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GECO: a global gravity model by locally combining GOCE data and EGM2008

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The EGM2008 model is nowadays one of the description of the global gravitational field at the highest resolution. It is delivered with two, not fully consistent, sources of information on its error: spherical harmonic coefficient variances and a geographical map of error variances, e.g. in terms of geoid undulation. In the present work, the gravity field information derived from a GOCE satellite-only global model is used to improve the accuracy of EGM2008 model in the low to medium frequencies, especially in areas where no data were available at the time of EGM2008 computation. The key issue is to set up the error covariance matrices of the two models for an optimal least-squares combination: the full error covariance matrix of GOCE spherical harmonic coefficients is approximated by an order-wise block-diagonal matrix, while for EGM2008, the point-wise error variances are taken from the provided geoid error map and the error spatial correlations from the coefficient variances. Due to computational reasons the combination is directly performed in terms of geoid values over a regular grid on local areas. Repeating the combination for overlapping areas all over the world and then performing a harmonic analysis, a new combined model is obtained. It is called GECO and extends up to the EGM2008 maximum degree. Comparisons with other recent combined models, such as EIGEN-6C4, and a local geoid based on new gravity datasets in Antarctica are performed to evaluate its quality. The main conclusion is that the proposed combination, weighting the different input contributions not only on a global basis but also according to some local error information, can perform even better than other more sophisticated combinations in areas where the input global error description is not reliable enough.

Ke y wo rd s: Earth's gravity models, satellite missions, geoid combination, Antarctica

1. INTRODUCTION

In the last decades different methods have been developed to produce global gravity models (see, among others, Lerch et al., 1972; Rapp, 1975, 1984; Balmino et al., 1976; Wenzel, 1985; Lemoine et al., 1998; Tscherning, 2001; Reigber et al., 2005; Pavlis et al.,

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2012, Shako et al., 2014). They describe the Earth gravity field as a truncated series of spherical harmonic coefficients, with the corresponding variances and sometimes covariances, and can be obtained from satellite observations only or combining satellite and ground data.

Satellite-only models are computed considering satellite tracking data and spatial observations from CHAMP (Challenging Mini-Satellite Payload for Geo-scientific Research and Applications program), GRACE (Gravity Recovery and Climate Experiment) and GOCE (Gravity field and steady-state Ocean Circulation Explorer) gravity missions (*Reigber et al., 1996*; *Drinkwater et al., 2003*; *Tapley et al., 2004*). They are independent from ground gravity observations, therefore they do not contain any biases related to height datum inconsistencies (*Gatti et al., 2013*; *Gerlach and Rummel, 2013*). Moreover they are based on observations almost uniformly covering all the Earth, apart from possible polar gaps (*Sneeuw and Van Gelderen, 1997*). On the other hand, due to satellite altitude, satellite-only models cannot reach a high resolution. Up to now a wide set of satellite-only global models is available (see the ICGEM website for more details: *http://icgem.gfz-potsdam.de/ICGEM*).

Among the most recent ones, there are models exclusively based on CHAMP, e.g. ULux-CHAMP2013s (*Weigelt et al., 2013*), on GRACE, as ITG-GRACE2010S (*Mayer Gürr et al., 2010*) and AIUB-GRACE03S (*Beutler et al., 2010*), and on GOCE observations. In this last case the ESA official global gravity models are derived by applying three different approaches (*Pail et al., 2011*) and among them the fifth release of the time-wise GOCE solution (here called GOCE-TIM-R5) going till degree 280 can be cited (*Brockmann et al., 2014*). A set of satellite-only models, computed combining data coming from different gravity missions, is also available, as for example the GOCO models (*Pail et al., 2010*) that integrate GRACE and GOCE data.

Combined global gravity models have instead a very high resolution, merging satellite with terrestrial gravity information. Satellite data provide the low frequency terms, while altimetry/gravimetry surface gravity data allow to describe the highest frequencies of the gravitational potential harmonic development. At the moment the most used ones are EGM2008 (*Pavlis et al., 2012*) and the EIGEN models (*Förste et al., 2014*), the last delivery EIGEN-6C4 reaching degree 2190, as EGM2008. No GOCE data are embedded into EGM2008, while the EIGEN series includes this satellite information from the EIGEN-6C model. EGM2008 is complete till degree and order 2159 with some coefficients of higher degrees, according to an anti-aliasing strategy (*Jekeli, 1988*) which corresponds to a spatial resolution of about 10 km (see Fig. 1). It has been the first combined global gravity field model at a so high resolution and it was computed from a global set of area-mean free-air gravity anomalies integrated with the information coming from the GRACE gravity mission. In particular this database was constructed by merging information coming from different sources, e.g. radar altimetry offshore and ground as well as airborne gravity observation onshore (for more details see *Pavlis et al., 2012*).

This work addresses the problem of merging EGM2008 and a GOCE satellite-only model, more precisely the GOCE-TIM-R5 global model (Fig. 2 shows the difference between the two models in terms of geoid undulation up to degree 280). GOCE is actually more informative than EGM2008 in the areas where no ground gravity data were available at the time of EGM2008 computation, such as Africa, South America and

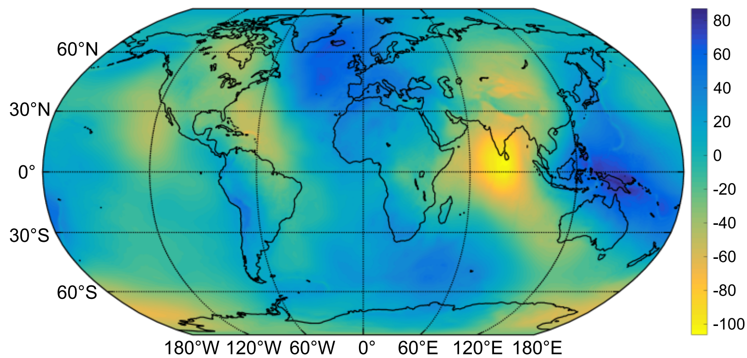

Fig. 1. EGM2008 high resolution geoid model up to degree 2190 (values in m).

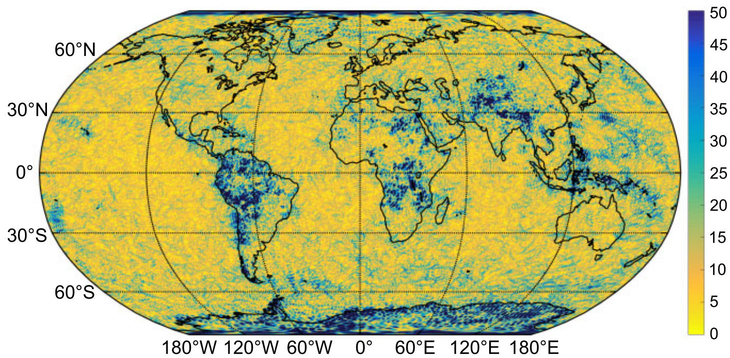

Fig. 2. Absolute differences between EGM2008 and GOCE-TIM-R5 geoid models up to degree 280 (values in cm, standard deviation $\sigma = 30.2$ cm). Values larger than 50 cm are saturated.

Antarctica. Spectrally speaking, GOCE is more accurate than EGM2008 in the medium frequencies, say up to degree 200 (see Fig. 3 which shows the full power error degree variances of the two models, i.e. the sum of all coefficient error variances per degree along orders, multiplied by the square of the Earth semi major axis); furthermore the GOCE model, being based on space observations only, is not affected by local biases and inconsistencies caused by the integration of different data sources.

Before illustrating the implemented combination strategy, it is important to remind how the accuracy of the two models is provided to the users.

EGM2008 error information is delivered in two different ways: spherical harmonic coefficient variances, which lead to a latitude dependent error shape (see Fig. 4) and a geographic error map, e.g. expressed in terms of geoid undulation (see Fig. 5). Errors are represented up to degree 359 because this is the degree range of interest of the combination here implemented (see Section 2). The truncation is straightforward in the case of coefficient error variances (no covariances are modelled), while the rescaling applied to the downloadable geographic error map (*http://earth-info.nga.mil /GandG/wgs84/gravitymod/egm2008*) is described in Section 2. The maximum degree of

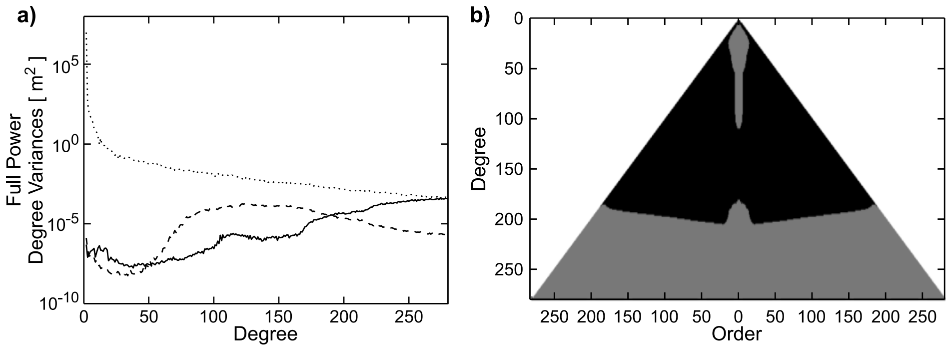

Fig. 3. **a)** Degree variances of the Earth's gravity signal (dotted curve), of the error of EGM2008 (dashed curve) and of the GOCE global model (solid curve); **b)** comparison between EGM2008 and GOCE coefficient errors: in black the degrees and orders for which GOCE coefficient error variances are smaller than those of EGM2008, in grey vice versa.

the GOCE model is 280 and no correlations of the combined coefficients with the GOCE information are expected above degree 360. Since the geographic error map describes the accuracy of the model according to the quality and the spatial distribution of the original gravity data, it provides a more realistic and local description of the EGM2008 error but completely disregards its spatial correlation. Note that the two error sources are different, even though the average of local errors is generally consistent with the coefficient variances (apart from Antarctica), as can be seen in Fig. 6.

The GOCE model is a set of spherical harmonic coefficients complete to degree and order 280 with the corresponding full error covariance matrix which is well approximated by an order-wise block-diagonal matrix (*Gerlach and Fecher, 2012*). This approximation makes the normal matrix numerically manageable providing estimates very close to the optimal ones (*Reguzzoni and Sansò, 2012*). Figure 7 shows the GOCE-TIM-R5 geoid error from the block-diagonal covariance matrix plus the omission error from degree 280 to degree 359 obtained from smoothed EGM2008 degree variances (σ = 14.3 cm). A detailed discussion on the error characteristics of the two models can be found in *Gilardoni et al. (2013)*.

This article outlines what was explained in *Gilardoni et al. (2013)*, where different strategies to improve EGM2008 using a GOCE-only global model are discussed in depth. The methods described in *Gilardoni et al. (2013)* are here applied at a global level and not just in a local area as the Mediterranean Sea. The purpose of this work is therefore to produce a new global gravity model, that is called GECO (GOCE and EGM2008 combined model), integrating EGM2008 and GOCE information. The peculiarity of the implemented strategy (illustrated in Section 2) consists in taking for EGM2008 the point-wise error variances from the provided geoid error map and the error spatial correlation from the coefficient variances. Due to computational reasons, the combination is not done at a global level in terms of spherical harmonic coefficients, but it is performed considering a functional of the gravitational potential, e.g. a grid of geoid values, dividing

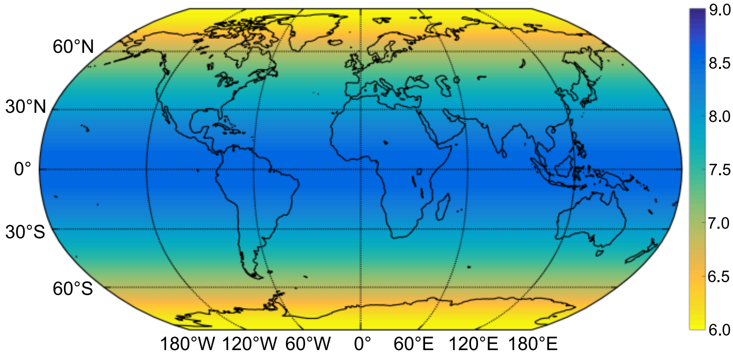

Fig. 4. EGM2008 geoid error from coefficient variances up to degree 359 (values in cm).

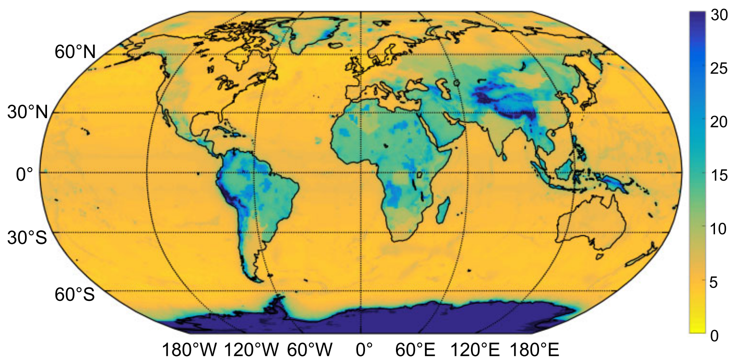

Fig. 5. EGM2008 geoid error up to degree 359 (values in cm). Values larger than 30 cm are saturated.

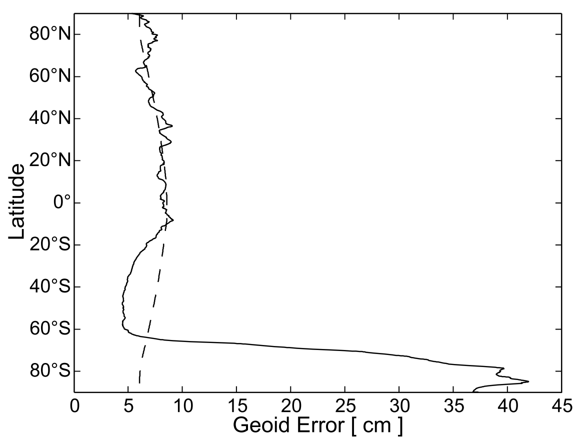

Fig. 6. EGM2008 geoid mean error up to degree 359 from coefficient variances (dashed line) and from the delivered error map (solid line). Below latitude 80°S geographic error is of about 40 cm.

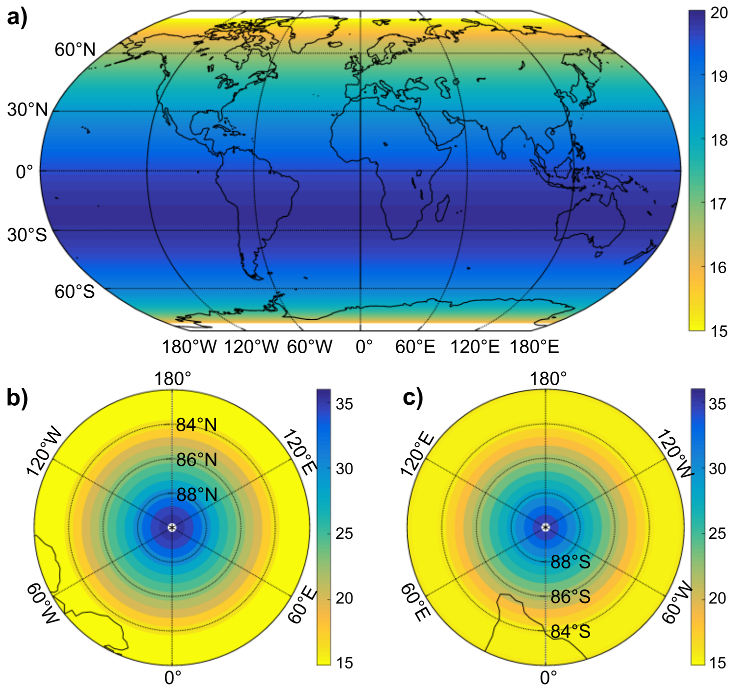

Fig. 7. GOCE-only geoid error up to degree 280 (values in cm) from block-diagonal covariance matrix plus the omission error from degree 280 to degree 359 (standard deviation $\sigma = 14.3$ cm): **a)** global map without polar gaps, **b)** north gap, **c)** south gap.

the Earth surface in different overlapping patches. In Section 3 an assessment of the GECO quality is done by comparing it with other high-resolution models and local geoids, while in Section 4 some final considerations are reported.

2. COMBINATION

The most direct way to combine two sets of spherical harmonic coefficients is to set up a least-squares adjustment using the available information on their error covariance matrices, i.e.

$$\boldsymbol{T}_{\mathrm{GE}} = \left[\boldsymbol{\Sigma}_{\mathrm{E}}^{-1} + \mathbf{B}_{\mathrm{G}}^{-1} \right]^{-1} \left[\boldsymbol{\Sigma}_{\mathrm{E}}^{-1} \boldsymbol{T}_{\mathrm{E}} + \mathbf{B}_{\mathrm{G}}^{-1} \boldsymbol{T}_{\mathrm{G}} \right] , \qquad (1)$$

where $\boldsymbol{T}_{\mathrm{GE}}$ is the resulting spherical harmonic coefficient vector, $\boldsymbol{\Sigma}_{\mathrm{E}}$ the diagonal covariance matrix of the EGM2008 error, $\mathbf{B}_{\mathrm{G}}$ the (order-wise) block-diagonal covariance matrix of the GOCE-only solution error, $\boldsymbol{T}_{\mathrm{E}}$ the EGM2008 and $\boldsymbol{T}_{\mathrm{G}}$ the GOCE-only model coefficient vector respectively (see Eq. (18) in *Gilardoni et al., 2013*). The model obtained in this way, i.e. using coefficient covariances (CC) only, meaning a diagonal matrix for EGM2008 and an order-wise block-diagonal matrix for GOCE, is called GECO-CC in the following. Figure 8 shows the coefficient standard deviations of EGM2008, GOCE and GECO-CC up to degree 359. The improvement in the medium degrees of GECO-CC with respect to EGM2008 is evident.

A more elaborated way to combine the two models is to improve the description of the EGM2008 error by adding local information from the EGM2008 geographical error map; it consists in taking the point-wise error variances from the provided geoid error map and the error spatial correlation from the coefficient variances

$$\mathbf{L}_{\mathrm{E}}^{NN} = \mathbf{S}_{\mathrm{E}} \left(\mathbf{D}_{\mathrm{E}}\right)^{-1} \mathbf{C}_{\mathrm{E}}^{NN} \left(\mathbf{D}_{\mathrm{E}}\right)^{-1} \mathbf{S}_{\mathrm{E}} \ , \tag{2}$$

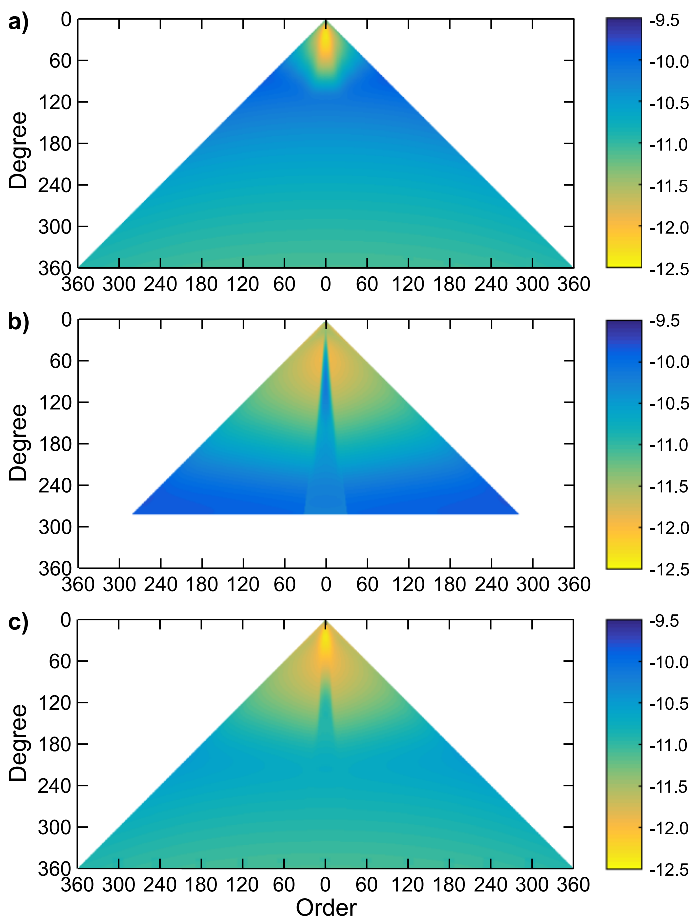

Fig. 8. **a)** EGM2008 coefficient error standard deviations, showing GRACE contribution below degree 100; **b)** GOCE-only coefficient error standard deviations showing polar gaps effects at low orders; **c)** GECO-CC coefficient error standard deviations, see Eq. (9), showing GOCE corrections to EGM2008 up to about degree 200. All the plots are in log10 scale.

where $\mathbf{L}_{\mathrm{E}}^{NN}$ is the localized covariance of $\boldsymbol{T}_{\mathrm{E}}$ and $\mathbf{S}_{\mathrm{E}}$ is a diagonal matrix such that the elements of its diagonal are just the standard deviations contained in the geoid error map $\sigma_N(\lambda,\vartheta)$, where ϑ is the colatitude and λ the longitude of the point. $\mathbf{D}_{\mathrm{E}}$ is a diagonal matrix containing the square root of the values of the main diagonal of $\mathbf{C}_{\mathrm{E}}^{NN}$, where $\mathbf{C}_{\mathrm{E}}^{NN}$ is the covariance matrix obtained propagating the error coefficient variances to the geoid, i.e.

$$C_{\mathrm{E}}\left(N_{ij}, N_{kh}\right) = \frac{1}{\gamma_i \gamma_k} \sum_{n,m} \sigma_{nm}^2 Y_{nm}\left(\vartheta_i, \lambda_j\right) Y_{nm}\left(\vartheta_k, \lambda_h\right) , \quad (3)$$

with γ the normal gravity, σ_{nm}^2 the coefficient error variances and Y_{nm} the spherical harmonic functions of degree n and order m (*Heiskanen and Moritz, 1967*). See Eq. (29) in *Gilardoni et al. (2013)* for further details.

This second approach has been applied to compute the GECO global model according to the following strategy. Due to computational limitations it is not possible to directly compute the GECO global model in terms of spherical harmonic coefficients. This would require to use Eq. (1) and to propagate the covariance matrix in Eq. (2) to spherical harmonic coefficients, leading to a full covariance matrix of coefficients up to degree and order 2190 (i.e. a matrix of about 2×10^{13} entries). Managing such a huge matrix would be of course numerically unfeasible. To overcome this problem a geoid model has been first computed on a global spherical grid at ground level ($R = 6378136.3$ m) with a resolution of $0.5° \times 0.5°$ and after that the corresponding spherical harmonic coefficients have been recovered by analysing the obtained geoid through numerical integration (*Colombo, 1981*; *Reguzzoni, 2004*). In this way it is possible to estimate the predicted model for a patch at a time, thus reducing the required computational burden. More precisely the world has been divided into 1152 patches of $37° \times 37°$ size with 15° overlap. For each patch, geoid grid values are predicted only for the central part of $7° \times 7°$ (Fig. 9 shows a patch example). The resulting geoid $\boldsymbol{N}_{\mathrm{GE}}$ has been computed by combining via least-squares adjustment the input geoids $\boldsymbol{N}_{\mathrm{E}}$ and $\boldsymbol{N}_{\mathrm{G}}$ obtained by synthetizing EGM2008 and the GOCE model respectively on the considered patch

$$\boldsymbol{N}_{\mathrm{GE}} = \left[\left(\mathbf{L}_{\mathrm{E}}^{NN}\right)^{-1} + \left(\mathbf{C}_{\mathrm{G}}^{NN}\right)^{-1}\right]^{-1} \left[\left(\mathbf{L}_{\mathrm{E}}^{NN}\right)^{-1} \boldsymbol{N}_{\mathrm{E}} + \left(\mathbf{C}_{\mathrm{G}}^{NN}\right)^{-1} \boldsymbol{N}_{\mathrm{G}}\right] . \quad (4)$$

In particular, the two input contributions with the corresponding error covariance information are:

- the EGM2008 geoid up to degree 359 with a locally adapted commission error covariance matrix $\mathbf{L}_{\mathrm{E}}^{NN}$ according to Eqs (2) and (3), in the sense that standard deviations $\mathbf{S}_{\mathrm{E}}$ come from the geoid error map up to degree 359. This requires to rescale the provided EGM2008 geoid error variance σ_N^2 of a factor ρ which is just

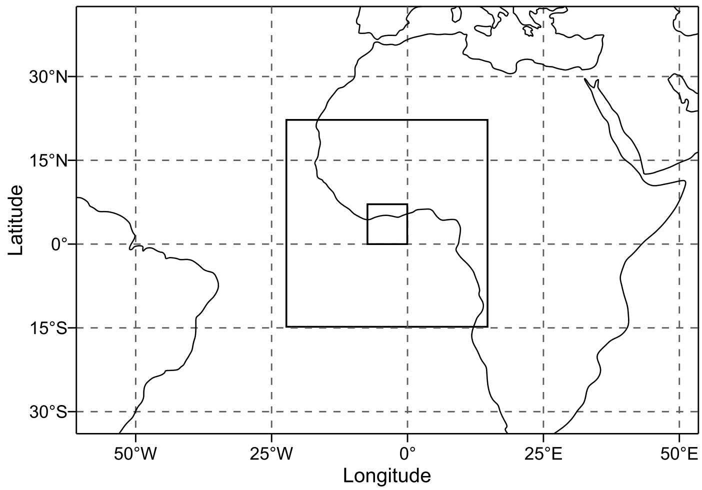

Fig. 9. Example of a patch definition: the big square represents the data patch while the small one the grid prediction area.

the ratio of the commission error of EGM2008 up to degree 359 with respect to the commission error of the complete model

$$\rho = \frac{\sum_{n=2}^{359} \sigma_{nm}^2}{\sum_{n=2}^{2190} \sigma_{nm}^2} , \tag{5}$$

with σ_{nm}^2 the coefficient variances of degree n and order m. This rescaling is naturally an approximation but it is done due to the limited information available;

- the GOCE geoid up to degree 280 with a commission error up to degree 280 computed as

$$C_{\mathrm{G}}^{\mathrm{comm}}\left(N_{ij}, N_{kh}\right) = \frac{1}{\gamma_i \gamma_k} \sum_{\ell,n,m} \sigma_{\ell n,m} Y_{\ell m}\left(\vartheta_i, \lambda_j\right) Y_{nm}\left(\vartheta_k, \lambda_h\right) , \tag{6}$$

with $\sigma_{\ell n,m}$ the error covariance between coefficients of degree ℓ and n, both of order m. Covariances between coefficients of different orders are neglected. The omission error from degree 281 up to degree 359 is evaluated as

$$C_{\mathrm{G}}^{\mathrm{omis}}\left(N_{ij}, N_{kh}\right) = \frac{1}{\gamma_i \gamma_k} \sum_n s_n^2 P_n\left(\psi_{ij,kh}\right) , \tag{7}$$

with s_n^2 the signal degree variances (estimated from EGM2008), P_n the Legendre polynomial of degree n and $\psi_{ij,kh}$ the spherical distance between points of coordinates $\left(\vartheta_i, \lambda_j\right)$ and $\left(\vartheta_k, \lambda_h\right)$. The global error covariance is therefore obtained as

$$C_{\mathrm{G}}\left(N_{ij}, N_{kh}\right) = C_{\mathrm{G}}^{\mathrm{comm}}\left(N_{ij}, N_{kh}\right) + C_{\mathrm{G}}^{\mathrm{omis}}\left(N_{ij}, N_{kh}\right) , \tag{8}$$

from which the covariance matrix $\mathbf{C}_{\mathrm{G}}^{NN}$ of Eq. (4) is populated.

The spherical harmonic coefficients are then computed by making an analysis of the combined geoid. The analysis is performed up to degree 359 (consistently with the $0.5° \times 0.5°$ resolution). From degree 360 to degree 2190 the GECO coefficients are directly taken from the EGM2008 model. The GECO coefficient errors are computed from the coefficient errors of EGM2008 and of the TIM R5 solution according to the first approach illustrated in Eq. (1), i.e from the diagonal of the matrix

$$\mathbf{B}_{\mathrm{GE}} = \left[\mathbf{\Sigma}_{\mathrm{E}}^{-1} + \mathbf{B}_{\mathrm{G}}^{-1}\right]^{-1} . \tag{9}$$

In other words, these are the error coefficient variances of the GECO-CC model. The GECO geoid and its associated geographical error map computed from the patch-wise least-squares adjustment are illustrated in Fig. 10 and 11, respectively. It can be noted that there are some east-west stripes in the GECO error map. Apart from the effect of the GOCE polar gaps at 83°, these stripes basically reflect the borders of the patchwork. If this border is close to an EGM2008 error discontinuity (like for example at about latitude 67°N), the stripe effect is more visible. This is of course a drawback of the method and a possible solution for a future improvement is proposed in the conclusions.

3. ASSESSMENT AND COMPARISONS

First of all, the GECO model is compared with EGM2008 in terms of geoid undulations. This shows that, as expected, the correction from GOCE information to EGM2008 is higher where there are no ground gravity data (e.g. the Himalayas, the Andes, the Congo basin) as illustrated in Fig. 12 up to degree 280 (for comparison with Fig. 2) and for all degrees in Fig. 13. Actually the GOCE contribution to the improvement of EGM2008 can be better evaluated by the difference between EGM2008 and GECO geoid error standard deviations (see Fig. 14), recalling that GECO error estimates are by definition always smaller than those of EGM2008. After that, one could ask whether there are significant differences between GECO and GECO-CC, i.e. the combination still based on a least-squares adjustment but not using the local geoid error information. As it can be seen from Fig. 15, the answer is that GECO and GECO-CC are generally similar apart from the areas with the highest geographic error of EGM2008 (Fig. 5) and therefore where the local error adaptation could provide additional information. Note that some differences are also present close to the poles, probably due to some numerical instability of the GECO combination procedure when applied in those areas.

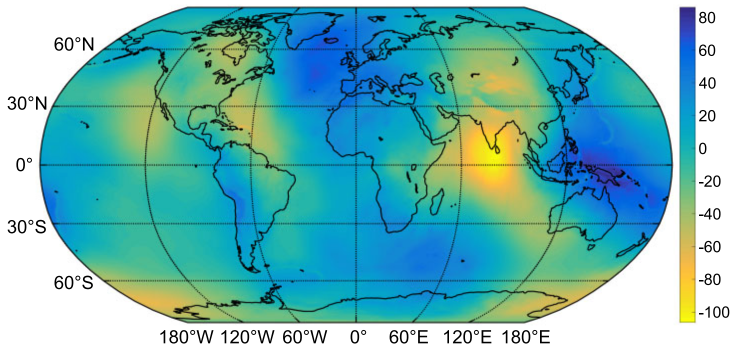

Fig. 10. GECO geoid model up to degree 359 (values in m).

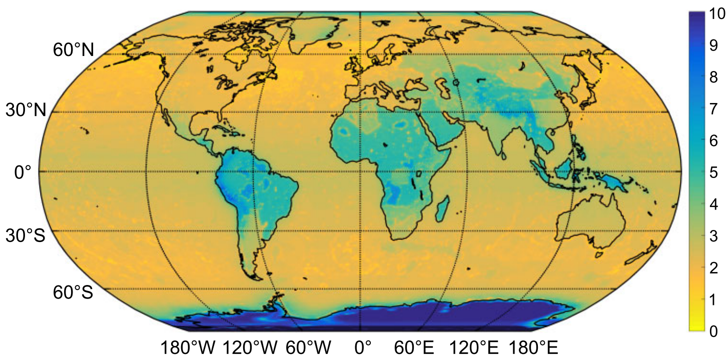

Fig. 11. GECO geoid model error up to degree 359 (values in m). Values larger than 10 cm are saturated.

The combined models have been first compared with the latest release of the EIGEN global gravity models (i.e. EIGEN-6C4) which integrates GRACE, GOCE as well as local gravity data and therefore can be considered as a reference for the evaluation of other models. Figure 16 shows the full power error degree variances of EGM2008, GOCE-TIM-R5, GECO and GECO-CC global gravity models with respect to EIGEN-6C4 taken as reference. Table 1 shows the root mean square (*rms*) difference between EGM2008, GECO and GECO-CC with respect to EIGEN-6C4 for the different areas of the globe as partitioned according to Fig. 17.

From Table 1 it can be inferred that globally the model that best approaches EIGEN-6C4 is GECO-CC. In the areas where there is a good quantity and quality of local gravity data (such as in Europe and North America) or in the oceans where altimetry is used, all the models are consistent with each other. However in the areas where there are no or few ground gravity data (such as in South America, Africa and Antarctica) the differences between models increase. In these areas and especially in Antarctica, excluding polar gaps, we guess that the "best" model is the one that best approaches the GOCE geoid,

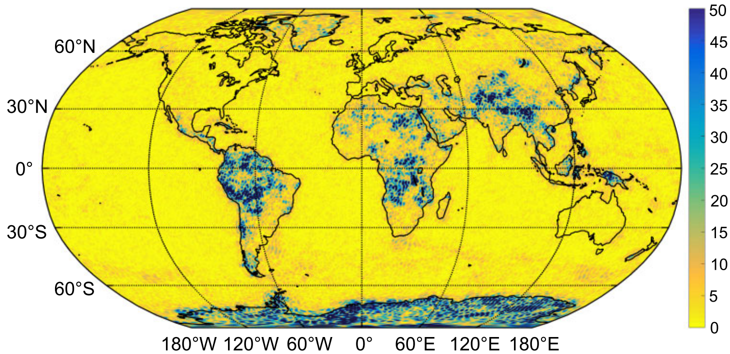

Fig. 12. Absolute differences between EGM2008 and GECO geoid up to degree 280, (values in cm, standard deviation σ= 18.6 cm). Values larger than 50 cm are saturated.

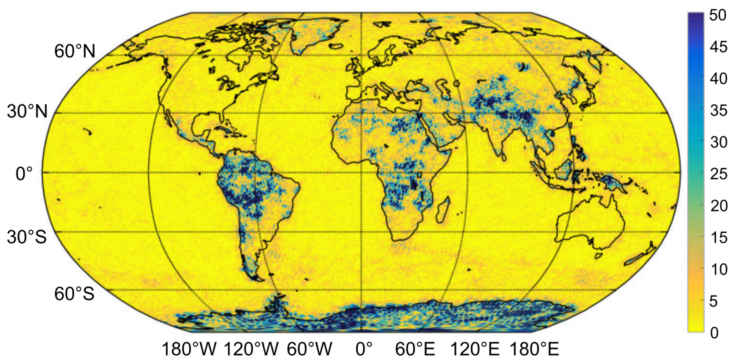

Fig 13. Absolute differences between EGM2008 and GECO geoid, (values in cm, standard deviation σ= 19.2 cm). Note that the coefficients of the two models are the same after degree 360. Values larger than 50 cm are saturated.

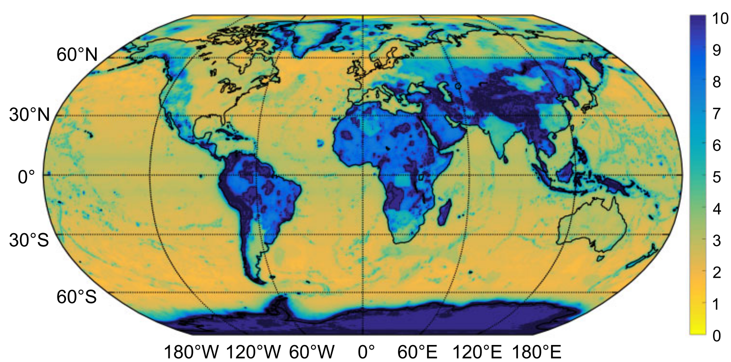

Fig. 14. Difference between EGM2008 and GECO geoid error up to degree 359 (values in cm). Values larger than 10 cm are saturated.

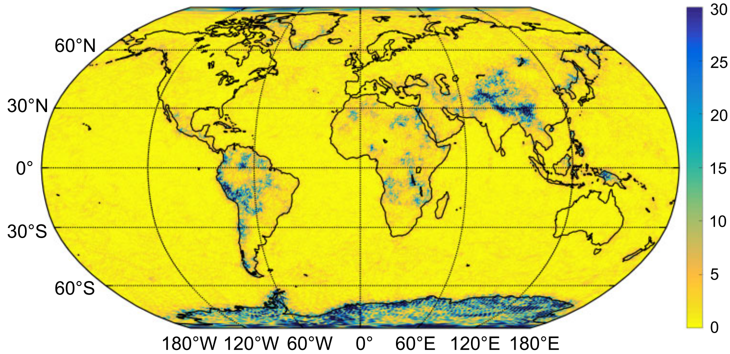

Fig. 15. Absolute differences between GECO and GECO-CC geoid up to degree 359, (values in cm, standard deviation σ= 11.1 cm). Values larger than 30 cm are saturated.

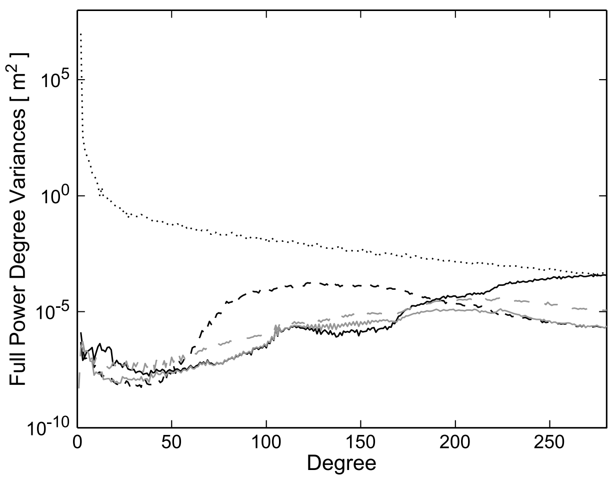

Fig. 16. Degree variances of the Earth gravity signal (dotted black curve), of the difference of EGM2008 (dashed black curve), GOCE-TIM-R5 (solid black curve), GECO (dashed grey curve) and GECO-CC (solid grey curve) with respect to EIGEN-6C4 as reference.

since this is the most important source of information, at least up to degree 180 (see Fig. 3). Evaluating the differences between the combined global models and GOCE (here the GOCE-TIM-R5 solution is considered) up to degree 180, it can be seen from Table 2 that GECO approaches GOCE better than EIGEN-6C4. The differences between GECO and EIGEN-6C4 in Antarctica (rms of the differences equal to 7.4 cm) could be attributed to the presence of much more GRACE data in the latter model. However, this is not the case since GECO-CC is much closer to EIGEN-6C4 in Antarctica (rms equal to 5.6 cm) although it incorporates as much GRACE data as GECO. This means that these differences in Antarctica depend more on the different combination strategy rather than the quantity of GRACE data. In this sense, we take the closeness of GECO to the GOCE model in Antarctica as a hint of its good quality, at least in that area.

Table 1. Root mean square difference (values in cm) between high resolution combined models and EIGEN-6C4 as reference. All the models are considered till their maximum degree $L_{max} = 2190$.

Area	Geoid Model		
	EGM2008	GECO	GECO-CC
All the world	12.8	16.3	9.1
Europe	3.9	2.5	2.0
North America	4.8	3.3	2.5
Oceans	7.6	7.6	6.3
Africa	22.6	6.9	2.5
South America	33.7	13.1	4.0
Antarctica	21.9	46.9	23.7

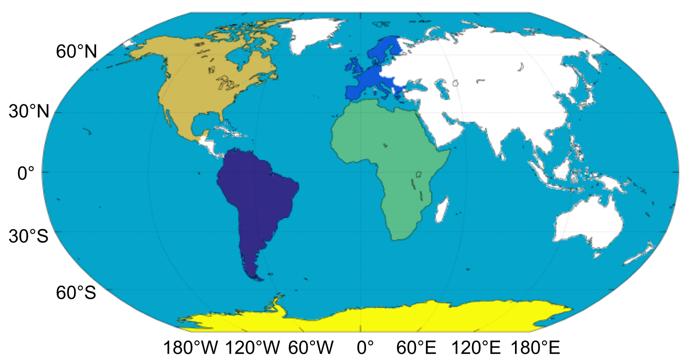

Fig. 17. Zones considered in geoid model comparisons and listed in Table 1.

In order to better understand if the GECO solution can bring some advantages with respect to the EIGEN-6C4 one in Antarctica (and in general in areas with few ground gravity data), a comparison has been performed in the Weddell Sea where a local geoid based on ground data is freely available from the ISG (International Service for the Geoid) website (*http://www.isgeoid.polimi.it/*). This regional geoid, represented in Fig. 18, has been computed using heterogeneous ground gravity data that are not present in the investigated global models and therefore it can be considered as a reference in the evaluation process (*Schwabe and Scheinert, 2014*). Looking at Table 3 it can be seen that in the Weddell Sea area GECO fits the local geoid better than EIGEN-6C4, as expected from the previous analysis. This is even emphasized in the mainland of this study area where the poorness of ground gravity data in the global models is stronger. This confirms the thesis that the EIGEN-6C4 combination is not optimal in the Weddell Sea and Table 2 suggests that this may hold also on the rest of Antarctica. The differences between the GECO and the EIGEN-6C4 geoid with respect to the regional model are shown in Figs 19 and 20, respectively.

Table 2. Root mean square difference (values in cm) between high resolution combined models and GOCE-TIM-R5 as reference (all the models up to degree 180). Only the area outside GOCE polar gaps is considered.

Area	Geoid Model			
	EGM2008	GECO	GECO-CC	EIGEN-6C4
All the world	12.0	1.4	1.7	2.9
Europe	4.1	1.0	0.7	1.1
North America	4.8	1.1	0.9	1.4
Oceans	6.0	1.4	1.4	2.2
Africa	22.6	1.3	1.5	2.3
South America	33.6	1.9	2.6	3.3
Antarctica	22.6	1.9	3.8	8.1

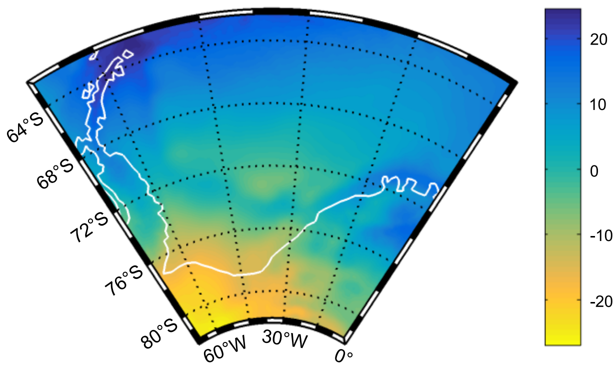

Fig. 18. Geoid model in the Weddell sea (values in m) used as a reference.

Table 3. Root mean square difference (values in cm) between global models and the regional geoid in the Weddell Sea taken as reference (all the models up to degree 2190).

Area	Geoid Model			
	EGM2008	GECO	GECO-CC	EIGEN-6C4
Mainland + ocean	52.1	33.6	35.7	45.1
Mainland	78.6	47.1	52.6	67.8
Ocean	27.5	22.7	20.9	24.3

Actually it can be observed from Table 3 that on the ocean GECO-CC approaches the regional model slightly better than GECO. This can be explained looking at the differences between GECO and GECO-CC over the ocean (Fig. 21): the two solutions should be quite similar, however there are significant differences close to the coast. This is due to the shape of the EGM2008 error map in this area (Fig. 22) that is the driver of the GECO combination. In particular, if the EGM2008 error map is somewhere inaccurate, this negatively affects the GECO combination weights. By looking at Fig. 22, it seems

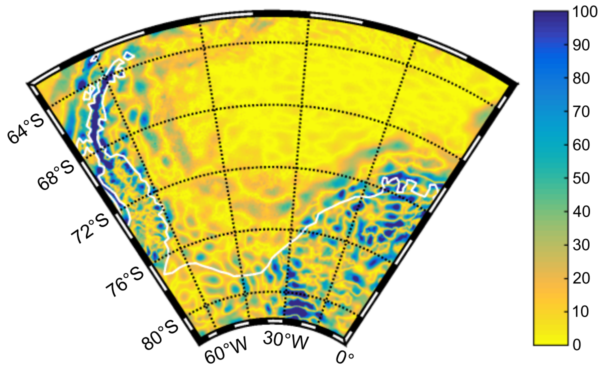

Fig. 19. Absolute differences in the Weddell Sea between GECO and the regional model (values in cm). Values larger than 100 cm are saturated.

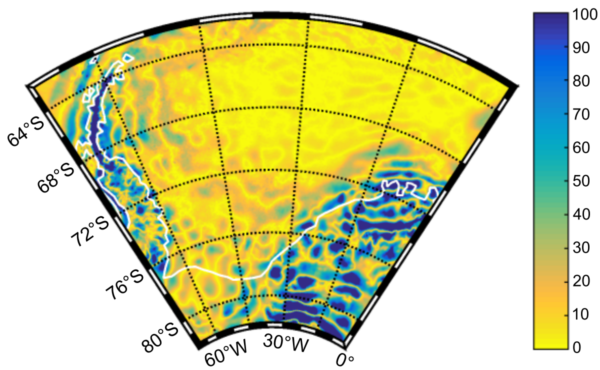

Fig. 20. Absolute differences in the Weddell Sea between EIGEN-6C4 and the regional model (values in cm). Values larger than 100 cm are saturated.

that the EGM2008 local error in the coastal area is a bit overestimated, leading to an overweighting of the GOCE contribution. Moreover, the error map shows some discontinuities (or sharp variations) that are correctly model by the GECO combination strategy for the variances but not for the correlations which in fact vary smoothly depending only on the EGM2008 coefficient variances.

4. CONCLUSIONS

In this paper a combination technique to insert a satellite-only model, like one of those coming from GOCE data, into an existing independent ultra-high resolution model is proposed. The novelty of the method is in the use of all the available error information about the two models. In fact, while for the satellite-only model a full error covariance matrix is generally given since it is expanded only to a moderate low degree, for the ultra-

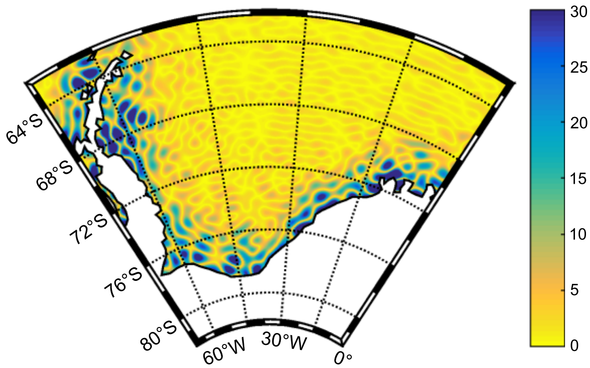

Fig. 21. Absolute differences in the Weddell Sea between GECO and GECO-CC (values in cm). Values larger than 30 cm are saturated.

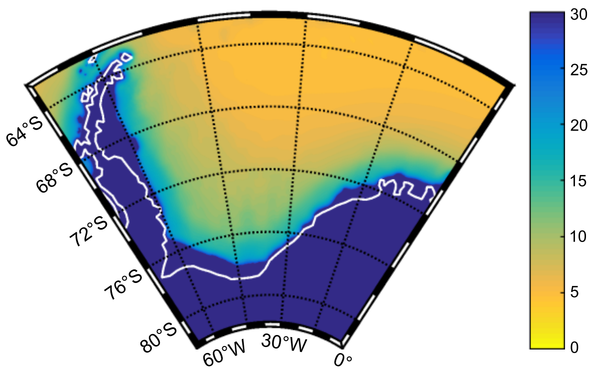

Fig. 22. EGM2008 geographic error map in the Weddell Sea up to degree 2190 (values in cm). Values larger than 30 cm are saturated.

high resolution model this is never the case but indeed different error descriptions (neither complete nor consistent to each other) are typically provided. This complex situation can be handled by implementing several local combinations over several overlapping patches in which the world has been previously subdivided. In particular the technique has been applied to merge EGM2008 with the latest release of the time-wise GOCE solution, giving rise to a model, called GECO, that seems to perform better than other GOCE-based combined global models, like EIGEN-6C4, in areas where few ground gravity data are available, like Antarctica. This assessment has been done by using a recent regional geoid model of the Weddell Sea as reference. However a more reliable and complete comparison of the performance of the different combined models should be based e.g. on independent GPS-leveling observations (well distributed all over the Earth surface), which at the moment are not publicly available. Here we just want to state that, as much sophisticated it could be, a direct combination of spherical harmonic coefficients driven by approximate global error covariance descriptions could not be the best choice in some areas of the world and that the GECO solution seems to be a realistic alternative to incorporate new satellite-only information into existing global models. On the basis of the achieved results, a possible future development of the presented combination method could be to subdivide the world into patches that have not a rectangular shape but delimit geographical areas with homogeneous (or quasi-homogeneous) EGM2008 error variances.

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