Fuzzy Sliding Mode Control Design for A Class of Disturbed Systems

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Abstract

This paper discusses the problem of the fuzzy sliding mode control for a class of disturbed systems. First, a fuzzy auxiliary controller is constructed based on a feedback signal not only to estimate the unknown control term, but also participates in the sliding mode control due to the fuzzy rule employed. Then, we extend our theory into the cases, where some kind of system information cannot be obtained, for better use of our theoretical results in real engineering. Finally, some typical numerical examples are included to demonstrate the effectiveness and advantage of the designed sliding mode controller.

Keywords: Fuzzy control, sliding mode control, disturbance

1. Introduction

Disturbance exists in real systems widely, which makes the corresponding control problem much more complicated. To solve this problem, the sliding mode control (SMC) provides a good solution. Up to now, many relevant researches have been carried out [1-13]. Based on SMC, the authors in [2] investigated the robust adaptive control problem for fuzzy systems with mismatched uncertainties. By using a high-gain observer, an output feedback model-reference variable structure controller is presented in [3] to achieve the global exponential stability with respect to a small residual set without generating peaking in the control signal. In [4], the subordinated reachability of the sliding motion is introduced to realize the control on a class of uncertain stochastic systems with time-varying delays. Through introducing a pseudo-inversion, the authors in [5] discussed the adaptive control for the uncertain discrete time linear systems preceded by hysteresis nonlinearity. In [6], a sufficient condition for existence of reduced-order sliding mode dynamics was derived to realize the SMC for a continuous-time switched stochastic system.

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For SMC, the insensitivity to disturbances is due to its switching action between the different sliding mode surfaces. Conventionally, the general SMC method is based on the upper norm bound of the time-varying disturbance. The difference between the time-varying disturbance and its upper norm bound may lead to the serious chattering problem. The most effective solution is to obtain the precise estimation of the time-varying disturbance. Nowadays there are some feasible tools such as neural networks, genetic algorithm, wavelet series and piecewise polynomials, can be used to solve this problem. However, the balance among the accuracy of the estimation, the amount of the calculation, and the complexity of the control system is always a big concern. How to design a controller for the disturbed system with good overall performance is a very challenging topic, and this motivates our research.

Since the pioneering work from Zadeh in 1965 [14], fuzzy science has received more and more attentions, corresponding researches can be seen in [15-29] and the references therein. Especially in 1992, Wang proved that fuzzy systems are universal approximators [30], fuzzy method soon became a powerful approximation tool and has wide application areas, SMC is one of them.

In this paper, an auxiliary fuzzy controller based on one feedback signal will be built to not only estimate the unknown disturbances, but also participate in SMC, features simple structure and high efficiency; then, we will extend our theory into the cases that some kind of system information is not accessible, to make our researches possess more practical engineering value; these above are the main contributions of the paper. At last, some typical simulation examples will be included to demonstrate the effectiveness of the designed controllers.

Notations used in this paper are fairly standard. Let R^n be the n-dimensional Euclidean space, $R^{n\times m}$ represents the set of $n\times m$ real matrix, * denotes the elements below the main diagonal of a symmetric block matrix, $(\cdot)^{(i)}$ denotes the *i*th derivative of $(\cdot)$, and the notation $A>0$ means that *A* is the real symmetric and positive definite.

2. **Problem Statement**

In this paper, the following disturbed system is considered

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t), \quad i<n \\ \dot{x}_n(t) &= f(x,t)+\Delta f(x,t)+g(t)+\Delta u(t)+b\cdot u(t) \end{aligned} \tag{1}$$

where $x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T\in R^n$ is the system state vector, $f(\cdot,t)$ is the nonlinear function, $\Delta f(\cdot,t)$ is the system parameter uncertainty, $g(t)$ is the system disturbance, $\Delta u(t)$ is the control parameter uncertainty, b is the nonzero control coefficient, and $u(t)$ is the control input.

Define the tracking error

$$E(t) = x(t) - x_r(t) \tag{2}$$

where $E = \left(e_1(t), e_2(t), \cdots e_n(t)\right)^T \in R^n$, $x_r(t) = \left(x_{r1}(t), x_{r2}(t), \cdots x_{rn}(t)\right)^T \in R^n$, and $x_{r1}(t)$ is the desired trajectory with

$$x_{ri}(t) = x_{r1}{}^{(i-1)}(t), \quad i \le n \tag{3}$$

where $(\cdot)^{(i-1)}$ denotes the $(i-1)$th derivative of $(\cdot)$.

Then the error dynamic system can be expressed by

$$\begin{aligned} \dot{e}_i(t) &= e_{i+1}(t) \\ \dot{e}_n(t) &= f(x,t) - x_{rn}(t) + p(t) + b \cdot u(t) \end{aligned} \tag{4}$$

with

$$p(t) = \Delta f(x,t) + g(t) + \Delta u(t) \tag{5}$$

The problem to be addressed in this paper is to design a controller such that the tracking error variable satisfies

$$\lim_{t\to\infty} \|E(t)\| = \lim_{t\to\infty} \|x(t) - x_r(t)\| \to 0 \tag{6}$$

Let us recall the following results which will be used throughout the paper.

Lemma 1[31]: If $w(t): R \to R$ is a uniformly continuous function for $t \ge 0$ and if $\lim_{t\to\infty} \int_0^t w(t)dt$ exists and is finite, then

$$\lim_{t\to\infty} w(t) \to 0 \tag{7}$$

Lemma 2. [32]: If there exists a positive scalar λ, satisfy

$$\dot{s}(\mathrm{t}) + \lambda s(\mathrm{t}) = 0 \tag{8}$$

Then, this dynamic system is exponential stable.

3. Design of fuzzy sliding mode controller

In this section, the fuzzy SMC method is introduced to realize the tracking control for the disturbed system.

First, the following sliding surface is introduced

$$s(\mathrm{t}) = C^T E(t) = e_n(\mathrm{t}) + \sum_{i=1}^{n-1} c_i e_i(\mathrm{t}) \tag{9}$$

where $C = [c_1, c_2, \cdots, c_{n-1}, 1]^T$ is chosen such that the distribution of the roots of characteristic equation $p^{n-1} + c_{n-1} p^{n-2} + \cdots + c_2 p + c_1 = 0$ is on the left side of complex plane to make the following system stable.

$$e_n(\mathrm{t}) + \sum_{i=1}^{n-1} c_i e_i(\mathrm{t}) = 0 \tag{10}$$

Then, we have

$$\begin{aligned} \dot{s}(\mathrm{t}) &= \dot{e}_n(\mathrm{t}) + \sum_{i=1}^{n-1} c_i e_{i+1}(\mathrm{t}) \\ &= f(x) - x_{rn}(t) + p(t) + b \cdot u(t) + \sum_{i=1}^{n-1} c_i e_{i+1}(\mathrm{t}) \end{aligned} \tag{11}$$

Introducing the following auxiliary sliding surface

$$\sigma(\mathrm{t}) = \dot{s}(\mathrm{t}) + \lambda s(\mathrm{t}) \tag{12}$$

where λ is a positive scalar.

We deduce that

$$\begin{aligned} \dot{\sigma}(t) &= \sum_{i=1}^{n-2} c_i e_{i+2}(t) + c_{n-1}\dot{e}_n(t) + \ddot{e}_n(t) + \lambda(\sum_{i=1}^{n-1} c_i e_{i+1}(t) + \dot{e}_n(t)) \\ &= \sum_{i=1}^{n-2} c_i e_{i+2}(t) + c_{n-1}(f(x) - x_{rn}(t) + p(t) + b \cdot u(t)) \\ &\quad + (\dot{f}(x) - \dot{x}_{rn}(t) + \dot{p}(t) + b \cdot \dot{u}(t)) \\ &\quad + \lambda(f(x) - x_{rn}(t) + p(t) + b \cdot u(t) + \sum_{i=1}^{n-1} c_i e_{i+1}(\mathrm{t}))] \end{aligned} \tag{13}$$

First, a fuzzy auxiliary controller $D(t)$ is built to estimate the SMC unknown term $d(t)$. Corresponding fuzzy rules are given by

IF $Sd(t) > 0$ THEN $D(t)$ should be increased
IF $Sd(t) < 0$ THEN $D(t)$ should be decreased

where

$$Sd(t) = \dot{s}(t) + \varepsilon \cdot \mathrm{sgn}(s(t))$$

It is noted that $D(t)$ can be very big. This may lead to some kind of serious control problem in practice. Therefore based on the following integral relationship with $D(t)$, $\Delta D(t)$ is introduced to carry out the fuzzy controller design.

$$D(t) = G\int_0^t \Delta D(s)ds \tag{14}$$

where G is the proportionality coefficient.

Let Sd denote the fuzzy input $Sd(t)$, and ΔD denote the fuzzy output $\Delta D(t)$. The fuzzy sets of the input and the output are defined as

$$Sd = \{NB, NM, ZO, PM, PB\}$$
$$\Delta D = \{NB, NM, ZO, PM, PB\}$$

where NB is negative and large, NM is negative and medium, ZO is zero, PM is the positive and medium, and PB is positive and large.

Select the following fuzzy rules

R1: IF Sd is PB THEN ΔD is PB
R2: IF Sd is PM THEN ΔD is PM
R3: IF Sd is ZO THEN ΔD is ZO
R4: IF Sd is NM THEN ΔD is NM
R5: IF Sd is PB THEN ΔD is NB

Then based on the Lyaponov method and the dynamic SMC theory, the following controller is presented.

Theorem 1. For $\varepsilon > 0$, system (1) can track the desired trajectory based on the following fuzzy dynamic sliding mode controller

$$\begin{aligned}\dot{u}(t) = \frac{1}{b}[&-\dot{f}(t) + \dot{x}_{rn}(t) - D(t) - \sum_{i=1}^{n-2} c_i e_{i+2}(t) - \sum_{i=1}^{n-1} \lambda c_i e_{i+1}(t)] \\ &+ (c_{n-1} + \lambda)(-f(t) + x_{rn}(t) + b \cdot u(t)) - \varepsilon \cdot \mathrm{sgn}(\sigma(t))\end{aligned} \tag{15}$$

where $D(t)$ is the auxiliary fuzzy controller, and used to estimate the SMC unknown term $d(t) = \dot{p}(t) + p(t)(c_{n-1} + \lambda)$.

Proof. Choose the Lyapunov functional candidate as

$$V(t) = \frac{1}{2}\sigma^2(t) \tag{16}$$

The time derivative of $V(t)$ along trajectories of the error model (4) is as

$$
\begin{aligned}
\dot{V}(t) &= \sigma(t)\dot{\sigma}(t) \\
&= \sigma(t)[\sum_{i=1}^{n-2} c_i e_{i+2}(t) + c_{n-1}(f(x) - x_{rn}(t) + p(t) + b \cdot u(t)) \\
&\quad + (\dot{f}(x) - \dot{x}_{rn}(t) + \dot{p}(t) + b \cdot \dot{u}(t)) \\
&\quad + \lambda(f(x) - x_{rn}(t) + p(t) + b \cdot u(t) + \sum_{i=1}^{n-1} c_i e_{i+1}(\mathrm{t}))]
\end{aligned} \tag{17}
$$

Substituting (15) into (17) yields

$$
\begin{aligned}
\dot{V}(t) &= \sigma(t)[\dot{p}(t) + (c_{n-1} + \lambda)p(t) - D(t) - \varepsilon\,\mathrm{sgn}(\sigma(t))] \\
&= -w(t)
\end{aligned} \tag{18}
$$

where $w(t) = \varepsilon|\sigma(t)|$

Consider $\varepsilon > 0$, we have $\dot{V} \le 0$. Integrating both sides of (18) from 0 to t leads to

$$
\lim_{t\to\infty} V(t) - V(0) \le -\lim_{t\to\infty} \int_0^t w(t)dt \tag{19}
$$

Since $V(t)$ is positive and $V(0)$ is finite, the following inequality can be concluded

$$
\lim_{t\to\infty} \int_0^t w(t)dt \le V(0) < \infty \tag{20}
$$

According to Lemma 1, we have

$$
\lim_{t\to\infty} w(t) = \lim_{t\to\infty} \varepsilon|\sigma(t)| \to 0 \tag{21}
$$

Hence we have $\lim_{t\to\infty} \sigma(t) \to 0$. According to Lemma 2, we obtain $\lim_{t\to\infty} s(t) \to 0$. Consider (9), we have $\lim_{t\to\infty} E(t) \to 0$. This means the achievement of the tracking control. The proof of Theorem 1 is thus completed. □

From Theorem 1, it can be seen that the proposed dynamic sliding mode controller needs the differential information of $f(t)$ and $x_{rn}(t)$. Suppose that we can not obtain these information, the following controller is presented.

Corollary 1. For $\varepsilon > 0$, system (1) can track the desired trajectory based on the following fuzzy dynamic sliding mode controller

$$
\begin{aligned}
\dot{u}(t) &= \frac{1}{b}[-D(t) - \sum_{i=1}^{n-2} c_i e_{i+2}(t) - \sum_{i=1}^{n-1} \lambda c_i e_{i+1}(t)] \\
&\quad + (c_{n-1} + \lambda)(-f(t) + x_{rn}(t) + b \cdot u(t)) - \varepsilon \cdot \mathrm{sgn}(\sigma(t))
\end{aligned} \tag{22}
$$

where $D(t)$ is the auxiliary fuzzy controller, and used to estimate the SMC unknown term $d(t) = \dot{p}(t) - \dot{f}(t) + \dot{x}_{rn}(t) + p(t)(c_{n-1} + \lambda)$.

From Corollary 1, we can see that the controller is based on the information of $f(t)$ and $x_{rn}(t)$. Suppose that we can not obtain $f(t)$ and $x_{rn}(t)$, the following controller is proposed.

Corollary 2. For $\varepsilon > 0$, system (1) can track the desired trajectory based on the following fuzzy dynamic sliding mode controller

$$\begin{aligned}\dot{u}(t) = \frac{1}{b(t)}[-D(t) - \sum_{i=1}^{n-2} c_i e_{i+2}(t) - \sum_{i=1}^{n-1} \lambda c_i e_{i+1}(t)] - \varepsilon \cdot \mathrm{sgn}(\sigma(t)) \\ + (c_{n-1} + \lambda) b \cdot u(t)\end{aligned} \tag{23}$$

where $D(t)$ is the auxiliary fuzzy controller, and used to estimate the SMC unknown term $d(t) = \dot{p}(t) - \dot{f}(t) + \dot{x}_{rn}(t) + (p(t) - f(t) + x_{rn}(t))(c_{n-1} + \lambda)$.

Theorem 1, Corollary 1 and Corollary 2 are all based on the fuzzy dynamic SMC theory. Finally, the following controller is proposed based on the fuzzy SMC theory.

Theorem 2. For $\varepsilon > 0$, system (1) can track the desired trajectory based on the following fuzzy sliding mode controller

$$u(t) = \frac{1}{b}[-f(x) + x_{rn}(t) - \sum_{i=1}^{n-1} c_i e_{i+1}(\mathrm{t}) - D(t) - \varepsilon \cdot \mathrm{sgn}\left(s(t)\right)] \tag{24}$$

where $D(t)$ is the auxiliary fuzzy controller, and used to estimate the SMC unknown term $d(t) = p(t)$.

Remark 1. From Theorems 1 and 2, it can be derived that, in theory, the dynamic sliding mode controller possesses the better anti-chattering performance through transferring the switching term $\varepsilon \cdot \mathrm{sgn}(\sigma(t))$ from $u(t)$ to $\dot{u}(t)$.

4. Numerical Examples

In this section, we will verify the proposed methodology by giving some illustrative examples.

First, we consider the following disturbed system

$$\begin{aligned}\dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= f(x,t) + p(t) + b \cdot u(t)\end{aligned}$$

where

$$f(x,t) = -0.5x_2(t) + x_1(t) - x_1^3(t), b = 133,$$
$$p(t) = 50\exp\left[-\frac{(t-1.5)^2}{2\cdot 0.2^2}\right] - 20\exp\left[-\frac{(t-3)^2}{2\cdot 0.1^2}\right]$$

For simulation purposes, we consider the step size 0.001 second, the initial state $x_0 = [-1,-1]^T$, the desired trajectory $x_r(t) = \sin(2\pi t)$, and the control parameters $\eta = 0.5, r = 0.5, G = 800, c = 150$. The membership function of the input and the output of the fuzzy auxiliary controller are shown in Figs. 1 and 2. First, we consider case 1 based on the general existing SMC method by fixing $D(t) = \max\{|d(t)|\} = 50$. Corresponding simulation results are shown in Figs. 3-5. Then, we consider case 2 based on our fuzzy SMC method in Theorem 2. Corresponding simulation results are shown in Figs. 6-8.

Remark 2. Figs. 3 and 6 show the time response of $d(t)$ and $D(t)$. Figs. 4 and 7 show the time response of the tracking error. Figs. 5 and 8 show the time response of the control input. In case 1 that based on the general existing SMC method, $D(t)$ is fixed as a constant of 50. Fig. 5 shows there is an obvious SMC chattering phenomenon when $d(t) \neq 50$, and the biggest amplitude is over 0.8. For this problem, $d(t)$ is estimated by the auxiliary fuzzy controller in case 2 that based on our fuzzy SMC method. From Fig. 8 we can see, the SMC chattering phenomenon is substantially reduced and the biggest chattering amplitude is less than 0.02, far less than 0.8. This demonstrates the advantage of our fuzzy sliding mode controller.

Next, we consider the following disturbed system

$$\dot{x}_1(t) = x_2(t), \quad i < n$$
$$\dot{x}_2(t) = f(x,t) + p(t) + b\cdot u(t)$$

where

$$f(x,t) = 2x_1 - 1.4x_2 - 0.8x_1^2, b = 1$$
$$p(t) = 20\exp\left[-\frac{(t-5)^2}{2\cdot 0.2^2}\right]$$
$$\dot{f}(x,t) = 2\dot{x}_1 - 1.4\dot{x}_2 - 1.6x_1\dot{x}_1$$
$$\dot{p}(t) = -\frac{20(t-5)}{0.25^2}\exp\left[-\frac{(t-5)^2}{2\cdot 0.2^2}\right]$$

The desired trajectory comes from the following nonlinear system

$$\dot{x}_{r1}(t) = x_{r2}(t)$$
$$\dot{x}_{r2}(t) = x_{r1}(t) - 0.2x_{r2}(t) - x_{r1}^3(t) - 0.32\cos(1.2t)$$

For simulation purposes, we consider the step size 0.001 second, the initial condition $x_0 = [-1, 2]^T$, $x_{r0} = [0.8942, -0.7008]^T$, and control parameters $\eta = 0.1, r = 0.1, G = 800$, $c = 150, \lambda = 2.0$. The membership functions of the input and the output of fuzzy system are same as above. First, we consider case 3 based on the general existing dynamic SMC method by fixing $D(t) = \max\{|d(t)|\} = 20$. Corresponding simulation results are shown in Figs. 9-12. Then, we consider case 4 based on the dynamic fuzzy SMC method in Theorem 1. Corresponding simulation results are shown in Figs. 13-16.

Remark 3. Figs. 9 and 13 show the time response of $d(t)$ and $D(t)$. Figs. 10 and 14 show the time response of the tracking error. Figs. 11 and 15 show the time response of the control input. Figs. 12 and 16 show the time response of $s(t)$ and $\sigma(t)$. In case 3, based on the general existing dynamic SMC method, $D(t)$ is fixed as a constant of 20. Figure 9 shows that the biggest amplitude of $d(t)$ is over 20. Figure 11 shows that the control chattering is transferred from $u(t)$ to $\dot{u}(t)$, and the biggest SMC chattering amplitude is over 5.0. However, from Fig. 10 we can see, $d(t)$ exceeds 20 at about 5 seconds, there appears a big tracking error, which can not meet the control requirement. For this problem, $d(t)$ is estimated by the auxiliary fuzzy controller in case 4 based on our dynamic SMC method. Fig. 15 shows that the biggest SMC chattering amplitude is diminished below 0.05, far less than 5.0. From Fig. 14 we can see the tracking control is realized within 4 seconds without any static and dynamic tracking error during the whole process, which demonstrate the advantage of our fuzzy dynamic sliding mode controller.

Remark 4. From the simulation examples, we can see the chattering phenomenon is reduced effectively by using the fuzzy controller, however there still exists the SMC switching term $\varepsilon \cdot \text{sgn}(s(t))$, though ε is a small constant. To further overcome the control chattering phenomenon, the switching term $\varepsilon \cdot \text{sgn}(s(t))$ is recommended to be replaced with $\varepsilon \cdot \text{sat}(s(t))$.

Remark 5. From examples 1 and 2, we can see the fuzzy dynamic sliding mode controller in Theorem 1 has better anti-chattering performance compared with the controller in Theorem 2 through transferring the switching term from $u(t)$ to $\dot{u}(t)$, and is more suitable for the situation with higher control performance requirement. However, the proposed controller in Theorem 2 has simpler structure and needs less system state information. Hence, it is more suitable for the situation with lower control performance requirement and high cost concerns.

Remark 6. The fuzzy auxiliary controllers presented in this paper are based on the feedback signal $Sd = \dot{\sigma}(t) + \varepsilon \cdot \text{sgn}(\sigma(t))$, and the fuzzy rule is designed to keep Sd at zero. Hence we have $\dot{V}(t) = \sigma(t)\dot{\sigma}(t) = -\varepsilon|\sigma(t)| \le 0$. This means that the fuzzy auxiliary controllers participate in the SMC meanwhile.

5. Conclusion

In this paper, the fuzzy control method and the sliding mode control method have been included to construct the controllers for disturbed systems. First, a fuzzy controller based on one feedback signal has been built not only to estimate the unknown control term, but also take part in the sliding mode control due to its fuzzy rule employed, and features simple structure and high efficiency. Then, we have extended our theoretical results into the cases, where some kind of system information is not accessible, to make our research more applicable to the real engineering. Finally some typical numerical examples have been given to demonstrate the effectiveness and advantage of the presented fuzzy sliding mode controller.

Acknowledgements

This work was partially supported by the Key Projects of Xihua University (Z1120946), the National Key Basic Research Program (973), China (No. 2012CB215202), the 111 Project (B12018), the National Natural Science Foundation of China (Nos. 61174058, 61170030), and the Engineering and Physical Sciences Research Council, UK (EP/F029195).

Appendix A

Proof of Theorem 2. Choose the Lyapunov functional candidate as

$$V(t)=\frac{1}{2}s^2(t) \tag{25}$$

The time derivative of $V(t)$ along trajectories of the error model (4) is as

$$\begin{aligned}\dot{V}(t)&=s(t)\dot{s}(t)\\&=s(t)[f(x)-x_{rn}(t)+d(t)+b\cdot u(t)+\sum_{i=1}^{n-1}c_ie_{i+1}(\mathrm{t})]\end{aligned} \tag{26}$$

Substituting (24) into (26) yields

$$\begin{aligned}\dot{V}(t)&=s(t)[d(t)-D(t)-\varepsilon\cdot\mathrm{sgn}(s(t))]\\&=-w(t)\end{aligned} \tag{27}$$

where $w(t)=\varepsilon\left|s(t)\right|$.

Consider $\varepsilon > 0$, we have $\dot{V}\le 0$. Integrating both sides of (27) from 0 to t leads to

$$\lim_{t\to\infty} V(t) - V(0) \le -\lim_{t\to\infty} \int_0^t w(t)dt \tag{28}$$

Since $V(t)$ is positive and $V(0)$ is finite, the following inequality can be concluded

$$\lim_{t\to\infty} \int_0^t w(t)dt \le V(0) < \infty \tag{29}$$

According to Lemma 1, we have $\lim_{t\to\infty} w(t) = \lim_{t\to\infty} \varepsilon |s(t)| \to 0$. Then considering (9), we derive $\lim_{t\to\infty} E(t) \to 0$. This means the achievement of the tracking control. The proof of Theorem 2 is thus completed. □

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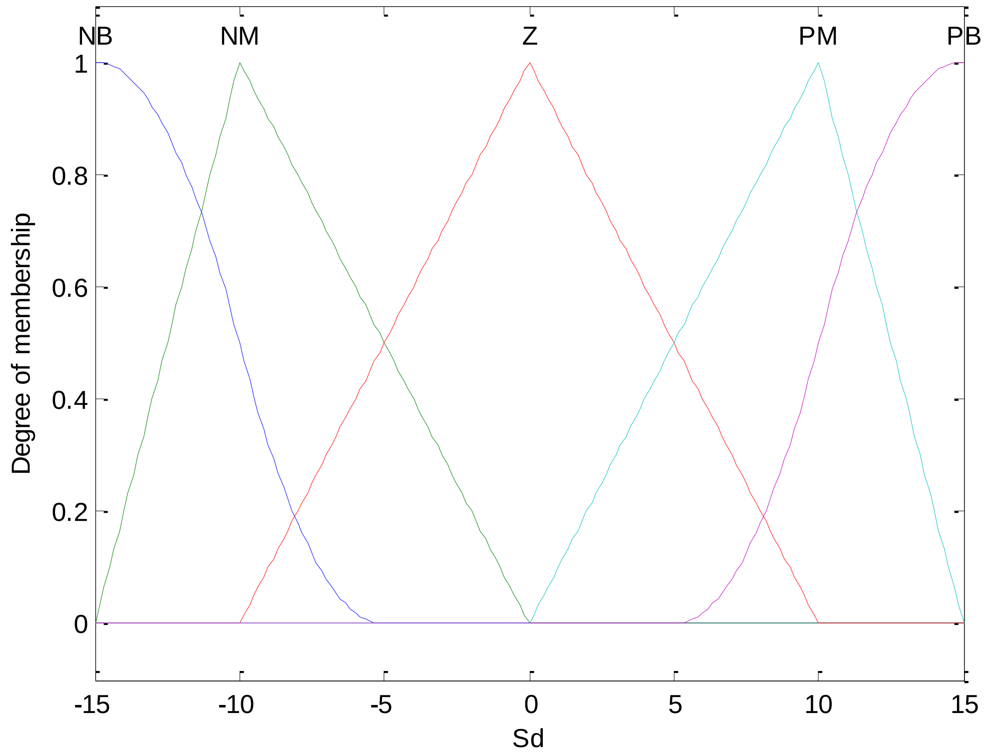

Fig.1. The membership function of the fuzzy input

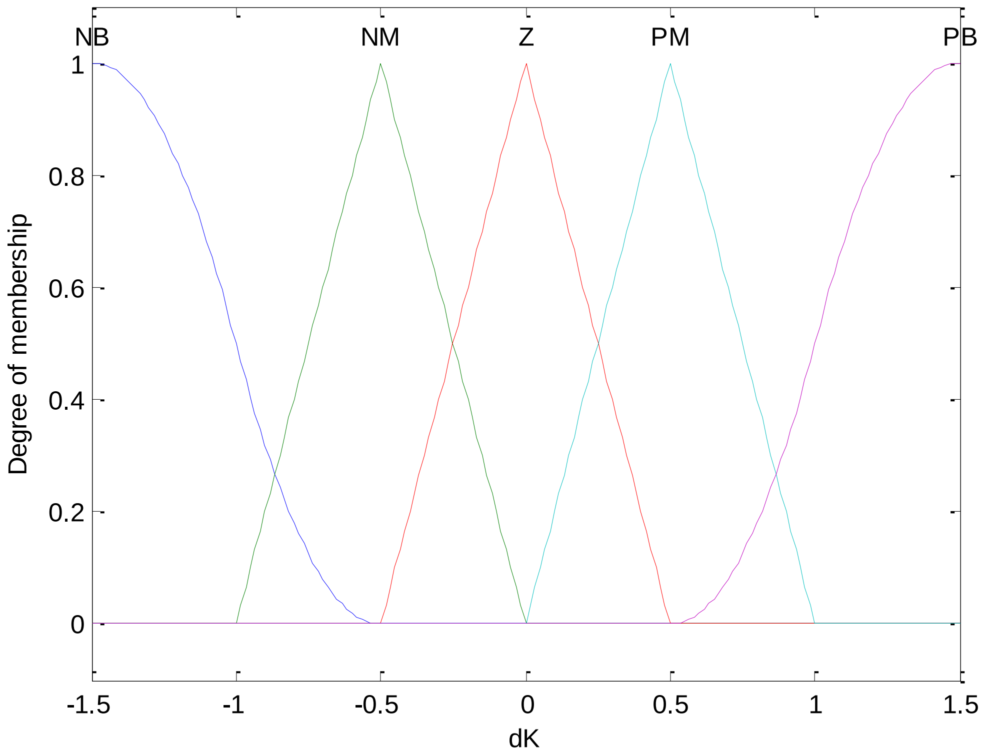

Fig.2. The membership function of the fuzzy output

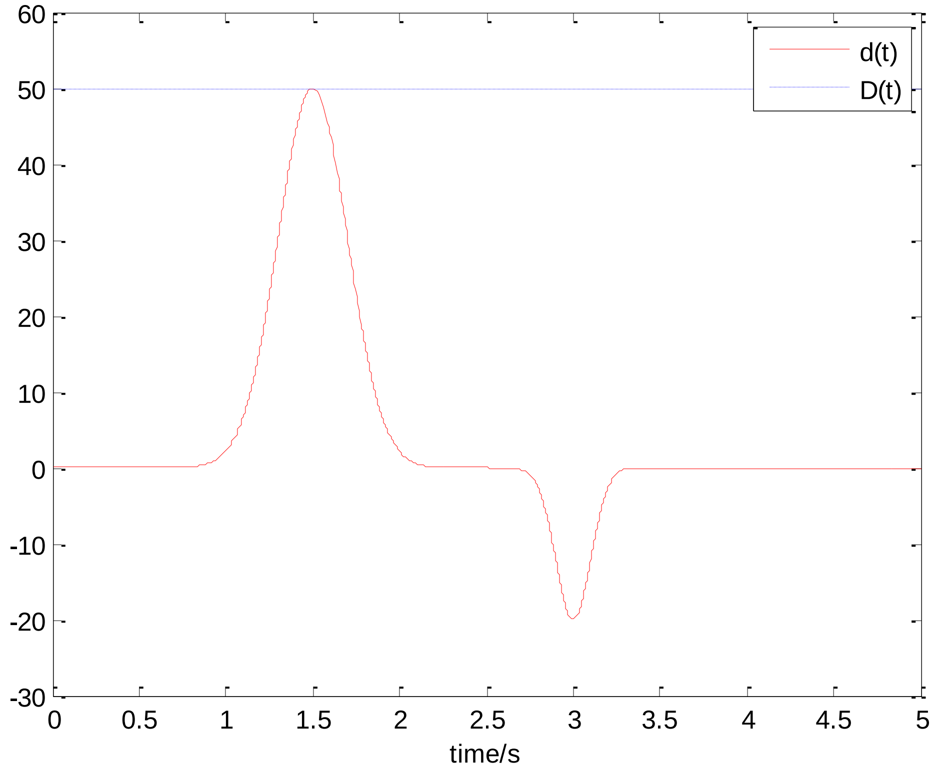

Fig.3. The time response of $d(t)$ and $D(t)$ for case 1

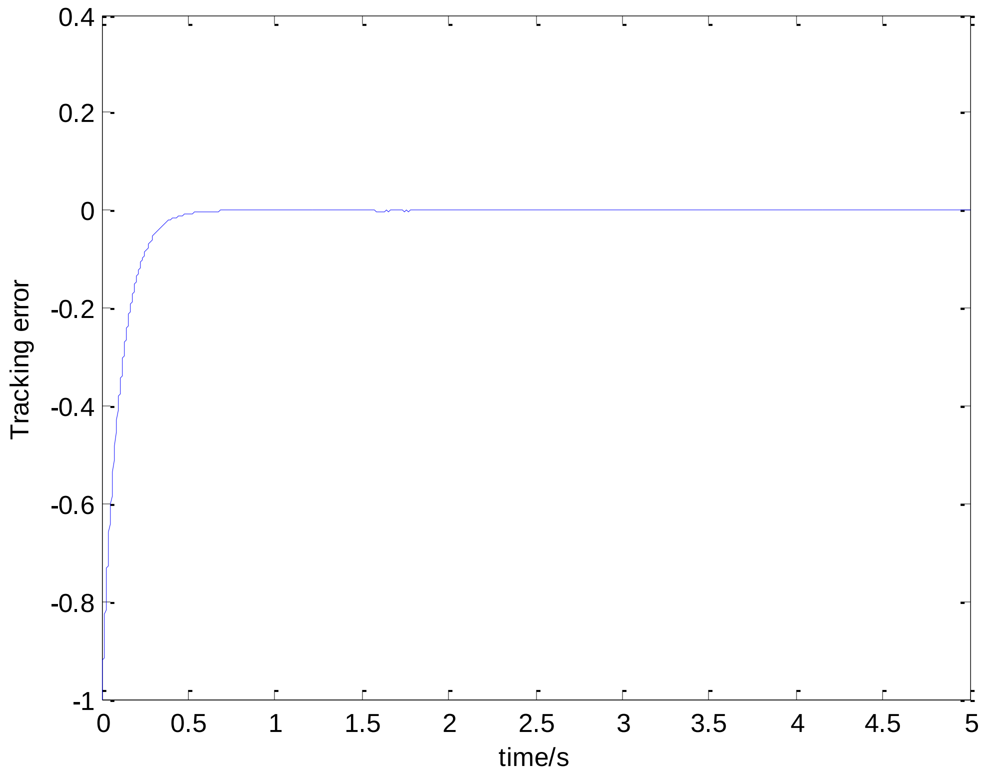

Fig.4. The time response of the tracking error for case 1

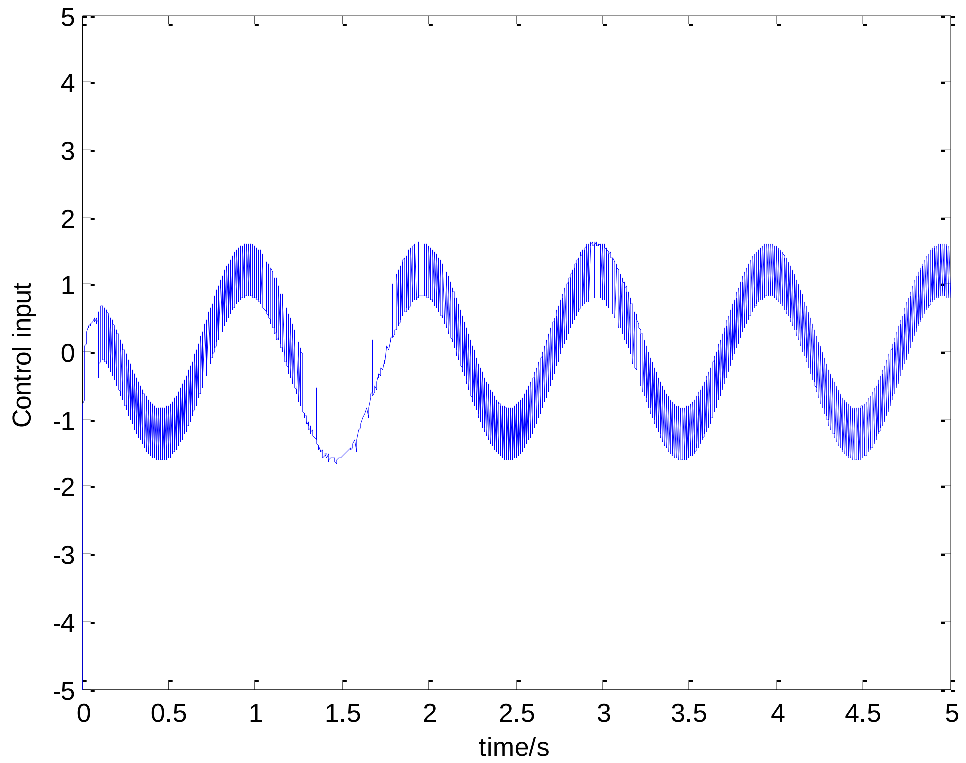

Fig.5. The time response of the control input for case 1

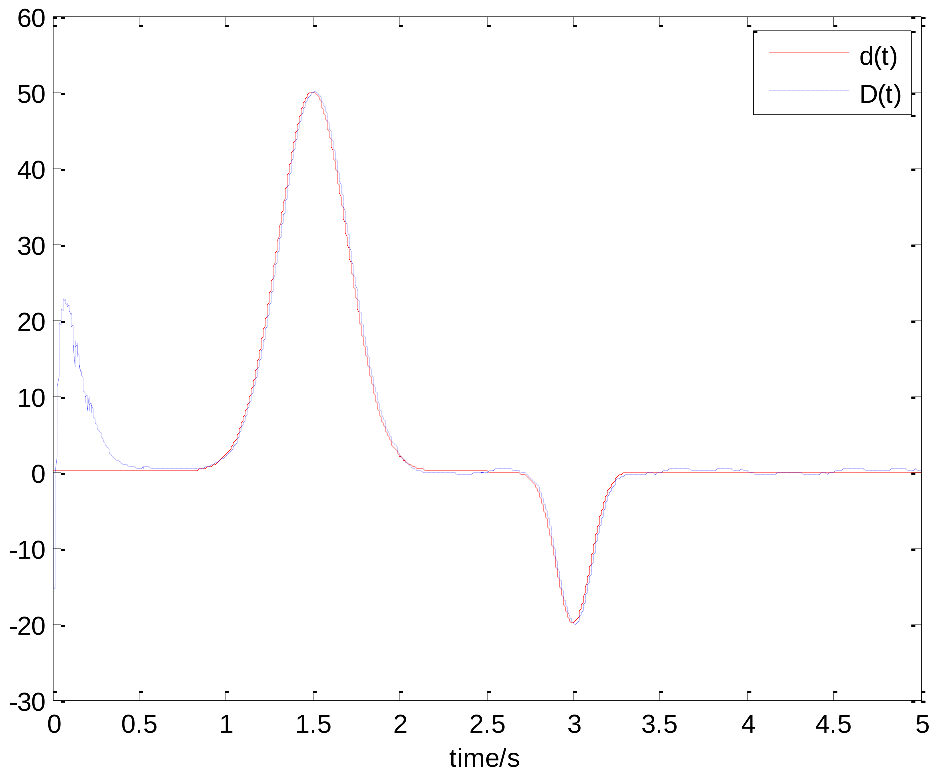

Fig.6. The time response of $d(t)$ and $D(t)$ for case 2

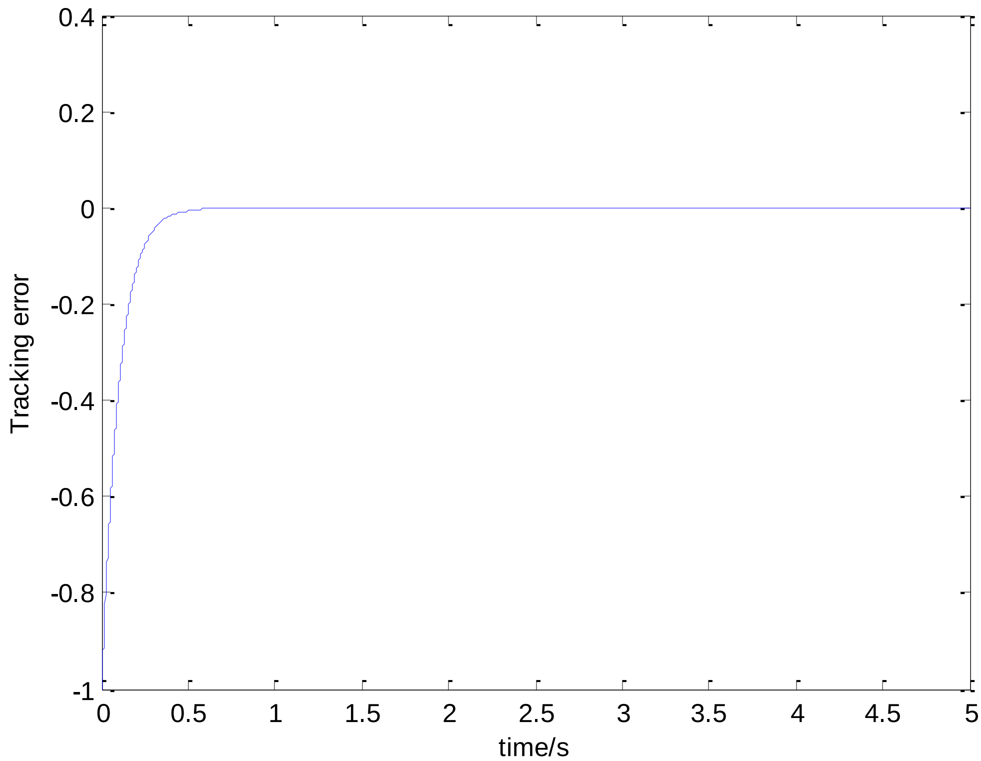

Fig.7. The time response of the tracking error for case 2

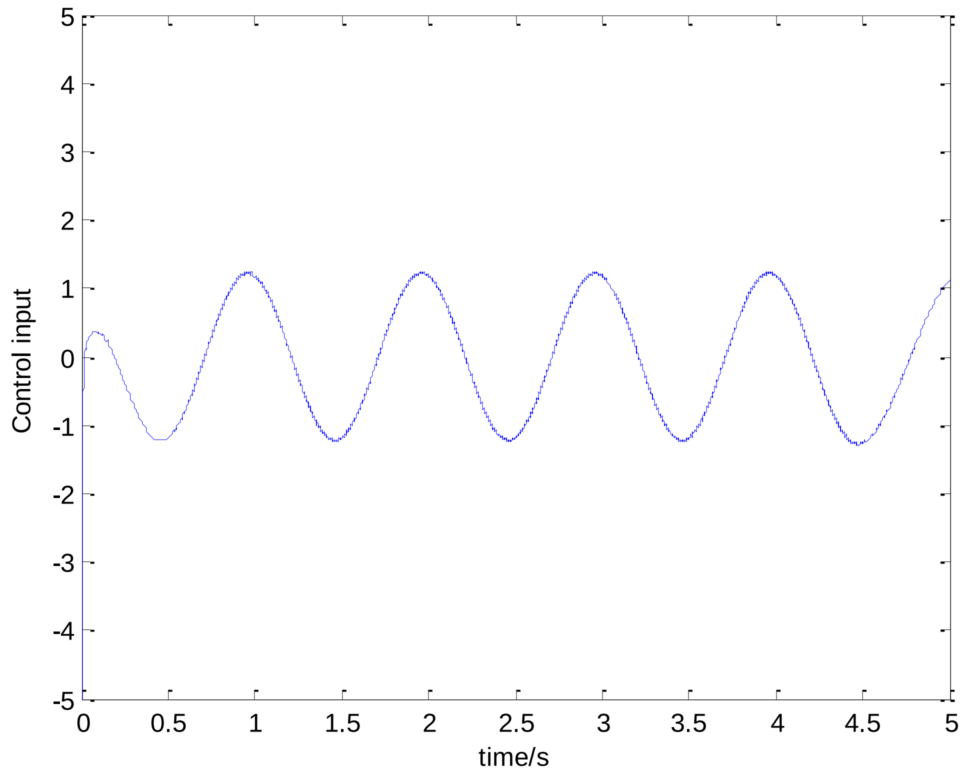

Fig.8. The time response of the control input for case 2

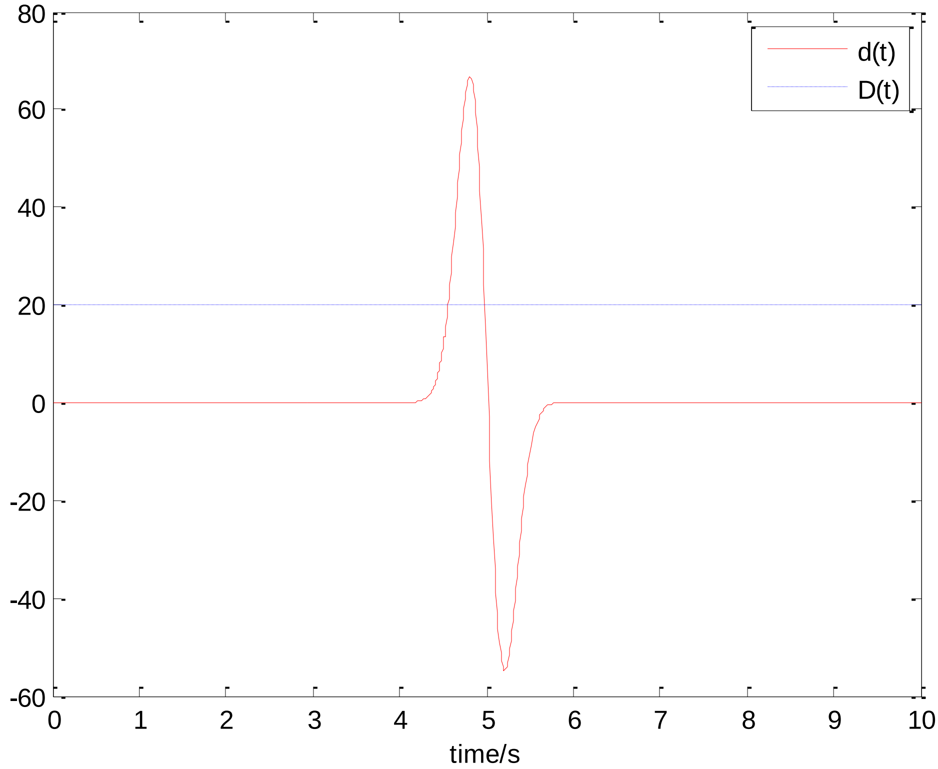

Fig.9. The time response of $d(t)$ and $D(t)$ for case 3

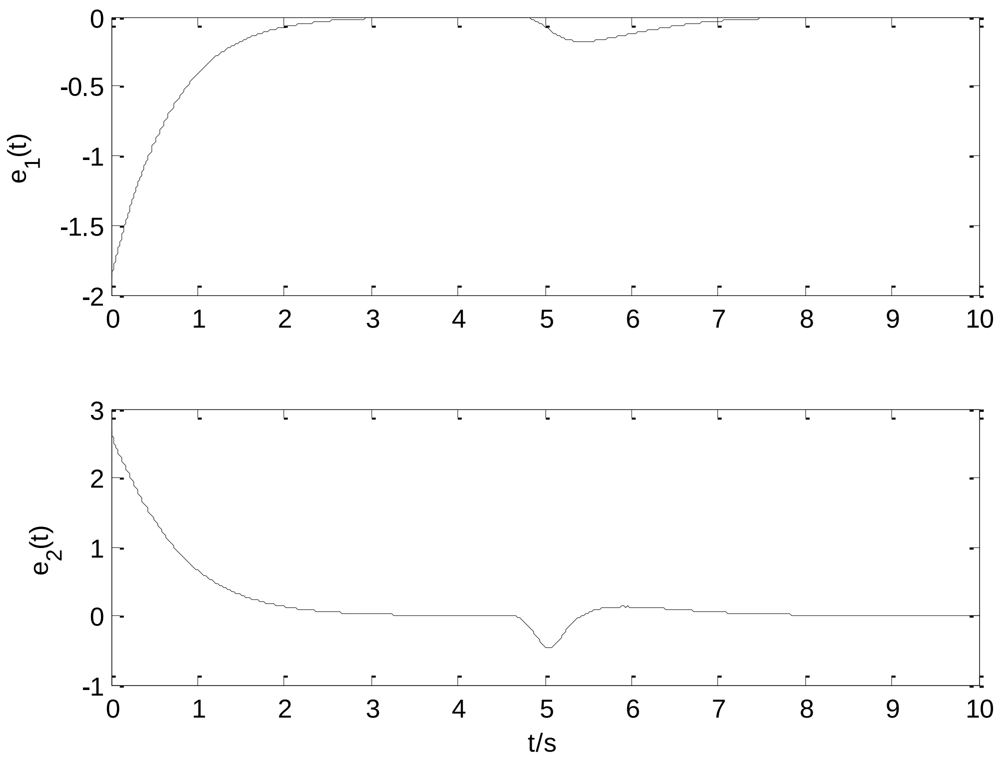

Fig.10. The time response of the tracking error for case 3

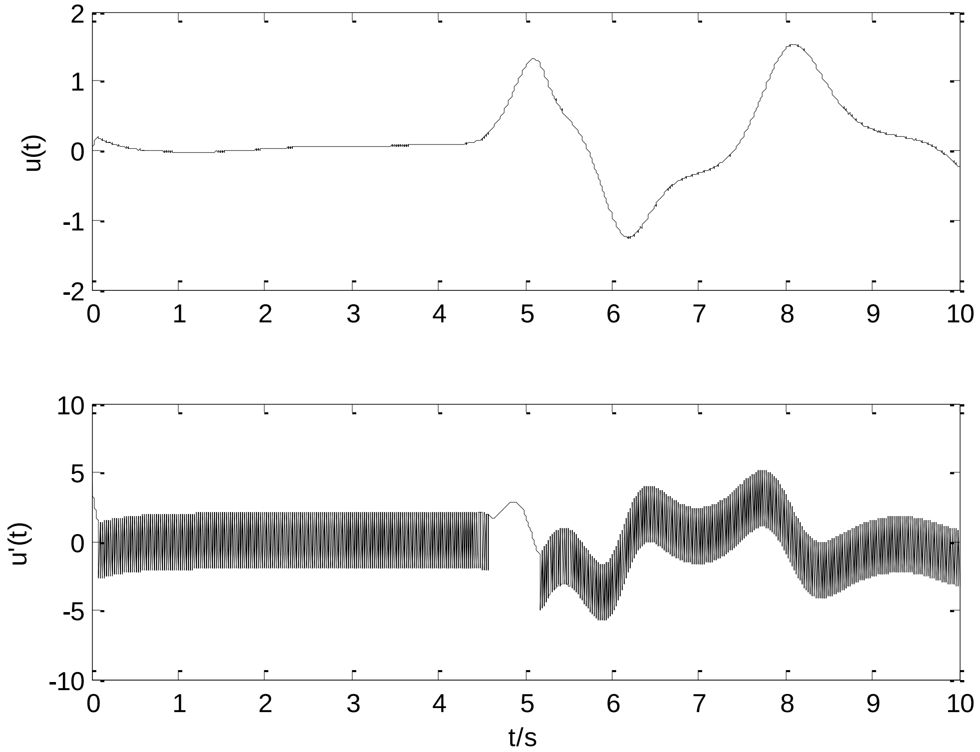

Fig.11. The time response of $u(t)$ and $\dot{u}(t)$ for case 3

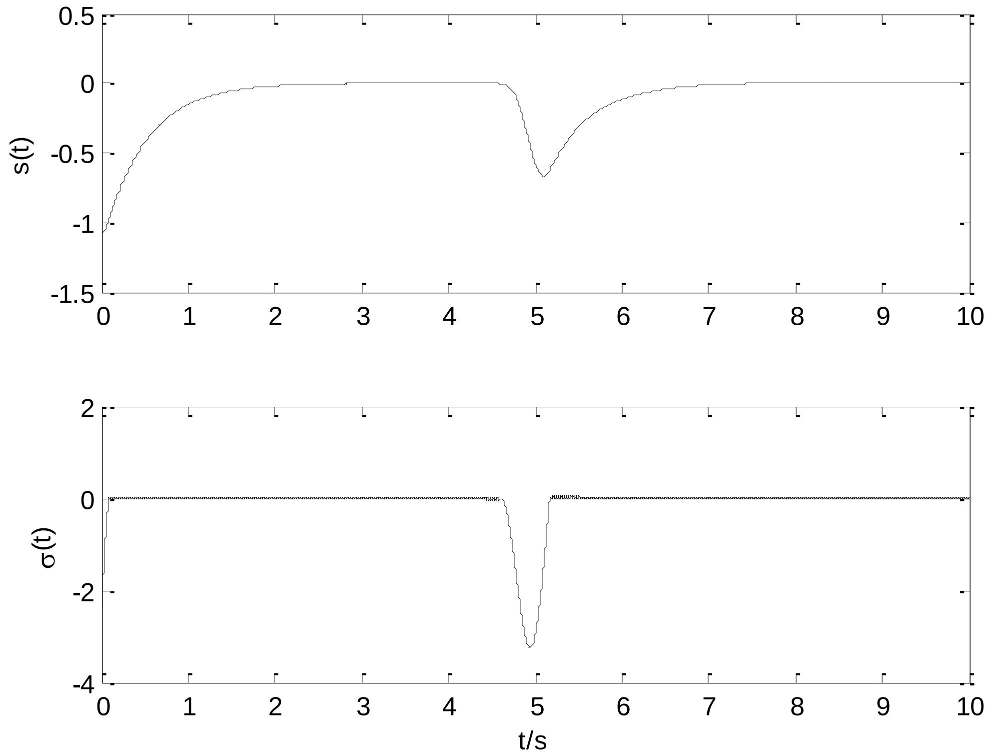

Fig.12. The time response of $s(t)$ and $\sigma(t)$ for case 3

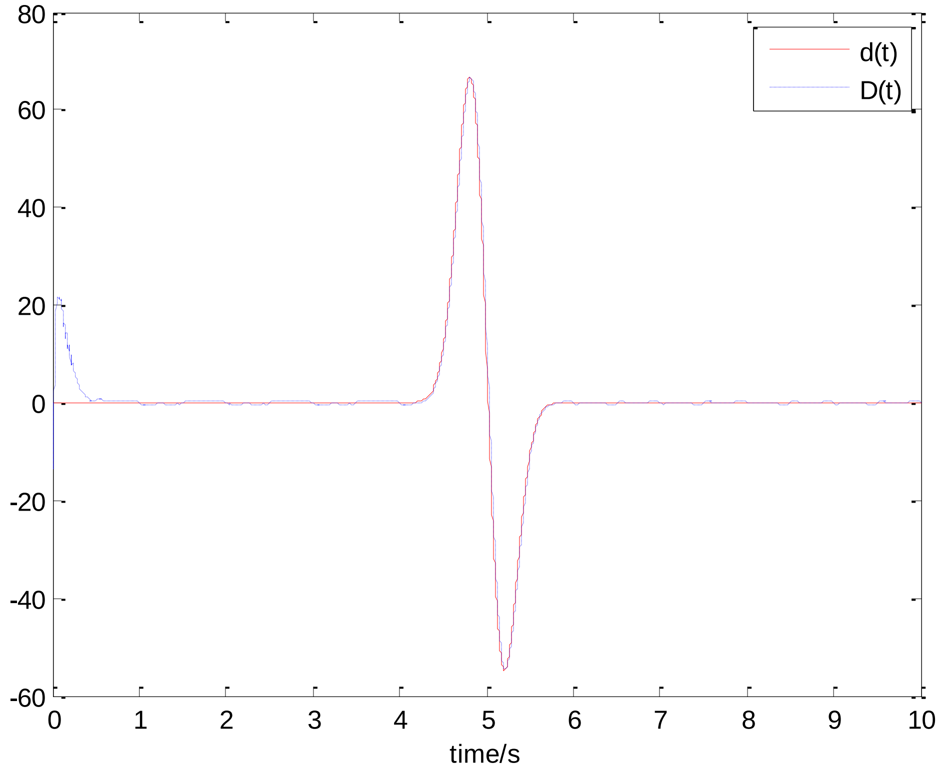

Fig. 13. Time response of $d(t)$ and $D(t)$ for case 4

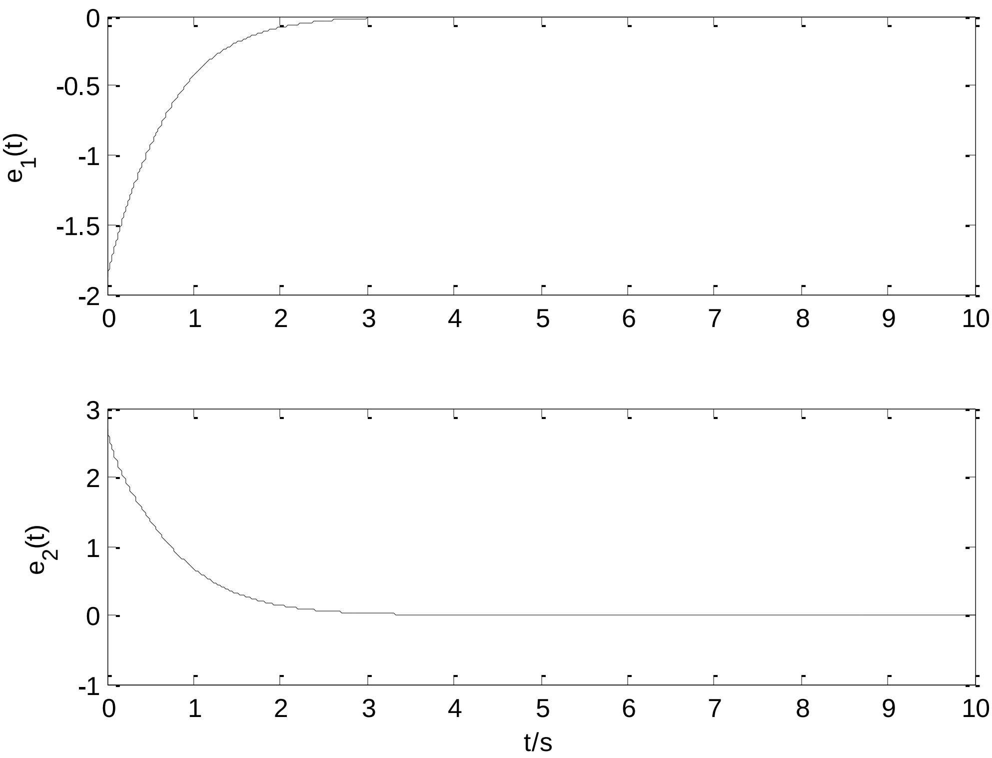

Fig. 14. Time response of the tracking error for case 4

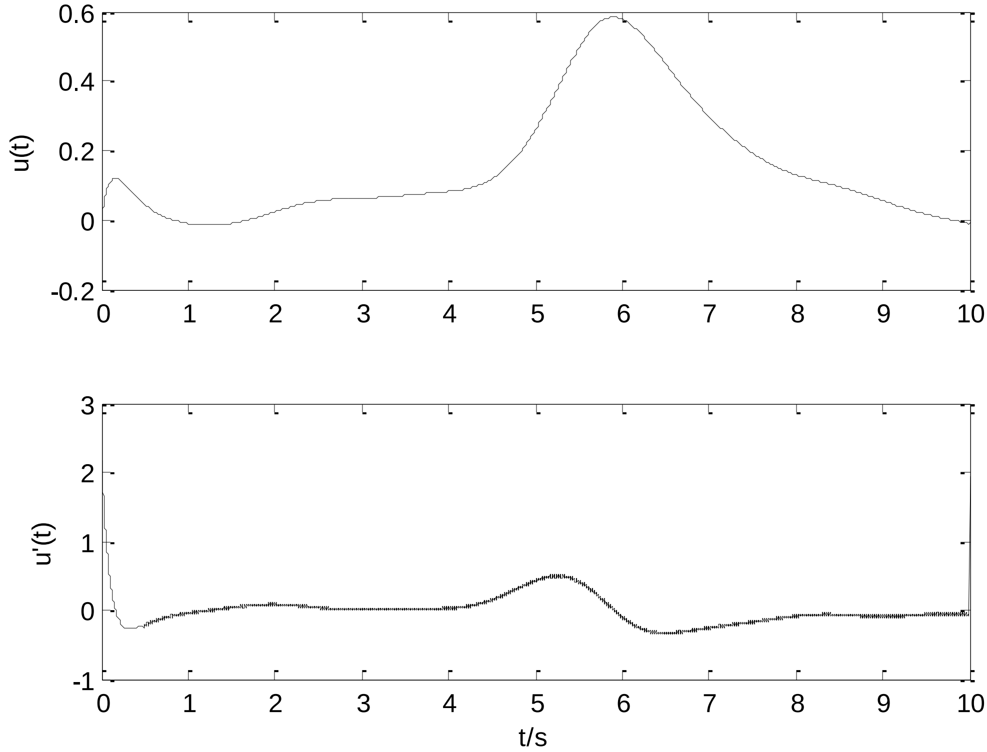

Fig. 15. Time response of $u(t)$ and $\dot{u}(t)$ for case 4

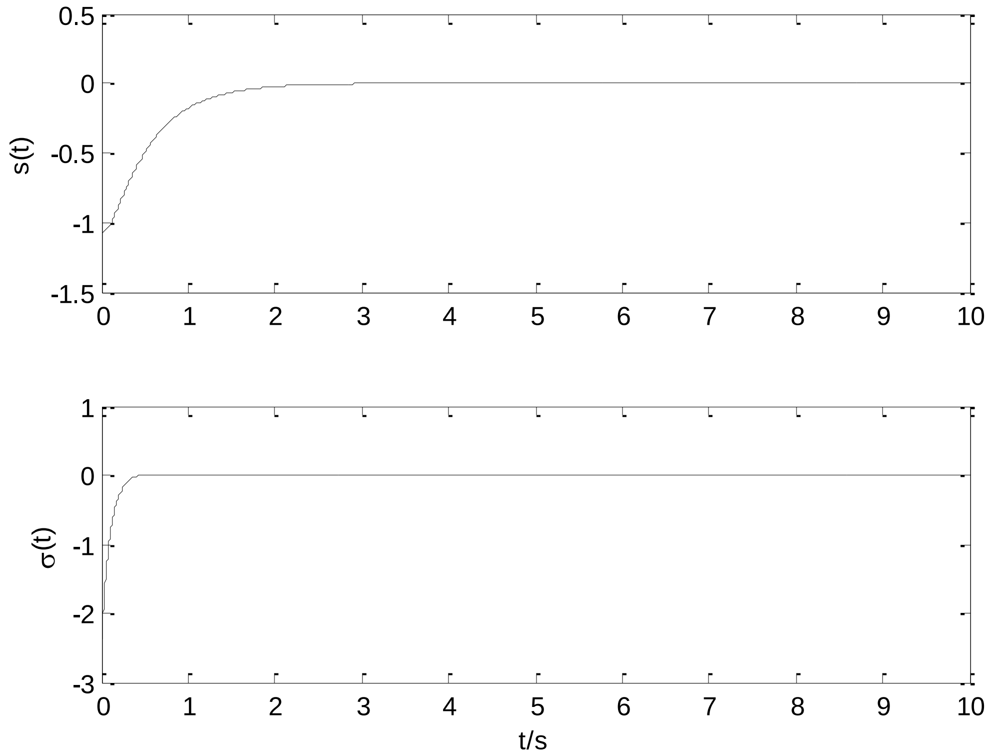

Fig. 16. Time response of $s(t)$ and $\sigma(t)$ for case 4