On Optimal Infrastructure Sharing Strategies in Mobile Radio Networks

Lorela Cano, Antonio Capone, Senior Member, IEEE, Giuliana Carello, Matteo Cesana, Member, IEEE, and Mauro Passacantando

Abstract—The rapid evolution of mobile radio network technologies poses severe technical and economical challenges to mobile network operators (MNOs); on the economical side, the continuous roll-out of technology updates is highly expensive, which may lead to the extreme, where offering advanced mobile services becomes no longer affordable for MNOs which thus, are not incentivized to innovate. Mobile infrastructure sharing among MNOs becomes then an important building block to lower the required per-MNO investment cost involved in the technology roll-out and management phases. We focus on a radio access network (RAN) sharing situation where multiple MNOs with a consolidated network infrastructure coexist in a given set of geographical areas; the MNOs have then to decide if it is profitable to upgrade their RAN technology by deploying additional small-cell base stations and whether to share the investment (and the deployed infrastructure) of the new small-cells with other operators. We address such strategic problems by giving a mathematical framework for the RAN infrastructure sharing problem which returns the “best” infrastructure sharing strategies for operators (coalitions and network configuration) when varying techno-economic parameters such as the achievable throughput in different sharing configurations and the pricing models for the service offered to the users. The proposed formulation is then leveraged to analyze the impact of the aforementioned parameters/input in a realistic mobile network environment based on LTE technology.

Index Terms—RAN sharing, heterogenous networks, 4G, mathematical programming, game theory.

I. INTRODUCTION

MOBILE telecommunications networks and services have been characterized by a dramatic uptake in the past two decades which is still to be over. According to [1], the penetration of mobile subscriptions has reached the amazing level of 96% worldwide in 2014, and the traffic delivered through mobile radio networks is expected to reach 49 Exabytes/month by 2021 [2] with a considerable share taken by bandwidth-eager services provided by aggressive Over The Top service providers.

To cope with such fast growing rate, the mobile networks have undergone, and are still undergoing, several technology migration phases cruising from the introduction of third generation (3G) and 3.5G wireless technologies on top of 2G networks to the standardization and deployment of the Long Term Evolution (LTE) with the recent launch of 5G initiatives [3]. The effect of such rapid evolution in the mobile networks technologies poses several technical and economical challenges to Mobile Network Operators (MNOs). On the technical side, the coexistence of multiple technologies in the Radio Access Network (RAN) calls for advanced radio resource orchestration procedures to cope with such heterogeneity. On the economical side, the combined effect of revenues of MNOs that tend to flatten and the network technology updates that are highly expensive may lead to the extreme where offering advanced mobile services becomes no longer affordable for MNOs which are not incentivized to innovate and migrate to new technologies [4].

In this context, the conventional model according to which each MNO retains complete control and ownership of its network is at odds with the large and frequent investments requested on the network infrastructure, and with the increased complexity in the management of the network components. Mobile infrastructure sharing among MNOs thus becomes an important building block to “break” such vertical and inflexible approach, by lowering the required per-MNO investment cost to cope in the technology roll-out and management phases.

Different forms of infrastructure sharing are already in place, ranging from basic unbundling and roaming, to site and spectrum sharing [5]. In these “classical” forms of sharing generally one MNO still retains ownership of the mobile network. On the other hand, we focus here on a RAN sharing scheme in which MNOs share a single radio infrastructure while maintaining separation and full control over the back hauling and respective core networks. In this work, we consider a scenario where multiple MNOs with a consolidated macro cells network infrastructure and consolidated market shares coexist in a given set of geographical areas; the MNOs have to decide if it is profitable to upgrade their RAN technology by deploying additional small-cell base stations and whether to share the investment (and the deployed infrastructure) of the new small-cells with other operators.

We address such strategic problem by providing a mathematical framework for the analysis of the RAN infrastructure sharing problem that takes into account both technical and economical aspects and provides the optimal sharing strategies for MNOs, that include coalitions with other MNOs and network configuration. The proposed infrastructure sharing problem is first tackled from the perspective of a regulatory
entity that can impose sharing configurations maximizing the
good of service perceived by all users and then from a
single MNO perspective, in order to account for MNOs as
profit-maximizing selfish entities. A Mixed Integer Linear
Programming (MILP) formulation is proposed to determine
sharing configurations maximizing the quality of service; this
formulation includes techno-economic parameters such as the
achievable throughput and the pricing models for the service
offered to the users. For representing the MNO perspective,
we propose a Non Transferable Utility (NTU) coalitional
game model. The proposed mathematical framework is then
leveraged to analyze the impact of the aforementioned para-
meters in a realistic mobile network environment based on
LTE technology for which numerical values for technical and
economic parameters are available. Note however that the
proposed approach is general and can be easily applied to
other scenarios with different small cell technologies.

The manuscript is organized as follows: Sec. II reviews
the mainstream literature in the field of infrastructure sharing
highlighting the main novelties of the proposed approach.
In Sec. III, we introduce the reference scenario describing
the techno-economic parameters involved in the infrastructure
sharing problem and the proposed mathematical framework
that allows to represent the problem from the two considered
perspectives. Sec. IV describes the considered scenarios and
cases while results and insights are reported in Sec. V, where
the strategic behavior of MNOs in several different realistic
scenarios is analyzed. Our concluding remarks are given
in Sec. VI.

II. RELATED WORK

The literature on infrastructure/resource sharing can be
viewed in two main research tracks: (i) works dealing with
techno-economic modeling of network sharing and (ii) works
on practical algorithms for management and allocation of
shared network resources. The first track mostly includes qual-
itative and quantitative studies of different sharing scenarios
and models for estimating capital and operational expendi-
tures. Particular attention is dedicated to the identification
of drivers and barriers to network sharing or possible new
organization of the mobile network value chain for sharing to
be viable.

Meddour et al. [6] suggest guidelines for MNO involved
in the sharing process and emphasize the need for subsidi-
dization and assistance from regulatory entities. Similarly,
Beckman et. al [7] show that the role of regulatory entities
is crucial to avoid the decline of market competition.

A recent work by Di Francesco et al. [8] introduces a
competition-aware network sharing framework in the context
of cellular network planning which allows to balance the
cost benefit of sharing and the push toward next-generation
technologies.

The authors of [9] model the capital and operational expendi-
tures for different levels of sharing and suggest outsourcing
as the solution to the challenges posed by network sharing.
In [10], the authors propose a benchmark-based model that
provides high-quality cost estimates for alternative delivery
options of the MNO processes such as “regionalization”,
“centralization” and “outsourcing”. Vaz et al. propose a frame-
work to evaluate the performance of heterogeneous network
deployment patterns in terms of net present value, capacity,
coverage, and carbon footprint [11]. By means of a technoe-
conomic analysis, the work in [12] addresses the cost/revenue
viability of different WLAN value network configurations
in the presence of MNOs and Service Application Providers and
the use cases for which there is incentive to share.

In the field of strategic modeling of resource/infrastructure
sharing, it is worth mentioning the works resorting to game
theory. Malanchini et al. [13] resort to non-cooperative games
to model the problems of network selection, when users can
choose among multiple heterogeneous wireless access, and
of resource allocation in which mobile network operators
compete to capture users by properly allocating their radio
resources. In [14], spectrum sharing among selfish MNOs in
unlicensed bands is modeled as a non-cooperative game. The
work in [15] and more extensively in [16] also use a non-
cooperative game to model the strategic decision of a MNO
regarding sharing its LTE infrastructure in a non–monopolistic
telecom market. Another example of 4G infrastructure shar-
ing is given in [17] which considers sharing LTE access
network femtocells with other access technologies such as
Wi–Fi. Cooperative game theory is used in [18] and [19];
in [18], the resource allocation problem in a shared network
is formalized in a two step problem: resource sharing among
the operators and resource bargaining among the users and
Mobile Virtual Network Operators of each operator; the work
in [19] considers not only sharing among MNOs but also
among operators of different wireless access technologies.

The research track on practical aspect of resource/infrastructure sharing focuses on algorithms and architectures
for managing shared resources. The work in [20] suggests
that radio resource management is handled by a third-party
service provider or an inter-connection provider to preserve
competition and reduce exposure. Anchora et al. ([21]) intro-
duce a ns-3 implementation to assess the performance of
spectrum sharing in a LTE multi-node/multi-MNO scenario,
where a virtual central entity is responsible for applying
the sharing policies to the common frequency pool. In [22],
virtualization of the wireless medium (spectrum sharing) is
proposed to exploit spectrum multiplexing and multi-user
diversity while allowing MNOs to remain isolated. Instead,
the authors in [23] introduce the Network without Borders
concept as a pool of virtualized wireless resources with a
shared radio resource manager. Along the same lines, Rahman et al. ([24]) introduce a novel architecture based on
wireless access network virtualization, where the key tenet is
to offload the baseband process from physical base station to
backend devices; in this way, the physical base stations can be
sliced into virtual base stations. In [25], instead, a 2-level radio
resource scheduling (among MNOs and for each MNO among
its user flows) BS virtualization scheme satisfying the 3GPP
SA1 RSE ([26]) requirements has been proposed. The work
in [27] proposes the necessary LTE architectural enhancements
to adopt capacity, spectrum and hardware sharing, and pro-
vides a simulation-based comparative performance analysis
of the proposed sharing scenarios and of no sharing case.
Johansson [28] provides an algorithm for fair allocation of the shared radio resource among multiple operators.

The aforementioned literature work either abstracts away technical aspects related to the mobile network performance to focus on more economic-oriented analysis and modeling, or the other way around. In our previous work [29], we focus on infrastructure sharing in a single and homogeneous geographical area. To the best of our knowledge, ours is one of the first attempts to strike a better balance between these two aspects of the sharing problem, by quantitatively modeling the relation between technical issues related to the radio communication at the access interface (area coverage, transmission rate, user density and quality observed by users) with economic issues (deployment costs and revenues) in mobile network infrastructure sharing. In this work we provide a more general framework which captures large-scale sharing scenarios featuring multiple geographical areas. Further, we consider two different perspectives: the single decision maker one, where the decision maker is a regulatory authority, and the multiple decision makers perspective, that accounts for the single MNO point of view.

III. MODELING THE PROBLEM OF MOBILE NETWORK INFRASTRUCTURE SHARING

We decided to explore two alternative infrastructure sharing configurations: socially optimal configurations providing the best service level for the users, which can be imposed by a regulatory authority1 and stable configurations representing a setting where MNOs act as selfish entities aiming to maximize their profits from upgrading their network. While a centralized approach allows to model the problem of determining socially optimal configurations, cooperative game theory is more suitable to determine stable configurations. In Section III-A, we introduce the techno-economic parameters representing the considered scenario and provide an MILP formulation for the centralized approach. In Section III-B, we discuss how an NTU cooperative game is adopted to determine stable configurations. We remark that in Sections III-A and III-B, we use the term coalition with a slight abuse of terminology to represent a set of MNOs which build a unique shared network, both when they decide to join the coalition based on their profit and when the coalition is suggested as a socially optimal choice. In III-A, the socially optimal coalitional structure (partition of the set of MNOs) is selected according to the regulator point of view and each MNO is assigned to its corresponding coalition. Instead, in III-B, each MNO joins the coalition that maximizes its individual profit; in other words, a coalition is stable when none of its members has an incentive to leave the coalition.

A. Socially Optimal Coalitional Structures-an MILP Formulation

We consider a set \( O \) of MNOs who have up and running 3G/4G networks over a set \( A \) of dense urban areas: each area \( a \in A \) is populated by \( N_a \) users and has a size \( A_a \). Parameter \( \sigma_i \) gives the share of users of MNO \( i \in O \) which is assumed to be equal in each area. The MNOs may consider investing to deploy additional LTE small-cells (HetNets) in some or all the areas. A MNO can either invest by itself or share the investment (and the deployed infrastructure) with a subset (or all) of the other MNOs. Let \( S \) denote the set of all possible coalitions that can be activated for the given set of MNOs (here we consider all possible non-empty subsets, thus \( |S| = 2^{|O|} - 1 \)). If a MNO invests by itself, the coalition is referred to as singleton. \( S_i \) is the set of coalitions MNO \( i \) can be part of. Each MNO inherits the customer base from its current network, assuming that users do not change their MNO but may subscribe to a new (LTE) data plan.

We consider the problem of determining the socially optimal sharing configurations, that is, how to partition MNOs in coalitions and how many small-cell base stations (BSs) each coalition of MNOs should activate in order to maximize the global service level provided to the users.

In each area a maximum number \( U_{max} \) of BSs can be activated by all coalitions.

Users are characterized by parameter \( \delta \) that represents their willingness to pay for 1 Mbps of LTE rate on a monthly basis and therefore the monthly price of 1 Mbps.

We consider an investment lifetime \( D \) (in months). The investment costs are then calculated over the whole \( D \) period. Both capital (e.g., site and BS acquisition) and operational (e.g., hardware and software maintenance, land renting and power supply) expenditures contribute to the overall costs of the infrastructure [6].

Let \( g_{\text{capex}} \) and \( g_{\text{opex}} \) denote the fixed CAPEX and annual OPEX components, respectively. \( g_{\text{capex}}^a \) is calculated as a fixed percentage (\( \zeta \)) of \( g_{\text{capex}} \), i.e., \( g_{\text{capex}}^a = \zeta g_{\text{capex}} \). We denote by \( g \) the cost of a single BS for the investment lifetime \( D \) which is determined as the sum of the fixed initial CAPEX and the OPEX accumulated during \( D \), i.e.,

\[
g = g_{\text{capex}} + \frac{1}{12}Dg_{\text{opex}}^a.
\]

The BSs installation cost of a coalition is then divided among the coalition members based on their market shares.

We assume that the same coalitional structure will apply to all areas, that is, MNOs will be assigned to the same coalition in all areas, as it can be easier for MNOs to coordinate with the same set of MNOs in all the areas.2 Table I recaps the problem’s parameters notation.

The partitioning of the set of MNOs \( O \) into a socially optimal coalitional structure is modeled as follows. Binary variables \( y_s \) represent the coalition activation: \( y_s \) equals one if coalition \( s \) is activated in all the areas \( a \in A \) and it invests (deploys BSs) in at least one of them. The binary variable \( x_{is} \) is equal to one if MNO \( i \) is assigned to coalition \( s \in S_i \) and \( s \neq S_i \).

\[1\] In the case of stable sharing configurations, as MNOs decide by themselves which coalition to join, selecting the same coalition (set of collaborating MNOs) in all the areas might also require less time for the sharing agreements to be approved by regulators. Nevertheless, we have also investigated the case in which MNOs are assigned/select a different coalition in each area, which overall does not provide significant gains with respect to forcing the same one over all areas.

\[2\]
invests, it equals zero if \( i \) is assigned to any other coalition in \( S_i \) but \( s \) or \( d \) does not invest. Constraints (2) guarantee that each MNO \( i \) is assigned to at most one coalition from \( S_i \). Constraints (3) make sure that if \( s \) is activated (\( y_s = 1 \)), all MNOs \( i \in s \) are assigned to \( s \).

\[
\sum_{s \in S} x_{is} \leq 1, \quad \forall i \in O, \quad (2)
\]

\[
x_{is} = y_s, \quad \forall s \in S, \quad \forall i \in s. \quad (3)
\]

If coalition \( s \) is activated, it will deploy a certain number of BSs for each area \( a \in A \), represented by a non-negative integer variable \( u^a_s \). If \( s \) is not activated or there is no investment (\( y_s = 0 \)), the corresponding variables \( u^a_s \), for each \( a \in A \), are forced to zero by means of Constraints (4). Conversely, a coalition is not active (\( y_s = 0 \)) if it does not deploy any BS in any of the areas (Constraint (5)); Constraint (6) limits the overall number of BSs deployed by all coalitions in each area.

\[
u^a_s \leq U_{\text{max}}^s, \quad \forall s \in S, \quad \forall a \in A, \quad (4)
\]

\[
y_s \leq \sum_{a \in A} u^a_s, \quad \forall s \in S, \quad (5)
\]

\[
\sum_{s \in S} u^a_s \leq U_{\text{max}}^a, \quad \forall a \in A. \quad (6)
\]

We assess the quality of service provided by MNOs through the average rate perceived by the users, which is an important indicator of the users’ level of satisfaction. This rate is different for each area \( a \in A \): firstly because we consider areas with different number of users (\( N_a \)) and size (\( A_a \)) and secondly because a different number of BSs (\( u^a_s \)) may be deployed in different areas. In the proposed model, we define two types of LTE user rate, namely nominal and average, for each coalition \( s \) in \( S \). The nominal user rate is the maximum achievable LTE rate for a certain level of Signal to Interference and Noise Ratio (SINR) and a given system bandwidth\(^3\) that a user perceives when assigned all downlink LTE resource blocks from its serving BS. The downlink SINR depends on the number of BSs activated by the coalition the user belongs to since a larger number of BSs results in the user being on the average closer to its serving BS, and thus receiving a stronger signal, but also closer to the interfering BSs.\(^4\) Thus, the nominal user rate of coalition \( s \) in area \( a \), represented by a non-negative continuous variable \( \rho_{s,a}^{\text{nom}} \), is a function of the number of deployed BSs \( u^a_s \). The behavior of \( \rho_{s,a}^{\text{nom}} \) as a function of \( u^a_s \) is investigated by simulating the deployment of the small cell BSs (see Subsec. IV-A).

Instead, the average user rate perceived by a user of coalition \( s \) in area \( a \) is represented by the continuous non-negative variables \( \rho_{s,a}^u \) and defined in terms of the nominal user rate (\( \rho_{s,a}^{\text{nom}} \)) and of the load of its serving BS as follows:\(^5\)

\[
\rho_{s,a}^u = \rho_{s,a}^{\text{nom}} (1 - \eta) \frac{\sum_{s \in S} y_s N_a}{u^a_s}, \quad \forall s \in S, \quad \forall a \in A, \quad (7)
\]

where parameter \( \eta \) is the user activity factor, that is, the probability that a user is actually active in his/her serving BS, \( \sum_{s \in S} y_s N_a \) is the total number of users that are served by members of coalition \( s \) in area \( a \), and the ratio \( \frac{\sum_{s \in S} y_s N_a}{u^a_s} \) is the average number of users served by one BS in area \( a \). As a result, \( \rho_{s,a}^{\text{nom}} \) is scaled down by the factor \((1 - \eta) \frac{\sum_{s \in S} y_s N_a}{u^a_s} \) which accounts for the average congestion level at a serving BS in \( a \).

In the MILP formulation, the nonlinearity of \( \rho_{s,a}^u \) in terms of \( u^a_s \) is handled by approximating \( \rho_{s,a}^u \) with a piecewise linear function described by the following constraints:

\[ral_{s,a}^i \leq y_s, \quad \forall s \in S, \quad \forall a \in A, \quad (8)
\]

\[
rho_{s,a}^u \leq R_{s,a}^{l,a} + a_{s,a}^{l,a+1} (u^a_s - U_{s,a}^{l,a}) + M (1 - \zeta_{s,a}^i), \quad (9)
\]

\[
z_{s,a}^i \geq 0, \quad \forall s \in S, \quad \forall a \in A, \quad (10)
\]

\[
\forall a \in A, \quad \forall s \in S, \quad \forall a \in A, \quad (11)
\]

where \( \zeta_{s,a}^i \) are binary variables that equal 0 if \( u^a_s = 0 \) (Constraints (7)) and therefore set to zero \( \rho_{s,a}^u \) when \( u^a_s = 0 \) (Constraints (9)). Constraints (8) model the piecewise linear functions approximating \( \rho_{s,a}^u \), for any \( s \in S, a \in A \), where \( L \) denotes the number of the linear pieces, \( a_{s,a}^{l,a} \) denotes the slope of the \( l \)-th linear piece, \( U_{s,a}^{l,a} \) and \( R_{s,a}^{l,a} \) are the coordinates (number of BSs and user rate, respectively) of the \((l+1)\) breakpoint whereas \( M \) is a big positive constant (see Appendix A for the details of the approximation).

Assuming that, in each area \( a \), users of any member of coalition \( s \) can be served by any of the BSs activated by \( s \) in \( a \), the average user rate provided by MNO \( i \) in area \( a \), represented by continuous non-negative variable \( q_{i,a}^u \), is equal to the average user rate of the coalition to which \( i \) is assigned, that is,

\[
q_{i,a}^u = \sum_{s \in S} \rho_{s,a}^u, \quad \forall i \in O, \quad \forall a \in A. \quad (10)
\]

As for the investment cost and revenues for the MNOs, it is reasonable to model the revenue\(^6\) per MNO \( i \) in area \( a \) as a continuous non-negative variable \( r_{i,a}^u \) which is linearly dependent on the MNO’s user rate \( q_{i,a}^u \) in that area as shown in (11): \( \delta q_{i,a}^u \) is the monthly revenue obtained from one user, which is then multiplied by the investment lifetime \( D \) and the number of users \( \sigma_i N_a \) of MNO \( i \) in area \( a \):

\[
r_{i,a}^u = \delta D \sigma_i N_a q_{i,a}^u, \quad \forall i \in O, \quad \forall a \in A. \quad (11)
\]

The cost incurred by MNO \( i \) in area \( a \), represented by non-negative continuous variable \( c_{i,a}^u \), is a linear function of the number of BSs activated in\( a \) by the coalition to which \( i \) is

\(^3\)The price per unit of service (\( \delta \)) represents the highest price all current users of each MNO are willing to pay for the new service. Therefore the number of users \( N \) is assumed independent of \( \delta \). Moreover, the proposed pricing model aims at translating the MNOs level of investment, which affects the service level perceived by users, into revenues. It is outside of the scope of the analysis we propose here to account for pricing models in line with those currently applied by MNOs which involve bundles of services, data caps etc. In the same lines, we do not account for the user migration among MNOs since it is generally determined by “non-technical” parameters such as special tariffs, bundle offers, brand fidelity and more in general marketing strategies.
TABLE I
SETS, PARAMETERS, AND CORRESPONDING VALUES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O$</td>
<td>Set of MNOs</td>
<td>${A,B,C}, {O}=3$</td>
</tr>
<tr>
<td>$A$</td>
<td>Set of Areas</td>
<td>${{1},{2},{3}}$</td>
</tr>
<tr>
<td>$S$</td>
<td>Set of coalitions</td>
<td>${A,B,C,AB,AC,BC,ABC}$</td>
</tr>
<tr>
<td>$S_i$</td>
<td>Set of coalitions MNO $i \in O$ can join</td>
<td>${\sigma \in \Sigma }$</td>
</tr>
<tr>
<td>$N_a$</td>
<td>Number of users of area $a \in A$</td>
<td>See Table IV</td>
</tr>
<tr>
<td>$A_a$</td>
<td>Size of area $a \in A$</td>
<td>See Table IV</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Market share of MNO $i \in O$</td>
<td>$M_i=(1/3,1/3,1/3)$</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>Max. number of BSs in the area</td>
<td>$4000$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Monthly price of 1 Mbps</td>
<td>Equidistant values in $[0.02,2]$€/Mbps</td>
</tr>
<tr>
<td>$D$</td>
<td>Investment lifetime [months]</td>
<td>$120$ [30],[28]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>User activity factor</td>
<td>$0.001$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>OPEX annual %</td>
<td>$15%$ [31]</td>
</tr>
<tr>
<td>$g_{\text{CAPEX}}$</td>
<td>CAPEX of BS cost</td>
<td>$30000€$</td>
</tr>
<tr>
<td>$g$</td>
<td>BS cost normalized for $D$</td>
<td>$7500€$</td>
</tr>
</tbody>
</table>

TABLE II
VARIABLE DOMAINS AND DESCRIPTION

| $\pi_a$ | $\in \{0,1\}$ if MNO $i \in O$ joins coalition $\sigma \in S$, in all areas, 0 otherwise |
| $\rho_s$ | $\in \{0,1\}$ if coalition $\sigma \in S$ is created in all areas, 0 otherwise |
| $u_s^a$ | Number of BSs activated by coalition $\sigma \in S$ in area $a \in A$ |
| $c_i^a$ | $\in \{0,1\}$ if coalition $\sigma \in S$ activates at least one BS in area $a \in A$, 0 otherwise |
| $\rho_a^{\text{min}}$ | Nominal user rate for coalition $\sigma \in S$ in area $a \in A$ |
| $\rho_s^{\text{up}}$ | User rate for coalition $\sigma \in S$ in area $a \in A$ |
| $q_i^{42}$ | User rate for MNO $i \in O \in A$ |
| $c_i^a$ | Costs of MNO $i \in O$ in area $a \in A$ |
| $r_i^{42}$ | Revenues of MNO $i \in O$ in area $a \in A$ |

assigned, divided among the coalition’s members proportionally to their number of users:

$$c_i^a = \sum_{s \in S} \sum_{j \in a} \sigma_{ij} u_s^a, \forall i \in O, \forall a \in A. \quad (12)$$

Although the *socially optimal* infrastructure sharing configurations provide the optimal service level for users, MNOs cannot be forced to undertake lossy investments. Therefore, Constraints (13) make sure that each MNO obtains a non-negative profit:

$$\sum_{a \in A} (c_i^a - c_i^a) \geq 0, \forall i \in O. \quad (13)$$

We consider two candidate objective functions to be maximized to determine the socially optimal sharing configurations:

$$\sum_{i \in O, a \in A} q_i^a, \quad (14a)$$

Objective (14a) favors efficiency by maximizing the sum of user rate over all MNOs and areas, whereas (14b) maximizes the smallest user rate (over all areas and MNOs), so as to privilege users’ fairness. We denote Objectives (14a) and (14b) by $\text{TOT}_Q$ and $\text{MIN}_Q$, respectively and use this notation throughout Section V. Sets and parameters describing the instances are recapped in Table I whereas variables in Table II.

In Appendix B, we prove that the decision version of the problem with objective $\text{MIN}_Q$ is NP-complete.

B. Stable Coalitional Structures - A Non Transferable Utility Cooperative Game Model

We now describe the problem of determining stable infrastructure sharing configurations. We assume that MNOs in a coalition will share their cost while each MNO will keep its individual revenue since the latter is incurred from its own share of users. As a result, the coalition worth, that is, the difference between the coalition global revenues and cost, cannot be redistributed among its members: therefore we adopt solution concepts of NTU cooperative games [32].

The game is formalized as a pair $(O, V)$, where the player set $O$ coincides with the set of MNOs and $V$ is a function that associates to each non-empty coalition $s \in S$ a subset of payoff allocation vectors $(\pi_i) \in O$, i.e.,

$$V(s) = \{ (\pi_i) | i \in O : \pi_i \leq p_i^s, \forall i \in s \},$$

where $p_i^s$ is the optimal payoff of player $i$ in coalition $s$.

Since each MNO is a self-interested entity that aims to maximize its individual profits from the investment, we define its optimal payoff $p_i^s$ from a given coalition as the largest profit (difference between total revenues and total cost) it can achieve if it becomes part of that coalition. Such payoffs are calculated in the following fashion: given a coalition $s \in S$, we determine the optimal number of BSs ($\tilde{u}_s^a$) activated in each area $a \in A$, calculate each member’s revenues and costs for each area and therefore calculate the MNO total profit.

The optimal number $\tilde{u}_s^a$ of BSs coalition $s$ can deploy in area $a$ is obtained solving the following problem:

$$\max \sum_{i \in s} r_i^a - c_i^a \quad (15)$$

$$\forall i \in s, \quad \pi_i^a = \delta D \sigma_i N_a \rho^a, \quad (16)$$

$$\forall i \in s, \quad c_i^a = \sum_{j \in a} g u_s^a, \quad (17)$$

$$\forall i \in s, \quad u_s^a \leq U_{\text{max}}, \quad (18)$$

$$\forall s \in S, \quad \rho^a \leq u_s^a, \quad (19)$$

$$\forall s \in S, \quad \rho_s^{\text{up}} \leq \rho_s^{\text{up}}, \quad (20)$$

$$\forall s \in S, \quad u_s^a \in \mathbb{Z}^+, \quad (21)$$

The objective function (15) can be rewritten as

$$\sum_{i \in s} \left( \delta D \sigma_i N_a \rho^a - \sum_{j \in a} \sigma_{ij} g u_s^a \right) \quad (22)$$

$$\sum_{i \in s} \left( \delta D \sigma_i N_a \rho^a - \sum_{j \in a} \sigma_{ij} g u_s^a \right) \quad (23)$$

where $\rho^a$ depends on $u_s^a$. As $\delta D \sigma_i N_a \rho^a - \frac{1}{\sum_{j \in a} \sigma_{ij}} g u_s^a$ is independent of the MNOs, the optimal number $u_s^a$ of BSs is the

7We remark that, in the problem we upper bound the number of BSs activated by each coalition in the area to $U_{\text{max}}$ (Constraint (18)) since, for the considered instances (see Section V), the total number of BSs activated by any partition of MNOs in the set $O$ does not exceed $U_{\text{max}}$, that is, the more stringent Constraint (6) which limits the number of BSs activated by all coalitions in the area to $U_{\text{max}}$ is never tight.
same for all the players and can be easily computed solving the above problem.

Therefore, the optimal payoff \( p^i_s \) of each MNO \( i \in s \) is

\[
p^i_s = \sum_{a \in A} \left( \delta D \sigma_i N_a \rho_s^a (\bar{a}^i_s) - \sum_{j \in s} \frac{\sigma_i}{\sigma_j} g^i_a \right)
\]

\[
= \frac{\sigma_i}{\sum_{j \in s} \sigma_j} \sum_{a \in A} \left( \delta D N_a \rho_s^a (\bar{a}^i_s) \sum_{j \in s} \sigma_j - g^i_a \right).
\]

In other words, the optimal payoff allocations \( p^i_s \) correspond to dividing the optimal worth of coalition \( s \), i.e.,

\[
\sum_{a \in A} \left( \delta D N_a \rho_s^a (\bar{a}^i_s) \sum_{j \in s} \sigma_j - g^i_a \right),
\]

among its members according to their relative market shares, i.e., \( \sigma_i / \sum_{j \in s} \sigma_j \).

In the following we look for stable infrastructure sharing configurations. We define a sharing configuration as a partition \( (s_1, \ldots, s_p) \) of the MNOs set \( S \), where coalitions \( s_1, \ldots, s_p \in S \). A configuration \( (s_1, \ldots, s_p) \) is said stable if for any \( j = 1, \ldots, p \) there is no nonempty subset \( s'_j \subset s_j \) such that

\[
p^i_{s_j} > p^i_{s'_j}, \quad \forall \ i \in s'_j,
\]

that is, for any coalition \( s_j \) no subset of MNOs has incentive to leave it.

IV. EXPERIMENTAL SETTINGS

We run several tests to evaluate how the coalitional structure, the level of investment, and therefore the performance indicators of both the socially optimal and stable configurations are affected by the user economic standpoint.

The MILP model (Section III-A) and problem (15)–(22) for any \( s \in S \) and \( a \in A \) (Section III-B) have been implemented in AMPL [33]. We have used Gurobi 6.0 [34] as a MILP solver. All tests were run on an Intel(R) Core(TM) i5-3230M CPU @2.6 Gzh. To keep the computational time limited, for some of the instances the acceptable relative MIP gap of Gurobi was set equal to 1e-6. When optimizing \( MIN_Q \), several equivalent optimal solutions may be found, which may not provide consistent values for the user rate of the non-bottleneck areas and MNOs. When needed, they have been computed in post-processing.

A. BS Deployment Simulation

A simulation environment was set up to derive the coalition user rate per area \( \rho^a_s \) as a function of each possible number \( u^a_s \) of BSs that coalition \( s \) can activate in area \( a \), i.e., from 1 up to \( U_{max} \). In details, the entire set of \( U_{max} \) BSs is uniformly distributed in a pseudo-random fashion on the considered square areas; 10 sample users are also randomly distributed over each area \( a \). The downlink SINR of each sample user in \( a \) for each coalition \( s \) (\( SINR^a_s \)) is calculated for each possible value of \( u^a_s \) as a function of: the signal power \( P_k \) the sample user receives from its serving BS \( k \) (i.e., the BS from which it receives the strongest signal), the signal power \( \sum_{j \neq k} P_j \) received from the interfering (non-serving) BSs and the white Gaussian noise signal power\(^8\) \( P_{noise} \). Since users are characterized by an activity factor \( \eta \), the captured interference is scaled down by the load of coalition \( s \) in area \( a \), i.e.,

\[
l^a_s = 1 - (1 - \eta) \frac{\rho_s^a}{\sigma^a_s} \cdot SINR^a_s
\]

is therefore calculated as follows:

\[
SINR^a_s = \frac{P_k}{l^a_s \left( \sum_{j \neq k} P_j \right) + P_{noise}}, \quad \forall \ s \in S, \forall \ a \in A. \quad (25)
\]

The received signal power \( P_{tx} \) has been calculated according to a three-parameter path loss model (transmitted signal power \( P_{tx} \), fixed path loss \( C_{pl} \) and path loss exponent \( \Gamma \)) defined within the GreenTouch Consortium [35]:

\[
P_{tx} = P_{tx} \left( 0.5 + 10 \log(d[km]) \right), \quad (26)
\]

where \( d \) is the sample user–BS distance. The calculated SINR is finally mapped to LTE nominal rate \( (\rho^a_s, nom) \) according to a multilevel SINR–to–rate scheme [35]. A single value for \( \rho^a_s, nom \) is obtained by averaging over the 10 sample users. An additional averaging is obtained by applying 100 iterations for each value of \( u^a_s \); \( \rho^a_s \) is then calculated analytically as the product \( \rho^a_s (1 - \eta) \frac{\sigma^a_s}{\rho^a_s} \), according to the definition in Section III-A.

B. Instances

We consider three square dense areas (their size and number of users are provided in Table III) and three MNOs (A, B and C) which is quite reasonable for the Italian (also European) telecom playground [16]. Assuming the dense urban areas belong to the same city, we consider the same distribution of users among MNOs in all of them. We report the results obtained for two such user distributions: \( M_1 \), MNOs have equal market shares (\( \sigma_A = \sigma_B = \sigma_C = 1/3 \)) and \( M_2 \), for which the market shares of A, B and C are 10%, 30% and 60%, respectively (\( \sigma_A = 0.1, \sigma_B = 0.3, \sigma_C = 0.6 \)).

The values of the user’s willingness to pay for 1 Mbps on a monthly basis \( \delta \) were deduced from current data tariff-plans applied by different Italian MNOs. We have considered 100 values in the range [0.02, 2] €/Mbps which were obtained discretizing the range uniformly with a 0.02 step.

The number of available sites for installing small cell BSs in a given geographical area is finite and most likely different for each area. We set \( U_{max} \) to 4000 for all the considered areas; such number of BSs it at least one order of magnitude larger than the minimum needed for coverage\(^9\) whereas deploying

\[
\begin{array}{|c|c|c|}
\hline
\text{Area} & \text{Number of users} & \text{Size} \\
\hline
Z_1 & N_1 = 20000 & A_1 = 4 \text{ km}^2 \\
Z_2 & N_2 = 20000 & A_2 = 0.5 \text{ km}^2 \\
Z_3 & N_3 = 10000 & A_3 = 1 \text{ km}^2 \\
\hline
\end{array}
\]

\( 8\)The white Gaussian noise signal power accounts for the considered system bandwidth.

\( 9\)If we consider small cells of 50 m range, the minimum number of small cell BSs for coverage would be roughly 500 for the largest area (Z1).
more BSs would result in only a marginal increase of the average user rate $\rho_s$ for the considered instances (see Figure 3).

The investment lifetime period $D$ is set to 120 months (see, e.g., [28], [30]) for all instances.

For the two user distributions we generate a scenario for each value of $\delta$, while the rest of parameters ($\omega$, $A$, $Na$, $A_o$, $g$, $U_{\text{max}}, \eta$, $D$) are fixed to the values provided in Table I.

V. Results

In this section, we examine the impact of the user economic standpoint (different values of $\delta$) and of the user distribution among MNOs ($\sigma_i$) on the coalitional structures and the level of investment first of the socially optimal configurations (Subsection V-A) and then of the stable configurations (Subsection V-B). The two configurations are then compared in Subsection V-C.

We recall that the user rate as a function of the number of deployed BSs for the different sharing configurations was obtained by means of simulation (Subsection IV-A) and that it behaves nonlinearly in the number of BSs; to obtain a MILP formulation of the problem, we have approximated the user rate functions with piecewise linear ones (see Subsection III-A, Appendix A). In order to account for the error introduced by the approximation, we investigate multiple configurations which perform very similarly. This allows us to identify general trends concerning the size and composition of the selected coalitional structures as we vary $\delta$ and the user distribution. For each value of $\delta$, we consider as socially optimal sharing configurations the ones selected by the optimal solution of problem (2)-(13), solved either under objective $TOT_Q$ or $MIN_Q$ (14a) or $MIN_Q$ (14b), and all configurations for which the objective function value is at most 0.5% smaller with respect to the optimal one. Similarly, for stable sharing configurations, we relax the stability condition as follows: we consider a configuration $(s_1, \ldots, s_p)$ to be stable if for any $j = 1, \ldots, p$ there is no nonempty subset $s'_j \subset s_j$ such that

$$\frac{p'_{s_j} - p_{s_j}}{p'_{s_j}} > 0.5\%, \quad \forall i \in s'_j.$$

The different outcomes are denoted by the following notation: $ABC$ represents the grand coalition, coalitional structures that consist of a singleton (i.e., a MNO investing alone) and a coalition of two MNOs are denoted by $A/BC$, $B/AC$ and $C/AB$,\(^\text{10}\) whereas the case when no sharing takes place, that is, when each MNO invests by itself, is denoted by $A/B/C$.

For each possible outcome, we report the values of $\delta$ for which the outcome is socially optimal under objectives $TOT_Q$ and $MIN_Q$ in Tables IV and V for user distributions $M_1$ and $M_2$, respectively. The results concerning the stable configurations are reported in Tables VIIIa and VIIIb for user distributions $M_1$ and $M_2$, respectively.

Concerning the level of investment, we report the number of BSs deployed by the sharing configurations only for a subset of the considered values of $\delta$ (i.e., {0.02, 0.04, 0.2, 0.4, 1, 2}) due to space limitations. For all values of $\delta$ for which we have identified multiple configurations (as illustrated in Tables IV, V, VIIIa and VIIIb), we report the results of the configuration selected by the optimal solution of the MILP model for the socially optimal configurations in Tables VI and VII, for user distributions $M_1$ and $M_2$, respectively. Similarly, when multiple configurations are stable, only one of them is reported in Tables IXa and IXb, for user distributions $M_1$ and $M_2$.

\(^\text{10}\)We remark that outcomes $A/BC$, $B/AC$ and $C/AB$ are equivalent for user distribution $M_1$ since MNOs have equal market shares.
the first number represents the number of BSs deployed by the coalition of two, whereas for outcome A/B/C, the number of BSs deployed by each MNO are reported in order (i.e., the first number corresponds to A, the second to B and third to C).

A. Socially Optimal Configurations

As a general rule, results show that as users are willing to pay more (i.e., for higher values of $\delta$) and, as a result, MNOs can afford a larger network cost, the socially optimal configurations consist of smaller and less congested coalitions in order to provide the best service level. Regarding the level of investment, the higher the value of $\delta$, the denser the network deployment as larger revenues make up for increasing network cost.

For very low and high values of $\delta$, results are very similar for both user distribution scenarios ($M_1$ and $M_2$). The grand coalition (ABC) outperforms the other configurations for $\delta = 0.02$ for $TOT_Q$ and for $\delta \leq 0.04$ for $MIN_Q$ for both $M_1$ and $M_2$. Although ABC is selected also for few other low values of $\delta$ for both objectives and user distributions, it performs similarly to other outcomes (Tables IV and V): e.g., for $M_2$, ABC is selected by $TOT_Q$ also for $\delta = 0.06$ but performs similarly to C/AB. Instead, A/BC, which represents the case when no sharing takes place, is always among the selected outcomes for $\delta \geq 0.06$ for both $TOT_Q$ and $MIN_Q$ for $M_1$ (Table IV) and for $\delta \geq 0.28$ for $TOT_Q$ and for $\delta \geq 0.26$ for $MIN_Q$ for $M_2$ (Table V).

However, for intermediate values of $\delta$, results seem more sensitive to the user distribution. For $M_1$, the equivalent outcomes A/BC, B/AC and C/AB are selected for almost all values of $\delta$ in $[0.06, 2]$ for $TOT_Q$ and for some values of $\delta$ in $[0.06, 0.22]$ for $MIN_Q$ (Table IV). However, since they are always selected alongside A/B/C, that is, they perform very similarly to the case when there is no sharing, there is practically no incentive for sharing also for intermediate values of $\delta$ for $M_1$. Instead for $M_2$, for $\delta$ in $[0.08, 0.26]$, the only socially optimal configurations selected by $TOT_Q$ are C/AB and, for a subset of the values of $\delta$ in this range, also B/AC (Table Va); similarly for $MIN_Q$ for $\delta$ in $[0.12, 0.24]$ (Table Vb). In C/AB and B/AC, both coalitions of two MNOs, AB and AC, involve A which has the smallest market share (10%) and therefore introduces the minimum level of interference to a coalition. Moreover, for low values of $\delta$, A benefits from being in a coalition since it cannot afford to invest sufficiently by itself given its small market share.\(^\text{11}\)

\(^{11}\)For instance, if all MNOs were to invest by themselves, for $\delta \leq 0.26$ users of MNO A would perceive the worst service level (user rate) due to A’s low level of investment. Instead, for $\delta \geq 0.28$, as A is able to densify its network, users of C perceive the lowest user rate since C is the largest/most congested MNO.

### TABLE VII
**Socially optimal Coalitional Structures and Corresponding Number of Activated BSs – User Distribution M2**

<table>
<thead>
<tr>
<th>$A/B$</th>
<th>0.02</th>
<th>0.04</th>
<th>0.2</th>
<th>0.4</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
</table>

*(a) $TOT_Q$*

### TABLE VIII
**VALUES OF $\delta$ FOR WHICH A COALITION STRUCTURE IS STABLE**

<table>
<thead>
<tr>
<th>Coalitional structure</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>[0.02, 0.1], [0.16, 0.22], 0.28</td>
</tr>
<tr>
<td>A/BC, B/AC, C/AB</td>
<td>[0.02, 0.52], 0.6, [0.98, 2]</td>
</tr>
</tbody>
</table>

*(a) User distribution $M_1$*

### TABLE IX
**Stable Coalitional Structures and Corresponding Number of Activated BSs**

<table>
<thead>
<tr>
<th>$A/B$</th>
<th>0.02</th>
<th>0.04</th>
<th>0.2</th>
<th>0.4</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z3</td>
<td>ABC</td>
<td>54</td>
<td>54</td>
<td>272</td>
<td>A/BC</td>
<td>178/274</td>
</tr>
</tbody>
</table>

*(a) User distribution $M_1$*

### TABLE VIII
**VALUES OF $\delta$ FOR WHICH A COALITION STRUCTURE IS STABLE**

<table>
<thead>
<tr>
<th>Coalitional structure</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>[0.02, 0.04], [0.1, 0.12], [0.18, 0.30]</td>
</tr>
<tr>
<td>A/BC</td>
<td>0.02, 0.06, [0.1, 0.14], [0.18, 0.36], [0.32, 0.54]</td>
</tr>
<tr>
<td>B/AC</td>
<td>[0.02, 0.08], [0.12, 0.16], [0.22, 0.52], [0.6, 2]</td>
</tr>
<tr>
<td>C/AB</td>
<td>[0.04, 0.06], [0.1, 2]</td>
</tr>
</tbody>
</table>

*(b) User distribution $M_2$*

### TABLE IX
**Stable Coalitional Structures and Corresponding Number of Activated BSs**

<table>
<thead>
<tr>
<th>$A/B$</th>
<th>0.02</th>
<th>0.04</th>
<th>0.2</th>
<th>0.4</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z1</td>
<td>ABC</td>
<td>74</td>
<td>169</td>
<td>700</td>
<td>C/AB</td>
<td>700/491</td>
</tr>
<tr>
<td>Z2</td>
<td>ABC</td>
<td>69</td>
<td>156</td>
<td>472</td>
<td>C/AB</td>
<td>700/465</td>
</tr>
<tr>
<td>Z3</td>
<td>ABC</td>
<td>12</td>
<td>66</td>
<td>287</td>
<td>C/AB</td>
<td>273/237</td>
</tr>
</tbody>
</table>

*(b) User distribution $M_2$*
Concerning the level of investment, in Tables VIa and VIb, we report the number of small cell BSs deployed in each area for the socially optimal sharing configuration selected by the optimal solution under $TOT_Q$ and $MIN_Q$, respectively, for a subset of the considered values of $\delta$ ($\{0.02, 0.04, 0.2, 0.4, 1, 2\}$) and user distribution $M_1$. Results concerning user distribution $M_2$ are reported in Tables VIIa and VIIb.

For most instances, both objectives $TOT_Q$ and $MIN_Q$ provide the same coalitional structures but slightly different number of deployed BSs. For instance, for user distribution $M_2$ and $\delta = 0.02$ (see Tables VIIa and VIIb), the grand coalition deploys 5 more BSs under $MIN_Q$ compared to $TOT_Q$ in the largest area (Z1), 16 more in the most congested/dense area (Z2), and 22 BSs less in area Z3 (smaller than Z1 and less congested than Z2). Since the overall profit of each MNO has to be non-negative, objective $MIN_Q$ achieves fairness by “redistributing” BSs across the areas so that the user rate of the worst served ones (Z1 & Z2) is increased at the expense of sacrificing the user rate of the better served one (Z3) (see also Figure 2 and observation (iv) in Section V-D).

Similar observations can be made for both user distribution scenarios concerning the impact of $\delta$ on the number of deployed BSs (Tables VI–VII). A little incentive from users (small $\delta$) forces MNOs to deploy only a small number of BSs in order to limit their cost and therefore guarantee an overall positive profit. For example, for user distribution $M_2$, $\delta = 0.02$, under objective $TOT_Q$ the grand coalition deploys 169 BSs in area Z1, 156 BSs in area Z2 and 110 BSs in area Z3 (Table VIIa). However, as users are willing to pay more (larger values of $\delta$), more BSs are deployed since higher revenues compensate the costs of deploying more BSs. In particular, all available sites per area ($U_{max}$) are used up in all the areas for user distribution $M_1$ under objective $TOT_Q$ when $\delta \geq 0.4$ (Tables VIa); instead, for $M_2$, the $U_{max}$ BSs are exhausted only in areas Z1 and Z2 when $\delta \geq 0.4$ whereas in Z3 the rate saturation is achieved by deploying less than $U_{max}$ BSs when $\delta \geq 0.46$ (Table VIIa).

B. Stable Configurations

Also for stable configurations, the higher the value of $\delta$, the smaller and less congested are the selected coalitions. For low values of $\delta$, MNOs prefer to collaborate with a larger number of MNOs so as to minimize the network cost. Instead, for higher $\delta$, i.e., higher revenues per unit of service provided, MNOs prefer to increase the service level, which in turn requires building less congested networks, i.e., either shared networks with fewer and smaller MNOs or individual ones.

For user distribution $M_1$ (see Table VIIa), when $\delta \leq 0.52$ there is always incentive for sharing, i.e., each MNO is better off building a shared network with at least one other MNO than investing alone. The grand coalition (ABC) is stable for all values of $\delta$ in $[0.02, 0.1]$ and a subset of values in $[0.16, 0.28]$ but it ceases to be stable when $\delta \geq 0.3$. The equivalent outcomes A/BC, B/AC and C/AB are stable for all $\delta$ in $[0.02, 0.52]$ but they become unstable for a subset of values of $\delta$ in $[0.54, 2]$ which in turn means that in such cases no sharing will take place and MNOs will build individual networks. However, for $\delta \geq 0.3$, A/BC, B/AC and C/AB perform very similarly to A/B/C.

For user distribution $M_2$ (see Table VIIb), as $\delta$ increases only configurations containing the least congested coalitions of two MNOs remain stable. The grand coalition (ABC) and outcome A/BC (which involves the largest coalition of two MNOs) are never stable for $\delta \geq 0.32$ and $\delta \geq 0.56$, respectively. For $\delta \geq 0.56$, C/AB and, for a subset of values of $\delta$, also B/AC are stable. In particular, outcome C/AB, in which the largest MNO C invests by itself whereas the smaller MNOs A and B collaborate, is always stable for $\delta \geq 0.1$.

Concerning the number of BSs deployed by the stable configurations (Tables IX), a little incentive from users (small $\delta$) forces MNOs to activate only a small number of BSs in order to limit their cost and therefore guarantee an overall positive profit. For example, for user distribution $M_2$ and $\delta = 0.02$, the grand coalition is stable and it activates 74 BSs in area Z1, 69 BSs in area Z2 and 12 BSs in area Z3 (Table IXb). However, as users are willing to pay more (larger values of $\delta$), more BSs are activated since higher revenues compensate the costs of activating more BSs.

C. Comparison

We now compare the behavior of the socially optimal and stable configurations. The impact of $\delta$ on the two configurations is overall very similar. However, there is incentive for sharing for a larger range of the values of $\delta$ in order to maximize the MNOs profits (i.e., for stable configurations) compared to maximizing the global/minimum user rate (i.e. for the socially optimal configurations). In other words, shared networks can be more beneficial from the MNOs perspective as sharing the network cost allows for larger profits but less beneficial from the user perspective due to the service level degradation experienced in more congested networks. Consider for instance user distribution $M_1$. The grand coalition ABC is socially optimal for $\delta \in [0.02, 0.04]$ for $TOT_Q$ and for $\delta \in [0.02, 0.06]$ for $MIN_Q$, but it is stable for a larger number of values of $\delta$ between 0.02 and 0.28. In general, under $MIN_Q$ sharing is selected as optimal strategy only for $\delta \leq 0.22$, while sharing configurations are stable for a wider range of values (up to $\delta = 2$), which means that for higher values of $\delta$ no sharing should takes place in order to provide the best service level, while there is incentive to share in order to maximize the MNOs’ profit.

Regarding the level of investment, the higher the value of $\delta$, the denser the network deployment for both configurations as larger revenues make up for increasing network cost. Nevertheless, for the same value of $\delta$ more BSs are deployed by the socially optimal configurations compared to the stable ones, as the former focus on the user rate whereas the latter, focusing on the profit, reflect the trade-off between increased revenues and cost. For instance, for $M_1$ and $\delta = 0.04$, the grand coalition is selected by $TOT_Q$ and it is stable; however, it deploys 443 BSs in area Z1, 448 in Z2 and 274 in Z3 under objective $TOT_Q$ (Tables VIa) whereas in order to maximize the MNOs profit, 157 BSs are deployed in area Z1, 163 in Z2 and 54 in Z3 (Table IXa).
D. Performance Indicators Analysis

We now analyze how different values of $\delta$ impact two key performance indicators for the users and the MNOs: the average user rate, $Q_{\text{avg}} = \frac{\sum_{i \in O} q_i}{|O|}$, and the average global profit, $P_{\text{avg}} = \frac{\sum_{i \in O} (v_i - c_a)}{|O|}$; when multiple configurations are selected for the same value of $\delta$ (as reported in Tables IV, V, VII), we average also over the different configurations. In particular, we analyze the “price” of imposing a fair coalitional structure (objective $\text{MIN}_Q$).

Results show that the socially optimal infrastructure sharing configurations outperform stable ones in terms $Q_{\text{avg}}$ and vice versa for $P_{\text{avg}}$. However, as users are willing to pay more, the two configuration types tend to provide very similar values of $Q_{\text{avg}}$ and $P_{\text{avg}}$.

As similar observations regarding the behavior of $Q_{\text{avg}}$ and $P_{\text{avg}}$ as a function of $\delta$ can be drawn for both user distributions $M_1$ and $M_2$, we report results concerning only $M_2$ in Figure 1.

As pointed out in Section V-A, the socially optimal configurations obtained applying objectives $\text{TOT}_Q$ and $\text{MIN}_Q$ are the same for most instances and they also provide very similar $Q_{\text{avg}}$ (the largest difference across all values of $\delta$ is approximately 1.1 Mbps) which can be observed by the overlap of their corresponding plots (see Figure 1). Therefore, solutions that are fair to all users in all the areas are also efficient.

More BSs are activated by the socially optimal configurations than by stable ones (see Subsection V-C) which is reflected in their corresponding $Q_{\text{avg}}$ and $P_{\text{avg}}$. The difference in the $Q_{\text{avg}}$ provided by the socially optimal configurations and stable ones for $\delta = 0.02$ is nearly 12.6 Mbps (45.8% gap); it goes down to 4.3 Mbps (8.1%) for $\delta = 1$ and eventually becomes nearly 1.8 Mbps (3.3%) for $\delta = 2$. Thus, for high $\delta$, the two types of configurations provide roughly the same quality of service to the users if they are very interested in the new service.

As far as $P_{\text{avg}}$ is concerned, for low values of $\delta$, the difference in the $P_{\text{avg}}$ provided by the two types of configurations is significantly different (see Figure 1). For $\delta = 0.02$, the configuration selected by $\text{TOT}_Q$ provides on the average only 55.2 € per MNO, whereas the stable configurations provide 262306.3 €. This suggests that solutions obtained from objectives $\text{TOT}_Q$ and $\text{MIN}_Q$ merely satisfy the constraint on having a positive profit while providing, on the average, a 12.6 Mbps higher user rate. However, with the increase of $\delta$, the difference in rate between the two types of configurations becomes negligible, and so does the difference in profit (only 2.8% for $\delta = 2$).

So far we have investigated the average performance indicators ($Q_{\text{avg}}$ and $P_{\text{avg}}$). We now analyze how the user rate per area and MNO ($Q$) and profit per area and MNO ($P$) are affected by the characteristics of MNOs (market share) and by the characteristics of the areas (size and population, reported in Table III) for both configurations.

Figure 2 illustrates the behavior of $Q$ with respect to $P$ in each area, for each MNO for the user distribution $M_2$. For high values of $\delta$, the two types of configurations provide roughly the same quality of service to the users if they are very interested in the new service.
M2 when \( \delta = 0.02 \). We recall that, when \( \delta = 0.02 \), the grand coalition (ABC) is socially optimal (for both \( TOT_Q \) and \( M1NQ \) objectives) and stable. For this scenario we can observe that: (i) the socially optimal configurations provide in every area higher user rates than the stable one, which in turn guarantee higher revenues, (ii) the grand coalition results in all MNOs providing the same user rate to users of the same area, while their profit follows their market shares (see Equations (12), (24)), (iii) in area Z3, MNOs obtain a negative profit under objective \( TOT_Q \), while the global profit for each MNO is positive, which indicates that a negative balance between costs and revenues can be accepted in some areas by the socially optimal configurations, (iv) the objective that favors fairness (\( M1NQ \)) improves the quality of service of the users of the largest area (Z1) and most congested area (Z2) at the cost of lowering the user rate of area Z3 and (v) since the user rate provided by a given coalitional structure in an area depends on the user density, on the size of the area and on the number of BSs activated in that area, a slightly higher user rate is achieved for the small, low user density area (Z3) by the socially optimal configurations as the LTE nominal rate is divided among less users and on the average users are closer to their serving BSs.

VI. CONCLUSIONS

This work analyzes the strategic situation in which MNOs have to decide whether to invest in LTE small cells in dense urban areas and whether to share the investment with other MNOs. A mathematical framework is proposed to address the problem of infrastructure sharing for the considered scenario. This framework accounts for techno-economic parameters such as the achievable throughput and a general pricing model for the LTE service. The problem has been tackled from two perspectives: the one of a regulatory entity which imposes infrastructure sharing configurations that optimize the quality of service perceived by all users and the MNOs perspective, which captures their competitive and profit-maximizing nature.

We propose an MILP formulation to determine socially optimal configurations (regulator perspective) and adopt concepts of cooperative game theory to determine stable configurations (MNOs perspective).

Results show that sharing configurations obtained under both perspectives are strongly affected by how much users are willing to pay for the new services but they also depend on the user distribution (MNOs market shares). Sharing is appealing from both perspectives when users are willing to pay little, regardless of the MNOs market shares as they all struggle with high infrastructure cost. Instead, if users were willing to pay more, there is generally more incentive to share from the MNO perspective and in particular when MNOs have significantly different market shares. For both perspectives, the selected configurations involve less congested coalitions, that is, coalitions of fewer and smaller MNOs, when the market shares are significantly different. When the focus is on the quality of service, such configurations behave very similarly to the case when no sharing takes place, that is, users are best served either by less congested coalitions or when all MNOs build individual networks.

The proposed mathematical framework has proved to be a flexible instrument of limited complexity to analyze in detail the possible strategies for different infrastructure sharing configurations under different techno-economic conditions. It can be further extended to incorporate spectrum management issues and therefore more elaborated game theory models, as well as different classes of users and heterogeneous technologies.

APPENDIX A

We recall that the nominal user rate \( \rho_s^{a,nom} \) is computed by means of the simulation described in Subsection IV-A whereas the average user rate \( \rho_s^a \) is derived from \( \rho_s^{a,nom} \) according to Equations (23).

\[
\rho_s^a = \rho_s^{a,nom} (1 - \eta) \sum_{i \in A} \eta_i N_i \rho_s^{a,nom}, \quad \forall \ s \in S, \forall \ a \in A. \tag{27}
\]

Figure 3 illustrates the simulated nominal user rate \( \rho_s^{a,nom} \), the average user rate \( \rho_s^a \) and the piece-wise linear function approximating \( \rho_s^a \) for coalition ABC in area Z1 (similarly for all the other considered areas and coalitions). In the following, we explain how the approximation was modeled in the MILP formulation.

As mentioned, \( L \) denotes the number of linear pieces (intervals) that approximate \( \rho_s^a \). We have considered equal values of \( L \) for all the coalitions \( s \in S \) and all the areas \( a \in A \). \( L \) was set to 11 for user distribution \( M_1 \) and to 10 for \( M_2 \). For each interval \( l \in \{1, \ldots, L\} \), coalition \( s \) and area \( a \), \([ U_s^{a,l-1}, U_s^{a,l} \] \) represents the range of the number of BSs that characterize the \( l^{th} \) interval, \( R_s^{a,l} \) is the average user rate when \( s \) activates \( U_s^{a,l} \) BSs in \( a \) and \( \alpha_s^{a,l} \) is the slope associated with the \( l^{th} \) interval. The average user rate \( \rho_s^a \) obtained by activating \( a_s^u \) BSs, with \( a_s^u \in [U_s^{a,l-1}, U_s^{a,l}] \), is therefore equal to \( R_s^{a,l-1} + \alpha_s^{a,l} (a_s^u - U_s^{a,l-1}) \). Equations (28) show how these parameters are related with one another.

\[
R_s^{a,0} = \rho_s^a (1), \quad \forall \ s \in S, \forall \ a \in A, \tag{28}
\]

\[
R_s^{a,l} = R_s^{a,l-1} + \alpha_s^{a,l} (U_s^{a,l-1} - U_s^{a,l-1}), \quad \forall s \in S, \forall a \in A, \forall l \in \{1, \ldots, L\}. \tag{28}
\]

In particular, \( U_s^{a,0} \) is equal to 1, whereas \( U_s^{a,L} \) is equal to \( U_{max} \), \( \forall s \in S \) and \( \forall a \in A \). Thus, the average user rate \( \rho_s^a \)
obtained by activating \( u^a_s \) BSs can be reformulated as:

\[
\rho^a_s = \min_{i \in \{0, \ldots, L-1\}} \{ R^{a}_{i,1} + a^{a}_{i+1} (u^a_s - U^{a}_{i,1}) \},
\]

\[ \forall s \in S, \forall a \in A. \] (29)

As \( \rho^a_s \) is maximized by any of the considered objective functions, Equations (29) can be replaced by Constraints (8). Notice that, the auxiliary binary variables \( z^a_s \) equal zero when either no BSs are activated by \( s \) in a \( u^a_s = 0 \) and therefore \( z^a_s = 0 \) due to Constraint (7)) or \( s \) is not active \( (y_s = 0 \) and therefore \( z^a_s = 0, \forall a \in A \) due to Constraints (4) and (7)). In turn, when \( z^a_s = 0 \), we should also have \( \rho^a_s = 0 \), which is guaranteed by Constraints (9) while Constraints (8) are made redundant by the term \( M (1 - z^a_s) \), where \( M = 1000 \).

\[ \text{APPENDIX B} \]

The optimization problem with objective \( M\text{IN}_Q \) and Constraints (2)–(13) will be denoted by Infrastructure Sharing Problem (ISP).

**Theorem:** The decision version of (ISP) is NP-complete.

**Proof:** The decision version of (ISP) can be formulated as:

Given a threshold \( \tilde{Q} > 0 \) on the quality, are there variables

\[
y_s, z^a_s, u^a_s \text{ and } \rho^a_s, \]

with \( s \in S \) and \( a \in A \), such that Constraints (2)–(13) are satisfied and \( M\text{IN}_Q \geq \tilde{Q} \)?

We will prove that the decision version of (ISP) is NP-complete by reduction from the Set Partitioning Problem (SPP) which is a well-known NP-complete problem (see, e.g., [36]). We recall the decision version of (SPP):

Given a universe \( \mathcal{U} \), a family \( \mathcal{C} \) of subsets of \( \mathcal{U} \) and a positive integer \( K \), is there a subset \( \mathcal{C}' \subseteq \mathcal{C} \) such that \( |\mathcal{C}'| \leq K \) and each element of the universe \( \mathcal{U} \) belongs to exactly one member of \( \mathcal{C}' \). We define the variables

\[
y_\mathcal{S} = z^a_\mathcal{S} = u^a_\mathcal{S} = \begin{cases} 1 & \text{if } s \in \mathcal{C}', \\ 0 & \text{otherwise}, \end{cases}
\]

\[
\rho^a_s = \begin{cases} R^0_s & \text{if } s \in \mathcal{C}', \\ 0 & \text{otherwise}. \end{cases}
\]

It is easy to check that Constraints (2)–(5) are satisfied. Constraint (6) is fulfilled since

\[
\sum_{s \in \mathcal{S}} u^a_s = |\mathcal{C}'| \leq K = U_{\max}.
\]

The values of variables \( z^a_s, u^a_s \) and \( \rho^a_s \) guarantee that Constraints (7)–(9) hold. Furthermore, Constraints (13) on the nonnegative profit of MNOs hold because

\[
\sum_{a \in A} (q^i - c^i) = \delta D \sigma_i N_0 q_i^a - \sum_{s \in \mathcal{S}} g \sum_{j \in s} \sigma_j u^a_j = \sum_{s \in \mathcal{S}} R^0_s u^a_s - \sum_{s \in \mathcal{S}} \left\lfloor \frac{|s|}{g} \right\rfloor u^a_s = 0.
\]

Finally, since \( \mathcal{C}' \) is a partition of \( \mathcal{O} \), any MNO \( i \) belongs to a unique coalition \( s_i \in \mathcal{C}' \) and \( q^a_i = \rho^a_i = R^0_i \geq \tilde{Q} \) for any \( i \in \mathcal{O} \), that is \( M\text{IN}_Q \geq \tilde{Q} \). Therefore, \( I_{ISP} \) is a YES instance.

Now, we prove the only if part. Assume that \( I_{ISP} \) is a YES instance, i.e., there are variables \( y_s, z^a_s, u^a_s \) and \( \rho^a_s \), with \( s \in S \) and \( a \in A \), such that all the Constraints (2)–(13) are satisfied and \( M\text{IN}_Q \geq \tilde{Q} \). For any \( i \in \mathcal{O} \) we have \( q^a_i \geq \tilde{Q} > 0 \), hence we get from Constraints (4), (7) and (9) that for any \( i \in \mathcal{O} \) there exists a unique coalition \( s_i \in \mathcal{C}' \) such that \( y_{s_i} = 1 \). Thus, \( u^a_{s_i} \geq 1 \) by Constraint (5). On the other hand, \( r^a_i = \delta D \sigma_i N_0 q_i^a = \rho^a_i \) and

\[
c^a_i = \sum_{s \in \mathcal{S}} g \sum_{j \in s} \sigma_j u^a_j = \sum_{s \in \mathcal{S}} \left\lfloor \frac{|s|}{g} \right\rfloor u^a_s = R^0_s u^a_s.
\]

Since \( 0 \leq r^a_i - c^a_i = \rho^a_i - R^0_s u^a_s \), we obtain from Figure 4 that \( u^a_{s_i} \leq 1 \). Thus, for any activated coalition
If we define

\[ C' = \{ s \in S : y_s = 1 \} \]

then \( C' \) is a partition of \( \mathcal{U} \) and \( \vert C' \vert = \sum_{s \in S} u_s^i \leq U_{\text{max}} = K \), therefore \( IS_{\mathcal{P}_P} \) is a YES instance.

### REFERENCES


---

Lorela Cano is currently pursuing the Ph.D. degree with DEIB, Politecnico di Milano. Her main research interests are in the area of techno-economic characterization of infrastructure sharing in networks based on game theoretical models.
Antonio Capone (S’95–M’98–SM’05) is currently a Full Professor with the Politecnico di Milano (Technical University of Milan), where he is also the Director of the ANTLab. His expertise is on networking and his main research activities include radio resource management in wireless networks, traffic management in software defined networks, network planning, and optimization. On these topics, he has published over 200 peer-reviewed. He was an Editor of the ACM/IEEE Transactions on Networking from 2010 to 2014. He serves in the TPCs of major conferences in networking, he is an Editor of the IEEE Transactions on Mobile Computing, Computer Networks, and Computer Communications.

Giuliana Carello has been an Assistant Professor with the Operation Research Group, Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, since 2005. She has published peer-reviewed papers in international journals and conference proceedings. Her research work interests are exact and heuristic optimization approaches, applied to integer and binary variable problems. Her research is mainly devoted to real life applications, such as telecommunication networks or health care management.

Matteo Cesana (S’01–M’04) received the M.S. degree in telecommunications engineering and the Ph.D. degree in information engineering from the Politecnico di Milano, Italy, in 2000 and 2004, respectively. From 2002 to 2003, he was a Visiting Researcher with the Computer Science Department, University of California at Los Angeles, Los Angeles, CA, USA. He is currently an Associate Professor with the Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano. His research activities are in the field of design, optimization, and performance evaluation of wireless networks with a specific focus on wireless sensor networks and cognitive radio networks. He is an Associate Editor of the Ad Hoc Networks journal.

Mauro Passacantando received the M.S. and Ph.D. degrees in mathematics from the University of Pisa, Pisa, Italy, in 2000 and 2005, respectively. From 2002 to 2012, he was an Assistant Professor with the Department of Applied Mathematics, University of Pisa. He is currently an Assistant Professor of Operations Research with the Department of Computer Science, University of Pisa. He has authored over 40 peer-reviewed papers in books, conference proceedings, and international journals. His research is mainly devoted to variational inequalities and equilibrium problems, concerning both theory and algorithms. In the last years, he was involved in non-cooperative game theoretic approaches to the service provisioning problem in cloud and multi-cloud systems.