty, the existence conditions of such controllers are proposed in terms of liner matrix inequalities; in [12], a reliable control for AHVs with both sensor and actuator failures is studied by utilising T–S fuzzy modelling technology. Most of the proposed methods are based on the linear model or the nominal linear model of AHVs, which limits the applicability of these methods. For a complex non-linear system, it is better to design a non-linear fault tolerant controller directly, but to the best of our knowledge, this problem has not been well discussed.

Non-linear tracking control methods have been widely studied [13, 14], but non-linear fault tolerant control (NFTC) is still in progress, much work should be done. For NFTC, the faults are treated as additive actuator faults [15, 16] at first, and the non-linear regulation theory is used to solve the NFTC problem; a Lyapunov reconstruction technique, which based on the a priori knowledge of a stabilising feedback for the nominal safe model and the knowledge of the associated Lyapunov function, is used to solve NFTC problem in [17]. In [18], a model predictive control (MPC)-based online reference reshaping and controller reconstruction method is presented. The proposed method has two main stages. Firstly, the reference command of the faulty system is reshaped online with respect to system faults; secondly, based on a non-linear MPC strategy, the control of the plant is reallocated according to the new reference command. Although the mentioned method needs real-time calculation, it provides an efficient way to the NFTC.

Motivated by the above discussions, in this paper, an NFTC strategy for AHVs will be proposed. After presenting the non-linear dynamics of AHVs, the fault model and the control objective of the paper are discussed. Since the fault model considered in this paper is a general form, and the dynamics of AHVs are really complex, we propose an MPC-based FTC strategy. The reference command is reshaped with respect to the faults, firstly. Then an optimal problem is obtained by MPC, and through solving the optimal problem online, the input of the plant can be obtained in real time. A simulation analysis is provided to confirm the effectiveness of the proposed control design approach. From the simulation results we can see that the proposed control strategy can guarantee a good tracking performance in the existence of faults. The main contributions of the paper can be summarised as follows:

(1) A non-linear FTC method is proposed for AHVs. Via reshaping the reference command online and reconstructing the input of the plant real timely, the FTC of AHVs can be solved.
(2) The proposed method can deal with not only the FTC problem, but also the input saturation of non-linear system.
(3) Though need real-time calculation, the proposed method is easily to be carried out in practice.

The rest of this paper is organised as follows. The non-linear model of AHVs, the fault model and the control objective of this paper are presented in Section 2. Section 3 gives the main results of the paper, the reference reshaping method and the MPC-based FTC strategy. In Section 4, a numeric simulation is given and we conclude this paper in Section 5.

Notation: The notations used throughout the paper are fairly standard. The superscript ‘T’ denotes matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; and diag{…} stands for a block-diagonal matrix. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

2 Problem formulation

2.1 Non-linear model of AHVs

The rigid-body equation of motion for AHVs considered in this paper is developed by NASA Langley Research Center [19]. The non-linear equations are described as follows

$$\begin{cases} \dot{V} = (T\cos\alpha - D)/m - \mu\sin\gamma/r^2 \\ \dot{\gamma} = (L + T\sin\alpha)/mV - (\mu - V^2 r)\cos\gamma/Vr^2 \\ \dot{h} = V\sin\gamma \\ \dot{\alpha} = q - \dot{\gamma} \\ \dot{q} = M_{yy}/I_{yy} \end{cases} \tag{1}$$

and the engine dynamics can be written as a second-order system

$$\ddot{\beta} = -2\xi\omega_n\dot{\beta} - {\omega_n}^2\beta + {\omega_n}^2\beta_c \tag{2}$$

where h and V represent the flight altitude and velocity of AHVs, respectively; α is the angle of attack of the vehicle, γ is the flight path angle and q represents the pitch rate. T, L, D and M_{yy} are the thrust, lift, drag and pitching moment, respectively. I_{yy} is the moment of inertia. Equation (2) represents the dynamics of the actuator, where ξ is the damping ratio of the actuator dynamics, ω_n is the natural frequency and β_c is the throttle setting.

The expressions of L, D, T and M_{yy} are modelled as

$$\begin{cases} L = \dfrac{1}{2}\rho V^2 S C_L \\ D = \dfrac{1}{2}\rho V^2 S C_D \\ T = \dfrac{1}{2}\rho V^2 S C_T \\ M_{yy} = \dfrac{1}{2}\rho V^2 S\bar{c}[C_M(\alpha) + C_M(\delta_e) + C_M(q)] \\ r = h + R_E \end{cases} \tag{3}$$

where

$$\begin{cases} C_L = 0.6203\alpha \\ C_D = 0.6450\alpha^2 + 0.0043378\alpha + 0.003772 \\ C_T = \begin{cases} 0.02576\beta, & \beta < 1 \\ 0.0224 + 0.00336\beta, & \beta > 1 \end{cases} \\ C_M(\alpha) = -0.035\alpha^2 + 0.036617\alpha + 5.3216\times 10^{-6} \\ C_M(\delta_e) = ce(\delta_e - \alpha) \\ C_M(q) = (\bar{c}/2V)q(-6.796\alpha^2 + 0.3015\alpha - 0.2289) \end{cases} \tag{4}$$

$\bar{c}$ represents mean aerodynamic chord and δ_e means elevator deflection of AHVs. This non-linear model is composed of five rigid-body state variables $x = [V, \gamma, \alpha, \beta, h]^T$, the output to be controlled is selected as the velocity V and the altitude h, then $y = [V, h]^T$. The control input $u = [\beta_c,\ \delta_e]^T$ does not appear explicitly in the equations. For a simple description, the following equations are used to represent the non-linear dynamics of AHVs in this paper

$$\begin{cases} \dot{x} = f(x, F, u) \\ y = h(x) \end{cases}$$

where F is a family of parameters included in (2)–(4).

2.2 FTC objective

Faults are inevitable, and may influence stability of AHVs. Faults may locate in the actuators, the components or the elevators of AHVs and may be caused by partial damage of the component or loss effectiveness of actuator fault. When faults occur, the faulty system can be described as

$$\begin{cases} \dot{x} = f_F(x_F, \Gamma_F, u_F(t)) \\ y = h(x) \end{cases}$$

Γ_F means that the aerodynamic coefficient drift from the nominal model and $f_F(\cdot)$ represents the component faults of the vehicle. The actuator fault $u_F(t)$ may have many forms, such as

$$u_{Fi}(t) = \lambda_i u(t) \tag{5}$$

where $0 < \lambda_i \leq 1$, $i = (\beta_c, \delta_e)$, and (5) represents loss of actuator effectiveness.

Besides, both of the control inputs are supposed to be constrained by a saturation value, expressed by

$$u_{i\min} \leq u_{Fi}(t) \leq u_{i\max} \tag{6}$$

which means that each of the inputs has a separate saturation limit. The saturation function $\mathrm{sat}(u_{Fi}(t))$ is defined as

$$\mathrm{sat}(u_{Fi}(t)) = \begin{cases} u_{i\max}, u_{i\max} > u_{Fi}(t) \\ u_{Fi}(t), u_{i\min} < u_{Fi}(t) < u_{i\max} \\ u_{i\min}, u_{Fi}(t) < u_{i\min} \end{cases}$$

Remark 1: The upper bound and the lower bound of the saturation function, $u_{i\max}$ and $u_{i\min}$ are not constrained to be symmetric [20], which is different from most of the linear anti-windup controller design methods.

Remark 2: $u_{i\max}$ and $u_{i\min}$ may not equal to the real limitation of the vehicle, since actuator may be locked in a small area. Then the upper bound and the lower bound will change according to the real situation.

Remark 3: From the above description we can see that the type of faults considering in this paper is a general case.

The control objective of AHVs is to track a command velocity and altitude vector $y_{\mathrm{com}}(t) = [V_{\mathrm{com}}(t), h_{\mathrm{com}}(t)]^{\mathrm{T}}$, such that the output tracking error achieves zero, that is

$$\lim_{t\to\infty} (y(t) - y_{\mathrm{com}}(t)) = 0 \tag{7}$$

With respect to the mentioned faults and actuator saturation, the tracking problem of FAHVs can be stated as follows: finding a bounded controller, such that

- the closed-loop system is robustly stable;
- the output of the system can track a command vector $y_{\mathrm{com}} = [V_{\mathrm{com}}(t), h_{\mathrm{com}}(t)]^{\mathrm{T}}$ without steady error;
- in the event of possible faults, the stability and the tracking performance of the system can still be guaranteed.

3 Main results

3.1 Robust non-linear control design

The non-linear equations (1) do not give direct relationships between inputs and outputs, so it is difficult to design a controller. Input/output linearisation uses full-state feedback to globally linearise the non-linear dynamics, and a nominal linear system can be constructed [21]. The classical linear controller design methods can then be utilised to regulate the closed-loop dynamics. As described in [5, 7], the nominal linearised model can be developed by repeated differentiation of V and h as

$$\begin{cases} \dot{V} = (T\cos\alpha - D)/m - \mu\sin\gamma/r^2 \\ \ddot{V} = \omega_1\dot{x}/m \\ \dddot{V} = (\omega_1\ddot{x} + \dot{x}^{\mathrm{T}}\Omega_2\dot{x})/m \end{cases} \tag{8}$$

and

$$\begin{cases} \dot{h} = V\sin\gamma \\ \ddot{h} = \dot{V}\sin\gamma + V\dot{\gamma}\cos\gamma \\ \dddot{h} = \ddot{V}\sin\gamma + 2\dot{V}\dot{\gamma}\cos\gamma - V\dot{\gamma}^2\sin\gamma + V\ddot{\gamma}\cos\gamma \\ h^{(4)} = \dddot{V}\sin\gamma + 3\ddot{V}\dot{\gamma}\cos\gamma - 3\dot{V}\ \dot{\gamma}^2\sin\gamma + 3\dot{V}\ddot{\gamma}\cos\gamma \\ -3V\dot{\gamma}\ddot{\gamma}\sin\gamma - V\dot{\gamma}^3\cos\gamma + V\dddot{\gamma}\cos\gamma \end{cases} \tag{9}$$

where

$$\begin{cases} \ddot{\gamma} = \pi_1\dot{x} \\ \dddot{\gamma} = \pi_1\ddot{x} + \dot{x}^{\mathrm{T}}\Pi_2\dot{x} \end{cases}$$

The expressions of ω_1, Ω_2 and π_1 can be found in [5, 7]. Defining

$$\begin{cases} \ddot{\alpha}_0 = \dfrac{1}{2}\rho V^2 S\bar{c}[C_M(\alpha) + C_M(\delta_e) - c_e\alpha]/Iyy - \ddot{\gamma} \\ \ddot{\beta}_0 = -2\xi\omega_n\dot{\beta} - \omega_n{}^2\beta \end{cases}$$

Then the expressions of $\ddot{\alpha}$ and $\ddot{\beta}$ can be viewed as two parts, control-independent and control-dependent

$$\begin{cases} \ddot{\alpha} = \ddot{\alpha}_0 + \left(\dfrac{c_e\rho V^2 S\bar{c}}{2Iyy}\right)\delta_e \\ \ddot{\beta} = \ddot{\beta}_0 + \omega_n{}^2\beta_c \end{cases} \tag{10}$$

From (10), the input $u = [\beta_c, \delta_e]^{\mathrm{T}}$ has been separate from the non-linear dynamics. Defining $\ddot{x}_0 = [\ddot{V}, \ddot{\gamma}, \ddot{\alpha}_0, \ddot{\beta}_0, \ddot{h}]$, then the non-linear equation (1) can be written as

$$\begin{cases} \dddot{V} = f_V + [b_{11} \quad b_{12}]u \\ h^{(4)} = f_h + [b_{21} \quad b_{22}]u \end{cases} \tag{11}$$

where

$$\begin{cases} f_V = (\omega_1\ddot{x}_0 + \dot{x}^{\mathrm{T}}\Omega\dot{x})/m \\ f_h = 3\ddot{V}\dot{\gamma}\cos\gamma - 3\dot{V}\dot{\gamma}^2\sin\gamma + 3\dot{V}\ddot{\gamma}\cos\gamma \\ \qquad -3V\dot{\gamma}\ddot{\gamma}\sin\gamma - V\dot{\gamma}^3\cos\gamma + V\dddot{\gamma}\cos\gamma + f_V \end{cases}$$

and

$$\begin{cases} b_{11} = \dfrac{\rho V^2 S c_\beta w_n^2}{2m}\cos(\alpha) \\ b_{12} = -\dfrac{c_e\rho V^2 S\bar{c}}{2mIyy}\left(T\sin\alpha + \dfrac{\partial D}{\partial\alpha}\right) \\ b_{21} = \dfrac{\rho V^2 S c_\beta w_n^2}{2m}\sin(\alpha+\gamma) \\ b_{22} = \dfrac{c_e\rho V^2 S\bar{c}}{2mIyy}\left(T\cos(\alpha+\gamma) + \dfrac{\partial L}{\partial\alpha}\cos\gamma - \dfrac{\partial D}{\partial\alpha}\sin\gamma\right) \end{cases}$$

The details of the diffeomorphism coordinate transform can be found in [5, 7]. By defining the tracking errors of the output as

$$e_V = V - V_{\text{com}}(t) \tag{12}$$

$$e_h = h - h_{\text{com}}(t) \tag{13}$$

and two virtual controls

$$v_V = f_V + [b_{11} \quad b_{12}]u \tag{14}$$

$$v_h = f_h + [b_{21} \quad b_{22}]u \tag{15}$$

a nominal linear system can be constructed

$$\begin{cases} \dot{\chi}_V = A_V \chi_V + B_V(v_V - \dddot{V}_{\text{com}}(t)) \\ \dot{\chi}_h = A_h \chi_h + B_h(v_h - h^{(4)}_{\text{com}}(t)) \end{cases} \tag{16}$$

where

$$\chi_V = [e_V \quad \dot{e}_V \quad \ddot{e}_V]^{\mathrm{T}}$$

$$\chi_h = [e_h \quad \dot{e}_h \quad \ddot{e}_h \quad \dddot{e}_h]^{\mathrm{T}}$$

$$A_V = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_V = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A_h = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_h = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Then the linear controller design methods can be utilised here to design the virtual controllers (v_V, v_h)

$$v_V = \dddot{V}_{\text{com}}(t) + K_V \chi_V \tag{17}$$

$$v_h = h^{(4)}_{\text{com}}(t) + K_h \chi_h \tag{18}$$

where K_V and K_h are controller gain matrices. After getting the virtual controllers, the real inputs of AHVs can be obtained through the following transformation

$$u = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}^{-1} \left(\begin{bmatrix} v_V \\ v_h \end{bmatrix} - \begin{bmatrix} f_V \\ f_h \end{bmatrix} \right) \tag{19}$$

3.2 Optimal trajectory reconfiguration

The virtual control (14) and (15) of AHVs are functions of $y_{\text{com}}(t)$ and its derivative, which are given or known. But in the existence of system faults or the actuator saturation, the controlled output may not track the command in real time, or even fail to track $y_{\text{com}}(t)$. Then the controller needs to be designed with respect to the system faults and the actuator saturation. It is a challenging task, especially for complex non-linear system. Since the controller is the function of $y_{\text{com}}(t)$ and its derivative, the controller can be changed to satisfy the constraints by adjusting the reference command. The reconstruction of the optimal reference command, marked as $y_d(t)$, will be discussed here.

The reconstruction method can be describe by the following minimising problem

$$\min J_1 = \int_{t_1}^{t_2} (y_d(t) - y_{\text{com}}(t))^{\mathrm{T}} Q (y_d(t) - y_{\text{com}}(t)) \, \mathrm{d}t$$

where $Q \in \mathbb{R}^{2\times 2}$ is a positive definite weighting matrix, t_1 is the initial interpolation time and t_2 is the final interpolation time. On the other hand, with respect to the actuator fault, a control weighting matrix R is defined and the minimising index can be written as

$$\min J_2 = \int_{t_1}^{t_2} u^{\mathrm{T}}(t) R u(t) \, \mathrm{d}t$$

where $R \in \mathbb{R}^{2\times 2}$ is a positive definite weighting matrix. Then the optimal trajectory reconfiguration problem can be transformed into the following optimal problem

$$\begin{aligned} \min J &= \min(J_1 + J_2) \\ &= \int_{t_1}^{t_2} [(y_d(t) - y_{\text{com}}(t))^{\mathrm{T}} Q (y_d(t) - y_{\text{com}}(t)) \\ &\quad + u^{\mathrm{T}}(t) R u(t)] \, \mathrm{d}t \end{aligned} \tag{20}$$

When solving the above optimal problem, the following constraints should be considered

$$\dot{x} = f_F(x, \Gamma_F, u_F) \tag{21}$$

$$y_d(t) = h(x) \tag{22}$$

$$y^{(k)}_{dj}(t_F) = y^{(k)}_{\text{com}j}(t_{\text{com}}), \quad j = (V, h), \quad k = 0, 1, \ldots, r \tag{23}$$

$$u_{i\min} \le u_{Fi}(t) \le u_{i\max} \tag{24}$$

$$t_2 \ge t_{\text{com}} \tag{25}$$

where t_2 is the final motion time of the faulty system and t_{com} is the final motion time for the reference command. r is the relative degree of the non-linear system.

Remark 4: The constraints (21) and (22) correspond to the faulty system equations; the constraint (23) means that the optimal trajectory should be the same as the reference command at the beginning and end time; then the constraint (24) is the saturation limit of the plant and the running time of the faulty system could be longer than the ordinary one, which is given in constraint (25).

For solving the above optimal problem, we make the following assumption:

Assumption 1: The optimal trajectory can be written in the canonical polynomial basis

$$y_{dj}(t) = \sum_{i=1}^{i=l+1} a_{ij} \left(\frac{t - t_1}{t_2 - t_1} \right)^{(i-1)}, \quad j = (V, h) \tag{26}$$

where l is the order of the polynomial, and a_{ij} are the interpolation coefficients.

Remark 5: With sufficient order, any smooth function can be approximated by a polynomial, so the above assumption always holds.

Remark 6: Through writing the output trajectories into canonical polynomial basis, the complexities of reference reconfiguration and MPC-based controller reconfiguration can be greatly reduced.

By considering the following equation

$$u_d = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}^{-1} \left(\begin{bmatrix} \dddot{V}_d(t) \\ h_d^{(4)}(t) \end{bmatrix} - \begin{bmatrix} f_V \\ f_h \end{bmatrix} \right) \tag{27}$$

Then the optimal problem (20) can be rewritten as

$$\min J(a_{ij}, t_2) \tag{28}$$

with the constraints

$$u_{i\min} \le u_{Fi}(t) \le u_{i\max}$$
$$t_2 \ge t_{\text{com}}$$

then the non-linear constrained optimisation is reduced to a more simple optimal problem of (a_{ij}, t_2).

3.3 MPC-based fault tolerant controller design

After getting the optimal trajectory of the reference command, the controller u should be reconfigurated. In equations (17) and (18), the expression of virtual control has been obtained. By replacing $y_{\text{com}}(t)$ with $y_d(t)$, a new virtual control can be constructed

$$\hat{v}_V = \dddot{V}_{\text{com}}(t) + K_V \hat{\chi}_V \tag{29}$$

$$\hat{v}_h = h_{\text{com}}^{(4)}(t) + K_h \hat{\chi}_h \tag{30}$$

where

$$\hat{\chi}_V = [V - V_d(t) \quad \dot{V} - \dot{V}_d(t) \quad \ddot{V} - \ddot{V}_d(t)]^{\text{T}}$$
$$\hat{\chi}_h = [h - h_d(t) \quad \dot{h} - \dot{h}_d(t) \quad \ddot{h} - \ddot{h}_d(t) \quad \dddot{h} - \dddot{h}_d(t)]^{\text{T}}$$

Then we reallocate the control $u(t)$ for the faulty system. To obtain a tracking performance of AHVs, the following MPC-based optimal problem is proposed (see (31))

where $Q_F \in \mathbb{R}^{2\times 2}$ is a positive definite weight matrix and t_H is a finite integration time. Since $0 < \rho < 1$, the tracking error will achieve zero.

3.4 Stability analysis

Owing to the contractive constraints on the tracking errors in the optimisation problems (31), the stability analysis of the above method is straightforward. This type of contractive constraints has already been used [22, 23]. Following [22, 23], we can drive the same stability conclusion when dealing with the MPC-based output tracking problem.

Before proceeding, we present the following lemma.

Lemma 1: If there exist constants ε_1 and ε_2, for all $y(t_0) \in \{y(t_0) \in \mathbb{R}^n, \|y(t_0) - y_d(t_0)\| \le \varepsilon_1\}$, and $\|y(t) - y_d(t)\| \le \varepsilon_2 \|y(t_k) - y_d(t_k)\|$, the optimal problems (31) have a solution, then the MPC-based fault tolerant controller design algorithm implies an exponential convergence of tracking error to be zero.

Owing to the page limit we omitted the proof, and the detailed proof can be found in [18].

From the above lemma, we can easily see that, for the above MPC-based optimal problem, the constants ε_1 and ε_2 always exist. So the MPC-based fault tolerant controller can guarantee global stability and the tracking performance of the closed-loop system.

4 Simulation results

In this section, some simulations are provided to evaluate the effectiveness of the proposed controller (31). In simulation, a climbing manoeuvre for altitude is considered, and the expressions are listed as follows (see (32))

where the final motion time $t_{\text{com}} = 60\,\text{s}$. The virtual controller gains K_V and K_h of (17) and (18) are chosen as, respectively

$$K_V = [20 \quad 145 \quad 25]$$
$$K_h = [2.25 \quad 5.5 \quad 6.3 \quad 308]$$

First, a normal tracking problem (without faults) is proposed to evaluate the MPC-based controller. The optimal problem is solved with $t_{\text{H}} = 0.5$, $\rho = 0.9$ and

$$Q_F = \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$$

The simulation results are listed in Figs. 1 and 2, where Fig. 1 is the responses of velocity and altitude with respect to (32), and Fig. 2 gives the control input of the plant. From these figures we can see that the proposed control method can provide a good tracking performance for the system without faults.

$$\text{Pro}(t_k, u_F(t)) \begin{cases} \min \displaystyle\int_{t_k}^{t_k+t_H} \begin{bmatrix} f_V + [b_{11} \quad b_{12}]u_F(t) - \hat{v}_V \\ f_h + [b_{21} \quad b_{22}]u_F(t) - \hat{v}_h \end{bmatrix}^{\text{T}} Q_F \begin{bmatrix} f_V + [b_{11} \quad b_{12}]u_F(t) - \hat{v}_V \\ f_h + [b_{21} \quad b_{22}]u_F(t) - \hat{v}_h \end{bmatrix} \text{d}t \\ \dddot{V} = f_V + [b_{11} \quad b_{12}]u_F(t) \\ h^{(4)} = f_h + [b_{21} \quad b_{22}]u_F(t) \\ u_{i\min} \le u_{Fi}(t) \le u_{i\max} \\ \|y(t_k) - y_d(t_k)\| \le \rho \|y(t_k+1) - y_d(t_k+1)\|, \quad t_k + 1 = t_k + T \\ T > 0, \ 0 < \rho < 1, \ k = 0, 1 \ldots \end{cases} \tag{31}$$

$$\begin{aligned} V_{\text{com}}(t) &= 0 \\ h_{\text{com}}(t) &= \begin{cases} 1000(6\,(t/t_{\text{com}})^5 - 15\,(t/t_{\text{com}})^4 + 10\,(t/t_{\text{com}})^3), & 0 \le t \le t_{\text{com}} \\ 1000, & t > t_{\text{com}} \end{cases} \end{aligned} \tag{32}$$

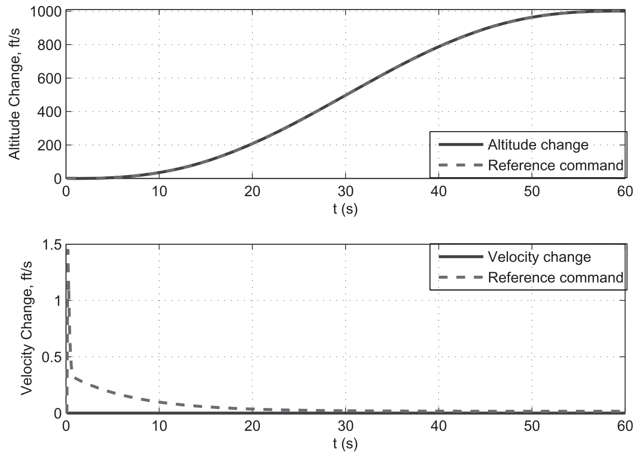

Fig. 1 *Tracking performance of normal case*

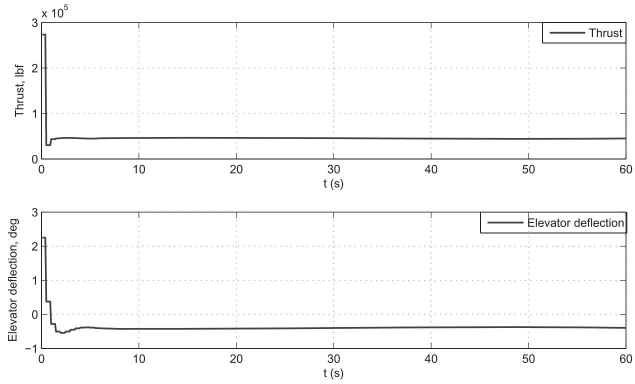

Fig. 2 *Input of normal case*

Then we assume that a fault occurs at the initial time $t_1 = 0$. The fault is a loss of effectiveness in the actuator, which can be modelled by a multiplicative coefficient as follows

$$u_{Fi}(t) = 0.5u_i(t)$$

what is more, we assume that the actuators of the plant suffer from saturation, and the upper bound, together with the lower bound of the saturation function is given as

$$0 \le T \le 4 \times 10^5$$
$$-4^\circ \le \delta_e \le 4^\circ$$

Then the viable input of the plant will be much smaller than the fault-free one.

Solving the non-linear optimal problem (31) by the MATLAB solver 'fmincon', a feasible solution can be obtained with the following optimal values (see equations at the bottom of the page)

The normal command and the optimal one are all listed in Fig. 3. Then by solving the MPC-based optimal problem

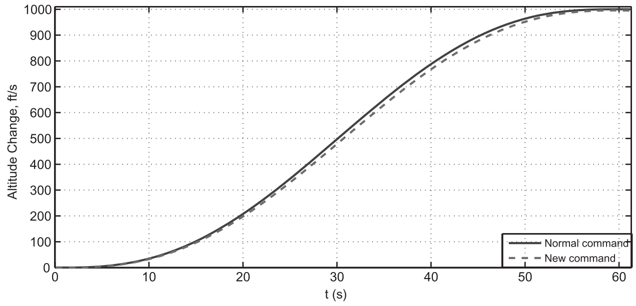

Fig. 3 *New command $h_d(t)$ and normal command $h_{com}(t)$*

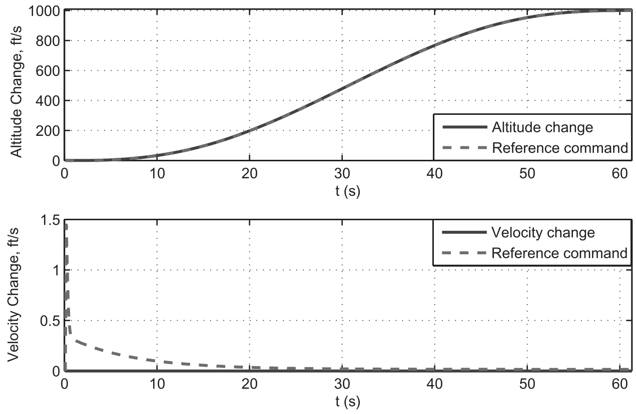

Fig. 4 *Tracking performance of fault case*

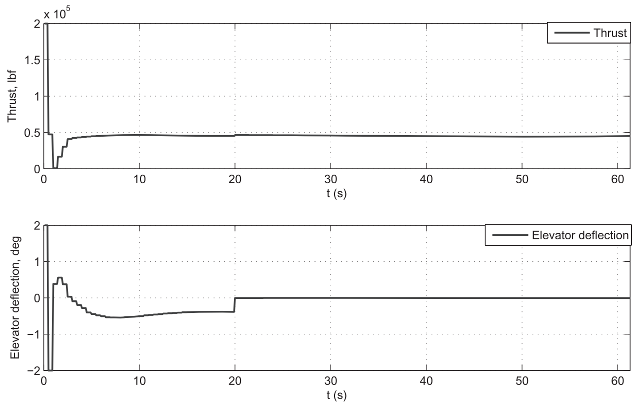

Fig. 5 *Input of fault case*

(31), the optimal controller can be obtained in real time. The simulation results are reported in Figs. 4 and 5.

5 Conclusion

In this paper, the non-linear FTC of AHVs has been addresses and a MPC-based online optimal method has been discussed. The AHVs are supposed to suffer from failures

$$t_2 = 61.3\ \mathrm{s}$$
$$V_\mathrm{d}(t) = 0$$
$$h_\mathrm{d}(t) = \begin{cases} 1000(6(t/t_2)^5 - 15(t/t_2)^4 + 10(t/t_2)^3 - 0.0005(t/t_2)^6), & 0 \le t \le t_2 \\ 1000, & t > t_2 \end{cases}$$

and then the reference command has been reshaped with respect to the faulty system. An MPC-based optimal control strategy has been proposed. The input of AHVs is computed online, according to the reshaped reference command. Simulation results have validated that the present control strategy can deal with the failures and saturation.

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