Roughness and Discharge Uncertainty in 1D Water Level Calculations

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1 Introduction

In hydraulics, water level predictions are necessary for a variety of purposes, such as design of flood protection measures, real-time flood forecasting, bridge maintenance and floodplain mapping. Knowledge of uncertainties in flow depth estimation is crucial for risk assessment and safe design of hydraulics structures. In recent years, modellers in the hydrological and hydraulic community are strongly promoting uncertainty analysis as standard practice in water resource research [1] and a risk-based approach to flood hazard assessment and management—where decision makers are provided with model predictions together with their associated uncertainties—which may be explicitly requested at institutional level [2–4].

Uncertainty analysis means the quantification of uncertainty in model outputs due to uncertainty in input data, parameters, model structure and modelling assumptions. A variety of statistical methods can be used to propagate input uncertainties through the model into output uncertainties. Monte Carlo methods [5] are the most widely used techniques in this general context, with many areas of applications, including water resources. These methods require random generation of a large ensemble of inputs from their probability distributions and successive deterministic model simulations to generate

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many realizations of the output. Monte Carlo methods are straightforward and robust techniques, which allow to deal with process nonlinearities and compute the whole probability density function of model outputs. As a drawback, Monte Carlo simulations are computationally demanding, especially when dealing with high quantiles; hence, their application is limited to simple models. Moreover, a major concern in the Monte Carlo simulation is that the accuracy of model output statistics and probability distributions depends on the number of simulations performed [6]. As a consequence, replication of Monte Carlo simulation runs for a given number of simulations is necessary to ensure convergence of the results. An extension of the Monte Carlo method is the generalized likelihood uncertainty estimation (GLUE) procedure, introduced by Beven and Binley [7] and widely used in hydrology. A less expensive alternative is the analytical treatment of the problem, i.e. the exact derivation of probability density functions (PDFs) of model outputs. The closed form solutions, only possible for simple relationships between uncertain inputs and predictions, are much less frequent in practical applications due to the functional complexity of most real life problems [6]. A suitable approximation of the statistical moments of the output variables can be determined by perturbation methods, as those based on the Taylor expansion of the model around the mean values of the inputs. These methods, like the first-order second-moment (FOSM) method [8, 9], are efficient and simple techniques widely used in civil engineering applications. However, when limited to low-order approximations, they cannot give information on the nonlinear behaviour of the systems and on high-order statistics of the model outputs.

Over the last decades, all these uncertainty methods have been applied to analyse and quantify the different sources of uncertainty in hydraulic models. The sources are manifold: hydraulic variables like roughness coefficient [10, 11], channel slope [12] and cross-sectional geometry [13] and hydrologic variables like extreme flood discharge [14–16] are the most important, but also discharge measurement errors [17, 18], stationarity assumption [19, 20] and parameter estimation methods [14] can be relevant. Moreover, flow-through bridges, irregular channel cross sections, meanders, seasonal variation in roughness and the model itself can all contribute significantly to the uncertainty in water level predictions. Overview of the sources of uncertainty in hydraulic models and in flood risk management can be found in the study of Pappenberger et al. [21] and Hall and Solomatine [22], respectively. Pappenberger et al. [23] also proposed general guidelines to choose the most appropriate uncertainty method for a given situation.

Early works on hydraulic model uncertainty were mostly based on a first-order analysis of the relationship between uncertain parameters and predictions. Lee and Mays [24] used a first-order analysis to determine the uncertainty of levee capacity as a function of the Manning roughness coefficient, friction slope, cross-sectional area and wetted perimeter. Cesare [25] and Mays and Tung [26] applied a first-order reliability method to incorporate the uncertainty of roughness coefficient into the Manning equation for discharge. More recently, Horritt used first- and second-order perturbation techniques to deal analytically with the uncertainty in river topography [12] and flow resistance [27]. In the case of simple functional relationships, analytical methods have also been used to obtain exact expressions of output uncertainties without approximation or extensive simulation. Analytical derivation of the exact PDFs in hydrologic and hydraulic problems has been carried out using both derived distribution techniques [28] and integral transform techniques [29]. With the increase in computational power, Monte Carlo simulations, GLUE procedures and sensitivity analysis (SA) methods are becoming more and more popular in the study of uncertainty in hydraulic models. Aronica et al. [30] investigated the uncertainty embedded in a two-dimensional (2D) flow propagation model using the GLUE procedure and focusing on the uncertainty of the roughness coefficient. Pappenberger et al. [23] used the GLUE methodology to investigate the sensitivity of flooding predictions to the uncertainty in input boundary conditions. Global sensitivity analysis has been used by Hall et al. [31] to assess the influence of variance in the Manning roughness coefficient on model predictions of flood outline and flow depth, and Pappenberger et al. [32] applied different methods of sensitivity analysis to a one-dimensional (1D) flood inundation model and compared the results on specific test cases.

In this study, we investigate the effect of two key uncertainty sources on water level calculations in 1D hydraulic models: roughness coefficient and upstream discharge. The uncertainties in these two variables are known to be the most relevant in flow depth predictions [21, 33, 34], in particular for extreme flood events. In these cases, hydraulic roughness is highly uncertain because flow measurements are not available or reliable for calibration and validation. Discharge is also uncertain because it results from the extrapolation of discharge frequency curves at very low exceedance probabilities. The present work investigates the effects of these uncertainties, both separately and combined, on the probability density functions of the predicted water levels. This is attained by analysing the uncertainty propagation problem for the uniform flow case and for a 1D steady flow model applied to two case studies (the Po River in Italy and the Garonne River in France). In the first study, the exact probability distribution of water levels is obtained in a closed form through direct propagation of the PDF of the input random variable (either discharge or roughness), while in the second case, the Monte Carlo method is used to propagate one or both the sources of uncertainty through the 1D hydraulic model.

The work focuses on the 1D hydraulic model because it is still the most widely used in practical river engineering [21]. Moreover, despite the 1D approach is limited in the representation of flow dynamics, it can predict floodplain inundation as well as 2D finite element models [35].

The present work aims to (1) quantify the effects of the individual and combined uncertainties of hydraulic roughness and upstream discharge on the water level predictions, (2) evaluate the relative contribution of the two random inputs on the output uncertainty, identifying where, when and how the model is more sensitive to discharge or roughness and (3) find out whether the symmetric distribution of discharge/roughness errors results in a non-symmetric distribution of the associated water depth anomalies and how big the asymmetry is.

The paper is organized as follows: in Section 2, we present the mathematical method. We start with the 1D shallow water equations and then give the details of the uncertainty analysis. In Section 3, we describe the river case studies and the Monte Carlo simulations. In Section 4, we present the results of the uniform flow case and the Monte Carlo simulations. Finally, in Section 5, we discuss the results and draw some conclusions.

2 Method

2.1 Hydraulic Modelling

One-dimensional shallow water equations are derived from two principles: the conservation-of-mass equation and the dynamic relation either for energy or momentum conservation. These equations were first cast by Jean-Claude Barré de Saint-Venant and so are known as the Saint-Venant equations. Neglecting lateral inflow, they can be written as

$$\begin{aligned} &\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+g\frac{dh}{dx}-g(i-J)=0 \\ &\frac{dA}{dt}+\frac{dQ}{dx}=0 \end{aligned} \quad (1)$$

where Q is the flow rate, h is the flow depth, U is the cross-sectional average velocity, A is the cross-sectional area, g is the gravity, i is the channel slope, and J is the friction slope. These equations are usually combined with an equation for the energy slope term J, like the Chézy formula

$$J=\frac{Q^2}{A^2C^2R} \quad (2)$$

where R is the hydraulic ratio, equal to the ratio of flow area A and wetted perimeter P, and C is the Chézy coefficient of flow resistance. The latter can be evaluated as a function of the Strickler roughness coefficient K_S by the following expression:

$$C=K_S R^{1/6} \quad (3)$$

Two different approximations of the Saint-Venant equations were used to propagate input uncertainties: the uniform flow and the steady flow models. Under the hypothesis of uniform flow, the Saint-Venant equations reduce to

$$i=J \quad (4)$$

so that the flow rate is given by

$$Q(h)=K_S\sqrt{i}A(h)R(h)^{2/3} \quad (5)$$

This relationship between the water level and the simultaneous flow discharge is known as stage-discharge relation or rating curve. In the case of rectangular channel of width B, the cross-sectional area is $A(h)=Bh$ and the hydraulic radius is $R(h)=Bh/(B+2h)$. Substituting $A(h)$ and $R(h)$ into Eq. (5), the following stage-discharge relationship results:

$$Q(h)=K_S\sqrt{i}B^{5/3}\frac{h^{5/3}}{(B+2h)^{2/3}} \quad (6)$$

This equation will be used for analytical derivation of the probability density functions of the water levels due to the Gaussian uncertainty of discharge or Strickler roughness coefficient.

For uncertainty propagation in real rivers, the steady shallow water equations

$$\begin{aligned} &U\frac{\partial U}{\partial x}+g\frac{dh}{dx}-g(i-J)=0 \\ &\frac{dA}{dt}+\frac{dQ}{dx}=0 \end{aligned} \quad (7)$$

were solved by the MASCARET open-source software, developed by Electricité De France Recherche & Développement (EDF R&D) in collaboration with the Centre d'Etudes Techniques Maritimes et Fluviales (CETMEF) over more than 25 years [36]. This modelling package is a set of numerical codes simulating 1D hydro-environmental problems through a network of open channels. The computational kernel is based on a well-balanced finite volume Roe scheme [37] developed by Goutal and Maurel [38] for applications such as dam-break wave simulation, reservoir flushing and flooding. Even if a steady flow analysis is a simplification of the full Saint-Venant equations, it appears reasonable when considering the time-space scales of the two river models [39, 40]. With particular reference to the Po River, similar considerations were also introduced by Dottori et al. [41].

2.2 Stochastic Modelling

Two sources of uncertainty of 1D hydraulic models were considered: upstream discharge and roughness parameter.

For uniform flow in rectangular channels, the probability distributions of water levels were calculated by treating either discharge or roughness coefficient as Gaussian random variables. In both cases, the channel slope and the river geometry were assumed to be deterministic variables. The exact PDFs of the water levels were analytically derived as a function of the known PDFs of discharge $p(Q)$ or roughness $p(K_S)$ via the derived distribution method [e.g. 26]. According to this approach, the relationship between the PDFs $f_X(x)$ and $f_Y(y)$ of two random variables X and $Y=g(X)$, where $g(X)$ is continuous in X and strictly monotonic, is

$$f_Y(y) = f_X\left[g^{-1}(y)\right]\left|\frac{dg^{-1}(y)}{dy}\right| \tag{8}$$

For uniform flow in rectangular channels (Eq. 6), the relations $Q(h)$ and $K_S(h)$ are strictly monotonic; hence, the probability density functions of water levels can be directly derived from the known PDFs of each random variable, Q or K_S, according to the following expressions:

$$p(h) = p(Q)\frac{dQ}{dh} \tag{9.1}$$

$$p(h) = p(K_S)\frac{dK_S}{dh} \tag{9.2}$$

Substituting the Gaussian PDFs $p(Q)$ and $p(K_S)$ and the first derivatives of the uniform flow Eq. (6) dQ/dh and dK_S/dh into Eqs. (9.1) and (9.2), respectively, the PDFs of the water levels can be easily obtained

$$p(h) = \frac{1}{3}\frac{K_S\sqrt{i}B^{5/3}h^{2/3}}{\sqrt{2\pi\sigma_Q^2}}\frac{(5B+6h)}{(B+2h)^{5/3}}\exp\left[-\frac{\left(K_S\sqrt{i}B^{5/3}\frac{h^{5/3}}{(B+2h)^{2/3}}-\mu_Q\right)^2}{2\sigma_Q^2}\right] \tag{10.1}$$

$$p(h) = \frac{1}{3}\frac{Q}{B^{5/3}\sqrt{i}\sqrt{2\pi\sigma_{K_S}^2}}\frac{(5B+6h)}{h^{8/3}(B+2h)^{1/3}}\exp\left[-\frac{\left(\frac{Q}{\sqrt{i}B^{5/3}}\frac{(B+2h)^{2/3}}{h^{5/3}}-\mu_{K_S}\right)^2}{2\sigma_{K_S}^2}\right] \tag{10.2}$$

where μ and σ indicate the mean and standard deviation of the input random variables.

For steady flow in real rivers, the relationship between water level and discharge or roughness is a differential (Eq. (7)) whose analytical manipulation does not yield a continuous and monotonic relationship between inputs and outputs. As such, it is not possible to derive the probability density function of water levels using the derived distribution method, even in the simplest case of a rectangular prismatic channel. Therefore, the Monte Carlo approach was used to propagate the uncertainty sources through the hydraulic models of the rivers studied here. In the Monte Carlo technique, a deterministic model is run several times with different sets of input variables, randomly generated from their probability distributions. This provides a sample of deterministic solutions, which are used to infer the statistical properties of the output distribution. In the case of steady flow in real rivers, we evaluated the effect of uncertainty in discharge and roughness, considered both alone and in combination.

The flow rate was assumed to be normally distributed, around its expected value, while a triangular probability density function was chosen for the roughness parameter. The coefficient of variation (CV) of the Q probability distribution was set equal to 6.7 %. If the Gaussian distribution were truncated at $\pm3\sigma$, this would lead to drawn values of discharge with a maximum deviation from the mean of about 20 %. The same uncertainty was assigned to the Strickler coefficient of the main channel, while the uncertainty of floodplain roughness was assumed to be larger (CV equal to 13.4 %). To achieve statistically reliable results, the number of Monte Carlo runs was determined by increasing the amount of experiments until convergence of the output statistical properties was reached. The resulting number of realization was equal to 10.000.

3 Case Study Descriptions

Two river case studies were considered in this work: the Po River in Italy and the Garonne River in France.

The Po River is the longest and most important Italian river, flowing across Northern Italy from the French border on the west to the Adriatic Sea on the east. The Po River is subject to destructive floods, threatening the population—nearly a third of all Italians—and the intensive agricultural and industrial activities which are located within its basin. For this reason, many studies focused on the middle-lower Po River to study the effects of different sources of uncertainty of hydraulic model predictions. For example, Di Baldassarre and Montanari [42] analysed and quantified the uncertainty of river discharge data, derived using the rating curve method, by means of a 1D hydraulic model (HEC-RAS) of the 330 km reach of the Po River from Isola Sant'Antonio to Pontelagoscuro. Similarly, Domeneghetti et al. [43] studied the propagation of rating curve uncertainty on the calibrated Manning coefficients using a quasi 2D hydraulic model of the Po River reach from Piacenza to Pontelagoscuro.

The study reach of the Po River is located in the lower part of the catchment, just upstream the delta region. It extends for approximately 80 km from Mantua to Ferrara. This portion of the river is characterised by a stable main channel confined by high levees, which can convey up to the 500-year flood. Channel width is highly variable and ranges from 400 to 2200 m with an average value of 850 m. The average slope of the reach is 10^{-4} m/m. A uniform value for the Strickler coefficient (K_S=40 $m^{1/3}/s$) has been set all along the channel, as after the calibration by Domeneghetti et al. [44], based on traditional and remotely sensed hydrometric data of the major flood in October 2000. The geometry of the selected reach was represented by 22 cross sections, as from the ground survey by the Interregional Agency for the Po River (AIPO) in 2005. The upstream boundary condition was defined as a constant discharge, while at the downstream end, a fixed water level was set. Uncertainty propagation in the Po River was investigated for two river discharges: 12.000 and 15.000 m^3/s, corresponding to the 100- and 500-year events, respectively. The peak flood quantiles $Q(T)$ were estimated using the index flood method, which requires the estimation of the mean annual flood of the catchment and the regional growth curve [45]. The latter is a dimensionless relationship between frequency and flood, normalized by an appropriate index flood (usually the mean annual flood). The Po River catchment can be divided into four homogeneous flood regions [46]. For each region, the parameters of the growth curve were obtained by De Michele and Rosso [47] using the method of the probability-weighted moments (PWMs) to fit the generalized extreme value distribution (GEV) to the normalized flow measurements. The index flood, i.e. the mean annual flood, was estimated as the mean of the maximum annual flood series, measured at the stream gauge of Pontelagoscuro, close to Ferrara.

The second case study is a 25-km reach of the Garonne River, approximately between Marmande and La Réole. The Garonne River is the third longest river in France with a length of about 600 km, from its source in the Pyrenean Massif to its mouth in the Atlantic Ocean. Earlier works on the Garonne River have been carried out by Goutal et al. [48] and Passoni et al. [49], who investigated discharge and roughness uncertainty propagation in a 1D hydraulic model.

The study reach is characterised by a main channel (150–350 m wide, with an average width of 215 m) and wide lateral floodplains with an average width of about 1200 m. Despite the studied reach has wide floodplains, it was shown that it can still be modelled accurately by a 1D hydraulic model, at least with regards to water level predictions [50]. The bankfull discharge is approximately equal to the mean annual discharge (1000 m^3/s), and the average channel slope is equal to 3.10^{-4} m/m. The river geometry was described by 47 cross sections derived by the ground surveys of the Direction Départementale de l'Equipement du Lot et Garonne. The Garonne model has been calibrated, using channel and floodplain roughness as free parameters, against steady state water surface profiles for bankfull discharge by Besnard and Goutal [50]. Two different Strickler coefficients were used to represent the main channel and the floodplains, equal to 33 and 12 $m^{1/3}/s$, respectively. A constant discharge at the upstream end and a stage-discharge relationship at the downstream end were used as boundary conditions. River discharges of 2600 and 3500 m^3/s were simulated.

4 Results

In this section, we present the results of the uncertainty propagation study for the case of uniform flow in prismatic rectangular channels and for steady flow in real rivers.

4.1 Uniform Flow Model

The concept of uncertainty propagation in uniform flow is illustrated in Fig. 1. The plot shows that the discharge probability distribution is mapped into the water level axis by the stage-discharge relationship. The slope and concavity of this curve are responsible for the statistical properties of the resulting distribution: the slope influences the variance of the output PDF, while the concavity affects its skewness. In particular, the water level probability density function has a lower coefficient of variation than the input PDF (for both inputs: discharge or roughness) and it is no longer Gaussian. Due to the nonlinearity of the stage-discharge relationship, the shape of the output distribution is somewhat distorted and it presents a heavier left tail (negative skewness) if discharge uncertainty is propagated and a heavier right tail (positive skewness) in the case of uncertainty in the roughness coefficient. It can also be noticed that the production of asymmetry—due to uncertainty propagation through the uniform flow equation—is more relevant for lower values of the mean input variable (μ_Q or μ_{K_S}). As a result, the skewness of the resulting PDFs will be higher, when uncertainty in roughness is considered, than in case of discharge uncertainty propagation.

For a given probability distribution of the input variable, the water level PDFs only depend on the channel width and slope and on discharge or roughness (depending on which variable is deterministic and which one is aleatory). These parameters are responsible for the slope and concavity of the curve, hence for variance and skewness of the water level distributions. The effect of these parameters and the probability distribution of the input random variable [$N(\mu_Q, \sigma_Q)$ or $N(\mu_{K_S}, \sigma_{K_S})$] was investigated by means of the analytical expressions of the water level PDFs, given by Eqs. 10.1 and 10.2, for discharge and roughness uncertainty, respectively.

We evaluated the probability density functions of the water levels (10.1 and 10.2) for a 100-m-wide channel, with a bed slope of 2.10^{-3} m/m (Table 1). In the case of discharge uncertainty, we set the Strickler coefficient to 33 $m^{1/3}/s$ and the expected value of discharge equal to 1800 m^3/s. In the case of roughness uncertainty, the discharge was set equal to 1800 m^3/s and the expected value of roughness equal to 33 $m^{1/3}/s$. For these parameters, we calculated the water level PDFs for different input distributions [$N(\mu_Q, \sigma_Q)$ or $N(\mu_{K_S}, \sigma_{K_S})$], characterised by coefficients of variation ranging from 1.6 to 16 %. These values correspond to the uncertainties between 5 and 50 % (obtained by truncating the normal distribution $N(\mu, \sigma)$ at $\pm 3\sigma$). A wide range was chosen because the coefficients of variation are generally assumed rather than known in probabilistic and reliability analysis [51].

Figure 2 shows how the coefficient of variation and the skewness coefficient of the water level distributions depend on the coefficient of variation of the input random variable. The effects of the input uncertainty on the resulting distributions are as expected, for the above-mentioned theoretical reasons: asthe uncertainty in Q or K_S increases, the uncertainty in h increases as well, and the PDFs become more and more asymmetric. The coefficient of variation of the output variable depends linearly on the coefficient of variation of discharge or roughness, and the water level probability distribution is less dispersed than the corresponding input PDF, consistently with the functional structure of the stage-discharge relationship (Eq. (6))

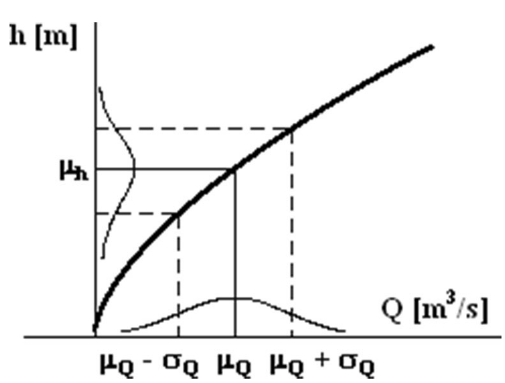

Fig. 1 Uncertainty propagation in uniform flow models

Table 1 Parameters of the uniform flow analysis

Uncertainty source	B (m)	i [−]	K_S ($m^{1/3}$ s^{-1})	Q (m^3 s^{-1})
Q	100	0.002	33	N (1800, 28.8/288)
K_S	100	0.002	N (33, 0.53/5.3)	1800

Q discharge uncertainty propagation, K_S roughness uncertainty propagation

Moreover, due to the nonlinearity of the uniform flow equation, the water level probability distributions are no longer Gaussian but are left or right skewed, depending on the source of uncertainty (discharge or roughness, respectively). The skewness coefficient of the resulting PDFs is directly proportional to the coefficient of variation of $p(Q)$ and $p(K_S)$. The asymmetry is higher (about five times) in the case of roughness uncertainty, because the concavity of the stage-discharge relationship is larger for low values of the expected input variable, as already noticed. From a practical viewpoint, the probability distribution of flow depth remains approximately Gaussian for discharge uncertainty propagation, even when the input uncertainty is large, while the asymmetry is evident when roughness uncertainty is considered. This indicates that for uniform flow and large roughness uncertainties, the skewness of the output distribution cannot be neglected. Finally, the sensitivity of the results to the deterministic variables of the output PDFs was investigated by changing the channel width and slope and either discharge or roughness, depending on the source of uncertainty (i.e. the former in the case of roughness uncertainty propagation and the latter when uncertainty in discharge was considered). The aim of the analysis was to determine if the shape of the water level distribution is preserved when the channel geometry and hydraulic properties change. The results indicate that the coefficient of variation and the skewness of the water level PDFs (shown in Fig. 2) are not substantially affected by the channel slope and width and the flow rate and roughness, as long as they are considered as deterministic inputs. Consequently, the results of the uncertainty analysis in uniform flow hold for a wide range of rectangular channels.

4.2 Real Case Studies

The results of the Monte Carlo simulations for the Po River and Garonne River were used to evaluate the main statistical

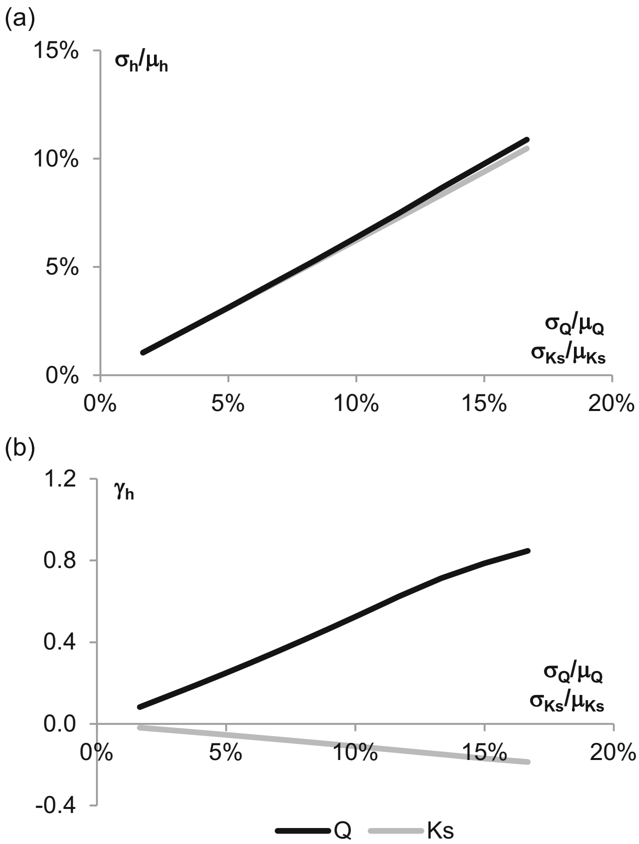

Fig. 2 Dependency of the coefficient of variation (**a**) and skewness coefficient (**b**) of water level PDFs on the coefficient of variation of the input random variable, for discharge uncertainty (*black line*) and roughness uncertainty (*grey line*) propagation in uniform flow

properties of the water level probability density functions and their sensitivity to the two sources of uncertainty, namely discharge and roughness.

Figures 3 and 4 show the coefficient of variation and the skewness coefficient of the water level PDFs along the Po River and Garonne River, respectively, for different uncertainty sources and different upstream discharges.

For the Po River, the uncertainty in water levels is much smaller in the case of discharge uncertainty than that in the case of roughness uncertainty, while for the Garonne River, the two sources have the same effect on the coefficient of variation of the output variable. For both rivers, the highest uncertainty in outputs results when the two uncertainty sources are propagated together through the steady flow model. The water level distributions have a lower coefficient of variation than the input PDFs, irrespective of which input uncertainty is considered (discharge or roughness). Moreover, the output PDFs have higher coefficients of variation for the Po River (between 1.9 and 5.4 %) than for the Garonne River (between 1.3 and 2.2 %). The differences between the coefficients of variation of the two simulated discharges are negligible for both the test cases.

Like in the uniform flow in rectangular channels, the output distributions are left skewed in the case of discharge

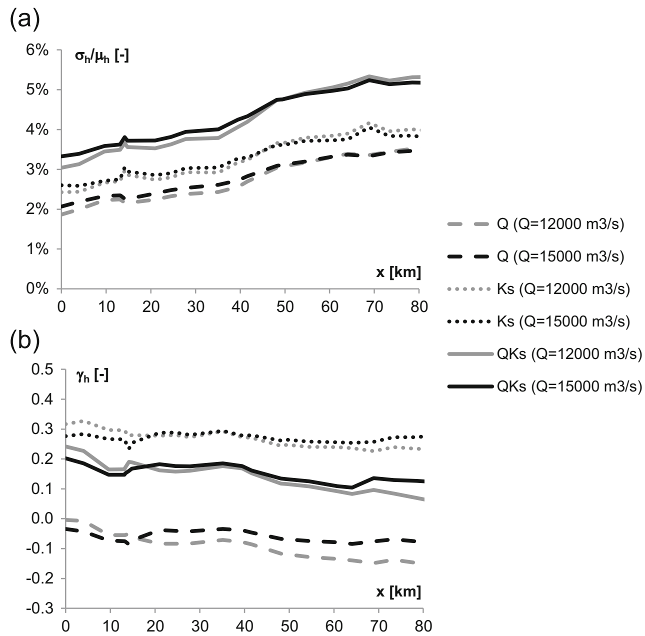

Fig. 3 Coefficient of variation (*a*) and skewness coefficient (*b*) of the water level probability density functions along the Po River, for different uncertainty sources (*Q* discharge uncertainty, K_S roughness uncertainty, QK_S combined discharge and roughness uncertainty) and different flow rates

uncertainty and right skewed in the case of roughness uncertainty, while when both uncertainties are considered, the skewness is very close to 0. A major difference between the two study cases is that the skewness coefficient is almost constant along the 80-km reach of the Po River, while it

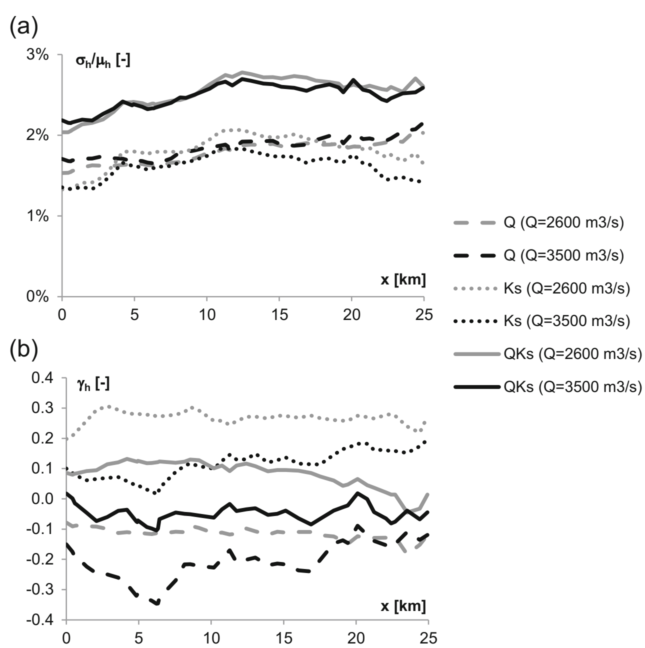

Fig. 4 Coefficient of variation (*a*) and skewness coefficient (*b*) of the water level probability density functions along the Garonne River, for different uncertainty sources (*Q* discharge uncertainty, K_S roughness uncertainty, QK_S combined discharge and roughness uncertainty) and different flow rates

fluctuates along the Garonne River. Moreover, in the Po River case, the asymmetry is almost the same for the two simulated discharges, while in the Garonne River, the skewness coefficients differ for the two cases. These differences may be explained considering their different geometries: the cross sections of the Garonne River are composite, while the study reach of the Po River is characterised by a main channel only. The greater complexity of the Garonne geometry may be responsible for the variations of the asymmetry along the river and between the two simulated discharges.

The degree of uncertainty of the water level predictions can be quantified by calculating the quantiles of the empirical PDFs obtained by the Monte Carlo procedure. This may give an insight on the importance of uncertainty in the water level calculations of 1D hydraulic models. For this purpose, we evaluated the 0.75, 0.9 and 0.95 quantiles of the water level probability density functions, all along the test rivers, and we compared them with the deterministic water levels (i.e. the water levels computed from the mean flow discharge and/or the mean Strickler coefficient). Table 2 reports the minimum, average and maximum differences between the quantiles and the deterministic water levels, computed along the rivers for the different uncertainty sources. From the data, one can conclude that the uncertainty in water level calculations is far from being negligible, especially when both discharge and roughness uncertainties are present. In this case, there is a 10% probability that the water levels are actually higher than their estimated value, by 0.85 and 0.57 m on average, for the Po River and Garonne River, respectively. There is also a small, but not negligible, probability (5 %) of estimating water levels lower than the actual value by more than 1 m for the Po River and about 0.7 m for the Garonne River. The quantile data can also be useful to identify the river cross sections where the water level calculations are subject to higher uncertainty and the locations which are more sensible to errors in discharge or roughness estimation.

Table 2 Minimum, average and maximum values along the Po River and Garonne River of the differences between the 0.75, 0.9 and 0.95 quantiles of the water level probability density functions (h_{q75}, h_{q90}, h_{q95}) and the deterministic water levels (h^), for different uncertainty sources

	$h_{q75}-h$^ (m)	$h_{q90}-h$^ (m)	$h_{q95}-h$^ (m)
Po River			
Q	0.25/0.29/0.32	0.47/0.55/0.61	0.60/0.71/0.78
K_S	0.25/0.34/0.38	0.48/0.65/0.72	0.61/0.83/0.92
QK_S	0.35/0.45/0.49	0.67/0.85/0.93	0.86/1.09/1.20
Garonne River			
Q	0.20/0.22/0.25	0.38/0.42/0.47	0.49/0.54/0.60
K_S	0.13/0.20/0.25	0.24/0.38/0.47	0.31/0.49/0.61
QK_S	0.24/0.30/0.34	0.46/0.57/0.65	0.59/0.73/0.83

Q discharge uncertainty, K_S roughness uncertainty, QK_S combined discharge and roughness uncertainty

To quantify the relative importance of the two uncertainty sources in determining the output uncertainty, the following sensitivity indices [52] have been computed:

$$S_Q = \frac{\rho_{Qh}^2}{\rho_{Qh}^2 + \rho_{K_S h}^2}$$
$$S_{K_S} = \frac{\rho_{K_S h}^2}{\rho_{Qh}^2 + \rho_{K_S h}^2} \qquad (11)$$

where ρ is the Spearman correlation coefficient between the vector of random inputs (Q or K_S) and the corresponding output vector (h).

Figure 5 shows the sensitivity indices along the Po River and Garonne River. It can be noticed that discharge is the key uncertainty source in proximity of the end of the reaches, probably due to the influence of the downstream boundary condition. As we move upstream, the sensitivity index of discharge decreases, and towards the upstream end, the roughness coefficient contributes most to the output uncertainty. The same behaviour is observed for the two study cases, but for the Po River, the sensitivity of the resulting water levels to roughness uncertainty is higher than that for the Garonne case.

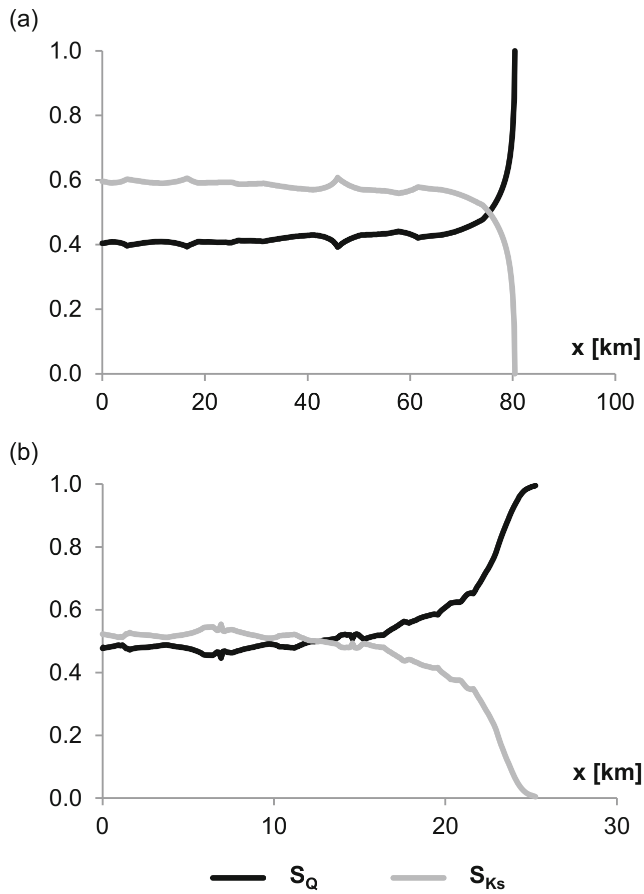

Fig. 5 Sensitivity indices along the Po River (*a*) and Garonne River (*b*)

5 Discussion and Conclusions

In this study, we investigated the effect of two key uncertainty sources on water level calculations in 1D hydraulic models: the roughness coefficient and the upstream discharge. The work examines the effect of these uncertainties, considered both individually and combined together, on the probability density functions of the predicted water levels. Uncertainty propagation is studied for the case of uniform flow in prismatic rectangular channels and for steady flow in two rivers: the Po River in Italy and the Garonne River in France.

In the uniform flow case, the exact probability density functions of the water levels are computed analytically through the derived distribution method. Two main conclusions are drawn about the statistical properties of the resulting PDFs. Firstly, the water level distributions have a lower coefficient of variation than the input PDFs, whatever the input is (discharge or roughness). This is due to the mathematical nature of the relationship between input and output, which determines a reduction of the coefficient of variation when the input is propagated through the model into the output. However, it is important to notice that epistemic uncertainties due to model structure and modelling assumptions have been neglected in this analysis. If considered, they would have led to higher output uncertainties.

Secondly, the propagation of symmetric input distributions through the uniform flow equations gives asymmetric output distributions, due to model nonlinearities. In particular, discharge uncertainty leads to left skewed output PDFs, while roughness uncertainty generates output distributions with heavier right tails. This indicates that symmetrical distributions cannot be used to represent water level uncertainty even if input PDFs are symmetrical. This is particularly relevant for roughness uncertainty because it implies underestimation of high quantiles. On the other hand, when discharge uncertainty is crucial, the assumption of symmetrical PDFs would lead to overestimation of high quantiles, hence to precautionary estimates.

The results of the Po River and Garonne River show that water level PDFs have a lower coefficient of variation than the input PDFs. This was already observed in the uniform flow case, and it is a result of the mathematical relationship between input and output. The results of the steady flow cases also confirm that water level distributions are left skewed in the case of roughness uncertainty and right skewed in the case of discharge uncertainty. When both uncertainty sources are propagated together, the skewness of the output PDF is close to 0.

The Po River results indicate that when roughness and discharge have the same level of uncertainty, the former leads to higher uncertainty of the water level predictions. Differently, in the Garonne case, the coefficient of variation of the water level PDFs is almost the same for both uncertainty sources. This is likely due to the different quantification of uncertainties for the Garonne River: uncertainty in Strickler coefficient is assumed to be higher than uncertainty in discharge, due to the presence of floodplains. Similarly, it can be observed that water level calculations are more sensitive to uncertainty in the Strickler coefficient for the Po case, while there is no clear dominating uncertainty source in the Garonne model.

The steady state results suggest that the probability of underestimating water levels (up to 1 m) is far from being negligible, especially when both discharge and roughness are subject to uncertainty. These results are undoubtedly associated with the study sites. However, it is important to notice that the conditions of the studied rivers can be considered representative for many alluvial European rivers.

Finally, it should be noticed that the statistical properties of the output variable are highly dependent on the coefficient of variation of the input, as shown by the uniform flow results and previous studies [e.g. 51]. A crucial issue in uncertainty analysis is represented by the quantification of parameter and input uncertainties, in terms of coefficients of variation and associated distributions. Information on the level of uncertainty of hydraulic and hydrologic variables is usually not available, and assumptions have to be made on the magnitude of the uncertainties. In this work, both discharge and channel roughness are assumed to have the same coefficient of variation, while the uncertainty in the floodplain Strickler coefficient was assumed to be twice as much, based on expert opinion. Other catchments could have been characterised by different levels of uncertainty of the hydraulic and hydrologic variables so that different conclusions could have been drawn with respect to the dominating uncertainty source. For example, discharge uncertainty may be significantly higher than roughness uncertainty in ungauged basins [53], while the opposite may occur when extreme flood events are simulated because no calibration data is available [54].

The present work is far from being complete. At this stage of the investigation, we only considered parameter and input uncertainty, disregarding other relevant sources of uncertainty. Future work should consider rating curve uncertainties and uncertainties in model structure, comparing 1D and 2D flood routing approaches and steady and unsteady flow analysis.

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