Asynchronous L_1 control of delayed switched positive systems with mode-dependent average dwell time

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A B S T R A C T

This paper investigates the stability and asynchronous L_1 control problems for a class of switched positive linear systems (SPLSs) with time-varying delays by using the mode-dependent average dwell time (MDADT) approach. By allowing the co-positive type Lyapunov–Krasovskii functional to increase during the running time of active subsystems, a new stability criterion for the underlying system with MDADT is first derived. Then, the obtained results are extended to study the issue of asynchronous L_1 control, where "asynchronous" means that the switching of the controllers has a lag with respect to that of system modes. Sufficient conditions are provided to guarantee that the resulting closed-loop system is exponentially stable and has an L_1-gain performance. Finally, two numerical examples are given to show the effectiveness of the developed results.

1. Introduction

Positive systems are dynamic systems whose state variables are constrained to be positive (at least nonnegative) at all times. Such systems abound in various fields, e.g., biomedicine [4], ecology [5], and TCP-like Internet congestion control [15]. During the past decades, switched systems have been investigated by many researchers due to the theoretical devel-opment as well as practical applications [21]. Several methods have been developed to study the stability of switched systems, such as the common Lyapunov function approach, the average dwell time (ADT) scheme, and the multiple Lyapunov functions method (see [1,10,25–27]). Recently, the mode-dependent average dwell time (MDADT) approach [37] has been proposed for the stability analysis and control synthesis of switched systems. It has been shown that the results obtained by the MDADT approach are more general than those derived by other methods.

Recently, switched positive linear systems (SPLSs) which consist of a family of positive linear subsystems and a switching signal governing the switching among them have received considerable attention due to their broad applications in congestion control [3] and communication systems [14]. Many useful results on stability and stabilization of such systems have appeared (see [2,8,9,12,13,31,32,36,38,39]). Because time-delay phenomena exist widely in engineering and social systems and often cause instability or bad system performance in control systems, time-delay systems have been extensively studied (see [11,12,16–20,28–30,39]). Some results on SPLSs with time-delays have been obtained [12,39].

On the other hand, the disturbance rejection problem has been a hot topic [7]. Some results on L_1-gain analysis for positive systems have been reported in [6,24]. The reason for this study is that the L_1-gain can provide a more useful

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description for positive systems because 1-norm gives the sum of the values of the components, which is more appropriate, for instance, if the values represent the amount of material or the number of animal in a species [6].

In almost all the aforementioned works on switched positive systems, a very common assumption in the state-feedback stabilization context is that the controllers are switched synchronously with the switching of system modes, which is quite unpractical. As pointed out in [27], there inevitably exists asynchronous switching in actual operation, i.e. the switching instants of the controllers exceed or lag behind those of the subsystems. Thus, it is necessary to consider asynchronous switching for realistic control. Some results on switched systems under asynchronous switching have been proposed in [22,23,25,26,33–35]. However, to the best of our knowledge, the asynchronous L_1 control problem for SPLSs, which constitutes the main motivation of the present study, has not been investigated yet.

The main contribution of this paper is threefold: (1) by constructing an appropriate co-positive type Lyapunov–Krasovskii functional, a new stability criterion is derived by using the MDADT method; (2) by allowing the Lyapunov–Krasovskii functional to increase during the running time of active subsystems, improved stability and L_1-gain analysis results are obtained; (3) the obtained results are extended to study the issue of asynchronous L_1 control.

The remainder of this paper is organized as follows. In Section 2, problem statements and necessary lemmas are given. In Section 3, based on the MDADT approach, stability and asynchronous L_1 control problems for SPLSs with time-varying delays are addressed, and sufficient conditions are also provided to guarantee the exponential stability of the closed-loop system. Two numerical examples are provided to show the effectiveness of the proposed approach in Section 4. Concluding remarks are given in Section 5.

Notation: In this paper, $A \succ 0 (A \succeq 0)$ means that all the elements of A are positive (nonnegative). $A \succ B$ $(A \succeq B)$ means that $A - B \succ 0$ $(A - B \succeq 0)$. R^n is the n-dimensional real vector space and R^n_+ is the set of n-dimensional vectors with nonnegative elements; $R^{n\times s}$ is the set of all real matrices of $(n \times s)$-dimension. A^T denotes the transpose of A. The vector 1-norm of $x \in R^n$ is denoted by $\|x\| = \sum_{l=1}^{n} |x_l|$, where x_l is the lth element of x. 1_q denotes the column vector with q rows containing only *1* entries; 1_f denotes the column vector with f rows containing only *1* entries; For scalars $y_a, y_{a+1}, \ldots, y_b$, $y_a y_{a+1} \ldots y_b$ is denoted by $\Pi_{i=a}^{b} y_i$; $\exp\{\cdot\}$ is the exponential operate; $L_1[t_0,\infty)$ is the space of absolute integrable vector-valued functions on $[t_0,\infty)$, i.e., we say $z{:}[t_0,\infty) \to R^q$ is in L_1 $[t_0,\infty)$ if $\int_{t_0}^{\infty} \|z(t)\| dt < \infty$.

2. Problem statements and preliminaries

Consider the following switched linear system with time-varying delay:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + G_{\sigma(t)}x(t-d(t)) + B_{\sigma(t)}u(t) + E_{\sigma(t)}w(t), \\ x(t_0+\theta) = \varphi(\theta), \theta \in [-\tau, 0], \\ z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}w(t), \end{cases} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^p$ and $z(t) \in R^q$ denote the system state, the control input, and the controlled output, respectively; $w(t) \in R^f$ is the disturbance input which belongs to $L_1[t_0,\infty)$; $\sigma(t){:}[t_0,\infty) \to \underline{N} = \{1, 2, \ldots, N\}$ is a piecewise constant function of time, called a switching signal; N is the number of subsystems; t_0 is the initial instant; A_i, G_i, B_i, C_i, D_i and E_i, $\forall i \in \underline{N}$, are known constant matrices with appropriate dimensions; $d(t)$ denotes the time-varying delay satisfying $0 \leqslant d(t) \leqslant \tau$ and $\dot{d}(t) \leqslant d < 1$ for known positive constants τ and d; $\varphi(\theta)$ is a continuous vector-valued initial function defined on interval $\tau, 0]$.

Next, we will give the definition of switched positive system (1).

Definition 1 [24]. System (1) is said to be an SPLS if for any switching signals $\sigma(t)$ and initial conditions $\varphi(\theta) \succeq 0$, $\theta \in [-\tau, 0]$, it satisfies $x(t) \succeq 0$ and $z(t) \succeq 0$, $\forall\, t \geqslant t_0$.

Definition 2 [8]. A is called a Metzler matrix, if its off-diagonal entries are non-negative.

Due to the asynchronous switching, the switching instants of the controllers do not coincide with those of the subsystems. Without loss of generality, the "asynchronous" means that the switching of the controllers has a lag with respect to that of system modes. Then, the real control input will become

$$u(t) = K_{\sigma(t-\Delta_\kappa)}x(t), \quad \forall t \in [t_\kappa, t_{\kappa+1}), \quad \kappa = 0, 1, \ldots \tag{2}$$

where $\Delta_0 = 0$, and $\Delta_\kappa < t_{\kappa+1} - t_\kappa$ represents the delayed period.

Remark 1. The period Δ_κ guarantees that switching instants of the controllers lag behind the switches of system modes, and also there exists a period during which the system mode and the controller operate synchronously.

Let the ith subsystem be activated at the switching instant t_κ, and the jth subsystem be activated at the switching instant $t_{\kappa+1}$. Then the corresponding controllers are activated at the switching instants $t_\kappa + \Delta_\kappa$ and $t_{\kappa+1} + \Delta_{\kappa+1}$, respectively. Upon applying the controller (2) to system (1), the resulting closed-loop system is given by

$$
\begin{cases}
\dot{x}(t) = \overline{A}_i x(t) + G_i x(t-d(t)) + E_i w(t), & \\
z(t) = C_i x(t) + D_i w(t), & \forall t \in [t_\kappa + \Delta_\kappa, t_{\kappa+1}) \\
\dot{x}(t) = \overline{A}_{i,j} x(t) + G_j x(t-d(t)) + E_j w(t), & \\
z(t) = C_j x(t) + D_j w(t), & \forall t \in [t_{\kappa+1}, t_{\kappa+1} + \Delta_{\kappa+1})
\end{cases} \tag{3}
$$

where $\overline{A}_i = A_i + B_i K_i$ and $\overline{A}_{i,j} = A_j + B_j K_i$.

Lemma 1 [24]. *System (3) is positive if and only if $\overline{A}_i$ and $\overline{A}_{i,j}$ are Metzler matrices, and $G_i \succeq 0$, $E_i \succeq 0$, $C_i \succeq 0$, $D_i \succeq 0$, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \neq j$.*

Definition 3 [24]. System (1) is said to be exponentially stable under the switching signal $\sigma(t)$, if there exist constants $\zeta > 0$ and $\rho > 0$ such that the solution of the system satisfies $\|x(t)\| \leqslant \zeta \|x(t_0)\|_c e^{-\rho(t-t_0)}$, $\forall t \geqslant t_0$, where $\|x(t_0)\|_c = \sup_{-\tau \leqslant \theta \leqslant 0} \|\varphi(\theta)\|$.

Definition 4. [37]For a switching signal $\sigma(t)$ and any $T_2 \geqslant T_1 \geqslant 0$, let $N_{\sigma i}(T_1, T_2)$ be the switching number that the ith subsystem is active over the interval $[T_1, T_2)$ and $T_i(T_1, T_2)$ be the total running time of the ith subsystem over the interval $[T_1, T_2)$, $i \in \underline{N}$. We say that $\sigma(t)$ has a mode-dependent average dwell time T_{ai} if there exist positive numbers N_{0i} (we call N_{0i} the mode dependent chatter bounds) and T_{ai} such that $N_{\sigma i}(T_1, T_2) \leqslant N_{0i} + T_i(T_1, T_2)/T_{ai}$ holds.

Definition 5. System (1) is said to have an L_1-gain performance level γ under the switching signal $\sigma(t)$, if the following conditions are satisfied:

(i) system (1) is exponentially stable when $w(t) \equiv 0$;
(ii) under zero initial conditions, i.e., $\varphi(\theta) = 0$, $\theta \in [-\tau, 0]$, the following inequality holds for all nonzero $w(t) \in L_1[t_0, \infty)$:

$$
\int_{t_0}^{\infty} e^{-\sum_{i=1}^{N} \{\alpha_i T_i(t_0,t)\}} \|z(t)\| dt \leqslant \gamma \int_{t_0}^{\infty} \|w(t)\| dt, \tag{4}
$$

where α_i is a given positive constant and $T_i(t_0, t)$ is the total running time of the ith subsystem over the interval $[t_0, t)$.

Remark 2. When $\alpha_i = \alpha$, $\forall i \in \underline{N}$, (4) will degenerate into (2) in [24]. Thus Definition 5 given here can be viewed as an extension of the one proposed in [24].

The aim of this paper is to design a state-feedback controller and a set of admissible switching signals with MDADT such that the resulting closed-loop system (3) is positive and exponentially stable with L_1-gain performance.

3. Main results

3.1. Stability and L_1-gain analysis

Before proceeding further, we present here the following results on the exponential stability for the SPLS (1) with $u(t) \equiv 0$ and $w(t) \equiv 0$ for later use.

Theorem 1. *Consider the SPLS (1) with $u(t) \equiv 0$ and $w(t) \equiv 0$. Let $\alpha_i > 0$ be given constants. If there exist vectors $\nu_i \succ 0$, $\upsilon_i \succ 0$ and $\vartheta_i \succ 0$ of appropriate dimensions, such that, $\forall i \in \underline{N}$,*

$$
A_i^T \nu_i + \alpha_i \nu_i + \upsilon_i + \tau \vartheta_i \preceq 0, \tag{5}
$$

$$
G_i^T \nu_i - (1-d) e^{-\alpha_i \tau} \upsilon_i \preceq 0, \tag{6}
$$

then the system is exponentially stable for any switching signal $\sigma(t)$ with the following MDADT

$$
T_{ai} > T_{ai}^* = \ln \mu_i / \alpha_i, \tag{7}
$$

where $\mu_i \geqslant 1$ satisfy

$$
\nu_i \preceq \mu_i \nu_j, \upsilon_i \preceq \mu_i \upsilon_j, \vartheta_i \preceq \mu_i \vartheta_j, \quad \forall (i,j) \in \underline{N} \times \underline{N} \tag{8}
$$

Proof. For any $T > 0$, let $t_0 = 0$ and denote by $t_1, t_2, \ldots, t_{\kappa-1}, t_\kappa, \ldots, t_{N_\sigma(0,T)}$ the switching instants on the interval $[t_0, T)$, where $N_\sigma(0,T) = \sum_{i=1}^{N} N_{\sigma i}(0,T)$. Let $T_i(0,T)$ be the total running time of the ith subsystem over the interval $[0, T)$.

Consider the following Lyapunov–Krasovskii functional for the ith subsystem:

$$V_i(t,x(t)) = x^T(t)\nu_i + \int_{t-d(t)}^{t} e^{\alpha_i(-t+s)}x^T(s)\upsilon_i ds + \int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\alpha_i(-t+s)}x^T(s)\vartheta_i ds d\theta \tag{9}$$

where $\nu_i \succ 0$, $\upsilon_i \succ 0$ and $\vartheta_i \succ 0$ are vectors to be determined.

For the sake of simplicity, $V_i(t,x(t))$ is written as $V_i(t)$ in this paper. Taking the derivation of the Lyapunov–Krasovskii functional along the trajectory of the ith subsystem yields:

$$\begin{aligned}
\dot V_i(t) =& \dot x^T(t)\nu_i - \alpha_i\int_{t-d(t)}^{t} e^{\alpha_i(-t+s)}x^T(s)\upsilon_i ds + x^T(t)\upsilon_i - (1-\dot d(t))e^{-\alpha_i\tau}x^T(t-d(t))\upsilon_i\\
&-\alpha_i\int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\alpha_i(-t+s)}x^T(s)\vartheta_i ds d\theta + \tau x^T(t)\vartheta_i - \int_{-\tau}^{0} e^{\alpha_i\theta}x^T(t+\theta)\vartheta_i d\theta\\
&= -\alpha_i V_i(t) + \alpha_i x^T(t)\nu_i + \dot x^T(t)\nu_i + x^T(t)\upsilon_i - (1-\dot d(t))e^{-\alpha_i\tau}x^T(t-d(t))\upsilon_i\\
&+\tau x^T(t)\vartheta_i - \int_{t-\tau}^{t} e^{\alpha_i(s-t)}x^T(s)\vartheta_i ds\\
&\leqslant -\alpha_i V_i(t) + x^T(t)(\alpha_i\nu_i + A_i^T\nu_i + \upsilon_i + \tau\vartheta_i)\\
&+x^T(t-d(t))\left(G_i^T\nu_i - (1-d)e^{-\alpha_i\tau}\upsilon_i\right) - \int_{t-d(t)}^{t} e^{-\alpha_i\tau}x^T(s)\vartheta_i ds\\
&\leqslant -\alpha_i V_i(t) + x^T(t)(\alpha_i\nu_i + A_i^T\nu_i + \upsilon_i + \tau\vartheta_i)\\
&+x^T(t-d(t))\left(G_i^T\nu_i - (1-d)e^{-\alpha_i\tau}\upsilon_i\right)
\end{aligned}$$

It can be obtained from (5) and (6) that

$$\dot V_i(t) \leqslant -\alpha_i V_i(t) \tag{10}$$

It follows that

$$V_{\sigma(t)}(t) \leqslant e^{-\alpha_{\sigma(t)}(t-t_\kappa)}V_{\sigma(t)}(t_\kappa), \quad t\in[t_\kappa, t_{\kappa+1}) \tag{11}$$

From (7) and (8), one has

$$\begin{aligned}
V_{\sigma(T)}(T) &\leqslant e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)})}V_{\sigma(t_{N_\sigma(0,T)})}(t_{N_\sigma(0,T)}) \leqslant \mu_{\sigma(T)}e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)})}V_{\sigma(t^-_{N_{\sigma(0,T)}})}(t^-_{N_\sigma(0,T)})\\
&\leqslant \mu_{\sigma(T)}e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)})}e^{-\alpha_{\sigma(t_{N_\sigma(0,T)-1})}(t_{N_\sigma(0,T)}-t_{N_\sigma(0,T)-1})}V_{\sigma(t_{N_{\sigma(0,T)-1}})}(t_{N_\sigma(0,T)-1}) \leqslant \cdots\\
&\leqslant V_{\sigma(0)}(0)\prod_{s=0}^{N_\sigma(0,T)-1}\mu_{\sigma(t_{s+1})}\exp\left\{\sum_{s=0}^{N_\sigma(0,T)-1}\left[-\alpha_{\sigma(t_s)}(t_{s+1}-t_s)\right] - \alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)})\right\}\Bigg)\\
&\leqslant V_{\sigma(0)}(0)\prod_{i=1}^{N}\mu_i^{N_{\sigma i}(0,T)}\exp\left\{\sum_{i=1}^{N}\left[-\alpha_i\sum_{s\in\phi(i)}(t_{s+1}-t_s)\right] - \alpha_{\sigma(T)}(T-t_{N_{\sigma(0,T)}})\right\}\Bigg)\\
&\leqslant \exp\left\{\sum_{i=1}^{N}N_{0i}\ln\mu_i\right\}\exp\left\{\sum_{i=1}^{N}T_i(0,T)\ln\mu_i/T_{ai} - \sum_{i=1}^{N}\alpha_i T_i(0,T)\right\}V_{\sigma(0)}(0)\\
&= \exp\left\{\sum_{i=1}^{N}N_{0i}\ln\mu_i\right\}\exp\left\{\sum_{i=1}^{N}(\ln\mu_i/T_{ai}-\alpha_i)T_i(0,T)\right\}V_{\sigma(0)}(0),
\end{aligned} \tag{12}$$

where $\phi(i)$ denotes the set of s satisfying $\sigma(t_s) = i$, $t_s \in \{t_0, t_1, t_2, \ldots, t_{\kappa-1}, t_\kappa, t_{\kappa+1}, \ldots, t_{N_{\sigma(0,T)}}\}$, $\mu_{\sigma(t_{s+1})}$, $\mu_{\sigma(T)} \in \{\mu_1, \mu_2, \ldots, \mu_N\}$, $\alpha_{\sigma(T)} \in \{\alpha_1, \alpha_2, \ldots, \alpha_N\}$.

Set $\varepsilon_1 = \min_{(r,i)\in\underline{n}\times\underline{N}}\{\nu_{ir}\}$, $\varepsilon_2 = \max_{(r,i)\in\underline{n}\times\underline{N}}\{\nu_{ir}\}$, $\varepsilon_3 = \max_{(r,i)\in\underline{n}\times\underline{N}}\{\upsilon_{ir}\}$ and $\varepsilon_4 = \max_{(r,i)\in\underline{n}\times\underline{N}}\{\vartheta_{ir}\}$, where ν_{ir}, υ_{ir} and ϑ_{ir} represent the rth elements of ν_i, υ_i and ϑ_i, respectively, $\underline{n} = \{1, 2, \ldots, n\}$. Then, one obtains

$$V_{\sigma(T)}(T) \geqslant \varepsilon_1\|x(T)\|$$

$$V_{\sigma(t_0)}(t_0) \leqslant \varepsilon_2\|x(t_0)\| + (\varepsilon_3 e^{-\tau\alpha_{\sigma(t_0)}} + \varepsilon_4\tau e^{-\tau\alpha_{\sigma(t_0)}})\int_{t_0-\tau}^{t_0}\|x(s)\|ds$$

It follows that

$$
\begin{aligned}
\|x(T)\| &\leqslant \frac{1}{\varepsilon_1}\exp\left\{\sum_{i=1}^{N} N_{0i}\ln\mu_i\right\}\exp\left\{\sum_{i=1}^{N}(\ln\mu_i/T_{ai}-\alpha_i)T_i(0,T)\right\}\left(\varepsilon_2\|x(t_0)\| + (\varepsilon_3 e^{-\alpha_{\sigma(t_0)}\tau} + \varepsilon_4\tau e^{-\alpha_{\sigma(t_0)}\tau})\int_{t_0-\tau}^{t_0}\|x(s)\|ds\right)\\
&\leqslant \frac{1}{\varepsilon_1}\exp\left\{\sum_{i=1}^{N} N_{0i}\ln\mu_i\right\}(\varepsilon_2 + \varepsilon_3\tau e^{-\alpha_{\sigma(t_0)}\tau} + \varepsilon_4\tau^2 e^{-\alpha_{\sigma(t_0)}\tau})\exp\left\{\max_{i\in\underline{N}}(\ln\mu_i/T_{ai}-\alpha_i)(T-t_0)\right\}\sup_{-\tau\leqslant\theta\leqslant 0}\|\varphi(\theta)\|
\end{aligned}
$$

Set

$$
\zeta = \frac{1}{\varepsilon_1}\exp\left\{\sum_{i=1}^{N} N_{0i}\ln\mu_i\right\}(\varepsilon_2 + \varepsilon_3\tau e^{-\alpha_{\sigma(t_0)}\tau} + \varepsilon_4\tau^2 e^{-\alpha_{\sigma(t_0)}\tau}),
$$

$$
\rho = \min_{i\in\underline{N}}(\alpha_i - \ln\mu_i/T_{ai}).
$$

It can be obtained from (7) that

$$
\|x(T)\| \leqslant \zeta e^{-\rho(T-t_0)}\|x(t_0)\|_c, \forall T \geqslant t_0, \tag{13}
$$

This completes the proof. □

Remark 3. In Theorem 1, we get a delay-dependent stability criterion by utilizing the MDADT method instead of the ADT method [24]. In [24], the parameters λ and μ are same for all subsystems, i.e., mode-independent. However, the parameters α_i and μ_i in this paper are mode-dependent, which would reduce the conservativeness existed in [24].

When $d(t)\equiv 0$, the SPLS (1) will generate to a delay-free system and the result in Theorem 1 reduces to the one proposed in Theorem 1 of [36].

Based on Theorem 1, we will present a stability result for the SPLS (3) with $w(t)\equiv 0$ by considering a class of Lyapunov–Krasovskii functionals allowed to increase with bounded increase rate during some intervals.

Theorem 2. *Consider the SPLS (3) with $w(t)\equiv 0$. Let $\alpha_i > 0$ and $\beta_i > 0$ be given constants. If there exist vectors $v_i \succ 0$, $\upsilon_i \succ 0$, $\vartheta_i \succ 0$, $v_{i,j} \succ 0$, $\upsilon_{i,j} \succ 0$ and $\vartheta_{i,j} \succ 0$ of appropriate dimensions, such that, $\forall (i,j)\in \underline{N}\times\underline{N}$, $i\neq j$,*

$$
\overline{A}_i^T v_i + \alpha_i v_i + \upsilon_i + \tau\vartheta_i \preceq 0, \tag{14}
$$

$$
G_i^T v_i - (1-d)e^{-\alpha_i\tau}\upsilon_i \preceq 0, \tag{15}
$$

$$
\overline{A}_{i,j}^T v_{i,j} - \beta_i v_{i,j} + \upsilon_{i,j} + \tau\vartheta_{i,j} \preceq 0, \tag{16}
$$

$$
G_i^T v_{i,j} - (1-d)\upsilon_{i,j} \preceq 0, \tag{17}
$$

then the system is exponentially stable for any switching signal $\sigma(t)$ with the following MDADT scheme

$$
T_{aj} > T_{aj}^* = \left(\Delta_{mj}(\alpha_j+\beta_j) + \ln(\mu_{0j}\mu_{1j}\mu_{2j})\right)/\alpha_j, \tag{18}
$$

where Δ_{mj} denotes the maximal delay period that the switching of the controller of the jth subsystem lags behind that of the subsystem, $\mu_{0j} = \mu_j = e^{\tau(\alpha_j+\beta_j)}$ and $\mu_{1j}\mu_{2j} \geqslant 1$ satisfy

$$
v_j \preceq \mu_{1j}v_{i,j}, \upsilon_j \preceq \mu_{1j}\upsilon_{i,j}, \vartheta_j \preceq \mu_{1j}\vartheta_{i,j}, \quad v_{i,j} \preceq \mu_{2j}\mu_{0j}v_i, \upsilon_{i,j} \preceq \mu_{2j}\upsilon_i, \vartheta_{i,j} \preceq \mu_{2j}\vartheta_i \tag{19}
$$

Proof. For any $T > 0$, let $t_0 = 0$ and denote by $t_1, t_2, \ldots, t_{\kappa-1}, t_\kappa, \ldots, t_{N_\sigma(0,T)}$ the switching instants in the interval $[t_0,T)$, where $N_\sigma(0,T) = \sum_{i=1}^{N} N_{\sigma i}(0,T)$. Let $T_i(0,T)$ be the total running time of the ith subsystem over the interval $[0,T)$.

Let the ith subsystem be activated at $t_{\kappa-1}$ and the jth subsystem be activated at t_κ, $(i,j)\in\underline{N}\times\underline{N}$, $i\neq j$. Construct the following Lyapunov–Krasovskii functional for the SPLS (3):

$$
V(t) = \begin{cases} x^T(t)v_i + \int_{t-d(t)}^{t} e^{\alpha_i(-t+s)}x^T(s)\upsilon_i ds + \int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\alpha_i(-t+s)}x^T(s)\vartheta_i ds d\theta, & \forall t\in[t_{\kappa-1}+\Delta_{\kappa-1}, t_\kappa) \\ x^T(t)v_{i,j} + \int_{t-d(t)}^{t} e^{\beta_j(t-s)}x^T(s)\upsilon_{i,j} ds + \int_{-\tau}^{0}\int_{t+\theta}^{t} e^{\beta_j(t-s)}x^T(s)\vartheta_{i,j} ds d\theta, & \forall t\in[t_\kappa, t_\kappa+\Delta_\kappa) \end{cases} \tag{20}
$$

When $w(t)\equiv 0$, by Theorem 1, we obtain from (14)–(17) that

$$
\dot V(t) \leqslant \begin{cases} -\alpha_i V(t), & \forall t\in[t_{\kappa-1}+\Delta_{\kappa-1}, t_\kappa) \\ \beta_i V(t), & \forall t\in[t_{\kappa-1}, t_{\kappa-1}+\Delta_{\kappa-1}) \end{cases} \tag{21}
$$

From (19) and (20), at the instants t_κ and $t_\kappa + \Delta_\kappa$, we have

$$
V(t_\kappa+\Delta_\kappa) \leqslant \mu_{1j}V((t_\kappa+\Delta_\kappa)^-), \quad V(t_\kappa) \leqslant \mu_{2j}\mu_j V(t_\kappa^-)
$$

For $T \geqslant t_{N_\sigma(0,T)} + \Delta_{N_\sigma(0,T)}$, we obtain by induction that

$$
\begin{aligned}
V(T) \leqslant & e^{-\alpha_{\sigma(T)}(T - t_{N_\sigma(0,T)} - \Delta_{N_\sigma(0,T)})} V(t_{N_{\sigma(0,T)}} + \Delta_{N_\sigma(0,T)}) \\
\leqslant & \mu_{\sigma(T)1} e^{-\alpha_{\sigma(T)}(T - t_{N_\sigma(0,T)} - \Delta_{N_\sigma(0,T)})} V((t_{N_\sigma(0,T)} + \Delta_{N_\sigma(0,T)})^-) \\
\leqslant & \mu_{\sigma(T)1} e^{-\alpha_{\sigma(T)}(T - t_{N_\sigma(0,T)} - \Delta_{N_\sigma(0,T)})} e^{\beta_{\sigma(T)}\Delta_{N_\sigma(0,T)}} V(t_{N_\sigma(0,T)}) \\
\leqslant & \mu_{\sigma(T)}\mu_{\sigma(T)1}\mu_{\sigma(T)2} e^{-\alpha_{\sigma(T)}(T - t_{N_\sigma(0,T)} - \Delta_{N_\sigma(0,T)})} e^{\beta_{\sigma(T)}\Delta_{N_\sigma(0,T)}} V(t^-_{N_\sigma(0,T)}) \\
\leqslant & \mu_{\sigma(T)}\mu_{\sigma(T)1}\mu_{\sigma(T)2} e^{-\alpha_{\sigma(T)}(T - t_{N_\sigma(0,T)} - \Delta_{m\sigma(T)})} e^{\beta_{\sigma(T)}\Delta_{m\sigma(T)}} V\left(t^-_{N_\sigma(0,T)}\right) \\
\leqslant & \cdots \\
\leqslant & \exp\left\{\sum_{i=1}^{N} \quad -\alpha_i \sum_{s\in\phi(i)} (t_{s+1} - t_s - \Delta_{mi}) + \beta_i \sum_{s\in\phi(i)} \Delta_{mi}\right)\Bigg\} \\
& V(t_0)\prod_{i=1}^{N} (\mu_i\mu_{i1}\mu_{i2})^{N_{\sigma i}(0,T)} \\
\leqslant & \exp\left\{\sum_{i=1}^{N} N_{0i} \ln(\mu_i\mu_{i1}\mu_{i2})\right\} \\
& \exp\Bigg\{\sum_{i=1}^{N} \ln(\mu_i\mu_{i1}\mu_{i2}) T_i(0,T)/T_{ai} \\
& + \sum_{i=1}^{N} (-\alpha_i(T_i(0,T) - \Delta_{mi}N_{\sigma i}(0,T)) + \beta_i\Delta_{mi}N_{\sigma i}(0,T))\Bigg\} V(t_0) \\
\leqslant & \exp\left\{\sum_{i=1}^{N} N_{0i}(\ln(\mu_i\mu_{i1}\mu_{i2}) + (\alpha_i + \beta_i)\Delta_{mi})\right\} \\
& \exp\left\{\sum_{i=1}^{N} (\ln(\mu_i\mu_{i1}\mu_{i2})/T_{ai} - \alpha_i + (\alpha_i + \beta_i)\Delta_{mi}/T_{ai}) T_i(0,T)\right\} V(t_0)
\end{aligned}
$$

where $\phi(i)$ denotes the set of s satisfying $\sigma\ (t_s) = i$, $t_s \in \{t_0, t_1, t_2, \ldots, t_{\kappa-1}, t_\kappa, t_{\kappa+1}, \ldots, t_{N_{\sigma(0,T)}}\}$, $\Delta_{m\sigma(T)} \in \{\Delta_{m1}, \Delta_{m2}, \ldots, \Delta_{mN}\}$, $\mu_{\sigma(T)} \in \{\mu_1, \mu_2, \ldots, \mu_N\}$, $\mu_{\sigma(T)1} \in \{\mu_{11}, \mu_{21}, \ldots, \mu_{N1}\}$, $\mu_{\sigma(T)2} \in \{\mu_{12}, \mu_{22}, \ldots, \mu_{N2}\}$, $\alpha_{\sigma(T)} \in \{\alpha_1, \alpha_2, \ldots, \alpha_N\}$ and $\beta_{\sigma(T)} \in \{\beta_1, \beta_2, \ldots, \beta_N\}$.

It follows from (18) that $V(T)$ converges to zero as $T \to \infty$. Then the exponential stability of the SPLS (3) with $w(t) \equiv 0$ can be deduced by following the proof line of Theorem 1.

The proof is completed. □

Remark 4. The proof of Theorem 2 is similar to the one of Theorem 1. Note that the Lyapunov–Krasovskii functional considered in Theorem 2 can be increasing both at switching instants and during the interval $[t_{\kappa-1}, t_{\kappa-1} + \Delta_{\kappa-1})$. However, the possible increment will be compensated by the more specific decrement (by limiting the lower bound of MDADT), therefore, the system exponential stability is still guaranteed.

Now, we are in a position to consider the L_1-gain analysis for the SPLS (3).

Theorem 3. *Consider the SPLS (3). Let $\alpha_i > 0$, $\beta_i > 0$ and $\gamma > 0$ be given constants. If there exist vectors $\nu_i \succ 0$, $\upsilon_i \succ 0$, $\vartheta_i \succ 0$, $\nu_{i,j} \succ 0$, $\upsilon_{i,j} \succ 0$ and $\vartheta_{i,j} \succ 0$ of appropriate dimensions, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \neq j$, such that*

$$\overline{A}_i^T \nu_i + \alpha_i \nu_i + \upsilon_i + \tau\vartheta_i + C_i^T 1_q \preceq 0, \tag{22}$$

$$\overline{A}_{i,j}^T \nu_{i,j} - \beta_i \nu_{i,j} + \upsilon_{i,j} + \tau\vartheta_{i,j} + C_j^T 1_q \preceq 0, \tag{23}$$

$$G_i^T \nu_i - (1-d)e^{-\alpha_i\tau}\upsilon_i \preceq 0, \tag{24}$$

$$G_j^T \nu_{i,j} - (1-d)\upsilon_{i,j} \preceq 0, \tag{25}$$

$$E_i^T \nu_i + D_i^T 1_q - \gamma 1_f \preceq 0, \tag{26}$$

$$E_j^T \nu_{i,j} + D_j^T 1_q - \gamma 1_f \preceq 0, \tag{27}$$

then the system is exponentially stable and has a prescribed L_1-gain performance level γ for any switching signal with the MDADT scheme (18), where μ_{0j}, μ_{1j} and μ_{2j} satisfy (19).

Proof. Choose the Lyapunov–Krasovskii functional (20) for the SPLS (3). By Theorem 2, the exponential stability of the SPLS (3) with $w(t) \equiv 0$ is ensured by (22)–(25).

We are now in a position to consider the L_1-gain performance.

Define $\Gamma(t) = \|z(t)\| - \gamma\|w(t)\|$. When $w(t)\neq 0$, it follows from (22)–(27) that

$$\dot{V}(t) \leqslant \begin{cases} -\alpha_i V(t) - \Gamma(t), & \forall t \in [t_{\kappa-1} + \Delta_{\kappa-1}, t_\kappa) \\ \beta_i V(t) - \Gamma(t), & \forall t \in [t_{\kappa-1}, t_{\kappa-1} + \Delta_{\kappa-1}) \end{cases} \tag{28}$$

Then, integrating both sides of (28), we have

$$V(t) \leqslant \begin{cases} e^{-\alpha_i(t-t_{\kappa-1}-\Delta_{\kappa-1})} V(t_{\kappa-1} + \Delta_{\kappa-1}) - \int_{t_{\kappa-1}+\Delta_{\kappa-1}}^{t} e^{-\alpha_i(t-s)}\Gamma(s)ds, & \forall t \in [t_{\kappa-1} + \Delta_{\kappa-1}, t_\kappa) \\ e^{\beta_i(t-t_{\kappa-1})} V(t_{\kappa-1}) - \int_{t_{\kappa-1}}^{t} e^{\beta_i(t-s)}\Gamma(s)ds, & \forall t \in [t_{\kappa-1}, t_{\kappa-1} + \Delta_{\kappa-1}) \end{cases} \tag{29}$$

where $\Gamma(s) = \|z(s)\| - \gamma\|w(s)\|$.

For $T \geqslant t_{N_\sigma(0,T)} + \Delta_{N_\sigma(0,T)}$, by Definition 4, (19) and (29), we can obtain by induction that

$$\begin{aligned}
V(T) \leqslant\, & e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)}-\Delta_{N_\sigma(0,T)})} V(t_{N_\sigma(0,T)} + \Delta_{N_\sigma(0,T)}) \\
& - \int_{t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}}^{T} e^{-\alpha_{\sigma(T)}(T-s)}\Gamma(s)ds \\
\leqslant\, & \mu_{\sigma(T)1} e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)}-\Delta_{N_\sigma(0,T)})} V((t_{N_\sigma(0,T)} + \Delta_{N_\sigma(0,T)})^-) \\
& - \int_{t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}}^{T} e^{-\alpha_{\sigma(T)}(T-s)}\Gamma(s)ds \\
\leqslant\, & \mu_{\sigma(T)1} e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)}-\Delta_{N_\sigma(0,T)})} \{ e^{\beta_{\sigma(T)}\Delta_{N_\sigma(0,T)}} V(t_{N_\sigma(0,T)}) \\
& - \int_{t_{N_\sigma(0,T)}}^{t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}} e^{\beta_{\sigma(T)}(t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}-s)}\Gamma(s)ds \} \\
& - \int_{t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}}^{T} e^{-\alpha_{\sigma(T)}(T-s)}\Gamma(s)ds \\
\leqslant\, & \mu_{\sigma(T)}\mu_{\sigma(T)1}\mu_{\sigma(T)2} e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)}-\Delta_{N_\sigma(0,T)})} e^{\beta_{\sigma(T)}\Delta_{N_\sigma(0,T)}} V(t^-_{N_\sigma(0,T)}) \\
& - \mu_{\sigma(T)1} e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)}-\Delta_{N_\sigma(0,T)})} \int_{t_{N_\sigma(0,T)}}^{t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}} e^{\beta_{\sigma(T)}(t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}-s)}\Gamma(s)ds \\
& - \int_{t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}}^{T} e^{-\alpha_{\sigma(T)}(T-s)}\Gamma(s)ds \\
\leqslant\, & \mu_{\sigma(T)}\mu_{\sigma(T)1}\mu_{\sigma(T)2} e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)}-\Delta_{m\sigma(T)})} e^{\beta_{\sigma(T)}\Delta_{m\sigma(T)}} V\left(t^-_{N_\sigma(0,T)}\right) \\
& - \int_{t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}}^{T} e^{-\alpha_{\sigma(T)}(T-s)}\Gamma(s)ds \\
& - \mu_{\sigma(T)1} e^{-\alpha_{\sigma(T)}(T-t_{N_\sigma(0,T)}-\Delta_{N_\sigma(0,T)})} \int_{t_{N_\sigma(0,T)}}^{t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}} e^{\beta_{\sigma(T)}(t_{N_\sigma(0,T)}+\Delta_{N_\sigma(0,T)}-s)}\Gamma(s)ds \} \\
\leqslant\, & \cdots \\
\leqslant\, & \exp\left\{\sum_{i=1}^{N} N_{0i} \ln(\mu_i\mu_{i1}\mu_{i2})\right\} \\
& \exp\left\{\sum_{i=1}^{N} \ln(\mu_i\mu_{i1}\mu_{i2}) T_i(0,T)/T_{ai}\right. \\
& \left. + \sum_{i=1}^{N} (-\alpha_i(T_i(0,T) - N_{\sigma i}(0,T)\Delta_{mi}) + \beta_i N_{\sigma i}(0,T)\Delta_{mi})\right\} V(t_0) \\
& - \int_{t_0}^{T} e^{\left\{\sum_{i=1}^{N} -\alpha_i(T_i(s,T)-N_{\sigma i}(s,T)\Delta_{mi})+\beta_i N_{\sigma i}(s,T)\Delta_{mi}\right\}} \Gamma(s) \prod_{i=1}^{N} (\mu_i\mu_{i1}\mu_{i2})^{N_{\sigma i}(s,T)} ds
\end{aligned}$$

where $\phi(i)$ denotes the set of s satisfying$\sigma(t_s) = i$, $t_s \in \{t_0, t_1, t_2, \ldots, t_{\kappa-1}, t_\kappa, t_{\kappa+1}, \ldots, t_{N_{\sigma(0,T)}}\}$, $\Delta_{m\sigma(T)} \in \{\Delta_{m1}, \Delta_{m2}, \ldots, \Delta_{mN}\}$, $\mu_{\sigma(T)} \in \{\mu_1, \mu_2, \ldots, \mu_N\}$, $\mu_{\sigma(T)1} \in \{\mu_{11}, \mu_{21}, \ldots, \mu_{N1}\}$, $\mu_{\sigma(T)2} \in \{\mu_{12}, \mu_{22}, \ldots, \mu_{N2}\}$, $\alpha_{\sigma(T)} \in \{\alpha_1, \alpha_2, \ldots, \alpha_N\}$ and $\beta_{\sigma(T)} \in \{\beta_1, \beta_2, \ldots, \beta_N\}$.

Under the zero initial condition, one has

$$0 \leqslant -\int_{t_0}^{T} e^{\left\{\sum_{i=1}^{N} -\alpha_i(T_i(s,T)-N_{\sigma i}(s,T)\Delta_{mi})+\beta_i N_{\sigma i}(s,T)\Delta_{mi}\right\}} \Gamma(s) \prod_{i=1}^{N} (\mu_i\mu_{i1}\mu_{i2})^{N_{\sigma i}(s,T)} \tag{30}$$

That is

$$\int_{t_0}^{T} e^{\left\{\sum_{i=1}^{N}-\alpha_i(T_i(s,T)-N_{\sigma i}(s,T)\Delta_{mi})+\beta_i N_{\sigma i}(s,T)\Delta_{mi}\right\}}\|z(s)\|\prod_{i=1}^{N}(\mu_i\mu_{i1}\mu_{i2})^{N_{\sigma i}(s,T)}ds$$
$$\leqslant \gamma\int_{t_0}^{T} e^{\left\{\sum_{i=1}^{N}-\alpha_i(T_i(s,T)-N_{\sigma i}(s,T))+\beta_i N_{\sigma i}(s,T)\right\}}\|w(s)\|\prod_{i=1}^{N}(\mu_i\mu_{i1}\mu_{i2})^{N_{\sigma i}(s,T)}ds \quad (31)$$

Multiplying both sides of (31) by $e^{-\sum_{i=1}^{N}(\alpha_i+\beta_i)N_{\sigma i}(t_0,T)\Delta_{mi}}\prod_{i=1}^{N}(\mu_i\mu_{i1}\mu_{i2})^{-N_{\sigma i}(t_0,T)}$ yields

$$\int_{t_0}^{T} e^{\sum_{i=1}^{N}\{-\alpha_i T_i(s,T)-(\alpha_i+\beta_i)N_{\sigma i}(t_0,s)\Delta_{mi}-N_{\sigma i}(t_0,s)\ln(\mu_i\mu_{i1}\mu_{i2})\}}\|z(s)\|ds \leqslant \gamma\int_{t_0}^{T} e^{\sum_{i=1}^{N}\{-\alpha_i T_i(s,T)-(\alpha_i+\beta_i)N_{\sigma i}(t_0,s)\Delta_{mi}\}}\|w(s)\|ds \quad (32)$$

It follows from (18) that

$$\int_{t_0}^{T} e^{\sum_{i=1}^{N}\{-\alpha_i T_i(s,T)-((\alpha_i+\beta_i)\Delta_{mi}+\ln(\mu_i\mu_{i1}\mu_{i2}))T_i(t_0,s)/T_{ai}\}}\|z(s)\|ds \leqslant \gamma\int_{t_0}^{T} e^{\sum_{i=1}^{N}-\alpha_i T_i(s,T)}\|w(s)\|ds \quad (33)$$

Integrating both sides of (33) from $T = t_0$ to ∞ leads to

$$\int_{t_0}^{\infty} e^{-\sum_{i=1}^{N}\{\alpha_i T_i(t_0,t)\}}\|z(t)\|dt \leqslant \gamma\int_{t_0}^{\infty}\|w(t)\|dt \quad (34)$$

This means that system (3) achieves a prescribed L_1-gain performance level γ.

This completes the proof. □

3.2. Asynchronous L_1 control

Based on the obtained stability and L_1-gain analysis results, the following theorem presents sufficient conditions for the existence of a state-feedback controller for the SPLS (1) in the presence of asynchronous switching such that the corresponding closed-loop system (3) is positive and exponentially stable with an L_1-gain performance level γ.

Theorem 4. *Consider the SPLS (1). Let $\alpha_i > 0$, $\beta_i > 0$ and $\gamma > 0$ be given constants. If there exist vectors $\nu_i \succ 0$, $\upsilon_i \succ 0$, $\vartheta_i \succ 0$, $\nu_{i,j} \succ 0$, $\upsilon_{i,j} \succ 0$, $\vartheta_{i,j} \succ 0$, and any matrices K_i of appropriate dimensions, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \neq j$, such that $\overline{A}_i = A_i + B_i K_i$ and $\overline{A}_{i,j} = A_j + B_j K_i$ are Metzler matrices, and (24)–(27) and the following inequalities hold*

$$A_i^T\nu_i + h_i + \alpha_i\nu_i + \upsilon_i + \tau\vartheta_i + C_i^T 1_q \preceq 0, \quad (35)$$
$$A_{i,j}^T\nu_{i,j} + K_i^T B_j^T\nu_{i,j} - \beta_i\nu_{i,j} + \upsilon_{i,j} + \tau\vartheta_{i,j} + C_j^T 1_q \preceq 0, \quad (36)$$

where $h_i \succeq K_i^T B_i^T \nu_i$, then the resulting closed-loop system (3) is positive and exponentially stable with an L_1-gain performance level γ for any switching signal with the MDADT scheme (18), where μ_{0j}, μ_{1j} and μ_{2j} satisfy (19).

Proof. Upon introducing vectors h_i satisfying $h_i \succeq K_i^T B_i^T \nu_i$, and substituting them into (22), the theorem can be directly obtained from Theorem 3.

This completes the proof. □

Remark 5. Differently from the result in [35], we get sufficient conditions for the existence of an L_1-gain performance level. Also, the result proposed in Theorem 4 is derived via the MDADT approach, which is different from those adopted in [33–35]. The parameters α_i and β_i are mode-dependent, which brings more flexibility to find feasible controllers. On the other hand, our result can cover the result of [24] as a special case, where the asynchronous switching is not considered.

Remark 6. It is noticed that (24), (25), (26), (27), (35) and (36) are mutually dependent. We can firstly solve (24), (26) and (35) to obtain the vectors ν_i, υ_i, ϑ_i, h_i. Then we can get K_i by $h_i \succeq K_i^T B_i^T \nu_i$. By substituting the obtained K_i into (36), and solving (25), (27) and (36), we can obtain these vectors $\nu_{i,j}$, $\upsilon_{i,j}$, $\vartheta_{i,j}$. In addition, it can be seen that a smaller α_i will be favorable to the feasibility of (24), (26) and (35), and a larger β_i will be favorable to the feasibility of (25), (27) and (36). In view of these, we put forward the following algorithm to obtain K_i.

Algorithm 1.

Step (1) For each $i \in \underline{N}$, choose a α_i (For the first time, we can choose a larger α_i), and solve (24), (26) and (35).
Step (2) If (24), (26) and (35) are unfeasible, then decrease α_i appropriately, and go to Step (1).

Step (3) If there exists a feasible solution, then get v_i, υ_i, ϑ_i, h_i. By $h_i \succeq K_i^T B_i^T v_i$, find a K_i such that $\overline{A}_i = A_i + B_iK_i$ and $\overline{A}_{i,j} = A_j + B_jK_i$ are Metzler matrices, and then substitute it into (36).
Step (4) Choose a β_i (For the first time, we can choose a smaller β_i), and solve (25), (27) and (36).
Step (5) If (25), (27) and (36) are unfeasible, then increase β_i appropriately, and go to Step (4).
Step (6) If there exists a feasible solution, then get $v_{i,j}$, $\upsilon_{i,j}$, $\vartheta_{i,j}$, and compute T^*_{aj} by (18) and (19).

4. Numerical examples

In this section, two examples will be presented to demonstrate the potential and validity of our developed theoretical results.

Example 1. Consider the switched linear systems consisting of two positive subsystems described by:

$$A_1 = \begin{bmatrix} -0.5302 & 0.0012 & 0.0873 \\ 0.2185 & -0.7494 & 0.5411 \\ 0.7370 & 0.1543 & -0.3606 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5136 & 0.4419 & 0.3689 \\ 0.1840 & -0.3951 & 0.0080 \\ 0.3163 & 0.6099 & -1.0056 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.01 & 0.001 & 0 \\ 0 & 0.01 & 0.1 \\ 0.05 & 0 & 0.01 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.012 & 0 & 0 \\ 0.014 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}.$$

It is obvious that the trajectories of such a switched system will remain positive if $x(0) \succeq 0$. Our purpose here is to find the admissible switching signals with MDADT such that the system is exponentially stable.

To illustrate the advantages of the proposed MDADT switching, we shall also present the design results of switching signals for the system with ADT switching for the sake of comparison. By different approaches and setting the relevant parameters appropriately, the computation results for the system with two different switching schemes are listed in Table 1.

It can be seen from Table 1 that the minimal MDADT are reduced to $T^*_{a1} = 6.8663, T^*_{a2} = 7.8472$ for given $\mu_1 = \mu_2 = 3$, and one special case of MDADT switching is $T^*_{a1} = T^*_{a2} = 7.8472$ by setting $\alpha_1 = \alpha_2 = 0.14$, which is the ADT switching, i.e., the designed MDADT switching is more general.

Example 2. Consider system (1) with parameters as follows

$$A_1 = \begin{bmatrix} -1 & 7 \\ 8.5 & -2.5 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix},$$

$$C_1 = [0.1 \quad 0.3], \quad D_1 = 0.3,$$

$$A_2 = \begin{bmatrix} -6.8 & 3.5 \\ 9.3 & -6.6 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix},$$

$$C_2 = [0.2 \quad 0.4], \quad D_2 = 0.2.$$

By Lemma 1, the trajectories of such a switched system will obviously remain positive if $\varphi(\theta) \succeq 0, \theta \in [-\tau, 0]$. Our purpose here is to design a set of stabilizing controllers and find the admissible switching signals with MDADT such that the resulting closed-loop system is exponentially stable with an L_1 disturbance attenuation performance level in the presence of asynchronous switching.

Taking $\Delta_{m1} = 1.0$, $\Delta_{m2} = 0.5$, $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, $\tau = 0.1$, $d = 0.1$ and $\gamma = 1$, and solving (24), (26) and (35) in Theorem 4 give rise to

Table 1
Computation results for the system with two different switching schemes

	ADT switching [38]	MDADT switching
Feasible solutions	$v_1 = [70.6769 \quad 26.0675 \quad 27.0391]^T$, $v_2 = [24.3130 \quad 75.9861 \quad 11.9570]^T$, $\upsilon_1 = [0.6826 \quad 1.2278 \quad 0.6271]^T$, $\upsilon_2 = [0.4213 \quad 0.6719 \quad 0.3204]^T$, $\vartheta_1 = [7.1928 \quad 10.0483 \quad 6.9808]^T$, $\vartheta_2 = [5.0049 \quad 5.5576 \quad 3.9734]^T$	$v_1 = [64.7231 \quad 22.8869 \quad 24.0552]^T$, $v_2 = [21.8226 \quad 67.5209 \quad 10.5841]^T$, $\upsilon_1 = [0.4451 \quad 1.0511 \quad 0.3853]^T$, $\upsilon_2 = [0.4103 \quad 0.5137 \quad 0.2134]^T$, $\vartheta_1 = [5.3587 \quad 9.0788 \quad 5.1133]^T$, $\vartheta_2 = [4.9672 \quad 4.7610 \quad 3.1748]^T$
Switching parameters	$\lambda = 0.14, \mu = 3$, $\tau^*_a = 7.8472$	$\alpha_1 = 0.16, \quad \alpha_2 = 0.14, \quad \mu_1 = \mu_2 = 3$ $T^*_{a1} = 6.8663, T^*_{a2} = 7.8472$

$$
\nu_1 = \begin{bmatrix} 0.5757 \\ 0.7741 \end{bmatrix}, \quad \nu_2 = \begin{bmatrix} 0.7487 \\ 0.5872 \end{bmatrix}, \quad \upsilon_1 = \begin{bmatrix} 1.2302 \\ 1.1821 \end{bmatrix}, \quad \upsilon_2 = \begin{bmatrix} 1.1858 \\ 1.2650 \end{bmatrix},
$$

$$
\vartheta_1 = \begin{bmatrix} 1.0186 \\ 1.0186 \end{bmatrix}, \quad \vartheta_2 = \begin{bmatrix} 1.0186 \\ 1.0186 \end{bmatrix}, \quad h_1 = \begin{bmatrix} -8.6846 \\ -5.0069 \end{bmatrix}, \quad h_2 = \begin{bmatrix} -3.1003 \\ -1.7068 \end{bmatrix}.
$$

By $h_i \succeq K_i^T B_i^T \nu_i$, the state feedback gain matrices can be obtained as follows:

$$
K_1 = [-20.4460 \quad -11.7876], \quad K_2 = [-16.1223 \quad -8.8759].
$$

Obviously, $\overline{A}_i = A_i + B_i K_i$ and $\overline{A}_{i,j} = A_j + B_j K_i$ are Metzler matrices. Then, choosing $\beta_1 = 0.5$ and $\beta_2 = 0.6$, and solving (25), (27) and (36), we obtain

$$
\nu_{2,1} = \begin{bmatrix} 0.7916 \\ 0.9499 \end{bmatrix}, \quad \nu_{1,2} = \begin{bmatrix} 0.4309 \\ 0.3613 \end{bmatrix}, \quad \upsilon_{2,1} = \begin{bmatrix} 0.9653 \\ 0.9630 \end{bmatrix}, \quad \upsilon_{1,2} = \begin{bmatrix} 0.9921 \\ 1.0477 \end{bmatrix},
$$

$$
\vartheta_{2,1} = \begin{bmatrix} 0.8413 \\ 0.8509 \end{bmatrix}, \quad \vartheta_{1,2} = \begin{bmatrix} 0.8680 \\ 0.8689 \end{bmatrix}.
$$

Then, according to (19), we have $\mu_{11} = 1.2744$, $\mu_{21} = 1.7375$, $\mu_{01} = 1.0942$, $\mu_{12} = 1.6187$, $\mu_{22} = 0.8863$ and $\mu_{02} = 1.0942$. From (18), it can be obtained that $T_{a1}^* = 4.4623$ and $T_{a2}^* = 3.0031$. Choosing $T_{a1} = 4.5$ and $T_{a2} = 3.1$, simulation results of the closed-loop systems are shown in Figs. 1 and 2, where the initial conditions are $x(0) = [0.1 \quad 0.2]^T$, and $x(t) = [0 \quad 0]^T, t \in [-0.1 \quad 0.$ It can be seen that the closed-loop system is positive and exponentially stable, which indicates that the proposed method is effective.

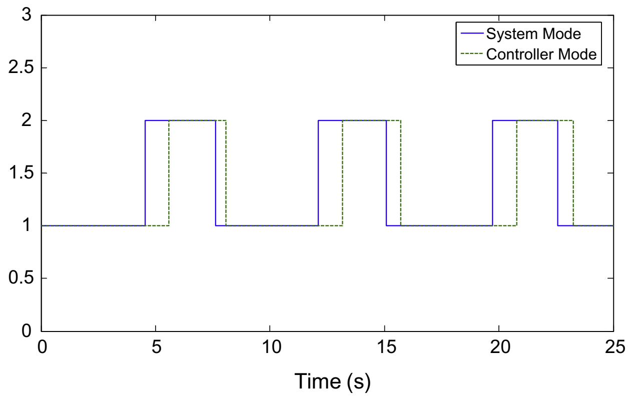

Fig. 1. Switching signal.

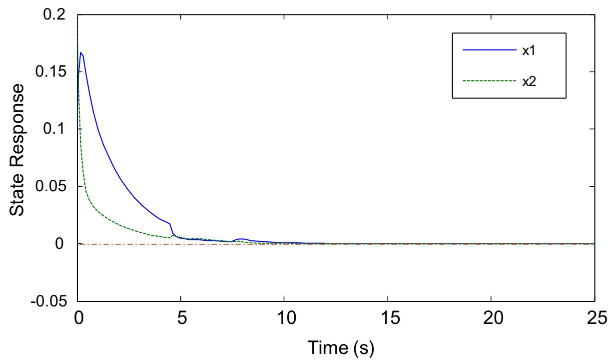

Fig. 2. State responses of the closed-loop system.

5. Conclusions

Using the MDADT approach, the stabilization problem of positive switched systems with time-varying delays under asynchronous switching has been investigated in this paper. We have designed a feedback controller and a class of switching signals such that the closed-loop system is exponentially stable and has a prescribed L_1-gain performance in presence of asynchronous switching. Our future work will focus on the L_1 fault detection observer design for positive switched systems under asynchronous switching.

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