Size and Composition of Public Investment, Sectoral Composition and Growth *

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Abstract

After providing some descriptive evidence on the relationship between the size and composition of public expenditure and the sectoral employment composition of the economy, this paper develops an endogenous growth model with two private sectors, where the government provides, as pure public goods, both infrastructure investment, directly affecting the productivity of private capital in the 'modern' sector, and a flow of goods and services, enhancing the productivity of the otherwise labour-intensive 'traditional' sector. Government productive expenditure affects the long-run growth rate through its size and composition, both directly, by enhancing the productivity of private factors, and indirectly, by changing the employment sectoral composition of the economy.

Keywords: Public Investment, Growth, Sectoral Composition

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1 Introduction

The role of public investment in fostering economic growth has been constantly present both in academic research, starting from the seminal contributions of Aschauer (1989) and Barro (1990), and in the policy debate (European-Commission, 2001; Besley and Van Reenen, 2013).[1] The initial strong positive effect of public investment on growth, found, among others, by Aschauer (1989) and Easterly and Rebelo (1993), was later questioned by other empirical studies, which addressed spurious correlation and reverse causality issues. The empirical evidence on the effect of total government expenditure on aggregate output growth is generally 'mixed', and the related debate still quite heated (Gramlich, 1994; Nijkamp and Poot, 2004; Bergh and Henrekson, 2011). Beyond the methodological aspects, what emerges clearly is that results strongly depend on the types of expenditure considered and the definition used. Distinguishing between productive and unproductive public expenditure, as a first step, seems to lead to the clearest results. Productive public expenditure, variously defined, is shown to positively affect growth by several contributions (Kneller et al., 1999; Angelopoulos et al., 2007; Romero-Avila and Strauch, 2008), but even in this case there is no general consensus owing to the differences in the magnitude and statistical significance of the effect, and sometimes even in the sign (Afonso and Furceri, 2010).

However, even when focus is limited to productive public expenditure only, this lack of robust evidence on the impact of public investment on growth may be due to a composition effect. Indeed several recent studies have shown, on the one hand, that there is a wide heterogeneity in the effects of public investment across regions and industries and, on the other hand, that the effects differ depending on the type of investment (Holtz-Eakin and Lovely, 1996; Devarajan et al., 1996; Romp and de Haan, 2007; Ghosh and Gregoriou, 2008; Florio, 2011).

This paper seeks to contribute to this literature by providing a theoretical framework disentangling the effect of different types of productive public expenditure and, unlike the past endogenous

[1] In this paper, we use public investment and productive public expenditure as synonyms referring to the total government expenditure devoted to the provision of public goods enhancing the productivity of the private sectors of the economy. Similarly, we use public infrastructure and public capital interchangeably, referring to the stock of public capital, resulting from the accumulation of the share of productive public expenditure devoted to infrastructure.

growth literature with public intervention (see Section 2), by simultaneously considering the interplay with the sectoral composition of the economy. This is relevant as productive public expenditure may affect the sectoral composition of employment and output. From the demand side, depending on the type of public investment, it drives an increase in the demand for industries using different technologies; from the supply side, public investment may unevenly affect the productivity of different industries, depending on each investment's scope (i.e., the composition of public expenditure) influencing, this way, relative prices and returns to private factors, with potential effects on the composition of private expenditure. In a nutshell, this paper aims at investigating the growth effects of public investments when they are mediated by the sectoral composition of the economy.

From a positive point of view, a higher public expenditure negatively affects the growth rate, being a large share of productive government expenditure devoted to the traditional sector of the economy, whose employment share accordingly increases (see Section 3). This may explain the very high positive effect of public expenditure on growth obtained from theoretical models not taking into account that productive public expenditure mostly generates demand for the traditional sector of the economy. This amounts to saying that a large positive effect of public expenditure on the productivity of private factors of production can be consistent with a low effect on aggregate growth, once the effect on the sectoral composition of the economy is taken into account. From a normative point of view, this approach provides the government with a new policy tool, i.e., the composition of public expenditure in terms of *types of goods* ('modern' vs. 'traditional'), to affect the growth rate of the economy.

Moreover, we add to the structural change literature (see Section 2) by introducing another potential determinants of an economy's sectoral structure, i.e., public expenditure composition, complementing those supply-side and demand-side factors highlighted by previous contributions where only private agents' decisions are considered and where structural composition is driven by fundamentals (technology and preferences). Our approach implies that the sectoral composition of an economy does not necessarily reflect the fundamentals, but also public expenditure level and composition, which may be driven by several other determinants.

In more detail, as a first step we provide some descriptive evidence on the relationship between the size and composition for 'traditional' and 'modern' goods of government productive spending and the sectoral composition of employment. Productive public expenditure's size and share spent for traditional goods both are significantly negatively associated with the employment share in the 'modern' sector. In order to explain this evidence, we develop an endogenous growth model with two private sectors: a 'modern' sector (e.g., manufacturing, business services) and a 'traditional' sector (e.g., personal services, construction), both producing goods which can be used for consumption and investment.[2] In this economy there are two publicly provided goods, which are purchased by the government and provided to enhance the productivity of the private sectors of the economy. The public goods are produced by different sectors, the 'traditional' sector and the 'modern' sector, and they unevenly affect the productivity in the two sectors. In this set up, the government has two policy instruments (in a balanced budget framework), namely the size and the composition of public expenditure, i.e. in term of 'traditional' and 'modern goods', both affecting the growth rate of the economy. We focus on productive public expenditure (i.e., the public expenditure affecting the productivity of private sectors) and its composition, since both types of public goods affect the supply side of the economy.

The main implications of our model can be summarized as follows. In the long-run the growth rate of the economy depends on government intervention, both directly, through the size of public expenditure and its composition and indirectly, through the long-run effects of government decisions on the employment sectoral composition.[3] We conduct two policy experiments. First, we investigate how the growth rate of the economy and the sectoral employment shares vary as the size of the public sector increases, holding constant the public expenditure composition. Second, we study the effect of changing the composition of public expenditures at an invariant size of the public sector. We single out a growth-maximizing size of public expenditure for any given composition of public expenditure. More interestingly, we single out a growth-maximizing public expenditure composition for any given size of total public expenditure, which is particularly relevant to the current political

[2]We define our 'modern' and 'traditional' aggregates following a technology criterion in the spirit of the recent contributions following Baumol (1967), mentioned in Section 2. The sectors constituting the two aggregates are assumed to use similar technologies even if producing goods or services with different uses. See Section 3 for the correspondent sectoral taxonomy used in the empirical analysis.

[3]In Section D we also study the transition path where the employment shares change overtime.

agenda, where a lot of emphasis has been put on public expenditure's containment rather than on its composition. We provide a comparison between our results and those of a one-sector economy with similar features.

This paper is structured as follows. Section 2 briefly reviews the literature, Section 3 provides some empirical motivation for our theoretical analysis. Section 4 introduces the basic framework of the model; in Section 5, we analyze the BGP and the transitional dynamics of the economy; in Section 6, we carry out some comparative statics, looking at the long-run effects of a change in both the tax rate and the share of expenditure devoted to infrastructure; Section 7 concludes and sketches the directions for future research.

2 Related literature

This paper is related to two main streams of theoretical literature. The first one is the endogenous growth literature following the seminal contributions of Barro (1990), analyzing the relationship between productive public expenditure and aggregate growth (Futagami et al., 1993; Turnovsky, 1997). A few contributions allow for the public expenditure's composition to play a role, by looking at the interplay either between public consumption expenditures and public productive expenditures (Baier and Glomm, 2001; Turnovsky, 2004; Angelopoulos et al., 2007; Economides et al., 2010), and/or between different types of government productive expenditures, as we do in this work, albeit in a different framework (Dioikitopoulos and Kalayvitis, 2008; Agénor, 2010; Carboni and Medda, 2011).[4]

Closer to our work, Ghosh and Roy (2004), in the spirit of Devarajan et al. (1996), combine a stock and a flow of public goods by including both public capital and public services in their production function and provide a full comparison with those contributions considering either of the two types of public expenditure.

In this stream of contributions, beyond the policy instruments governing the total size of public

[4] In Dioikitopoulos and Kalayvitis (2008), public expenditure can be devoted either to 'new' public investment or to existing public capital maintenance, similarly to Rioja (2003), Kalaitzidakis and Kalayvitis (2004). In Agénor (2010) public expenditure can be devoted to infrastructure capital and health or unproductive activities.Carboni and Medda (2011) consider public capital as a stock which they split into two components with different productivity.

intervention (i.e. tax rates), the government also controls the public expenditure shares. However, this literature introduces public expenditure in a one- (private) sector[5] economy framework, implying that the sector directly benefiting from the public capital's spillovers is also the one producing the public investment good. But this is not necessarily the case. In general, this literature does not investigate how the growth effects of public good provision may be mediated by the changes it induces in the sectoral composition of the economy.

A second stream of literature related with our work analyses the determinants and the growth-effects of structural change.[6] The recent contributions in this literature investigate the main drivers of structural change underlining either the role of the supply side of the economy, with heterogeneous technology and low substitutability in demand (Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008; Moro, 2012; Herrendorf and Valentinyi, 2015) or the role of the demand side, in particular the presence of an Engelian effect in demand (Echevarria, 1997; Kongsamut et al., 2001; Bonatti and Felice, 2008). Other contributions assess the relative weight of the two drivers (Herrendorf et al., 2013; Boppart, 2014; Comin et al., 2015). Among these recent works, in particular, are those investigating whether the transition towards technologically stagnant sectors in developed economies may hinder growth in the long run (Echevarria, 1997; Kongsamut et al., 2001; Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008; Bonatti and Felice, 2008; Moro, 2015). The contributions belonging to this literature do not generally take into account the potential role of government intervention (through public expenditure) in affecting the sectoral composition of the economy.[7]

3 Empirical motivation

Public sector's expenditure is not evenly distributed across sectors. As a consequence, a government could affect an economy's structure in terms of sectoral production and employment by choosing

[5] In particular, one consumption good.

[6] For a comprehensive review of this literature, see Herrendorf et al. (2014).

[7] Interestingly, Herrendorf and Valentinyi (2015) in a recent paper considering structural change with sector specific endogenous technological progress show that there is room for optimal industrial policy to increase the pace of structural change.

the composition of its public expenditure. In this section, we simply report some descriptive evidence, whose only aim is to show how different components of public expenditure are significantly associated with the employment sectoral composition of an economy. The latter is measured by the share of employment in the 'modern' sector (manufacturing and business services) over total employment.

In the empirical literature on the relationship between public expenditure and growth (see Section 1), several different public expenditure decompositions have been used, all of them relying on the two main classifications of public expenditure provided by national statistical offices: the functional classification (COFOG), which beaks down total public expenditure across functions of government[8] and the Economic Classification, which distinguishes different type of public expenditures based on their economic function (investment, intermediate consumption, compensations, social benefits, social transfers). In order to get closer to our theoretical intuition on the role of public expenditure composition for different sectors, we choose another strategy with respect to previous contributions and to build our public expenditure components we use the International Input-Output tables from the WIOD dataset, recently released within a project founded by the Seventh Framework Programme of the European Commission. The database is built on national accounts statistics, national input-output tables and national supply-use tables for 40 countries (among which are the EU27 countries), for the period 1995-2011 (Timmer et al., 2015). In particular, it provides domestic and international input-output flows at two-digit industry level. It provides statistics for the government consumption for different goods harmonized across countries. For the sectoral employment we rely on the Groningen 10 sector database and the EUKLEMS database (2009). We use Penn World Tables (8.1) and IMF data for the real gdp per capita and total public expenditure. As for the sectoral classification, the 'modern' sector includes agriculture, mining, manufacturing and financial, insurance, real estate and business services (firb), while the 'traditional' sector includes the rest of the economy (services, public utilities, construction). Our classification therefore coincides with the dichotomy tradable vs. non-tradable goods (see for instance Valentinyi and Herrendorf, 2008) except for the fact that we include financial and business

[8](i) General public affairs, (ii) economic affairs,(iii) housing, (iv) education, (v) social protection, (vi) health care,(vii) defence, (viii) culture, recreation and religion,(ix) environment protection,(x) public order and safety.

services in the tradable sector. We think that this classificaion is more suitable to the aim of this analysis, where 'modern' and 'traditional' sectors differ in terms of technology (capital intensity and productivity) and tradability. Nevertheless the empirical results reported below do not change when we consider the classical dichotomy between tradable and non-tradable goods. Figures A1-A3 show the time trends in the employment share of the 'modern' sectors for different groups of countries: advanced economies, Eastern-European countries and developing countries. Although the weight of the 'modern' sector is smaller in advanced economies, in all cases its share on total employment falls overtime.

We estimate a linear regression, where the dependent variable is the share of employment in the 'modern' sector of country c in year t (EMP_{ct}^{M}) and the independent variables are the logarithm of lagged real GDP per capita (gdp_{ct-1}), the lagged share of total public expenditure on GDP (G_{ct-1}), the lagged share of productive public expenditure over GDP ($PRODG_{ct-1}$), the lagged ratio between the productive public expenditure devoted to 'traditional' goods and total public productive expenditure ($TRADG_{ct-1}$), and year fixed effects ($Year$). As for the definition of the various public expenditure variables, total public expenditure is total public expenditure including also public expenditure for social transfers and is taken from IMF data, productive public expenditure is the total government consumption expenditure included in Input-Output tables. We split the productive public expenditure into two categories: the expenditure devoted to the 'modern' sector and the one devoted to the 'traditional' one. We then include in the regression $TRADG_{ct-1}$, the ratio between the productive public expenditure devoted to 'traditional' goods and total public productive expenditure ($PRODG_{ct-1}$). The linear specification reads as follows

$$EMP_{ct}^{M} = \alpha_0 + \alpha_1 gdp_{ct-1} + \alpha_2 G_{ct-1} + \alpha_3 PRODG_{ct-1} + \alpha_4 TRADG_{ct-1} + Year + \epsilon_{ct}. \qquad (1)$$

where ϵ_{ct} is an error term.

Since our aim is not to assess causality but simply report statistical associations, we neglect potential endogeneity issues and report OLS estimates of equation (1). As we said, the whole point of these estimates is just to show that the public sector can potentially affect an economy's sectoral composition by choosing the composition of its expenditure in terms of types of goods. Our sample includes an unbalanced panel of 26 countries[9] observed for the period 1996–2011, for which data on all variables are available.[10] It is worth noting that we have excluded from equation (1) country fixed effects. Although country fixed effects are useful to control for time-invariant country's unobservable variables which may be correlated with the level and composition of public expenditure and the employment share in the 'modern' sector, the latter is extremely persistent overtime. Indeed, the unconditional correlation coefficient between current and one-year lagged employment in the 'modern' sector is 0.997. Thus, in the relatively short period spanned by our data, we think it is unfeasible to control for country fixed effects or to include the lagged employment share (estimating a dynamic model), as these variables would practically explain all the variability in the dependent variable, leaving nothing else to be explained by our regressors of interest.[11] For the same reason, given the very slow changes observed in sectoral composition, we think our data are much more useful to investigate the interactions between the public sector and employment sectoral composition than between the public sector and structural change (i.e. changes in this composition overtime).

We obtain the following OLS estimate[12]

$$EMP_{ct}^{M} = \underset{(.343)}{1.747} - \underset{(.025)}{.070}\ gdp_{ct-1} + \underset{(.15)}{.218}\ G_{ct-1} - \underset{(.322)}{.650}\ PRODG_{ct-1} - \underset{(.359)}{.819}\ TRADG_{ct-1} \quad (2)$$

where standard errors in parentheses are clustered at the country level to allow for (unspecified) serial correlation in the error term, and are robust to heteroskedasticity.

The negative sign of the coefficient of the per-capita GDP should capture the role of income effects in demand in affecting the sectoral composition of the economy, consistently with previous literature. Therefore, the coefficients of public expenditure components capture what is left over

[9] Australia, Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Hungary, India, Ireland, Italy, Japan, Latvia, Luxembourg, Malta, Mexico, Netherlands, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, United Kingdom.

[10] We consider this relatively short time span because we have to rely on the World Input Output Tables to split the total productive public expenditure according to the sectoral taxonomy that we propose as relevant for growth.

[11] After including country fixed effects, only within-country variation overtime (i.e. structural change) would be exploited in the regression.

[12] Year fixed effects are not reported for brevity.

and above the income effect in consumption. Our estimates suggest that an increase in the share of public productive expenditure over GDP and an increase in the share of public productive expenditure spent in 'traditional' goods over public productive expenditure both contribute to reducing the share of employment in the 'modern' sector.[13] This result suggests that the composition of public expenditure matters for the sectoral structure of the economy. The effect may depend on demand and supply channels. Indeed, the government expenditure for different goods will directly affect the composition of demand for goods produced by the different sectors and therefore the employment composition. On the other hand, by providing public goods the government affects, possibly unevenly, sectoral productivity and therefore relative prices. As a consequence, household consumption composition may also be affected, depending on the degree of elasticity of substitution across goods, and have a feedback on employment composition. Moreover, by increasing the productivity of the private factors, the government may affect the private investment and therefore the sectoral composition. The negative sign of the coefficient of total productive public expenditure is due to the fact that in our sample the share of public expenditure in the 'traditional' sector is very high.

If different sectors of an economy are characterized by different levels of productivity, size and composition of public expenditure can also impact on the growth rate of the economy. Although the direct impact of the public sector on economic growth has been widely investigated, both theoretically and empirically (see Sections 1 and 2), its indirect effects mediated by sectoral composition (in a multi-sector economy) are still unexplored.

In what follows, we will investigate the effect of size and composition of public expenditure on an economy's structural composition and growth rate from a theoretical point of view using an endogenous growth model, while a more precise quantification of the empirical relevance of our theoretical model is left for future research.

[13] The $t-$statistics are -2.28 and -2.02, respectively.

4 The model

The main features of the model, which will be presented below in this section, are the following. There are two private sectors in the economy: a 'modern' sector M, whose productivity benefits from the provision of public capital (i.e. infrastructure) and a flow of public goods; and a 'traditional' sector S, whose technology benefits from the provision of the flow of public goods only.

We allow for a spillover from the accumulation of private capital in the 'modern' sector, as in Kalaitzidakis and Kalayvitis (2004). In this very stylized economy, the 'modern' sector produces medium/high technology goods or services. It may be thought of as composed of manufacturing industries and advanced services (e.g., business services, but also R&D labs), thus producing goods for final consumption C_{Mt} (e.g., durables), for private investments I_t (e.g., machinery and equipments, but also knowledge intensive goods such as IT goods used either in production or in R&D labs), and, finally, intermediate goods G_{Ft} (e.g., new materials in construction, software for administration services), purchased by the government and publicly provided to enhance the productivity of both the sectors. The 'traditional' sector S may be thought of as composed of service industries and construction industries, producing goods and services characterized by a traditional labour-intensive technology, whose productivity can be improved through the public provision of intermediate goods G_{Ft}. The 'traditional' sector S produces goods that can be used for both final consumption, C_{St}, and public investment, G_{It}, purchased and publicly provided by the government, which accumulates in a stock of public capital enhancing the productivity of the 'modern' sector.

This framework is suitable for investigating the growth effects of the public provision of the so called 'core' infrastructure (Gramlich, 1994) such as transport networks (e.g. roads, highways, railways, airports, waterways), the network side of telecommunications and infrastructure for energy provision, taking into account, on the one hand, that their technology is construction (i.e., 'traditional' sector) intensive and, on the other hand, that they unevenly affect the productivity in different sectors of the economy. The 'core' infrastructure that we are considering typically enhances the productivity and efficiency of private sectors, by increasing the tradability of goods and services (e.g., reducing time and transport costs and/or making their tradability technologically 'feasible'), as extensively underlined by the New Economic Geography literature (Krugman, 1991;

Ottaviano, 2008).[14] This gives an intuition as to why they should unevenly affect the productivity of 'modern' (typically, tradable) and 'traditional' (typically, non-tradable) sectors (Holtz-Eakin and Lovely, 1996). On the other hand, 'core' infrastructure is typically a non-tradable good. Consistently with the evidence provided in Section 3, our work is focused on the role of public intervention in transitional and developed economies, where the agricultural sector is a shrinking 'modern' sector, while it may not be suitable to study the role of public investment in helping underdeveloped economies' take off.

The public capital and the flows of public intermediate goods are pure public goods, directly provided by the government, in a balanced budget framework, by levying taxes on income (see Section 4.1).[15] Private agents, firms and households, take fiscal policies as given when making private optimal decisions. This model can be seen as an extension of Ghosh and Roy (2004) where both stock and flows of public goods are considered in an endogenous growth framework in a one-sector economy. Except for allowing for the private capital spillover, the 'modern' sector in this model displays the same technology as in Ghosh and Roy (2004). Unlike them we introduce a second sector in the economy, the 'traditional' sector producing a consumption good and the public investment good.

4.1 Government

Government total expenditure splits into two different flows of goods: G_{It} is the flow of investment goods purchased by the government from the 'traditional' sector S, which can be accumulated into the public capital (i.e. infrastructure) K_{Gt} used in the 'modern' sector M; G_{Ft} is the flow of goods

[14]This literature points out that the main pathways through which public infrastructure positively affects growth rest, more or less directly, on trade costs reduction, allowing for the exploitation of economies of scale or for increasing firms' efficiency through higher competition.

[15]We do not include here the comparison between different provision modes. Moreover, we consider all the different types of productive public expenditure, the stock (i.e. the infrastructure) and the flow (i.e. the intermediates goods), as pure public goods, abstracting from the potential congestion of public capital, differently from Dioikitopoulos and Kalayvitis (2008), Economides et al. (2010) and Chatterjee and Ghosh (2011). In other words, in our framework public infrastructure corresponds to a non-rival, non-excludable good, available equally to each firm, independent of the size of the economy.

or services, purchased by the government from the 'modern' sector M, provided to sector S and M in order to enhance their productivity.

The government directly provides G_{Ft} and K_{Gt} to firms in sector S and in sector M, respectively, and total public expenditure is financed by a constant proportional tax on capital and labour income, τ, which is exogenous.

We assume that all tax revenues are spent and the government runs a balanced budget:

$$G_t \equiv G_{Ft} + p_t G_{It} = \tau(w_t + r_t K_t), \qquad 0 < \tau < 1 \tag{3}$$

The shares of G_t that the government spends, respectively, on investment goods (v_I) and on intermediates (v_F) are constant and exogenous

$$G_{Ft} = v_F G_t \tag{4}$$

$$p_t G_{It} = v_I G_t. \tag{5}$$

The government budget in (3), together with (4) and (5), implies $v_F + v_I = 1$. From now on, we define $v_I = v$, the share of public expenditure on infrastructure, and $v_F = 1 - v$, the share on intermediate goods.

Tying total expenditure G_t to tax revenues for a given tax rate, and assuming that the government runs a balanced budget, is essentially like assuming that the ratio of total public expenditure to GDP is set arbitrarily.[16]

Public capital does not depreciate [17] and the public capital stock evolves according to

$$K_{Gt+1} - K_{Gt} = G_{It}. \tag{6}$$

[16] Government expenditure is parametrized in a similar fashion by, for instance, Futagami et al. (1993), Dioikitopoulos and Kalayvitis (2008), Agénor (2010).

[17] This assumption simplifies the analysis and appears to be innocuous once we assume it holds for private capital too, and that the depreciation rate is exogenously given.

4.2 Production

There is a large population (normalized to one) of firms producing in sector M or in sector S. They are identical within each sector and in each period t they produce good M and good S, respectively. Good M is the numeraire of the economy and its price is normalized to one. Firms produce good M and good S with different technologies and hire capital and labour from households at each point in time.

Firms in sector M produce with the following technology

$$Y_{Mt} = (A_t L_{Mt})^{\alpha} K_t^{1-\alpha}, \;\; 0 < \alpha < 1, \tag{7}$$

with

$$A_t = [K_t^{1-\gamma}(K_{Gt}^{\beta} G_{Ft}^{1-\beta})^{\gamma}], \;\; 0 < \beta < 1, 0 < \gamma \leq 1 \tag{8}$$

where K_t is the individual firm's stock of private capital and L_{Mt} the labour share used to produce the good M at time t, respectively; K_{Gt} is the per capita stock of public infrastructure; G_{Ft} is the flow of public expenditure accruing to the individual firm.[18] When $\gamma = 1$ we are back to a technology with no spillover from the sectoral stock of private capital. In this case the only engine of growth of the economy is the public intervention. In each t, firms in sector M employ labour and rent capital in order to maximize their profits, given by

$$\pi_{Mt} = Y_{Mt} - w_t L_{Mt} - r_t K_t, \tag{9}$$

where w_t and r_t are, respectively, the wage rate and the rental rate of capital at time t.

Firms in sector S produce with the following technology

[18] K_{Gt} and G_{Ft} may be interpreted as contributing to the Total Factor Productivity in the technology. Public capital acts at the level of sector M: an increase in the stock of public goods shifts the production function upward, enhancing the marginal product of the factor inputs. K_G and G_{Ft} may also be considered as a third factor of production. As pointed out by Romp and de Haan (2007) given the Cobb-Douglas technology, the two interpretations are equivalent.

$$Y_{St} = G_{Ft} L_{St}, \tag{10}$$

where L_{St} is the labour share used in production in sector S and G_{Ft} is define as above. The production of the S good requires combining government spending in a flow of intermediate goods or services, broadly considered, and labour. As already mentioned, G_{Ft} are goods purchased by the government from sector M and which may be provided in order to enhance the productivity of labour in sector S.[19] Thus, sector S does not directly benefit from the services of the infrastructure, but it does so indirectly, through the flow of public intermediate goods produced by sector M.

In each t firms in sector S employ labour in order to maximize their profits, given by

$$\pi_{St} = p_t Y_{St} - w_t L_{St}, \tag{11}$$

where w_t is the wage rate and p_t is the price of good S (in units of good M).

The representative firm in sector M takes A_t as given and the representative firm in sector S takes G_{Ft} as given, when taking their private optimal decisions.

From the firm's First Order Conditions in sector M and S, and taking into account the government constraints, (3) and (4), given that labour is homogeneous and perfectly mobile across sectors so that the intersectoral efficiency condition must hold, we derive the rental rate of private capital

$$r_t = \{(1-\alpha) L_{Mt}^{\alpha} Z_t^{a} [(1-\nu)\tau(\frac{\alpha}{L_{Mt}(1-\alpha)} + 1)]\}^{\frac{1}{1-b}}, \;\; a = \beta\gamma\alpha, b = (1-\beta)\gamma\alpha \tag{12}$$

where $Z_t \equiv \frac{K_{Gt}}{K_t}$ is the public to private capital ratio, and L_{Mt} is the employment share in the modern sector, at time t, and where τ and v govern the size and the composition of public expenditure; w_t, the wage rate in the economy is

$$w_t = \frac{\alpha K_t}{(1-\alpha) L_{Mt}} r_t, \tag{13}$$

[19] G_F can be thought of as a second factor of production. See the previous footnote.

and p_t the relative price

$$p_t = \frac{\alpha Z_t^a (1-\nu)\tau(\frac{\alpha}{L_{Mt}(1-\alpha)}+1)^{b-1}}{r_t L_{Mt}^{\alpha-1}}. \tag{14}$$

The relative price in this framework depends not only on the parameters representing both technology and preferences (through the employment shares), but also on those representing the public policy. It is worth noting that the returns to factors and the relative price depend on the employment share in the 'modern' sector, this implying that they are changing as long as the employment shares change (see also Section 5.1).

4.3 Households

The economy is populated by a large number of infinitely lived households. Population is assumed to be constant and normalized to unity.[20] Individuals supply labour inelastically in the two sectors, they receive both capital and labour income, they pay taxes on their income and they decide intertemporally the share of income to be devoted to the two consumption goods and their savings in order to maximize their utility over an infinite time horizon.

The preference structure of the household is represented by a utility function showing constant elasticities both across goods and over time

$$u_t = \frac{[(\omega_t C_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega_t) C_{St}^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}]^{1-\sigma} - 1}{1-\sigma}, \qquad \sigma > 0, \varepsilon > 0, \omega > 0 \tag{15}$$

where u_t is the period utility, and C_{Mt} and C_{St} are, respectively, the quantities of M good and S good consumed by the representative household in period t. If $\sigma = 1$ and $\varepsilon = 1$ the utility in (15) collapses into the logarithmic utility with unitary elasticities both across goods and over time. This utility specification allow us to disentangle the channel induced by the reaction of the consumption

[20]The assumption of constant population is not entirely neutral. In this framework, we assume that there is no congestion in benefiting from the infrastructure. This assumption may turn out to be less plausible with growing population. In a model where population growth is allowed for, congestion costs should be introduced.

of private composition to a change in the relative price when $\varepsilon \neq 1$, along the lines of the, so called, supply side drivers of structural change.[21]

The period budget constraint for the representative household can be written as

$$K_{t+1} + p_t C_{St} + C_{Mt} = (1 + r_t(1-\tau))K_t + w_t(1-\tau)L_t, \tag{16}$$

where K_t is the total amount of private capital owned in t by the representative household, L_t is the quantity of labour supplied (which is fixed and normalized to unity) by the representative household, τ is the exogenous tax rate on capital income, p_t, w_t and r_t are the relative price, the wage rate and the return to private capital, respectively.

In each t, the representative household chooses the sequence $\{K_{n+1}\}_{n=t}^{\infty}$, $\{C_{Mn}\}_{n=t}^{\infty}$ and $\{C_{Sn}\}_{n=t}^{\infty}$ in order to maximize its discounted sequence of utility.

$$\sum_{n=t}^{\infty} \theta^{n-t} u_n, \qquad 0 < \theta < 1 \tag{17}$$

subject to (16), where θ is a time-preference parameter.

The representative household problem in (17) can be solved by maximizing

$$\mathcal{L} = \sum_{n=t}^{\infty} \theta^{n-t} \left\{ u_n + \lambda_t[(1 + r_t(1-\tau))K_t + w_t(1-\tau)L_t - K_{t+1} - p_t C_{St} - C_{Mt}] \right\}, \tag{18}$$

with respect to the control variables C_{St} and C_{Mt}, the state variable K_{t+1} and the costate variable λ_t.

By eliminating the costate variable, we obtain

$$C_{Mt} = C_{St} p_t^{\varepsilon} (\frac{\omega}{1-\omega})^{\varepsilon}, \tag{19}$$

[21] This preference structure is used by Ngai and Pissarides (2007) to show that a elasticity of substitution lower than one generates an increase in the employment share in the service sector in a model where sectors differ in the exogenous rate of productivity growth and the service sector is assumed to display a lower rate of productivity growth.

and the Euler equation

$$\frac{C_{St+1}}{C_{St}} = \left[\theta(1 + r_{t+1}(1-\tau))\frac{p_t}{p_{t+1}}\right]^{\frac{1}{\sigma}} \left[\frac{\omega p_t^{\varepsilon-1}(\frac{\omega}{1-\omega})^{\varepsilon-1} + (1-\omega)}{\omega p_{t+1}^{\varepsilon-1}(\frac{\omega}{1-\omega})^{\varepsilon-1} + (1-\omega)}\right]^{\frac{1-\sigma\varepsilon}{(1-\varepsilon)\sigma}} . \tag{20}$$

Private capital does not depreciate[22] and its stock evolves according to the following law of motion:

$$K_{t+1} = I_t + K_t. \tag{21}$$

5 The dynamic system in the decentralized economy

All markets in this economy are competitive.

Labour market clearing, given that we have normalized the population to one and that labour is supplied inelastically, implies

$$L_{Mt} + L_{St} = 1 \tag{22}$$

Equilibrium in the market for good M requires

$$Y_{Mt} = C_{Mt} + I_t + G_{Ft}, \tag{23}$$

where we assume, as mentioned at the beginning of this section, that good/service M can be consumed (C_{Mt}) or used as the private investment good accumulated in sector M (I_t), or used as an intermediate good in sector S and M (G_{Ft}).

Equilibrium in the market for good S requires

$$Y_{St} = C_{St} + G_{It}, \tag{24}$$

[22] See footnote 17 above.

where we assume that the good/service S can be either consumed (C_{St}) or used as the public investment good that will accumulate to constitute the public infrastructure in sector M (G_{It}).

We study the decentralized equilibrium of this economy. In this setting, we define:

Definition 1 *A competitive equilibrium is a set of infinite sequences for the quantities $\{K_{n+1}, K_{Gn}, C_{Mn}, C_{Sn}\}_{n=t}^{\infty}$, together with prices $\{p_n, w_n, r_n\}_{n=t}^{\infty}$, such that, for given fiscal policies, that is, for given values of the tax rate, τ, and the public expenditure shares, v: household utility, (17), is maximized subject to the budget constraint (16); the firm's problem is solved in both sectors, i.e., (12), (13), and (14) hold; all markets clear, i.e., (22), (23), and (24) are satisfied; the government's budget constraint in (3) is satisfied for given expenditure shares (4) and (5); and public and private capital evolve according to their own laws of motion, (6) and (21).*

The economy is governed by a dynamic system which can be reduced to a system of two difference equations, (25) and (27) below, in two variables: $Z_t \equiv \frac{K_{Gt}}{K_t}$ and L_{Mt}.

By using the labour market equilibrium condition in (22), the market clearing condition for good S (24), the technology in sector S (10), we rewrite the Euler equation in (20):

$$\Lambda(L_{Mt}, L_{Mt+1}, Z_t, Z_{t+1}) = 0 \tag{25}$$

where

$$\Lambda(L_{Mt}, L_{Mt+1}, Z_t, Z_{t+1}) : \frac{r_t(\frac{\alpha}{(1-\alpha)L_{Mt}})[(1-\nu)(1-L_{Mt}) - \frac{\nu}{p_t}]}{r_{t+1}(\frac{\alpha}{(1-\alpha)L_{Mt}})[(1-\nu)(1-L_{Mt}) - \frac{\nu}{p_{t+1}}]} \times$$
$$[\theta(1 + r_{t+1}(1-\tau))\frac{p_t}{p_{t+1}}]^{\frac{1}{\sigma}} \left[\frac{\omega p_t^{\varepsilon-1}(\frac{\omega}{1-\omega})^{\varepsilon-1} + (1-\omega)}{\omega p_{t+1}^{\varepsilon-1}(\frac{\omega}{1-\omega})^{\varepsilon-1} + (1-\omega)}\right]^{\frac{1-\sigma\varepsilon}{(1-\varepsilon)\sigma}} - \mu_{Kt} - 1, \tag{26}$$

and p_t and r_t are the relative price and the rental rate of private capital derived from the firms' First Order Conditions as in (14) and (12), respectively, and μ_{Kt} is the growth rate of private capital. By substituting in the market clearing condition for good M, (23), the technology equations in the two sectors, (7) and (10), the household's intra-temporal condition in (19), the market clearing condition for the labour market, (22), and for good S, (24), the law of motion for private and public capital (21) and (6), and imposing the government budget constraints (3) and (4), the aggregate resource constraint of the economy is derived:

$$\Omega(L_{Mt}, L_{Mt+1}, Z_t, Z_{t+1}) = 0 \tag{27}$$

where

$$\Omega(L_{Mt}, L_{Mt+1}, Z_t, Z_{t+1}) : L_{Mt}^{\alpha} Z_t^{a} \tau(1-v) r_t(\frac{\alpha}{L_{Mt}(1-\alpha)}+1)[\tau(1-\nu) r_t(\frac{\alpha}{L_{Mt}(1-\alpha)}+1)]^{b-1} - \tau r_t(\frac{\alpha}{L_{Mt}(1-\alpha)}+1)[(1-v)(1-L_{Mt}) - \frac{\nu}{p_t}] p_t^{\varepsilon}(\frac{\omega}{1-\omega})^{\varepsilon} - 1 - \mu_{Kt} \tag{28}$$

and where μ_{Kt} is the growth rate of the private capital obtained from the law of motion of public capital, (6), taking into account the government's budget constraints (3) and (5):

$$\mu_{Kt} = (\frac{Z_t}{Z_{t+1}}) + \frac{\nu \tau r_t(\frac{\alpha}{L_{Mt}(1-\alpha)}+1)}{Z_{t+1} p_t} - 1. \tag{29}$$

Therefore, along an equilibrium path, (25) and (27) must be satisfied, together with the transversality condition

$$Lim_{t\to\infty}\left(\theta^t \lambda_t K_t\right) = 0, \tag{30}$$

to ensure that the intertemporal resource constraint is met.

5.1 Balanced growth path

In order to study the behaviour of this economy in the long run, we characterize a BGP by setting $Z_{t+1} = Z_t = Z$ and $L_{Mt+1} = L_{Mt} = L_M$.

Definition 2 *The Balanced Growth Path of this economy is a set of sequences* $\{Z_n, L_{Mn}\}_{n=t}^{\infty}$ *, for a given tax rate* τ *and spending share* v*, such that (25), (27), (29), and the transversality condition (30), are satified.*

Under the above definition, the existence of a BGP implies that in the long run (i.e., in a mature economy) the ratio between public and private capital and the employment shares stabilize. If a BGP exists for the dynamic system (25)–(27) it is characterized by Z and L_M solving the system (31)-(32):

$$\frac{\nu\tau r[\frac{\alpha}{L_M(1-\alpha)}+1]}{p} - [\theta(1+r(1-\tau))]^{\frac{1}{\sigma}} + 1 = 0 \tag{31}$$

and

$$L_M^{\alpha} Z^{a} \tau(1-v) r(\frac{\alpha}{L_M(1-\alpha)}+1)[\tau(1-\nu)r(\frac{\alpha}{L_M(1-\alpha)}+1)]^{b-1}\tau r(\frac{\alpha}{L_M(1-\alpha)}+1)\times \\ [(1-v)(1-L_M) - \frac{\nu}{p}]p^{\varepsilon}(\frac{\omega}{1-\omega})^{\varepsilon}[\theta(1+r(1-\tau))]^{\frac{1}{\sigma}} = 0. \tag{32}$$

Along the BGP the growth rate of private capital is given by

$$\mu_K = \frac{\nu\tau r[\frac{\alpha}{L_M(1-\alpha)}+1]}{p}. \tag{33}$$

Along the BGP, the employment shares and the public to private capital ratio stabilize. Both the relative price p, as appears from (14), and the rental rate of private capital r (12) stabilize when the employment shares and the ratio of public to private capital do. The stock of private capital, the stock of public capital, the aggregate GDP, the total public expenditure, the output in both sectors and wages grow at the same steady rate ($\mu_K = \mu_{KG} = \mu_{GDP} = \mu_G = \mu_{Y_M} = \mu_{Y_S} = \mu_w = \mu$).[23]

The growth rate depends on all the parameters of the economic system through the equilibrium employment share, i.e., through the long-run sectoral composition. In particular, the long-run rate is

[23] See Section C in the Appendix.

affected by government intervention through public expenditure size and composition, both directly, i.e. positively affecting the productivity of the private sectors and negatively affecting private investment (crowding out), and indirectly, through the long-run effects of public intervention on the employment shares, i.e., through sectoral composition. In previous contributions, the aggregate growth rate is affected by public intervention only directly, without taking into account its effect on the sectoral composition, that is to say, without taking into account how public intervention affects the employment shares which the growth rate depends on.

On the other hand, differently from previous contributions on the determinants of structural composition where changes in the latter are driven by technology and/or preferences, here the long-run sectoral composition also depends on public policy instruments, which the government can directly affect.

The above nonlinear system is solved numerically. For a large range of economically relevant parameter values, the conditions required to have a well defined equilibrium, with $0 < L_M < 1$ and $Z > 0$, satisfying the transversality condition are met.

We use the following baseline parameter values: $\theta = 0.94$; $\alpha = 0.37$; $\gamma = 0.3$; $\beta = 0.7$; $\sigma = 1$; $\epsilon = 0.85$; $\omega = 0.4$; $\tau = 0.2$; $\nu = 0.8$ (see Table 1). For these parameter values the economy displays a BGP characterized by $L_M = 0.2337$, $Z = 0.2997$ and the growth rate $\mu = 0.0184$, which is saddle path stable. We study the transitional dynamics and show the changes in the main variables along the transition in Section D in the Appendix. We set (τ), the share of total productive public expenditure on GDP, equal to 0.2, consistently with the values for mature economies in our data in Section 3 (see also European-Commission (2012)). We set $\nu = 0.8$ to match the share of productive public expenditure devoted to the 'traditional' sector in mature economies as in our data. The parameter entering the production function in sector M, α, has been set consistently with the evidence reported in the literature for private capital and labour output shares in the manufacturing and taking into account that here the 'modern' sector includes business services.[24] We set $\gamma = 0.3$ and $\beta = 0.7$, by using firms' FOCs, in order to be consistent with a long run value of the public to private capital ratio and that of the employment share in the 'modern' sector for a

[24] The value assigned to α corresponds to the capital share in the Tradable sector in producer prices as estimated in Valentinyi and Herrendorf (2008).

mature economy in the long run: $Z = 0.29$ which is close to the value reported in Turnovsky (2004) and $L_M = 0.23$, which is close to the value for a mature economies in our sample (see Section 3). We nevertheless run our simulations for several other sets of parameter values. In particular, we also set $\gamma = 1$ to consider the theoretical case in which the spillover from the stock of private capital is not at work. $\omega = 0.4$ is set to match the household expenditure share for the modern good in mature economies in our data in Section 3. We set $\epsilon = 0.85$, following the estimated value for ϵ in Herrendorf et al. (2013), to allow for substitutability lower than one across goods in consumption since we are dealing with broad aggregates of goods. A time preference parameter θ in the interval [0.85-0.95] is standard in the literature. In the benchmark simulation and in all the numerical results analyzed below, for theoretical purposes, we set the value of the parameter $\sigma = 1$, to be in line with what is often used in the literature and in particular with most of the theoretical literature on public capital and growth. Nevertheless, in order to get a transition path closer to what observed in the data we also run our simulation with $\sigma < 1$ and $\epsilon = 0.75$ (see Section D in the Appendix).

TABLE 1 ABOUT HERE

There are two main channels through which the government intervention affects the sectoral composition and the long-run growth of this economy. First, a demand channel, by buying goods from the two sectors, namely G_F and G_I. Second, a supply channel, since the public goods enter asymmetrically the two private sectors, this way affecting the relative price of goods. By considering homothetic preferences with different degrees of substitutability, we do not include in our analysis the Engelian effect in demand potentially affecting the sectoral composition of the economy as income grows, but we consider the effect of a change in the relative price, induced by the government intervention, on the composition of household consumption.

TABLE 2 ABOUT HERE.

Table 2 reports the long-run solutions comparing the equilibrium values of the growth rate (μ)[25] and the employment share in the 'modern' sector (L_M) when demand is price inelastic ($\epsilon = 0.85$)

[25]When not differently specified with growth rate we mean the aggregate growth rate of the economy (μ).

and when it displays unit elasticity ($\epsilon = 1$), considering both when a spillover from the stock of private capital is at work ($\gamma = 0.3$) and when it is not ($\gamma = 1$). In the latter case the long-run growth rate of the economy is driven by the public intervention only. Both the equilibrium growth rate of the economy and the employment share in the 'modern' sector are lower under low substitutability ($\epsilon = 0.85$) than under a unit elasticity of substitution in private consumption. This is due to a larger share of private consumption of the S sector under $\epsilon = 0.85$. With no spillover from the stock of private capital, that is to say when the public intervention is the only engine of growth, the growth rate is not surprisingly lower while the employment share in the 'modern' sector is larger.

RESULT 1. The growth rate (μ) depends on the government intervention both directly and through the effect on the sectoral composition of the economy. The equilibrium growth rate and the employment in the 'modern' sector (L_M) are lower when demand is price inelastic.

6 Long-run growth effect of fiscal policies

We now consider the effect of changes in fiscal policy rules. The growth effect of fiscal policies is a relevant issue since in many circumstances the aggregate growth rate in the long run is the target of public intervention. Moreover, the growth rate turns out to be relevant for the government whenever the sustainability of public expenditures becomes an issue. This is the reason why the contributions belonging to the literature analyzing the role of public expenditure in a growth framework investigate the effects of a change in the government intervention on the long run growth rate of the economy. Nevertheless, with only the few exceptions mentioned in Section 2, this literature usually limits the analysis to the role of a change in the tax rate, i.e., in the size of total public expenditure.[26] Since the total public expenditure, on the one hand, has a positive effect on the growth rate by enhancing the productivity of capital and labour, but on the other hand, reduces the resources for the accumulation of private capital, the analysis consists in looking for the growth-maximizing ratio between total public expenditure and private capital returns, that is the growth-maximizing τ. Our framework differs from previous contributions for two reasons.

[26] It is worth noting that, in line with other contributions (see footnote 16), our assumptions imply that when considering the effect of a change in the tax rate we are dealing with a change in the ratio of total productive public expenditure to GDP.

As claimed at the end of Section 5.1, on the one hand, in our two-sector framework the above mentioned trade-off is complicated by the effect of public expenditure on the sectoral composition of the economy. On the other hand, our framework allows us to consider two independent policy instruments affecting the growth rate of the economy: τ, representing the size of total public expenditure, and ν, representing its composition. Whenever it is constrained in increasing its total size, the government can affect the growth rate by modifying the composition of public expenditure ν.

There are previous contributions considering a similar combination of policy instruments (Baier and Glomm, 2001; Turnovsky, 2004; Ghosh and Roy, 2004; Dioikitopoulos and Kalayvitis, 2008), some of them showing the suboptimality of the Barro (1990) growth maximizing tax rate. Nevertheless, the mechanisms highlighted by these contributions work along different lines and in particular are not related to the effect of the policy parameters on the sectoral composition.

We conduct two policy experiments by showing numerically the growth and sectoral composition effects of a change in the government size (τ) for a given expenditure composition, and those of a change in the expenditure composition (ν) for a given government size. We consider both the cases with and without spillover from the private capital accumulation, and both types of preferences structure ($\epsilon = 0.85$ and $\epsilon = 1$). The case with no spillover is included mainly for comparison with previous literature and robustness purposes, but, it is worth noting that this case does not reproduce the main features of an economic system since with no private capital spillover the only engine of growth in the economy would be the public intervention. Figures 1 and 2 report the results of the first policy experiment.

FIGURE 1 ABOUT HERE

When ν is low ($\nu = 0.2$) the employment share in the 'modern' sector is increasing with τ under both assumptions on preferences ($\epsilon = 1$ and $\epsilon = 0.85$) and on the private capital spillover ($\gamma = 1$ and $\gamma = 0.3$); the relationship between τ and L_M is instead negative in both cases when ν is high ($\nu = 0.8$), for economically meaningful values of τ.[27]

[27] A inverted U-shape relationship emerges only in the case of no-spillover from the private capital and price inelastic demand, in presence of very low values of τ, implying that the relationship is in general negative for economically meaningful parameter values.

Changes in τ impact on employment shares through different channels: from the supply side, an increase in τ raises the productivity of private factors, this way potentially reallocating employment to the 'modern' sector producing the private capital. On the other hand, from the demand side, the effect of a variation in τ on the employment composition depends on the public expenditure composition for the two sectors. For high ν, the demand-side effect goes in the opposite direction with respect to the supply-side effect, shifting the employment share towards the 'traditional' sector, and the negative effect is more likely to prevail. For low ν, the demand-side effect and the supply-side effect go in the same direction increasing the employment share in the 'modern' sector. As a consequence, the effect of a variation in τ on the employment composition for high ν is much lower than in the case of low ν, since in the former case the demand effect counterbalances the supply side one.

FIGURE 2 ABOUT HERE

As shown in Figure 2 the growth rate of the economy displays, with the exceptions of some cases, an inverted U-shape as the size of the public sector (measured by τ) increases, under both types of preferences. The inverted U-shape relationship between μ and τ occurs because of the trade-off produced by the provision of public capital, as highlighted by the previous literature (see Section 2). Low tax rates imply low growth, since public investment is the engine of growth in this economy. Higher tax rates increase the growth rate by increasing the productivity of private factors. Nevertheless, when tax rates are too high private investments are crowded out and the growth rate declines. The growth maximizing τ for given ν is much lower when there is a private capital spillover owing to the fact that the crowding-out effect is amplified by the spillover. As a consequence, the shape of the relationship between τ and the growth rate is different depending whether a private capital spillover exists or not. When there is a spillover, the growth rate is maximized for low values of τ. In particular, for $\nu = 0.8$, the growth maximizing τ is 0.3 under low-substitutability in consumption ($\epsilon = 0.85$). Under unit elasticity of substitution in consumption ($\epsilon = 1$), for the range of values for which there is a well defined equilibrium, a negative relationship between the growth rate and τ emerges. When $\nu = 0.2$, the growth maximizing τ is 0.4 under both preference structures. Although for a large set of τ values the relationship is monotonically decreasing, it is nevertheless worth noting that in the vicinity of the value of τ observed in the data ($\tau = 0.2$) the relationship is positive.

The opposite happens when no spillover is at work from the stock of private capital: the growth maximizing τ is very high. For $\nu = 0.8$ the relationship between τ and the growth rate is always positive (when the equilibrium is well defined). For $\nu = 0.2$ the growth maximizing τ is 0.8 under both preference structures. Therefore for most of the τ values the relationship is monotonically increasing. This is due to the fact that, without a private capital spillover, the only engine of growth is public intervention and the crowding out effect is small. We can conclude that in the case that is closest to the actual features of economic systems, with a spillover from the stock of private capital, low substitutability in consumption and high ν, the growth rate of the economy is increasing in the interval $\tau = 0.2 - 0.3$. To make an example, increasing τ from 0.2 to about 0.3, which means a 50% increase in the share of productive public expenditure in GDP, would raise the growth rate of the economy by 5%. It means that for small, economically meaningful and feasible changes in τ, it would be possible to obtain an increase in the growth rate.

RESULT 2. For a high (low) level of public expenditure share devoted to the 'traditional' sector (ν), a negative (positive) relationship between the size of public expenditure (τ) and the employment share in the 'modern' sector (L_M) emerges. The relationship between the size of public expenditure (τ) and the growth rate of the economy (μ) is increasing for economically meaningful values of τ, when a spillover from the private capital is at work and there is low-substitutability in consumption.

FIGURE 3 ABOUT HERE

Figure 3 reports the results of the second policy experiment, where we assess the effects of a change in the expenditure composition (ν) for a given government size. It shows that the employment share in the 'modern' sector M is monotonically decreasing, as the expenditure share devoted to the public infrastructure increases, while the growth rate of the economy displays an inverted U-shape. For given τ, as the expenditure share devoted to the public infrastructure increases, the demand for the public investment good increases and therefore the employment share in the S sector, which produces it, increases too. The growth rate initially increases as a consequence of the positive effect of the public capital on the productivity of private factors. Nevertheless, above a certain threshold of ν, the negative effect of the shrinking employment share in the 'modern' sector M on the growth rate prevails. This framework allows us to point out a growth maximizing expenditure composition for any given government size. This result is driven by the sectoral composition effects of the government intervention. The growth maximizing ν is 0.3 under both preference structures when a private capital spillover is at work, while it is 0.4 under both preference structures when there is no spillover. This implies that for most of the economically meaningful values of ν the growth rate is decreasing with ν. This is because higher ν implies higher expenditure for the 'traditional' good. As in the previous experiment, and for the same reason, the growth maximizing ν is larger without a private capital spillover, in both preferences structures. This analysis shows that, under both preference structures and alternative assumptions on the existence of a private capital spillover, the growth maximizing ν is much lower than the values commonly observed in the data, where ν is often between 0.8 and 0.9. From the normative point of view, lowering the share of public expenditure devoted to the 'traditional' sector would allow to increase considerably the growth rate of the economy without affecting the total amount of productive expenditure. For instance, with price inelastic demand and a private capital spillover, for given $\tau = 0.2$, a decrease in ν from 0.8 to 0.7, which corresponds to a decrease of about 13% would imply an increase in the growth rate of the economy of more than 40% (from 1.8% to 2.6%).

RESULT 3. A negative relationship emerges between the share of public expenditure devoted to the 'traditional' sector (ν) and the employment share in the 'modern' sector ($Ł_M$) given the size of public expenditure (τ). An inverted U-shape relationship emerges between the growth rate (μ) and the share of public expenditure devoted to the 'traditional' sector (ν). For economically meaningful parameter values, the relationship is monotonically decreasing. The growth maximizing level of ν for given τ is 'low' (in the interval 0.3-0.4) with respect to that observed in the data. Changes in ν for given τ have a large impact on the growth rate.

A general comment should be made with regards to the two policy experiments described above. For a given composition of the public expenditure, i.e. for given ν, the size of the public intervention (τ) maximizing the growth rate of the economy appears to be quite close to the one observed in the data when demand is price inelastic and a spillover from the stock of private capital exists.

On the other hand, for a given size of the public intervention (τ), the composition of the public expenditure, ν maximizing the growth rate of the economy is much lower than the one observed in the data. This implies that there is room for policies affecting the composition of the public expenditure; in particular, even small changes are capable to produce large effects on the growth rate of the economy.

FIGURE 4 ABOUT HERE

Our model represents an extension of the one-sector model where the whole economy is constituted by the 'modern' sector. We introduce the 'traditional' sector S as the sector which is producing the public investment good. In Figure 4 we compare the results of the two policy experiments in the two-sector model case with the ones of a one-sector model as derived in Section B in the Appendix. The shape of the relationship between the growth rate of the economy and the changes in the values of τ for given ν is similar in the cases of a one-sector economy and a two-sector economy. The size of public expenditure maximizing the growth rate is slightly lower in the case of the one-sector economy ($\tau = 0.2$ in the one-sector economy vs.$\tau = 0.3$ in the two-sector economy, with price inelastic demand and a private capital spillover). This is due to the two sectors - two flows of public expenditure structure of this economy. The size of the public sector has to be such that the marginal benefits of the public intervention are equal to its marginal costs. Nevertheless, in a two-sector framework the benefit no longer coincides with the marginal productivity of public capital in the 'modern' sector only; at the same time τ is no longer the cost of public capital provision, since tax revenues are also devoted to the provision of a flow of intermediate goods to the 'traditional' sector. But what is relevant to stress is that the share of public expenditure devoted to the 'traditional' sector ν which maximizes the growth rate of the economy is much higher in the one-sector model than in the two-sector one ($\nu = 0.7$ and $\nu = 0.3$, respectively, when demand is price inelastic and there is a private capital spillover; $\nu = 0.7$ and $\nu = 0.4$, respectively, when demand is price inelastic and no spillover is at work) because in the former the negative effect on growth due to an increase in the expenditure for the 'traditional' sector is not taken into account. This implies that despite the relationship between the growth rate and the values of ν is inverted U-shape in both the cases of a two-sector economy and a one-sector economy, in the former the relationship is monotonically decreasing for most of the economically meaningful values of ν, while in the latter it is monotonically increasing for most of the values of ν.

RESULT 4. The growth maximizing τ for a given level of ν is slightly higher in the one-sector economy. The growth maximizing ν for a given level of τ is higher in the one-sector economy.

TABLE 3 ABOUT HERE

Table 3 compares our results with those of the one-sector economy model (B) corresponding to the three cases of Barro (1990), Futagami et al. (1993) and Ghosh and Roy (2004). We consider the case in which growth is driven by the public sector only, as in those models, and we compare the case in which (i) public expenditure takes the form of both a stock and a flow as in Ghosh and Roy (2004), (ii) is only a flow as in Barro (1990) and (iii) is only a stock as in Futagami et al. (1993).[28] Table 3 shows that one-sector economy models predict larger growth rates than a two-sector model. In particular our model generates a substantially lower growth rate in equilibrium when compared with the one of Ghosh and Roy (2004), even when allocating the same share of public expenditure to stock and flows as in their model ($\nu = 0.2$) and if stock and flows combine in the production function of the 'modern' sector in the same way ($\beta = 0.1$). This is due to the presence of a 'traditional' labour-intensive sector in the economy.

RESULT 5. One-sector economy models of public induced endogenous growth predict much larger growth rates (μ) than a two-sector model.

7 Conclusion and further work

So far, the main scientific contributions analysing the relationship between public investment and growth have been developed in a one-(private) sector economy framework. Moreover, the growth literature investigating the determinants and effects of inter-industry relocation of resources usually disregards the role of government expenditure as a determinant of sectoral composition.

[28] We choose the parameter values for fiscal policies to match the numerical analysis in Ghosh and Roy (2004).

This paper represents a first attempt to fill this gap, by considering the role of public investment in an endogenous growth framework with structural change between technologically different industries and by providing some new descriptive evidence on the relationship between government expenditure and the sectoral composition of employment. We first show empirically that productive public expenditure' size and share spent for traditional goods both are significantly negatively associated with the employment share in the 'modern' sector.

Then, we build a model in which we allow for the government to provide both public capital and a flow of intermediate goods (in a balanced budget framework) in an economy with two private sectors, a 'modern' sector and a 'traditional' sector, both producing consumption and investment goods. In this set up, the government has two policy instruments, namely, the size and the composition of public expenditure for different types of goods, to affect the growth rate of the economy.

The growth rate of the economy in the long run depends on government intervention both directly, through public expenditure size and composition, and indirectly, through the impact of government expenditure on the long run employment shares. We conduct two policy experiments. First, we investigate how the growth rate of the economy and the sectoral employment shares vary as the size of the public sector increases, holding constant the public expenditure composition. Second, we study the effect of changing the composition of public expenditures at an invariant size of the public sector. The employment share in the 'modern' sector is decreasing (increasing) in the size of public sector when the share of expenditure for the 'traditional' sector is high (low), and it is always monotonically decreasing with the public expenditure composition. We show that for economically meaningful parameter values, a feasible policy intervention could increase the growth rate, namely, an increase in the size of productive investment (τ), in the vicinity of the actual one, given the composition (ν), or, alternatively a decrease in the share of expenditure devoted to 'traditional' sectors (ν), given the size (τ). We compare our findings with those of a one-sector economy constituted by the 'modern' sector only. We show that the growth maximizing size of expenditure is lower in the one-sector than in our two-sector framework, while the growth maximizing composition of expenditure for stock and flows of public goods is higher in the former, which does not take into account that the stock of public capital is produced by a traditional sector.

We also show that one-sector models with productive public expenditure tend to predict a larger growth rate in the long run.

Our analysis is relevant to the current political debate focused on the contraction of public expenditure, showing that changes not only in the size, but also in the allocation of public expenditures to different goods/services and for different sectors can be needed to achieve sustained economic growth in the long run.

In this paper, we have been focusing on the role of government in the long run, in particular, investigating the role of different components of productive public expenditure, i.e. those components entering the supply side of the economy. An interesting extension in order to better analyze the role of government in affecting the pace and speed of structural change along the transitional dynamics (and to properly fit the data) should be developed in a theoretical framework including other sources of growth and structural change and other types of government spending.

A second natural development of this work, given the role that 'core' infrastructure plays in a globalized world in affecting the international division of labour and trade flow composition, would be to extend the analysis to an open economy framework, where the growth and trade balance effects of public infrastructure provision should be treated taking into account the changes in the sectoral composition between internationally tradable and non-tradable goods.

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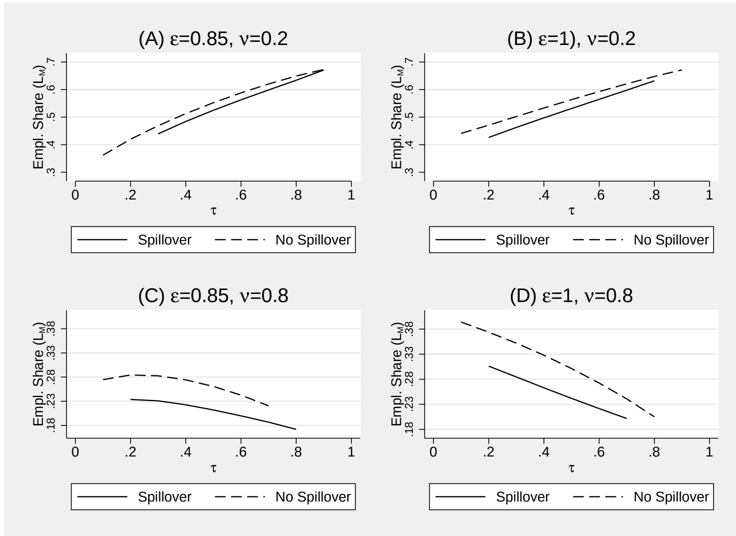
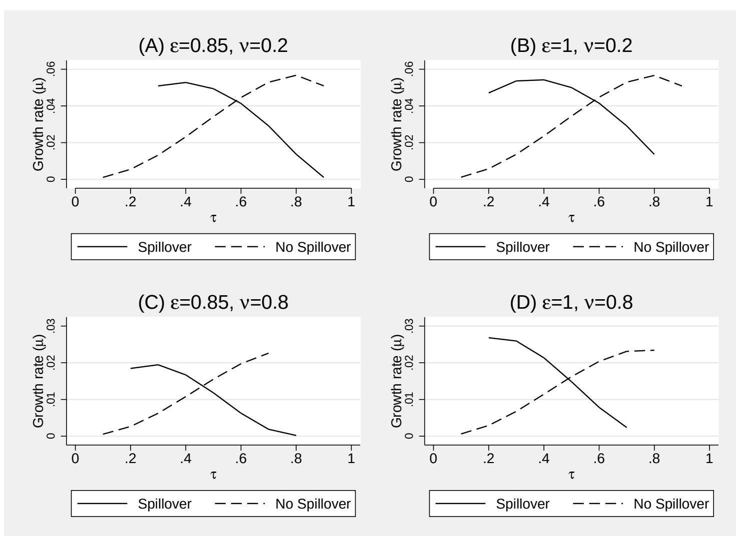
Figure 1: Changing expenditure size given its composition. Employment shares.



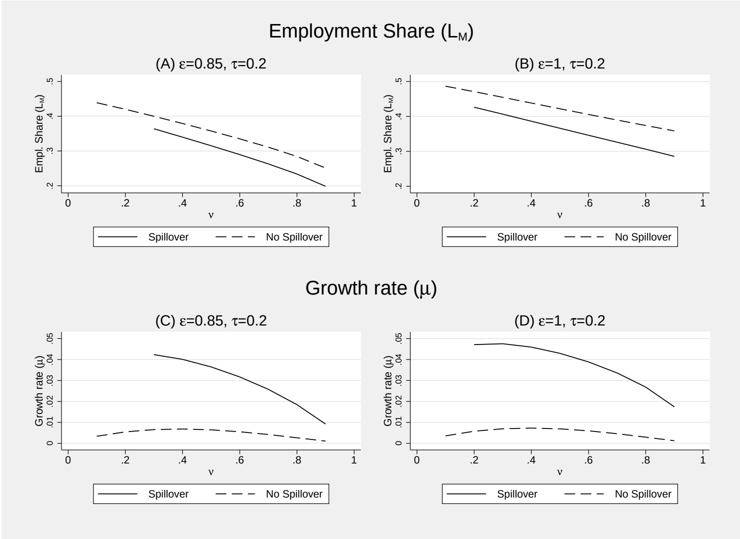
Note. $\beta = 0.7$; $\alpha = 0.37$; $\theta = 0.94$; $\sigma = 1$; $\omega = 0.4$. Spillover with $\gamma = 0.3$ (No spillover with $\gamma = 1$) refers to the existence (absence) of a private capital spillover.

Figure 2: Changing expenditure size given its composition. Growth rates.



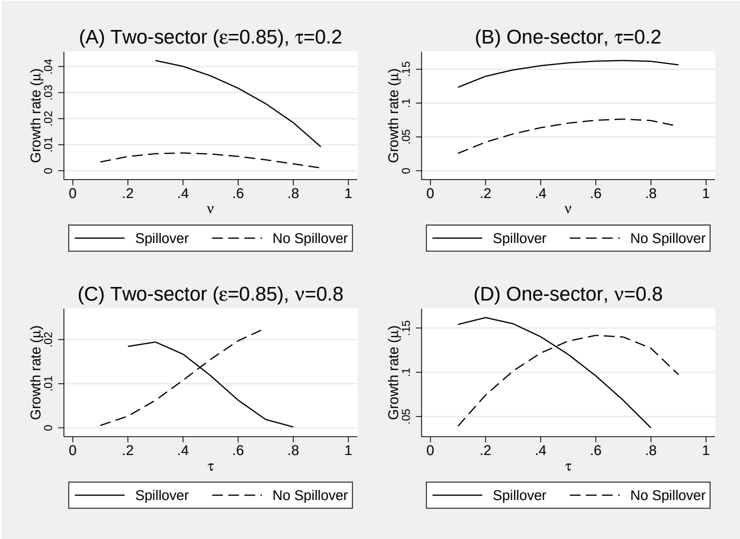
Note. $\beta = 0.7$; $\alpha = 0.37$; $\theta = 0.94$; $\sigma = 1$; $\omega = 0.4$. Spillover with $\gamma = 0.3$ (No spillover with $\gamma = 1$) refers to the existence (absence) of a private capital spillover.

Figure 3: Changing expenditure composition given its size. Employment shares and Growth rates.



Note. $\beta = 0.7$; $\alpha = 0.37$; $\theta = 0.94$; $\sigma = 1$; $\omega = 0.4$. Spillover with $\gamma = 0.3$ (No spillover with $\gamma = 1$) refers to the existence (absence) of a private capital spillover.

Figure 4: Two-sectors vs one-sector models



Note. $\beta = 0.7$; $\alpha = 0.37$; $\theta = 0.94$; $\sigma = 1$; $\omega = 0.4$. Spillover with $\gamma = 0.3$ (No spillover with $\gamma = 1$) refers to the existence (absence) of a private capital spillover.

Table 1: Parameter values

α	0.37
β	0.7
γ	0.3; 1
ϵ	0.85; 1; (0.75)
ω	0.4
θ	0.94
σ	1; (0.15)
τ	0.2
ν	0.8; 0.2

Note. Values used in Figure D2 are indicated in parentheses.

Table 2: Long-run equilibria

	γ=0.3		γ=1	
	μ	L_M	μ	L_M
two sectors (ϵ=1)	0.027	0.306	0.003	0.374
two sectors (ϵ=0.85)	0.018	0.234	0.003	0.284

Note. $\tau = 0.2$; $\nu = 0.8$; β= 0.7; $\alpha = 0.37$; $\theta = 0.94$; $\sigma = 1$; $\epsilon = 0.85$; $\omega = 0.4$. L_M and μ are the employment share in the modern sector and growth rate, respectively.

Table 3: Growth rates in one sector and two-sector models

(i)	(ii)	(iii)	This model
$\tau = 0.5$	τ=0.5	τ=0.5	$\tau = 0.5$
$\nu = 0.2$	$\nu \longrightarrow 0$	$\nu \longrightarrow 1$	$\nu = 0.2$
$\beta = 0.1$	β =0	β =1	$\beta = 0.1$
$\gamma = 1$	γ =1	γ =1	$\gamma = 1$
$\alpha = 0.4$	α=0.4	α=0.4	$\alpha = 0.4$
μ=0.027	μ=0.02	μ=0.32	μ=0.010

Note. Parameter values used in the two-sectors economy: θ=0.96; σ=1; ϵ=0.85; ω=0.4. Models in columns (i)-(iii) are one-sector models: (i) Public expenditure takes the form of both a stock and a flow as in Ghosh and Roy (2004); (ii) public expenditure is only a flow as in Barro (1990); and (iii) public expenditure is only a stock as in Futagami et al. (1993).

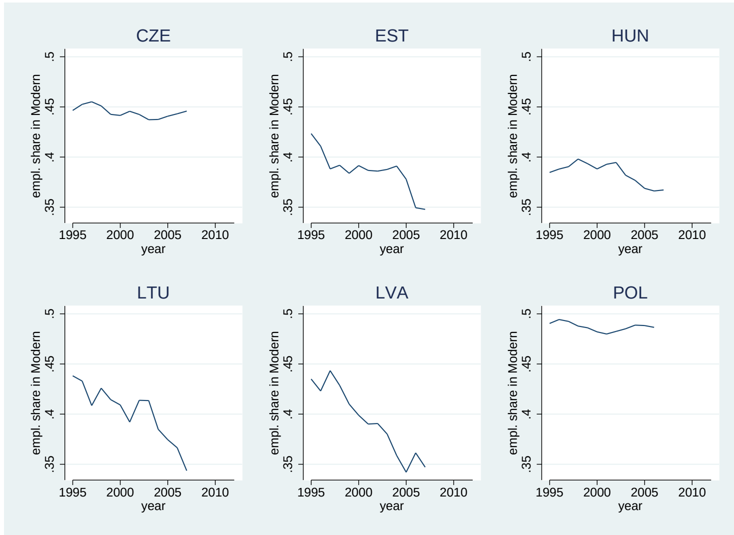
Appendices

A Employment shares in the 'modern' sector

Figure A1: Employment share in the 'modern' sector: Advanced economies

Note. AUS: Australia; DEU: Germany; ESP: Spain; FRA: France; ITA: Italy; JPN: Japan; NLD: the Netherlands; SWE: Sweden; USA: US.

Figure A2: Employment share in the ‘modern’ sector: Eastern-European countries



Note. CZE: Czech Republic; EST: Estonia; HUN: Hungary; LTU: Lithuania; LVA: Latvia; POL: Poland.

Figure A3: Employment share in the ‘modern’ sector: Developing countries

Note. ARG: Argentina; BRA: Brazil; CHN: China; IDN: Indonesia; IND: India; MEX: Mexico.

B One-sector economy

In what follows we describe the economic system with only one-sector and public intervention through stock and flows of public goods. The government behaves as in Section 4.1.

$$G_t \equiv G_{Ft} + G_{It} = \tau(w_t + r_t K_t), \qquad 0 < \tau < 1 \tag{34}$$

The shares of G_t that the government spends, respectively, on investment goods (v_I) and on intermediates (v_F) are constant and exogenous

$$G_{Ft} = v_F G_t \tag{35}$$

$$G_{It} = v_I G_t. \tag{36}$$

Public capital accumulates according to (6).

Firms produce with the following technology

$$Y_t = [K_t^{1-\gamma}(K_{Gt}^{\beta} G_{Ft}^{1-\beta})^{\gamma}]^{\alpha} L_t^{\alpha} K_t^{1-\alpha}, \;\; 0 < \alpha < 1, . \tag{37}$$

We normalize $L_t = 1$. Preferences are represented by a constant elasticity function:

$$U = \frac{C^{1-\sigma} - 1}{1 - \sigma}. \tag{38}$$

.

By solving the household problem we obtain the Euler equation

$$\frac{C_{t+1}}{C_t} = [\theta(1 + r_{t+1})(1 - \tau)]^{\frac{1}{\sigma}} \tag{39}$$

.

Private capital accumulates according to (21). From firms' FOCs and the government budget constraint in (4) and (3) we obtain:

$$r_t = \left\{ (1-\alpha) Z_t^a \left[\tau (1-\nu) \left(\frac{\alpha}{1-\alpha} + 1 \right) \right]^b \right\}^{\frac{1}{1-b}} \tag{40}$$

where $a = \beta\gamma\alpha$ and $b = (1-\beta)\gamma\alpha$.

Market clearing requires:

$$Y_t = C_t + I_t + G_{Ft} + G_{It}, \tag{41}$$

We derive the following dynamic system governing the economy

$$\Psi(H_t, H_{t+1}, Z_t, Z_{t+1}) = 0 \tag{42}$$

where

$$\Psi(H_t, H_{t+1}, Z_t, Z_{t+1}) : \frac{H_t}{H_{t+1}} \left\{ \theta \left[1 + r_{t+1}(1-\tau) \right] \right\}^{\frac{1}{\sigma}} - 1 - \mu_{Kt} \tag{43}$$

and

$$Z(H_t, H_{t+1}, Z_t, Z_{t+1}) = 0 \tag{44}$$

where

$$Z(H_t, H_{t+1}, Z_t, Z_{t+1}) : r_t \tau \left(\frac{\alpha}{1-\alpha} + 1 \right) \left[Z_t^a (1-\nu) \left(\frac{\alpha}{1-\alpha} + 1 \right)^{b-1} - 1 \right] - H_t - \mu_{Kt} \tag{45}$$

and the growth rate of private capital is given by

$$\mu_{Kt} = \frac{Z_t}{Z_{t+1}} + \frac{1}{Z_{t+1}} \nu \tau r_t \left[\frac{\alpha}{1-\alpha} + 1 \right] - 1, \tag{46}$$

where $Z_t \equiv \frac{K_{Gt}}{K_t}$ and $H_t \equiv \frac{C_t}{K_t}$.

C Growth rates in the BGP

From (31), (14) it is straightforward to show that when the employment shares stabilize, Z, p and r also do.

Since Z, the ratio of public to private capital, stabilizes along the BGP, we have that $\mu_K = \mu_{KG}$; moreover one can show that along the BGP the growth rate of aggregate output equals the growth rate of public and private capital.

In this economic system the aggregate output is given by

$$GDP_t = K_t \left\{ L_{Mt}^{\alpha} Z_t^{a} \left[\tau(1-v) r_t \left(\frac{\alpha}{L_{Mt}(1-\alpha)} + 1 \right) \right]^b + \frac{\alpha Z_t^b (1-v)\tau(1-L_{Mt}) \frac{\alpha}{L_{Mt}(1-\alpha)}}{\left[(1-v)\tau \left(\frac{\alpha}{L_{Mt}(1-\alpha)} + 1 \right) \right]^{1-b} L_{Mt}^{1-\alpha}} \right\} \tag{47}$$

from which it is easy to check that along the BGP $\mu_{GDP} = \mu_K = \mu$.

From (3), (5) and (4), by using (13) it is straightforward to derive

$$Gt = \tau K_t r_t \left[\frac{\alpha}{L_{Mt}(1-\alpha)} + 1 \right] \tag{48}$$

and

$$\mu_{Gt} = \mu_{G_F t} = \mu. \tag{49}$$

From (7), (3), (4) we derive

$$y_{Mt} = K_t L_{Mt}^{\alpha} Z_t^{a} \left[\tau(1-v) r_t \left(\frac{\alpha}{L_{Mt}(1-\alpha)} + 1 \right) \right]^b \tag{50}$$

while from (10) and (3), (4), (13), (14) we obtain

$$y_{St} = (1-v)\tau K_t r_t \left(\frac{\alpha}{L_{Mt}(1-\alpha)} + 1 \right)(1-L_{Mt}). \tag{51}$$

From (3), (5) and (4)

$$G_I = \frac{v\tau K_t r_t \left(\frac{\alpha}{L_{Mt}(1-\alpha)} + 1\right)}{P_t}. \tag{52}$$

From (13), (50), (51), (52)

$$\mu_{Y_M} = \mu_{Y_S} = \mu_w = \mu_{GI} = \mu. \tag{53}$$

D Local Stability and Transitional Dynamics

What we are looking at when analysing the economic system along its BGP is a long run path of what we may call a 'mature' economy where the main structural adjustments have already taken place. In order to investigate the characteristics of these adjustments in the proximity of the long run equilibrium, we must turn our attention to the transitional dynamics of the system.

Since the dynamic system in (25) and (27) is nonlinear, in order to examine the local stability we linearize the system around its BGP as reported in Section 5.1, where the parameter values and the correspondent BGP values of the main variables are: $\theta = 0.94$; $\alpha = 0.37$; $\gamma = 0.3$; $\beta = 0.7$; $\sigma = 1$; $\epsilon = 0.85$; $\omega = 0.4$; $\tau = 0.2$; $\nu = 0.8$; $L_M = 0.2337$, $Z = 0.2997$ and the growth rate $\mu = 0.0184$.

$$\begin{bmatrix} L_{Mt+1} - L^* \\ Z_{t+1} - Z^* \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} L_{Mt} - L^* \\ Z_t - Z^* \end{bmatrix}. \tag{54}$$

Since the linearized system in (54) has one unstable eigenvalue, $q_1 > 1$, and one stable eigenvalue, $q_2 < 1$, the BGP is saddle path stable.In particular, its characteristic roots are $q_1 = 1.3185$ and $q_2 = 0.9694$.[29]

Since the system is in two variables, one of which, L_{Mt}, is a jump variable, and the other, Z_t, a sticky variable whose initial value is not predetermined, we can set the initial condition for Z_0 and

[29]The system is too complex to derive a condition for saddle path stability; so we have to rest on numerical analysis. Nevertheless, we have carried out simulations over a wide range of parameters' values economically meaningful, and in all of them we found saddle path stability.

the constant c can be determined in order to analyse the stable path. Starting from a given initial ratio of the two types of capital, Z_0, the stable solution of the decentralized economy is given by

$$L_{Mt} - L_M^* = q_2^t Q_{21} c \tag{55}$$

$$Z_t - Z^* = q_2^t Q_{22} c \tag{56}$$

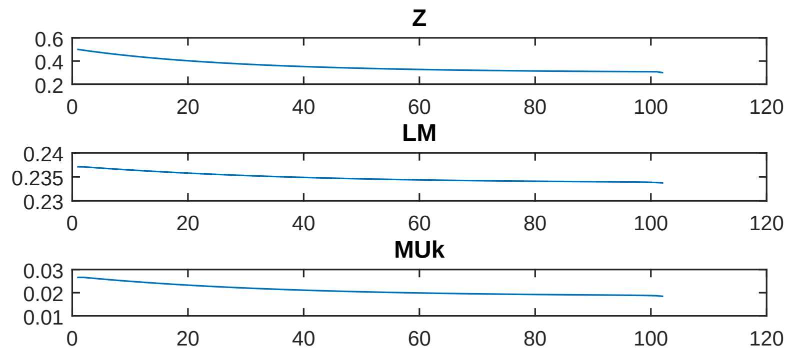
where q_2 is the stable eigenvalue, Q_{21}, Q_{22} the elements of the corresponding eigenvector, and c is the constant.

Setting the initial value of public to private capital ratio above its long-run equilibrium, for instance, $Z_0 = 0.5$, the resulting constant is $c = 0.2003$ and the share of labour in the M sector at time 0 is $L_{M0} = 0.2382$, that is, slightly above its long run equilibrium. The eigenvector corresponding to the stable root is given by $Q_{21} = 0.9997$ and $Q_{22} = 0.0225$ (see Figure D1). As mentioned in the text, in order to get in this framework a transitional dynamics quantitatively closer to the changes in the employment shares observed in the data a different parametrization is needed with σ lower than one and a slightly lower ϵ. We therefore study the transitional dynamics under the following set of parameter values: $\theta = 0.94$; $\alpha = 0.37$; $\gamma = 0.3$; $\beta = 0.7$; $\sigma = 0.15$; $\epsilon = 0.75$; $\omega = 0.4$; $\tau = 0.2$; $\nu = 0.8$; the correspondent BGP values are $L_M = 0.2129$, $Z = 0.1274$ and the growth rate $\mu = 0.0386$. In this case the characteristic roots are $q_1 = 1.0755$ and $q_2 = 0.8813$; therefore the equilibrium is saddle path stable and q_2 is the stable eigenvalue. Setting the initial value of public to private capital ratio above its long-run equilibrium, for instance, $Z_0 = 0.3$, the resulting constant is $c = 0.1922$ and the share of labour in the M sector at time 0 is $L_{M0} = 0.2975$, above its long run equilibrium. The eigenvector corresponding to the stable root is given by $Q_{21} = 0.8977$ and $Q_{22} = 0.4405$.

Along the unique path converging to the long run BGP, the share of labour in sector M, the public to private capital ratio, and the aggregate growth rate of the economy all are monotonically decreasing (see Figure D2). One can show that the opposite happens whenever starting from

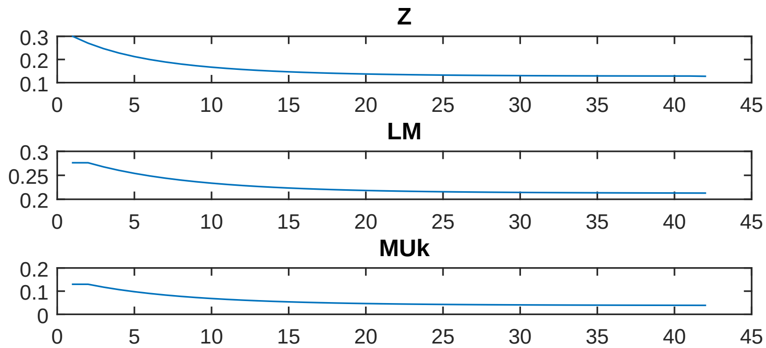
$Z_0 < Z$; that is to say, a public to private capital ratio lower than the long run equilibrium implies an increasing public to private capital ratio along the transition, as well as an increasing employment share in the 'modern' sector M together with an increasing aggregate output growth. It is worth underlining that since we are dealing here with a linearized system, the transition we are looking at is that of an economy 'close' to its steady growth long-run equilibrium.

Figure D1: Transitional dynamics (local)



Note. $t = time$. The parameter values are those used in the numerical example above, namely $\theta = 0.94$; $\alpha = 0.37$; $\gamma = 0.3$; $\beta = 0.7$; $\sigma = 1$; $\epsilon = 0.85$; $\omega = 0.4$; $\tau = 0.2$; $\nu = 0.8$; $Z_0 = 0.5$; where LM stands for L_M and MUk for μ, the employment share in the modern sector and growth rate of private capital, respectively.

Figure D2: Transitional dynamics (local)



Note. $t = time$. The parameter values are those used in the numerical example above, namely $\theta = 0.94$; $\alpha = 0.37$; $\gamma = 0.3$; $\beta = 0.7$; $\sigma = 0.15$; $\epsilon = 0.75$; $\omega = 0.4$; $\tau = 0.2$; $\nu = 0.8$; $Z_0 = 0.3$, where LM stands for L_M and MUk for μ, the employment share in the modern sector and growth rate of private capital, respectively.

E Tables of policy experiments

Table E1: Policy experiment: Changing expenditure size given its composition

	(ϵ=0.85)				(ϵ=1)			
	γ=1	γ=1	$\gamma = 0.3$	$\gamma = 0.3$	γ=1	γ=1	$\gamma = 0.3$	$\gamma = 0.3$
	μ	L_M	μ	L_M	μ	L_M	μ	L_M
	ν=0.8							
τ=0.1	0.0005	0.2747			0.0006	0.3941		
τ=0.2	0.0026	0.2844	0.0185	0.2337	0.0029	0.3739	0.0268	0.3061
τ=0.3	0.0062	0.2827	0.0194	0.2309	0.0067	0.3520	0.0259	0.2846
τ=0.4	0.0108	0.2743	0.0167	0.2227	0.0114	0.3279	0.0213	0.2629
τ=0.5	0.0155	0.2607	0.0118	0.2120	0.0162	0.3014	0.0148	0.2418
τ=0.6	0.0197	0.2447	0.0063	0.2000	0.0204	0.2725	0.0078	0.2215
τ=0.7	0.0222	0.2205	0.0019	0.1867	0.0231	0.2408	0.0024	0.2016
τ=0.8			0.0002	0.1719	0.0234	0.2051		
τ=0.9								
	ν=0.2							
τ=0.1	0.0011	0.3621			0.0012	0.4411		
τ=0.2	0.0055	0.4200			0.0058	0.4709	0.0471	0.4263
τ=0.3	0.0132	0.4690	0.0509	0.4394	0.0136	0.5021	0.0536	0.4627
τ=0.4	0.0232	0.5128	0.0528	0.4843	0.0236	0.5332	0.0542	0.4974
τ=0.5	0.0341	0.5524	0.0494	0.5250	0.0344	0.5635	0.0500	0.5313
τ=0.6	0.0446	0.5881	0.0414	0.5628	0.0447	0.5927	0.0415	0.5646
τ=0.7	0.0529	0.6204	0.0292	0.5987	0.0529	0.6209	0.0291	0.5979
τ=0.8	0.0567	0.6493	0.0137	0.6338	0.0566	0.6476	0.0136	0.6319
τ=0.9	0.0509	0.6731	0.0011	0.6711	0.0509	0.6715		

Note. $\beta = 0.7$; $\alpha = 0.37$; $\theta = 0.94$; $\sigma = 1$; $\omega = 0.4$. Entries are missing in case solutions are not well defined.

Table E2: Policy experiment: Changing expenditure composition given its size

	(ϵ=0.85)				(ϵ=1)			
	γ=1	γ=1	γ=0.3	γ=0.3	γ=1	γ=1	γ=0.3	γ=0.3
	μ	L_M	μ	L_M	μ	L_M	μ	L_M
ν=0.1	0.0034	0.4389			0.0035	0.4866		
ν=0.2	0.0055	0.4200			0.0058	0.4709	0.0471	0.4263
ν=0.3	0.0065	0.4000	0.0423	0.3640	0.0069	0.4547	0.0475	0.4064
ν=0.4	0.0068	0.3792	0.0401	0.3399	0.0073	0.4383	0.0459	0.3862
ν=0.5	0.0064	0.3576	0.0365	0.3151	0.0069	0.4219	0.0429	0.3660
ν=0.6	0.0055	0.3350	0.0317	0.2896	0.0060	0.4056	0.0388	0.3460
ν=0.7	0.0042	0.3110	0.0258	0.2629	0.0046	0.3896	0.0336	0.3261
ν=0.8	0.0026	0.2844	0.0185	0.2337	0.0029	0.3739	0.0268	0.3061
ν=0.9	0.0011	0.2510	0.0092	0.1986	0.0012	0.3585	0.0174	0.2856

Note. τ=0.2; $\beta = 0.7$; $\alpha = 0.37$; $\theta = 0.94$; $\sigma = 1$; $\omega = 0.4$. Entries are missing in case solutions are not well defined.

Table E3: Policy experiments: Two-sector vs. one-sector model

	Two-sectors (ϵ=0.85)		One-sector			Two-sectors (ϵ=0.85)		One-sector	
	γ=1	γ=0.3	γ=1	γ=0.3		γ=1	γ=0.3	γ=1	γ=0.3
ν=0.8	μ	μ	μ	μ	τ=0.2	μ	μ	μ	μ
τ=0.1	0.0005		0.0392	0.1540	ν=0.1	0.0034		0.0257	0.1234
τ=0.2	0.0026	0.0185	0.0742	0.1618	ν=0.2	0.0055		0.0422	0.1397
τ=0.3	0.0062	0.0194	0.1014	0.1548	ν=.3	0.0065	0.0423	0.0544	0.1490
τ=0.4	0.0110	0.0167	0.1217	0.1401	ν=0.4	0.0068	0.0401	0.0637	0.1553
τ=0.5	0.0155	0.0118	0.1353	0.1201	ν=0.5	0.0064	0.0365	0.0704	0.1595
τ=0.6	0.0197	0.0063	0.1417	0.0960	ν=0.6	0.0055	0.0317	0.0747	0.1620
τ=0.7	0.0226	0.0019	0.1398	0.0682	ν=0.7	0.0042	0.0258	0.0763	0.1630
τ=0.8		0.0002	0.1272	0.0373	ν=0.8	0.0026	0.0185	0.0742	0.1618
τ=0.9			0.0973		ν=0.9	0.0011	0.0092	0.0656	0.1565

Note. $\beta = 0.7$; $\alpha = 0.37$; $\theta = 0.94$; $\sigma = 1$; $\omega = 0.4$. Entries are missing in case solutions are not well defined.