EU AND US DESIGN APPROACHES FOR STEEL STORAGE PALLET RACKS WITH MONO-SYMMETRIC CROSS-SECTION UPRIGHTS

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ABSTRACT

Design, fabrication and erection of industrial steel storage rack systems can nowadays take place at different locations, potentially separated by thousands of kilometers. Consequently, manufacturing engineers need to understand the main code provisions adopted in the country where the rack will be in-service. Frequently, design is carried out in accordance with the European (EU) or United States (US) rack codes, which are the most commonly adopted standards for industrial storage systems. As appraisal of the key differences on structural performance associated with the different code provisions in term of storage performance, a previous study of the Authors was focused on storage pallet racks comprised of bi-symmetric cross-section uprights (vertical members). Now, attention is paid to racks with mono-symmetric uprights, typically influenced by relevant warping effects, which are traditionally neglected by rack provisions and, as a consequence, by manufacturing engineers. The design approaches already considered and compared for bi-symmetric uprights now appear to be inadequate and have been necessarily improved, as suggested by the Authors, including at least the contribution due to the bimoment acting along mono-symmetric uprights. Research outcomes, which are discussed in the paper, are related to a parametric analysis on several racks differing for configurations, geometry of components and degree of rotational stiffness of joints. The associated results regarding the four EU and two US considered alternatives are presented and compared directly to each other to allow for a concrete appraisal of the most relevant differences between the admitted design approaches. In order to highlight the importance of warping effects, which can be evaluated only by means of refined finite element (FE) analysis software, design has been undertaken using also the more traditional FE *beam* formulation implemented in the commercial analysis packages most frequently used in manufacturing offices. Finally, Appendix A presents a complete design example to be used as benchmark, where all the discussed design options are applied and compared to each other.

Keywords: *semi –continuous unbraced frames, adjustable selective steel storage pallet racks, open thin-walled cross-section, warping effects, safety index.*

1. INTRODUCTION

One of the main results of increasing globalization is that nowadays design, fabrication and erection of industrial steel structures for the storage of materials and products can take place at different locations, potentially separated by several thousands of kilometres. As a consequence, rack owners might require the use of widely accepted steel design codes and manufacturing designers might be consequently familiar with specifications that have substantial differences between each other. Due to the great interest in the comparison of different standard codes, a research project has been developed on the design methods according to European (EU) and United States (US) steel provisions. At first, attention has been focused on the more traditional carpentry steel frames made by hot-rolled I-shaped members: similarities and differences between the EN 1993-1-1 [1] and AISC360-10 [2] have been discussed by Bernuzzi et al. [3] summarizing the main results of a comparative parametric study carried out on more than 700 multi-story semi-continuous planar frames. In the second step, the approaches adopted for adjustable pallet racks have been considered and the design alternatives admitted by EU [4] and US [5] codes have been discussed and applied with reference to selective pallet racks realized by bi-symmetric cross-section members [6,7]. The third and last phase of the research, which is dealt with by the present paper, is focused on the influence of the mono-symmetry of the upright cross-sections most commonly used in design (figure 1).

Figure 1: typical mono-symmetric cross-section uprights used in industrial storage systems.

It is worth mentioning that rules currently adopted in Europe for the verification checks of pallet racks under monotonic loading derive from those proposed for hot-rolled bi-symmetric cross-section members. As a consequence, with the exception of design developed according to the Australian rack design standard [8], key features associated with mono-symmetric cross-sections typically used for

1　vertical (uprights) members are considered in a very simplified way, or frequently ignored. The direct

2　consequence, from the design point of view, is the erroneous assessment of the set of internal forces,

3　moment and displacements and of the rack performance, which in few cases reflects directly in a

4　structural system characterized by a level of reliability lower than the one required by standard codes.

5　In more detail, the limited availability of FE analysis packages appropriates to simulate via *beam*

6　elements the effects associated with warping torsion and the low confidence of manufacturing

7　engineers with the application of the thin-walled beam theory [9,10] produce the result that, in routine

8　design, no differences can be observed in case of bi- or mono-symmetric cross-section members. It

9　is worth noting that, the introduction of the 7th DOF in the structural analysis make the rack more

10　flexible but at the same time the presence of bi-moment could change remarkably the coefficient of

11　utilization, or equivalently the safety index, that is a parameter, ranging from 0 to 1

12　(resistance/stability limit is achieved), of great interest for practical design purposes. As better

13　explained in the following, its definition is associated with the effects of the design actions over the

14　resistance. Also the values of the bending moments should be different from those obtained by

15　neglecting warping.

16　The presence of the bimoment is currently neglected, as well as the influence of the coupling between

17　bending and torsion on the rack deformability and on the overall buckling mode. Very general

18　requirements are provided by codes, without practical indications for designers which expect from

19　them uniquely applicable procedures. In recent research papers edited by Cardoso and Rasmussen

20　[11] and Sangle et al. [12], the problem of warping in the FE model have been avoided by modelling

21　the racks with shell elements. This solution, that gives quite accurate results, is not easily replicable

22　by the structural engineer, due to the complexity of the modelling phase.

23　On the other hand, Teh et al. [13] using a *beam* finite element have studied the warping influence on

24　the global behavior of the racks giving, indications also about the influence on the buckling analysis

25　results. From the experimental point of view dynamic test was made by Dev and Ralukdar [14] on

26　isolated column, carrying out the warping influence on the modal parameters. Some preliminary

27　studies on the stress distribution across cross-sections and on the overall buckling conditions carried

28　out by Bernuzzi et al. [15-17] indicated that the effects associated with the use of mono-symmetric

29　cross-section members (i.e. warping torsion, Wagner coefficients and the eccentricity between the

30　shear center and the cross-section centroid) play an important role on the performance of medium-

31　rise pallet racks and cannot ever be neglected in a safe and reliable rack design.

32　No comparative analyses related to rack provisions adopted in different countries are available for

33　framed systems made up of mono-symmetric uprights. In the present paper, owing to the practical

34　interest of this matter, key features of four EU and two US design alternatives are briefly discussed.

In particular, a parametric study has been carried out on 216 medium-rise racks differing for the frame upright cross-section, longitudinal layout and connection performances. Structural analyses have been carried out by means of two different finite element (FE) analysis packages, differing in the FE *beam* formulation. The results of a traditional commercial FE formulation based on the use of the Timoshenko *beam* element [18] have been compared with a more refined one including the warping of the cross-section as an additional (7th) degree of freedom (DOF) [19]. In total, more than 3000 design cases have been developed focusing attention on the uprights, for which more relevant differences are expected with reference to pallet beams and lacings owing to the direct influence of warping effects with regard to the permissible design alternatives. Furthermore, Appendix A presents a complete benchmark example, where all the design paths admitted by both codes are applied and compared to one another, focusing attention not only on the performance of the different racks but also on the key features due to the presence of one axis of symmetry.

2. THE ADMITTED DESIGN ALTERNATIVES

In the case of steel storage systems, as well for carpentery frames, routine design is carried out in two phases.

- structural analysis of the frame, aimed at evaluating the set of displacements, internal forces and moments on members and joints;
- verification checks of members for displacements, resistance and instability and of joints for resistance and rotation demand.

It is worth noticing that remarkable differences associated with the use of 6 or 7 DOFs FE analysis packages can always be detected in each phase of the design process, starting from the values of the set of displacements, internal forces and moments used to verify the serviceability and ultimate limit states.

In the following, only the essential contents of the codes are briefly revisited, refering readers to previous papers [3,6] also for similarities and differences regarding the effective cross-section properties or directly to the code itself for a more detailed discussion about the principles of the alternatives offered to designers. Independent of the cross-section type, steel storage pallet racks are very flexible structures under lateral loads, owing to various factors such as the high slenderness of the uprights, the modest degree of rotational stiffness of beam-to-column joints and base-plate connections [20] and the absence of vertical bracing in the down-aisle direction, due to the need to provide the maximum storage capability. As a consequence, a second-order analysis is often required in routine design, which could also be carried out on occasion via approximated approaches, now obsolete for the relevant progresses of the commercial FE analysis packages . Moreover, owing to the extensive use of thin-walled cold-formed members, the traditional design methods for the

1 structural analysis developed for frames comprised of bi-simmetric hot-rolled members cannot be

2 directly adopted, despite it being proposed for storage systems. An open problem, which is currently

3 neglected in pallet rack design, is related to the influence of warping torsion on the load carrying

4 capacity of the uprights, as well as on the performance of the whole framed system. The equations

5 proposed for the design verification checks are in fact only efficient for bi-symmetric cross-section

6 members. When mono-symmetric uprights are employed, the set of displacements and of internal

7 forces and moments are significantly influenced by the cross-section warping. Owing to the

8 importance of this contribution, the bimoment must be accounted for in the design procedures, as

9 herein discussed. The Authors propose an improvement of the equations traditionally adopted for

10 routine design, requiring the assesment of the additional contribution of the bimoment stresses on the

11 considered cross-section, which can be adressed only by means of FE *beam* formulation characterized

12 by the cross-section warping.

13 **2.1 The EU approaches**

14 Structural analysis of racks according to the European practice, should be carried out via one of the

15 following methods, already identified by Bernuzzi [6]:

16 • EU-DAM: *Direct Analysis Method,* (specified in EN15512 sub-chapter 10.1.3);

17 • EU-RAM: *Rigorous Analysis Method,* (specified in EN15512 sub-chapter 9.7.6);

18 • EU-GEM: *General Method,* (specified in EN1993-1-1 sub-chapter 6.3.4).

19 • EU-IRAM: *Improved Rigorous Analysis Method (*proposed by Authors*).*

20 **EU-DAM**. The *direct analysis method (DAM)* requires an advanced three-dimensional analysis,

21 including both overall rack and member imperfections and joint eccentricities, where relevant.

22 Furthermore, as clearly stated by EN15512 in very general terms, rack design has to be carried out

23 by means of refined FE analysis packages able to capture accurately the coupling between flexure

24 and torsion and the influence of warping deformations on torsional, and consequently flexural-

25 torsional buckling, warping torsion and shear centre eccentricity. Therefore, only resistance checks

26 is required and in absence of indications by the codes, it appears necessary to include also the

27 bimoment contribution, as recently recommended by the Australian Code [8]. In the more general

28 case of beam-columns subjected to axial load (N_{Ed}), bending moments along the axis of symmetry, *y*,

29 and along the other principal axis, *z*, ($M_{y,Ed}$ and $M_{z,Ed}$, respectively) and bimoment (B_{Ed}), reference

30 should be conveniently made to the following definition of the safety index, SI_7^{EU-DAM}:

1
$$SI_7^{EU-DAM} = \left(\frac{\dfrac{N_{Ed}}{A_{eff} \cdot f_y}}{\gamma_{M0}} + \frac{\dfrac{M_{y,Ed}}{W_{eff,y} \cdot f_y}}{\gamma_{M0}} + \frac{\dfrac{M_{z,Ed}}{W_{eff,z} \cdot f_y}}{\gamma_{M0}} \right) + \left(\frac{B_{Ed}}{B_{Rd}} \right) = SI_6^{EU-DAM} + \left(\frac{B_{Ed}}{B_{Rd}} \right) \leq 1 \qquad (1a)$$

2   where A_{eff} and W_{eff} indicate the area and the section modulus of the effective cross-section,

3   respectively, f_y is the material yield strength, γ_{M0} is the material safety factor and B_{Rd} is the resisting

4   bimoment, which is defined as:

5
$$B_{Rd} = \frac{I_w \cdot f_y}{\omega_{max} \cdot \gamma_{M0}} \qquad (1b)$$

6   where, with reference to the gross cross-section, I_w is the warping constant and ω_{max} is maximum

7   value of the sectorial area.

8   For a better understanding of the paper's contents, it should be noted that eq. 1a) is proposed in a

9   form that clearly distinguishes between the contribution due to the traditional design approach

10 neglecting warping (subscript 6) and that due to the bimoment.

11 **EU-RAM.** The *Rigorous Analysis Method* (RAM) takes into account the lack-of-verticality

12 imperfections neglecting the effects of the out-of-straightness of members. EN15512 declares that

13 the *structure shall be considered a no-sway frame and buckling lengths shall be put equal to system*

14 *(geometrical) lengths*. This *a priori* imposition of a constant value of the effective length factor does

15 not allow one to efficiently account for the coupling between flexural and torsional buckling. At the

16 same time it ignores the influence of the joint stiffness as well as the possible benefits associated with

17 lean-on effects [21].

18 Stability checks of uprights are carried out with reference to the following equation:

19
$$SI^{EU-RAM} = \frac{N_{Ed}}{\dfrac{\chi_{min} \cdot A_{eff} \cdot f_y}{\gamma_{M1}}} + \frac{k_{LT} \cdot M_{y,Ed}}{\dfrac{\chi_{LT} \cdot W_{eff,y} \cdot f_y}{\gamma_{M1}}} + \frac{k_z \cdot M_{z,Ed}}{\dfrac{W_{eff,z} \cdot f_y}{\gamma_{M1}}} \leq 1 \qquad (2)$$

20 Where, in addition to the terms already introduced, χ_{min} and χ_{LT} are the reduction factors for axial

21 and lateral buckling, respectively.

22 The term χ_{min} depends directly on relative slenderness $\bar{\lambda}$ defined as:

23
$$\bar{\lambda} = \sqrt{\frac{A_{eff} \cdot f_y}{N_{cr}}} \qquad (3a)$$

24 where N_{cr} is the elastic critical axial buckling load for compressed members evaluated on the basis

25 of the well-established theoretical approaches applied to the gross cross-section.

6

1 It is worth mentioning that, according to this method, the flexural buckling loads must always be
2 evaluated considering the system length and the torsional buckling load as to be based on an effective
3 length usually assumed equal to 0.7 the height of the upright panel. As a consequence, the flexural-
4 torsional buckling load depends solely on the rack geometry, independent of the presence of bracing
5 systems and/or by the degree of rotational stiffness of the joints.

6 In the same way, χ_{LT} is the suitable reduction factor accounting for the lateral stability related to the
7 relative slenderness $\overline{\lambda}_{LT}$ defined as:

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_{eff} \cdot f_y}{M_{cr}}} \tag{3b}$$

9 where M_{cr} is the elastic critical bending moment for lateral buckling.

10 Furthermore, this approach has been presented focussing on the stability checks but the resistance
11 checks according to eq. 1) have also always to be carried out, which generally lead to smaller less
12 severe values of the SI, especially with reference to the fully loaded conditions, which is the case
13 considered in the parametric study herein presented.

14 **EU-GEM.** The *General Method* (GEM) takes into account only the lack-of-verticality imperfections.
15 This approach, which appears as very promising owing to its simplicity and efficiency when applied
16 to racks, allows warping effects to be accounted for in both resistance and buckling checks. A frame,
17 or equivalently a rack, is safe when:

$$\frac{\chi_{op}\alpha_{ult.k}}{\gamma_M} \geq 1 \tag{4a}$$

19 With reference to the symbols already presented, the verification criterion can be more conveniently
20 expressed as:

$$SI^{EU-GEM} = \frac{\gamma_M}{\chi_{op}\alpha_{ult.k}} \leq 1 \tag{4b}$$

22 where $\alpha_{ult,k}$ is the minimum load multiplier based on the cross-section resistance, χ_{op} is the buckling
23 reduction factor in reference to the overall structural system and γ_M is the material safety factor.

24 The ultimate load multiplier for resistance has n to account for key features associated with the use
25 of mono-symmetric cross-section members and hence is herein indicated as $\alpha_{ult,k,7}$. It can be
26 determined as:

$$\frac{1}{\alpha_{ult,k,7}} = \left(\frac{N_{Ed}}{\dfrac{A_{eff} \cdot f_y}{\gamma_{M0}}} + \frac{M_{y,Ed}}{\dfrac{W_{eff,y} \cdot f_y}{\gamma_{M0}}} + \frac{M_{z,Ed}}{\dfrac{W_{eff,z} \cdot f_y}{\gamma_{M0}}} \right) + \left(\frac{B_{Ed}}{B_{Rd}} \right) = \frac{1}{\alpha_{ult,k,6}} + \left(\frac{B_{Ed}}{B_{Rd}} \right) \tag{5}$$

1   Also in this case, it has been decided to split the term associated with the traditional design approaches

2    $\alpha_{ult,k,6}$ from the second term $\left(\dfrac{B_{Ed}}{B_{Rd}}\right)$, proposed by Authors, to suitably account for the presence of

3   mono-symmetric cross-section members. It is worth noting, that, despite eqs. 1) and 5) are formally

4   identical, the design values (subscripts Ed) are related to load conditions differing for imperfections:

5   local and global in DAM approach and only global in the GEM approach.

6   **EU-IRAM.** The main difference from the EU-RAM approach regards the upright stability check:

7   reference has to be made to eq. 2) but the evaluation of the critical axial load N_{cr} of each column is

8   based on the overall critical load multiplier (α_{cr}) directly obtained from a buckling analysis of the

9   rack. For the upright subjected to the design axial load N_{Ed}, the critical axial load can be directly

10   evaluated as:

$$N_{cr} = \alpha_{cr} \cdot N_{Ed} \qquad (6)$$

12   In the case of 7DOFs *beam* formulations, the assessment of the critical load multiplier ($\alpha_{cr}{}^{7}$) allows

13   for a direct evaluation of the critical load considering flexural-torsional buckling modes. Otherwise,

14   if the critical load multiplier is obtained from a 6DOFs overall buckling analysis ($\alpha_{cr}{}^{6}$) that

15   considering only the flexural buckling mode, the flexural-torsional buckling load is evaluated in a

16   different way by defining the effective length for torsional buckling on the basis of the geometry of

17   the frame.

18   **2.2 The US approaches**

19   As to the US rack design practice, RMI [5] states that all computations for safe loads, stresses and

20   deflections, have to be made in accordance with the conventional methods for structural design. As

21   to the design of medium-rise pallet racks, as clearly stated also by Sarawit and Pekoz [22], two

22   methods can be adopted:

23   • US – NOLM: *Notional Load Method,* (defined in AISC360 sub-chapter C1.1);

24   • US – ELM: *Effective Length Method,* (defined in AISC360 sub-chapter C1.2 and discussed

25   in Appendix 7.2).

26   For the evaluation of the performance of the uprights, in addition to the resistance checks according

27   to eq. 1), stability verification checks are required in both methods, differing from one another mainly

28   with regard to the value of critical buckling load or, equivalently, for the effective length factor K. A

29   common equation is proposed for both methods but the values of the internal forces and moments are

30   not equal due to the different set of imperfections, as discussed in detail in [7]. In particular, the safety

31   index for the kth method can be defined in a general form, as:

8

1
$$SI^{US-k} = \frac{P_r}{0.9 \cdot P_n} + \frac{M_x}{0.9 \cdot M_{nx}} + \frac{M_y}{0.9 \cdot M_{ny}} \leq 1 \qquad (7)$$

2 where P_n is the no3minal compression member capacity and M_{nx} and M_{ny} are the nominal flexural

3 capacities.

4 It should be noted that the *x*- and *y*- axes identifying principal cross-section axes in US codes

5 correspond to the *y*- an *z*- axes, respectively, according to the EU symbols. In order to avoid

6 confusion, reference can be made to table B1 proposing the equivalence between EU and US symbols

7 identifying the geometric cross-section parameters.

8 **US – NOLM.** The Notional Load Method is the main suggested method and it requires a geometric

9 non-linear analysis, considering all second-order effects, together with flexural, shear and axial

10 member deformations and modelling the geometric initial imperfections via horizontal forces

11 (notional loads) applied on each floor. All steel properties contributing to the elastic stiffness have to

12 be multiplied by 0.8 imposing a 20% reduction of the structural stiffness to lateral load [22,23],

13 mainly to account for the simplified approach used to evaluate the effective length of uprights, which

14 is based on the system length, i.e. *K*=1 as in the EU-RAM.

15

16 **US – ELM.** The Effective Length Method requires a second-order analysis, like in US-NOLM, but

17 without any stiffness reduction and imperfections are taken into account only via notional loads. The

18 evaluation of the member strengths is based on the effective length factor *K* for moment resisting

19 (sway) frames obtained from alignment charts or, more conveniently, from the use of FE buckling

20 analysis of the overall rack, as in the EU-IRAM.

21 **2.3 Comparison of the approaches**

22 The discussed EU and US design approaches for rack design present similarities and differences and

23 to this end reference can be made to table 1 for the aspects of relevant importance. As previously

24 introduced, these alternatives are proposed by the EU and US rack codes and independent of the

25 selected alternative, a second-order analysis is strongly recommended. A common remark is that key

26 features associated with the presence of mono-symmetric cross-section uprights are practically

27 neglected in both codes with regard to resistance checks. As a consequence, the Authors suggestion

28 is that, independent of the design alternatives and/or the considered code, all resistance checks have

29 to be executed mandatory according to eq. 1), in order to include also the bimoment contribution.

30 *Table 1: similarities and differences between the EU and US rack design approaches.*

Feature	EU DAM	US NOLM	EU RAM	US ELM	EU IRAM	EU GEM	US ELM

Lack of-verticality (sway imperfections)	YES		YES		YES
Member Out-of straightness imperfections	YES	NO	NO		NO
Stiffness reduction	NO	YES	NO		NO
Stability checks (K=effective length factor)	NO	YES ($K = 1$)	YES ($K = 1$)	YES ($K = K(N_{cr})$)	YES ($K = K(N_{cr})$)

1   It is worth noticing that the main differences between the EU-IRAM, EU-GEM and US-ELM

2   approaches are in the definition of the member imperfections and in the equations checking stability.

3   As to the key features associated with the use of mono-symmetric cross-section uprights, only few of

4   these methods appear adequate for design purposes: the EU-IRAM, EU-GEM and US-ELM consider

5   the presence of mono-symmetric cross-sections in the evaluation of the buckling load of the whole

6   framed system while EU-RAM and US-NOLM are based on the definition of the effective length for

7   flexural-torsional buckling depending solely on the geometry of the rack. Furthermore, the EU-GEM

8   approach seems more than adequate for pallet racks [24], taking adequately into account warping

9   effects in both resistance and stability checks and allowing for consideration of the presence of the

10   perforations [25], which are necessary to accept the end joints of pallet beams in the case of adjustable

11   storage systems.

12   **3. THE CONSIDERED RACKS**

13   The parametric study for the approach comparison has been focused on medium-rise double-entry

14   racks, unbraced in the down-aisle direction with four equal bays of 2.78 m length that typically allows

15   for the location of 3 pallet units per bay: rack depth is 1.00 m and upright frames present Z-panels

16   guaranteeing stability to cross-aisle loads. Four configurations (fig. 2) have been defined, differing

17   for the number of load levels (LL) and for the inter-storey height (h_i): two (_2LL with $h_i = 2500$ mm),

18   three (_3LL with $h_i = 2200$ mm), four (_4LL with $h_i = 1800$mm) and five (_5LL with $h_i = 1330$ mm)

19   storeys.

20   Three lipped channels (identified as MM_, DD_ and TT_ types) have been considered for uprights,

21   each having the same cross-section thickness. With reference to the gross cross-section, the value of

22   the area (A), second moments of area (I_y and I_z) and section modulii (W_y and W_z) are reported in table

23   2, together with the uniform (I_t) and warping (I_w) torsion constants and the maximum value of the

24   sectorial area (ω_{max}). Furthermore, on the basis of data provided by manufactures, in the same table

25   the Q_reduction factors associated with the results of stub-column tests, equal in both EU (Q_{EU}^N) and

1   US (Q_{US}^{N}) codes, and with bending tests along the two principal axes required by the EU design codes

2   (Q_{EU}^{My} and Q_{EU}^{Mz}) are also reported.

3   It should be noted that, despite the fact that the uprights cross-section commonly used are those

4   presented in figure 1, lipped channels are however used for pallet as well as cladding racks. In

5   addition, please note that the input data for this cross-section require the definition of the eccentricity

6   between the shear center and the cross-section centroid as well as the Wagner's constants, like for the

7   other mono-symmetric cross-section. Furthermore, this upright cross-section choice makes the

8   research outcomes however of great interest for the rack practice, being analysis and design checks

9   based on the use of 7DOFs *beam* formulation. A rather exhaustive overview of the cases most

10   frequently encountered in routine rack design is proposed: in fact, the ratio between the second

11   moments of area ranges from 1.0 to 3.0 and the ratio associated with section moduli from 1.0 to 1.5,
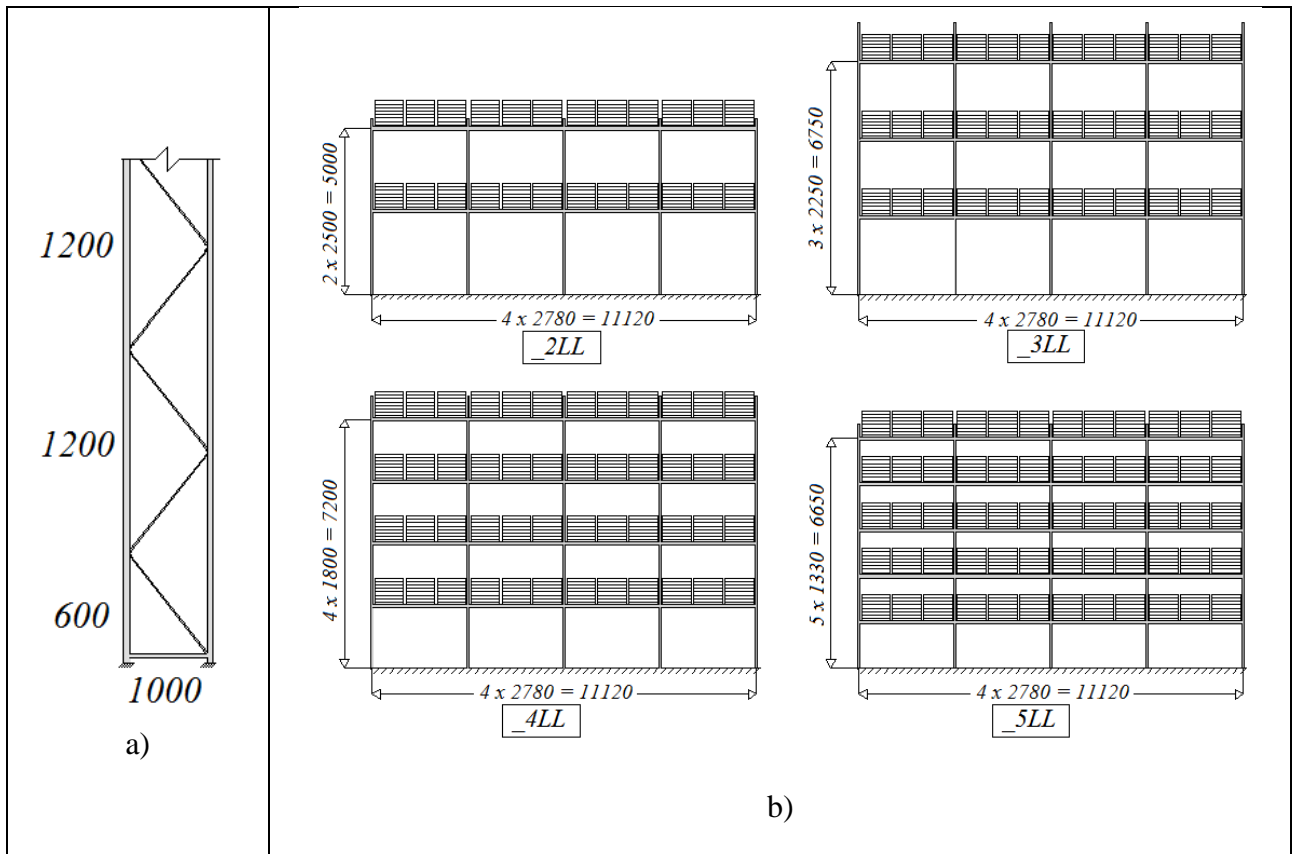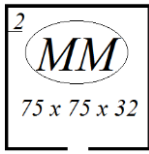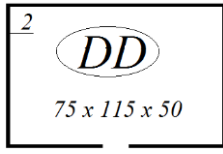
12   approximately.
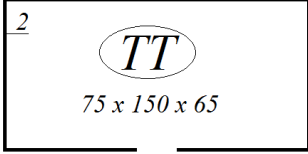
Figure 2: the considered pallet racks: view along the cross-aisle (a) and down-aisle (b) direction (dimension in millimetres).

13   Rectangular hollow sections (160x40x1.3 mm *RHS*) and square hollow section (35x35x2 mm *SHS*)

14   elements have been used for pallet beams and lacings of upright frames, respectively. All these

15   structural components are in S355 steel grade [26], with a yielding strength f_y = 355 N/mm^2.

Table 2: key features of the considered upright cross-sections.

	$\boxed{\overset{2}{MM}}$ 75 x 75 x 32	$\boxed{\overset{2}{\textit{DD}}}$ 75 x 115 x 50	$\boxed{\overset{2}{TT}}$ 75 x 150 x 65
$A\ (A_g)[mm^2]$	$5.63 \cdot 10^2$	$7.07 \cdot 10^2$	$8.38 \cdot 10^2$
$W_y\ (S_x\ )[mm^3]$	$13.70 \cdot 10^3$	$24.03 \cdot 10^3$	$35.09 \cdot 10^3$
$W_z\ (S_y\ )\ [mm^3]$	$12.65 \cdot 10^3$	$17.33 \cdot 10^3$	$21.63 \cdot 10^3$
$I_y\ (I_x\ )[mm^4]$	$50.99 \cdot 10^4$	$136.87 \cdot 10^4$	$261.42 \cdot 10^4$
$I_z\ (I_y\ )\ [mm^4]$	$48.93 \cdot 10^4$	$66.64 \cdot 10^4$	$83.54 \cdot 10^4$
$I_t\ (J)[mm^4]$	$0.088 \cdot 10^4$	$0.11 \cdot 10^4$	$0.13 \cdot 10^4$
$I_w\ (C_w\ )[mm^6]$	$1509.81 \cdot 10^6$	$5173.80 \cdot 10^6$	$11400.00 \cdot 10^6$
$\omega_{max}\ [mm^2]$	$42.78 \cdot 10^2$	$72.08 \cdot 10^2$	$90.02 \cdot 10^2$
$Q^N = Q_{EU}^N = Q_{US}^N$	0.850		
$Q_{EU}^{My} = Q_{EU}^{Mz}$	0.925		

1 The case of fully loaded racks has been considered with pallet units acting as a uniform load on all

2 pallet beams. In the numerical study, attention has been focussed on the following parameters:

3   •   the *degree of flexural stiffness associated with beam-to-column joints*: the selected values of

4   rotational stiffness $(S_{j,btc})$ of interest for practical applications have been expressed as

5   multiples (by means of term $\rho_{j,btc}$) of a reference stiffness value $S_{j,btc}^{EC3-LB}$ via the relationship:

6
$$S_{j,btc} = \rho_{j,btc} \cdot S_{j,btc}^{EC3-LB} \tag{13}$$

7   where $S_{j,btc}^{EC3-LB}$ is the stiffness associated with the lower bound of the semi-rigid domain, which

8   is the value corresponding to the transition between the domains of flexible (pinned) and semi-

9   rigid joints according to the classification criteria of part 1-8 of Eurocode 3 [27]. The

10   parameter $\rho_{j,btc}$ has been assumed to range from 1 to 10, and in addition, values of $\rho_{j,btc}$ equal

11   to 2, 4, 5 and 8 have been considered.

12   •   the *degree of flexural stiffness associated with base-plate connections*: as for beam-to-column

13   joints, the values of the base rotational stiffness $S_{j,base}$ have been selected as multiple, by means

14   of term $\rho_{j,base}$, of the transition stiffness $\left(S_{j,base}^{EC3-UB}\right)$ between the region of semi-rigid and rigid

15   joints, defined as:

16
$$S_{j,base} = \rho_{j,base} \cdot S_{j,base}^{EC3-UB} \tag{14}$$

17   Three values have been considered ($\rho_{j,base} = 0.15$, $\rho_{j,base} = 0.30$ and $\rho_{j,base} = 0.45$) to

18   characterize the rotational behavior of the base-plate having different steel details to connect

19   the rack with the foundation system or to the floor slab.

- the *warping restraint at the upright base-plate*: because of the different steel details used in the connection of the steel rack with the supporting base, two different ideal cases for base-plate restraint have been considered, indicated in the follow as WF (warping free) or WP (warping prevented).

As already mentioned, two commercial finite element analysis packages have been used for the structural analysis, SAP2000 [18], offering the classic *Timoshenko beam* element formulation with 6DOFs per node, and ConSteel [19] where the more refined *Eulero-Bernoulli* 7DOFs *beam* formulation is implemented.

In order to propose design cases comparable to each other and research outcomes of actual interest for routine design, prior to the design phase, a buckling analysis has been carried out for each rack. In particular, in the Authors' opinion, it appears necessary to propose research outcomes directly comparable in parametric studies similar to the one described in the present paper. For this reason, owing to the differences in the component performances, the multiplier of the 6DOFs buckling analysis (α_{cr}^6) was used to define the load condition multiplier as $0.56 \cdot \alpha_{cr}^6$, common to each considered design case.

As to the modelling of the geometrical imperfections, it has been decided to use notional loads instead of modelling racks with an imperfect geometry, due to the presence of sway and/or bow defects. In particular it has to be noted that:

- the lateral loads applied on each rack level to simulate the uprights out-of-plumbness correspond to 0.28% and 0.5% of the resulting vertical floor load, according to EU and US code, respectively;

- the bow imperfections, which are considered only by the EU-DAM approach, have been simulated by a uniform lateral load distributed along the upright and equilibrated in correspondence of the floor by lateral concentrated forces, as recommended by EN 1993-1-1.

Figure 3: layout of the parametric study.
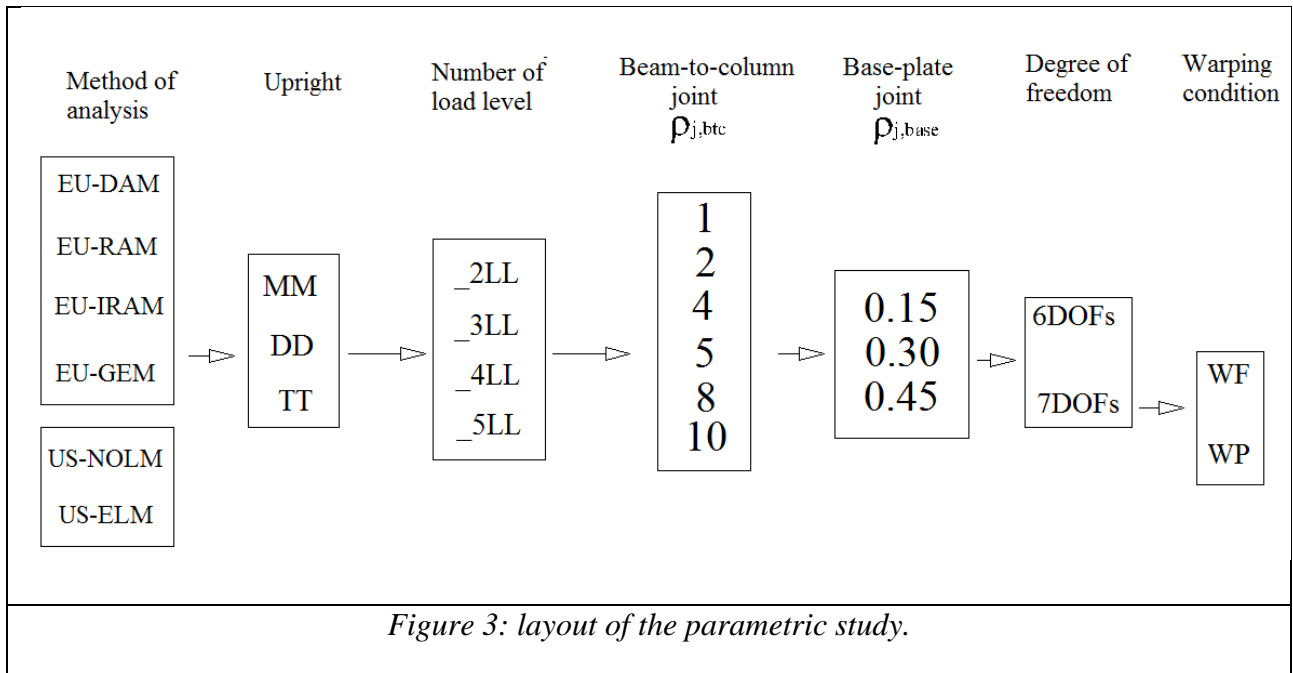
1   The layout summarizing the key parameters considered in this study is represented in figure 3: in total

2   more than 3500 design analysis have been carried out on 216 racks, modelled via two different FE

3   software packages and by applying 4 EU approaches and 2 US design procedures.

4   Owing to the large amount of data and to the need to clearly identify main research outcomes, the

5   numerical study has been carried out focusing the attention on the uprights, and neglecting joint and

6   pallet beam verifications, which are not expected to have a significant impact. Furthermore, a

7   preliminary investigation showed that distortional buckling phenomenon on uprights is not of interest

8   for the considered rack layout and as a consequence, it has been neglected.

9   **4.   TRADITIONAL DESIGN: INFLUENCE OF THE METHODS OF ANALYSIS**

10   As already mentioned, the design approaches introduced and discussed in section 2 have been quite

11   recently applied to racks with bi-symmetric cross-section uprights [7] and it was concluded that the

12   EU-DAM and EU-RAM approaches lead to SI values very close to one another and significantly

13   lower than those associated with the other approaches. Furthermore, it has been noted that a quite

14   accurate prediction of the rack performance seems possible via the EU-IRAM, EU-GEM and US-

15   ELM, approaches that are largely equivalent to one another, leading to reasonably similar values of

16   the load carrying capacity, despite the fact that they differ significantly in the input data for the overall

17   geometrical imperfections and for the equations used for member verification checks. As to racks

18   with mono-symmetric cross-section uprights, it is worth mentioning that in the case of traditional

19   6DOFs FE analysis packages, the presence of bimoment cannot be evaluated in the output analysis

20   data. Furthermore, the available design approaches, with the exception of EU-DAM, require

21   consideration of the stability check for beam-columns and for the flexural-torsional failure buckling

22   mode. The additional set of design cases considered in the present paper also allows for the evaluation

14

of the influence of this buckling mode that is expected to govern the design in several cases for which the shear centre coincides with the centroid.

Key results of the considered method of analysis can be evaluated and directly compared, in terms of safety index making reference to the more highly stressed upright, which is, in general, in the internal position of the rack, in the first bay where lateral loads simulating imperfections are applied. It is worth mentioning that increasing the degree of stiffness of both beam-to-column joints and base-plate connections also causes the value of the safety index to increases, independent of the adopted method of analysis. The results related to the EU approaches are sketched in figures 4 (MM_racks), 5 (DD_racks) and 6 (TT_racks). In particular, each figure, where different symbols are used to make a distinction on the basis of the number of load levels, is divided in four parts:

- part a), comparing the EU-RAM and EU-IRAM approaches;
- part b), comparing the EU-RAM and EU-DAM approaches;
- part c), comparing the EU-GEM and EU-IRAM approaches;
- part d), comparing the EU-GEM and EU-DAM approaches.

The bi-sector line is related to cases associated with equal SI and hence corresponds to a 0% difference between the two compared approaches. Two sub-domains are defined by the bi-sector line and the reference axis, allowing for the direct identification of the method providing the maximum value of the safety index, which is directly indicated by the reference axis itself. Furthermore, in each reference system, a dashed boundary line is also reported to specify the maximum value of the difference between the compared approaches. It can be pointed out also that:

- for the lowest beam-to column joint stiffness $\left(\rho_{j,btc} \leq 5.0\right)$, the lowest values of the SI are always associated with the EU-DAM and EU-RAM approaches, and both approaches lead to large differences from the EU-GEM (or equivalently EU-IRAM) approach, up to 70% in case of EU-GEM vs EU-DAM for DD_racks;

- when the beam-to column joint stiffness increases, the differences between all the EU approaches decrease and EU-DAM always gives the lowest value of SI;

- as already observed for bi-symmetric cross-section uprights, also in these cases the EU-IRAM and EU-GEM approaches lead to very limited differences in the SI values, never greater than 10%, independent of the geometry of the racks and components;
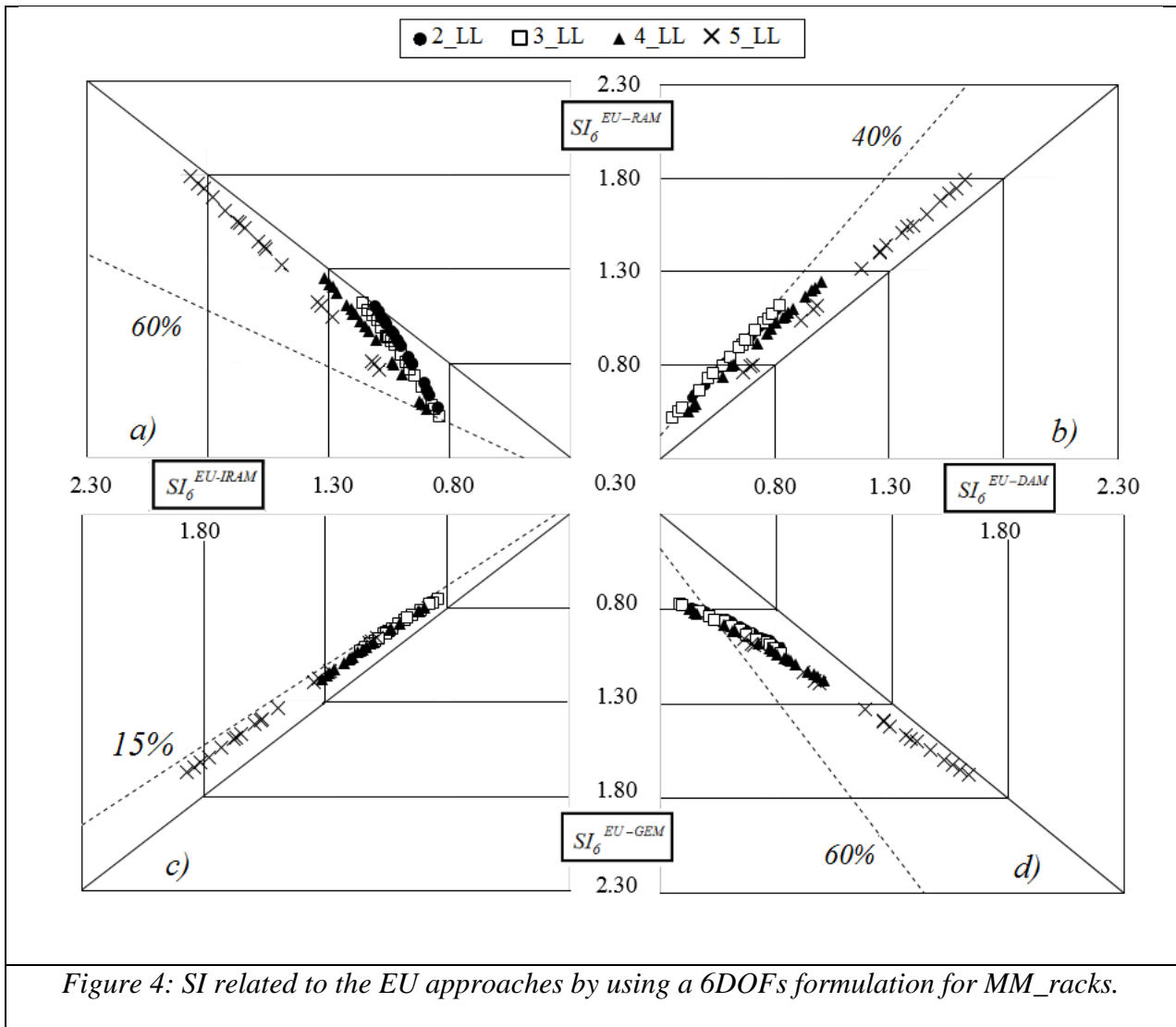
Figure 4: SI related to the EU approaches by using a 6DOFs formulation for MM_racks.

1 • contrary to what happens in cases of bi-symmetric cross-sections, the EU-DAM and EU-RAM
2 approaches lead in a few cases to significantly different results, especially in the case of
3 MM_racks (up to 40%). Moreover, the EU-RAM method, despite being based on an incorrect
4 assessment of the buckling load, includes the presence of the coupling between torsion and
5 bending at critical buckling conditions, reducing the differences with the EU-IRAM and EU-
6 GEM approaches;

Figure 5: SI related to the EU approaches by using a 6DOFs formulation for DD_racks.
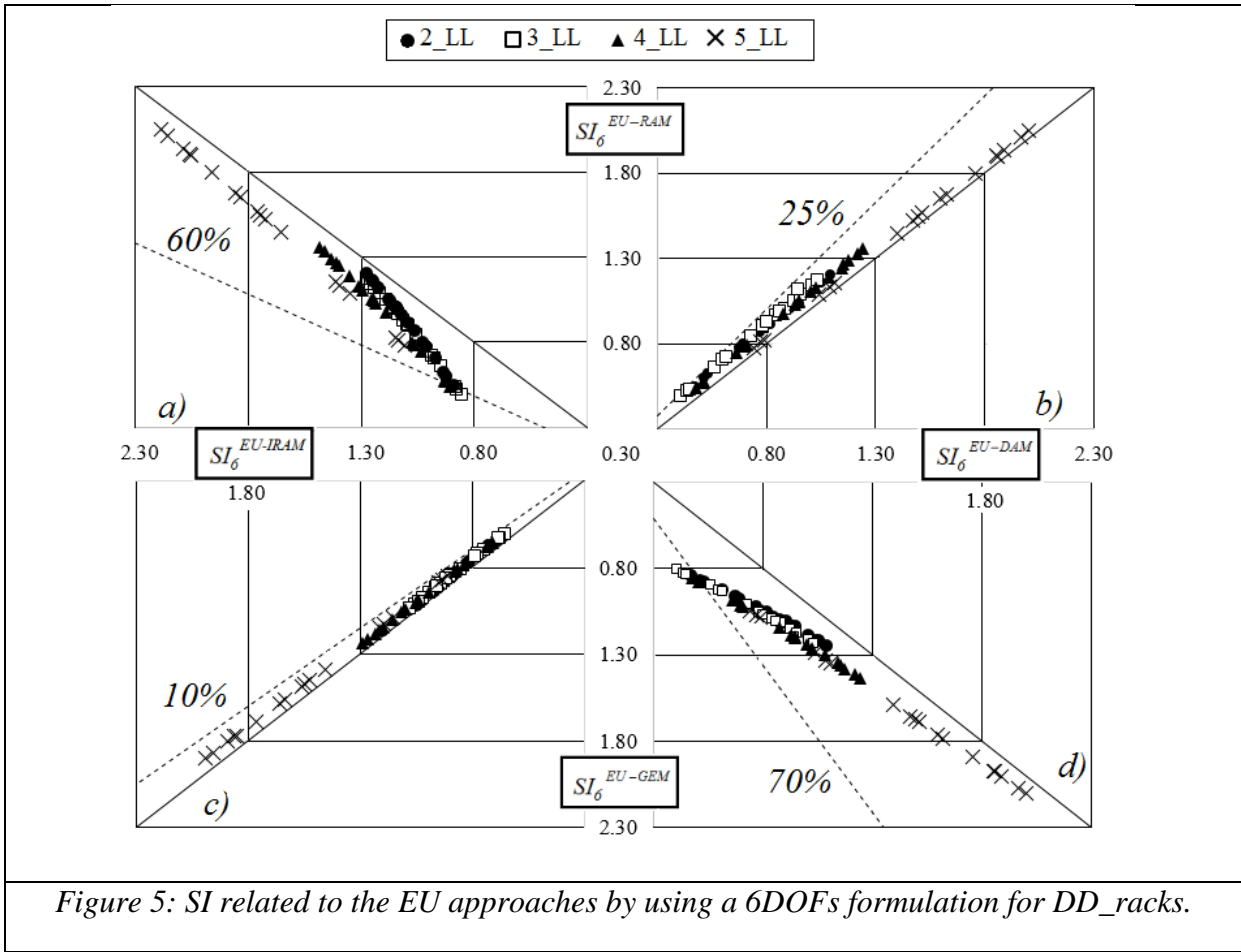
1   Furthermore, it can be concluded that, basing analyses on a 6DOFs *beam* element formulation,

2   two sets of SI values (EU-DAM plus EU-RAM and EU-IRAM plus EU-GEM) can be identified

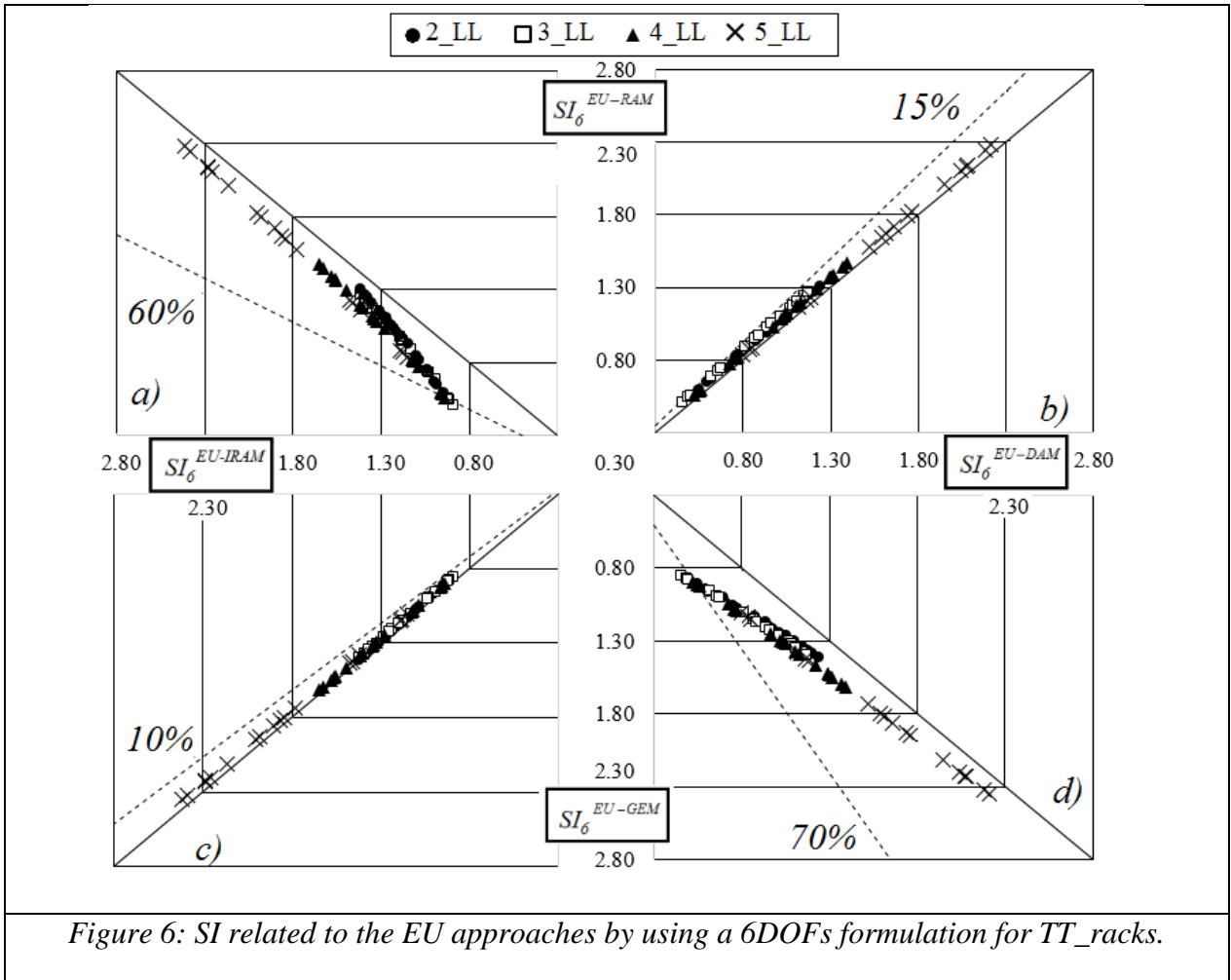3   which, lead to rack performance extremely different to each other.

Figure 6: SI related to the EU approaches by using a 6DOFs formulation for TT_racks.

1 As to the US design approaches, reference can be made to figure 7, where the value of the safety

2 index according to the two permitted alternatives can be directly compared. The figure is divided in

3 four parts, each of them related to one of the four considered rack geometries differing for the number

4 of floor levels and divided in two sub-domains, defined by the bisector. Also in these cases, the dashed

5 line allows for identifying directly the maximum differences associated with the method indicated in

6 the reference axes. It can be noted that generally, the safety index associated with the US-ELM

7 approach is greater than the that associated with US-NOLM and the differences are non-negligible,

8 ranging from 30% (_5LL cases) up to 60% (_3LL cases). In a very limited number of cases the US-

9 NOLM approach results in the more conservative SI, up to 5% in the case of the stiffer MM_racks (

10 $\rho_{j,btc} \geq 5.0$). In particular, the differences between the two approaches are very large when $\rho_{j,btc}= 1.0$

11 and $\rho_{j,base} = 0.15$, but increasing the stiffness of the joints decreases the differences significantly.

12 Furthermore, it also appears that the number of load levels influences the SI values: increasing the

13 interstorey height of the racks the differences between SI decreases.
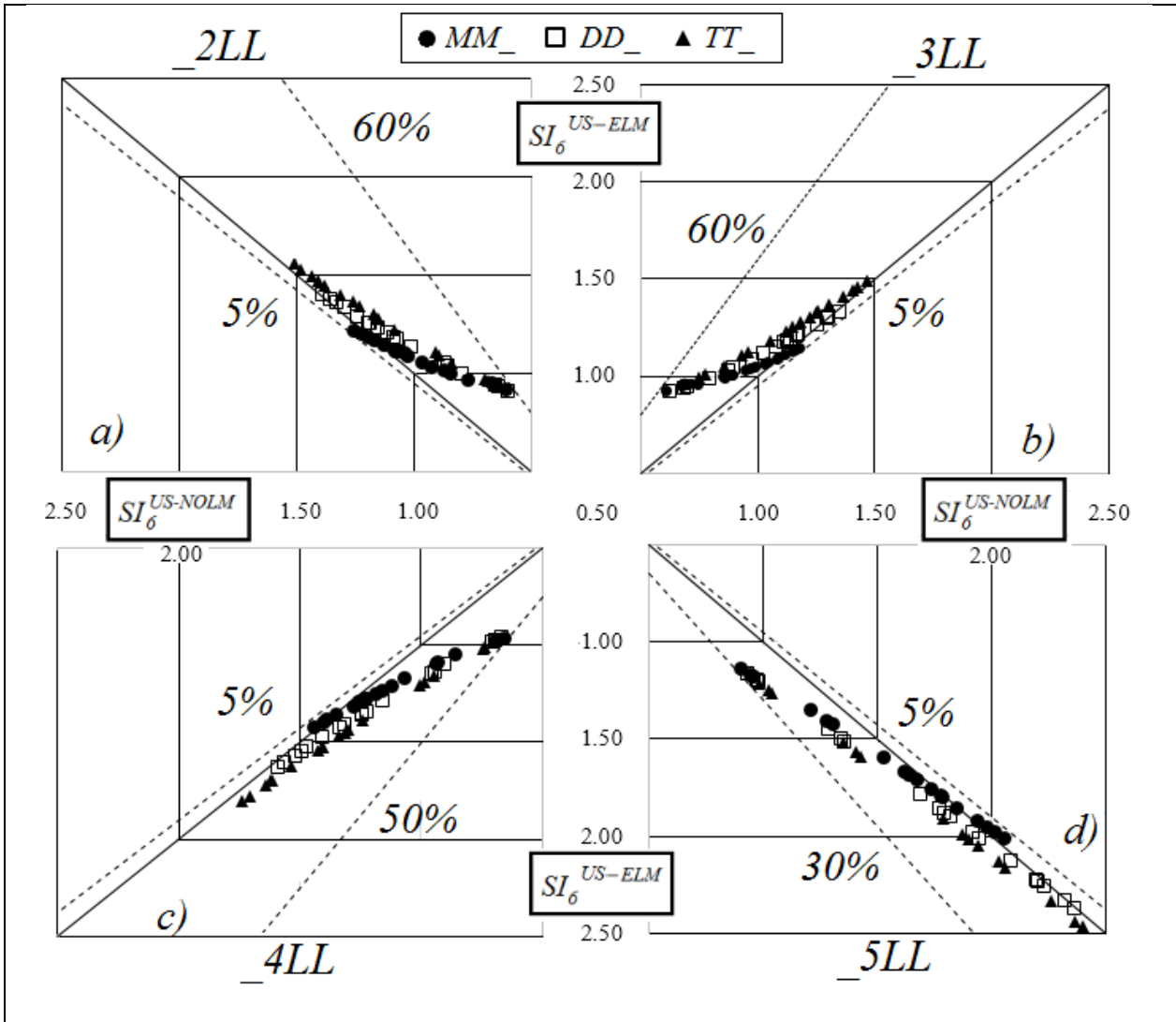
18

Figure 7: SI related to the US approaches, by using a 6DOFs formulation.

1 To better appreciate the differences between the EU and US approaches, table 3 can be considered,

2 where all the data have been grouped and summarized only on the basis of the upright cross-section

3 type, independent of the number of load levels and joint stiffness; in particular, the mean value

4 (*mean*), standard deviation (*st. dev.*), maximum (*Max*) and minimum (*min*) values are presented for

5 the methods that are compared in the same manner used for table 1. Differences between EU-IRAM,

6 EU-GEM and US-ELM approaches are quite limited as it appears from the ratios $\frac{SI_6^{US-ELM}}{SI_6^{EU-IRAM}}$ and $\frac{SI_6^{US-ELM}}{SI_6^{EU-GEM}}$

7 . In cases of US-ELM and EU-IRAM approaches, the associated SI values differ in mean of 6% and

8 up to 1.11, with a moderate dispersion of the corresponding ratio. If the US-ELM and EU-GEM

9 approaches are considered, slightly greater differences can be observed, especially with reference to

10 the MM_racks, having a mean value of $\frac{SI_6^{US-ELM}}{SI_6^{EU-GEM}}$ equal to 1.19 with a maximum value of 1.22. As to

11 the direct comparison between the EU-IRAM and EU-GEM approaches, reference can be made to

12 the part c) of figures 4-6 showing as already discussed, that the safety indices associated with the EU-

19

1 IRAM are slightly greater, up to 10% of the EU-GEM ones, with a mean value based on all the data
2 of 6%.

Table 3: comparison between the EU and US approaches for 6DOFs analysis.

		$\frac{SI_6^{US-ELM}}{SI_6^{EU-IRAM}}$	$\frac{SI_6^{US-ELM}}{SI_6^{EU-GEM}}$	$\frac{SI_6^{VM}}{SI_6^{EU-IRAM}}$	$\frac{SI_6^{VM}}{SI_6^{EU-GEM}}$	$\frac{SI_6^{VM}}{SI_6^{EU-ELM}}$	$\frac{SI_6^{VM}}{SI_6^{EU-DAM}}$	$\frac{SI_6^{VM}}{SI_6^{EU-RAM}}$	$\frac{SI_6^{VM}}{SI_6^{US-NOLM}}$
MM_	mean	1.06	1.19	0.98	1.10	0.93	1.52	1.17	1.03
	st. dev.	*0.0188*	*0.0224*	*0.0064*	*0.0085*	*0.0109*	*0.2844*	*0.1596*	*0.1314*
	Max	1.11	1.22	1.00	1.11	0.95	2.37	1.63	1.38
	min	1.02	1.14	0.97	1.09	0.91	1.14	0.98	0.90
DD_	mean	1.06	1.11	1.01	1.05	0.95	1.39	1.25	1.07
	st. dev.	*0.0242*	*0.0238*	*0.0094*	*0.0086*	*0.0135*	*0.2330*	*0.1741*	*0.1328*
	Max	1.10	1.14	1.02	1.06	0.97	2.09	1.76	1.43
	min	1.02	1.07	0.99	1.03	0.93	1.11	1.07	0.94
TT_	mean	1.07	1.09	1.01	1.04	0.95	1.35	1.26	1.08
	st. dev.	*0.0251*	*0.0237*	*0.0099*	*0.0085*	*0.0140*	*0.2003*	*0.1654*	*0.1164*
	Max	1.10	1.12	1.03	1.05	0.98	1.95	1.75	1.40
	min	1.01	1.05	0.99	1.03	0.93	1.11	1.08	0.96
All	mean	1.06	1.13	1.00	1.07	0.94	1.42	1.23	1.06
	st. dev.	*0.0227*	*0.0233*	*0.0086*	*0.0086*	*0.0128*	*0.2392*	*0.1664*	*0.1269*
	Max	1.11	1.22	1.03	1.11	0.98	2.37	1.76	1.43
	min	1.01	1.05	0.97	1.02	0.91	1.11	0.98	0.90

4

5 Furthermore, with reference to these three methods for design, the mean value SI_6^{VM} has been

6 computed, which can be assumed to define the real rack performances. In the same table, the $\frac{SI_6^{VM}}{SI_6^{EU-IRAM}}$

7 , $\frac{SI_6^{VM}}{SI_6^{EU-GEM}}$ and $\frac{SI_6^{VM}}{SI_6^{EU-ELM}}$ ratios are reported, which are comprised between 0.91 and 1.11, having mean

8 values ranging from 0.93 up to 1.10. In general, the ratios associated with the EU-IRAM approach

9 differ moderately from unity. If the EU-GEM approach is considered, the $\frac{SI_6^{VM}}{SI_6^{EU-GEM}}$ ratio is slightly

10 greater than unity whilst the US-ELM ratios are less than unity. Maximum differences in terms of

11 mean value of the safety index can be determined with reference to the EU-DAM approach: the ratio

12 $\frac{SI_6^{VM}}{SI_6^{EU-DAM}}$ ranges between 1.11 and 2.37 with a mean value of 1.52 related to the MM_racks. If the

13 comparison is made with the EU-RAM approach it can be observed that mean values of the $\frac{SI_6^{VM}}{SI_6^{EU-RAM}}$

14 ratio are significantly lower than those for EU-DAM, ranging from 1.17 (MM_racks) to 1.26

15 (TT_racks). Furthermore, in several cases the ratio is very elevated, up to 1.76 for DD_racks, which

16 allows for a safe use of this approach in routine design.

1   For what concerns the $\dfrac{SI_6^{VM}}{SI_6^{US-NOLM}}$ ratio, it can be noted that it appears acceptable from a design point of

2   view, with a mean value from 1.03 to 1.08, but presents very high maximum values, up to 43%. As

3   also observed for bi-symmetric cross-sections, when the racks are flexible, i.e. for the lowest values

4   of $\rho_{j,btc}$, the US-NOLM approach gives value of SI that are too low when compared to SI^{VM}, leading

5   to a quite conservative design. As to the EU-RAM method, the $\dfrac{SI_6^{VM}}{SI_6^{EU-RAM}}$ ratios are always greater than

6   those associated with the three equivalent approaches, despite being significantly lower than the ones

7   corresponding to the EU-DAM approach.

8   **5. ADVANCED DESIGN: INFLUENCE OF THE METHODS OF ANALYSIS**

9   As already highlighted, according to the basic principles of steel structure theory, structural analysis

10   and design of members having cross-sections with one axis of symmetry must be based on the use of

11   an adequate FE *beam* formulation that takes into account the presence of warping [19]. Owing to the

12   importance of this matter, attention is herein initially focused on warping restraint at the column bases

13   and then the results associated with the different methods of analysis are discussed and compared.

14   **5.1 Influence of the warping base restraints**

15   With both EU-DAM and EU-GEM approaches being based on the assessment of a resistance safety

16   index, it appears of paramount importance in the case of mono-symmetric uprights, to also focus

17   attention on the distribution of the bimoment. As expected, the value of safety index associated with

18   resistance check changes along the uprights section by section. In particular, in the case of EU-DAM,

19   the maximum value of the bimoment is generally located at the middle of the first story and not at the

20   top or bottom of the upright, as it is in the EU-GEM method where only global imperfections are

21   modelled. As an example of the different distributions of bimoment along the central and lateral

22   uprights, which is due to the different types of imperfections, reference can be made to the figures

23   A1 and A3 in Appendix A. It can be stated, in general, that the influence of warping depends mainly

24   on the cross-section types and on the number of load levels; and it is relatively independent of the

25   values of the flexural stiffness of both beam-to-column joints and base-plate connections. For better

26   appreciating the contribution of the bimoment at the end and at the top of the more stressed internal

27   upright, reference can be made to table 4. For the free warping (WF) and prevented warping (WP)

28   cases, the mean value of the contribution of the bimoment $\Delta SI_{BM}^{EU-DAM} = \left(\dfrac{B_{Ed}}{B_{Rd}}\right)$ is presented over the

29   safety index associated with the use of the DAM approach (SI_7^{EU-DAM}), already defined in eq. 1). All

30   data have been grouped and summarized on the basis of the number of the load levels and joint

1 stiffness. It appears immediately that the bimoment can never be neglected for design purposes when

2 this approach, which is based solely on the resistance check, is considered. Further key remarks are:

3 *Table 4: influence of the warping modelling on the bimoment for the EU-DAM approach.*

| $\dfrac{\Delta SI_{BM}^{EU-DAM}}{SI_7^{EU-DAM}}$		Top _WF	Top_WP	Bot_WP
MM_2LL DD_2LL TT_2LL	mean	0.20	0.10	0.22
	st. dev.	*0.0314*	*0.0180*	*0.0316*
	Max	0.25	0.13	0.28
	min	0.12	0.07	0.15
MM_3LL DD_3LL TT_3LL	mean	0.24	0.11	0.23
	st. dev.	*0.0484*	*0.0410*	*0.0306*
	Max	0.31	0.18	0.28
	min	0.16	0.04	0.15
MM_4LL DD_4LL TT_4LL	mean	0.26	0.11	0.24
	st. dev.	*0.0999*	*0.0607*	*0.0466*
	Max	0.41	0.21	0.34
	min	0.13	0.03	0.15
MM_5LL DD_5LL TT_5LL	mean	0.24	0.091	0.262
	st. dev.	*0.1276*	*0.0667*	*0.0696*
	Max	0.45	0.21	0.40
	min	0.09	0.02	0.15
All	mean	0.24	0.10	0.24
	st. dev.	*0.0881*	*0.1210*	*0.0935*
	Max	0.45	0.21	0.40
	min	0.09	0.02	0.14

4 • in the WF cases, the bimoment term contributes from 9% to 50% to the value of the SI, with

5 a mean value of 0.24 and a very low standard deviation (0.0881). The lowest values of the

6 ratio are generally associated with the large interstorey height. Increasing the number of load

7 levels causes the influence of the bimoment to increase. Maximum values of the $\dfrac{\Delta SI_{BM}^{EU-DAM}}{SI_7^{EU-DAM}}$

8 ratio range from 0.24 (DD_2LL racks) to 0.45 (TT_5LL racks).

9 • in the WP cases, warping influence is always maximum at the base level (bottom section): the

10 mean value of $\dfrac{\Delta SI_{BM}^{EU-DAM}}{SI_7^{EU-DAM}}$ is quite similar to the values associated with the WF ones.

11 It can be concluded that, for the EU-DAM approach the bimoment contribution is really important

12 for design purposes, with the contribution to the overall SI being up to 40% and never lower than 5%,

13 with a mean value related to all the cases equal to 24%.

14 Furthermore, it is worth mentioning that in the EU-IRAM, US-ELM and EU-GEM approaches, where

15 the stability checks are more severe than the resistance ones, the value of the elastic critical load

1   multiplier influences the safety index and column bases restraining warping cases a slight increase in

2   buckling resistance, i.e. α_{cr}^{WP} is always greater than α_{cr}^{WF}. As an example, reference can be made

3   to figure 8 where the distribution of the ratio $\dfrac{\alpha_{cr}^{WP}}{\alpha_{cr}^{WF}}$ is presented in terms of relative frequency. The

4   ratio is always greater than unity, increasing with the beam-to-column joint stiffness with a high

5   concentration of occurrences ranging between 1.06 and 1.16. Adequate attention must therefore be

6   paid to the correct modelling of the upright base restraints.

Figure 8: Distribution of the $\dfrac{\alpha_{cr}^{WP}}{\alpha_{cr}^{WF}}$ ratio in terms of relative and cumulative frequency.
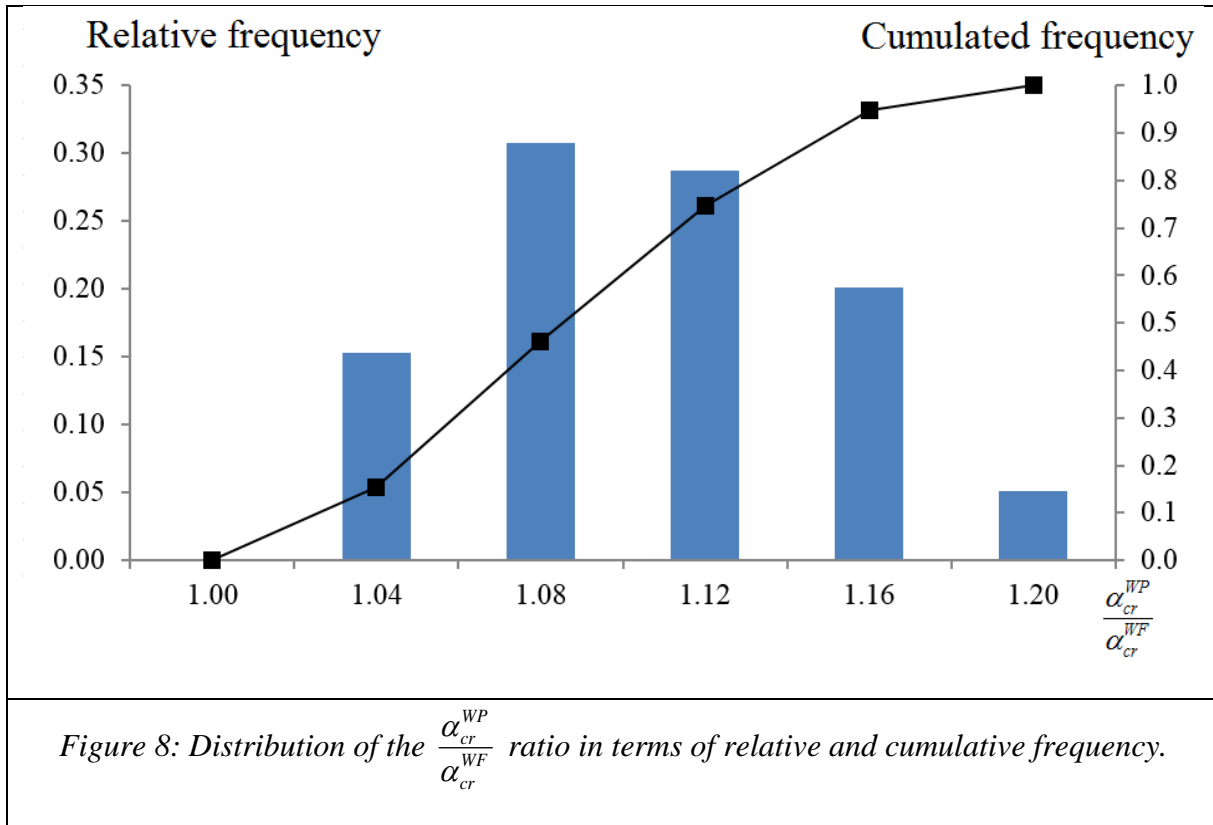
7

8   The previously discussed influence of the warping base restraints impacts the SI values as can be

9   observed in table 5 where the $\dfrac{SI_{WP}^{k}}{SI_{WF}^{k}}$ ratio in terms of *mean* value and standard deviation are presented

10   together with the maximum and minimum values for the kth design approach.

11

12   *Table 5: influence of the warping modelling on the values of the safety index.*

		$\dfrac{SI_{WP}^{EU-DAM}}{SI_{WF}^{EU-DAM}}$	$\dfrac{SI_{WP}^{EU-RAM}}{SI_{WF}^{EU-RAM}}$	$\dfrac{SI_{WP}^{EU-IRAM}}{SI_{WF}^{EU-IRAM}}$	$\dfrac{SI_{WP}^{EU-GEM}}{SI_{WF}^{EU-GEM}}$	$\dfrac{SI_{WP}^{US-NOLM}}{SI_{WF}^{US-NOLM}}$	$\dfrac{SI_{WP}^{US-ELM}}{SI_{WF}^{US-ELM}}$
MM_2LL	mean	1.35	1.04	1.02	1.05	1.05	1.03

23

DD_2LL TT_2LL	st. dev	0.0625	0.0377	0.0256	0.0425	0.0460	0.0288
	Max	1.55	1.15	1.10	1.15	1.19	1.11
	min	1.27	1.00	1.00	1.00	1.00	1.00
MM_3LL DD_3LL TT_3LL	mean	1.33	1.03	1.02	1.05	1.04	1.02
	st. dev	0.0588	0.0284	0.0187	0.0332	0.0334	0.0208
	Max	1.53	1.13	1.07	1.13	1.14	1.08
	min	1.25	1.00	1.00	1.01	1.00	1.00
MM_4LL DD_4LL TT_4LL	mean	1.27	1.03	1.02	1.06	1.04	1.03
	st. dev	0.0333	0.0283	0.0198	0.0326	0.0356	0.0229
	Max	1.35	1.13	1.09	1.13	1.17	1.11
	min	1.22	1.00	1.00	1.01	1.00	1.00
MM_5LL DD_5LL TT_5LL	mean	1.25	1.04	1.03	1.08	1.05	1.03
	st. dev	0.0408	0.0394	0.0302	0.0481	0.0479	0.0342
	Max	1.32	1.17	1.13	1.22	1.20	1.14
	min	1.17	1.00	1.00	1.02	1.00	1.00
All	mean	1.30	1.04	1.02	1.05	1.04	1.03
	st. dev	0.0489	0.0334	0.0235	0.0361	0.0408	0.0267
	Max	1.55	1.17	1.13	1.22	1.20	1.14
	min	1.17	1.00	1.00	1.00	1.00	1.00

It can be noted that the influence of warping restraint on the SI is quite modest for methods using stability verification criteria, but, as expected from the already discussed data in table 4, it becomes significant when EU-DAM, first column of the table, is considered. Considering the other approaches, the influence of the base warping restraint is more limited. It is slightly greater for the EU-GEM approach but not relevant for design purposes. For this reason, and owing to the need of limiting the number of variables affecting the research outcomes, the decision has been taken to continue the comparison between various methods referring to only the WP cases, which are always associated with the higher SI values.

5.2 Influence of methods of analysis

In case of FE *beam* formulations including warping effects, which have to be used to model mono-symmetric uprights, the safety index for the resistance checks must be evaluated accounting for the bimoment contribution, as recommended also by the Australian standard [8]. Figures from 9 to 12 are equivalent to figures 4-7 but related to the results obtained by means of the 7DOFs design approaches and hence are based on sets of displacements, internal forces and moments that differ from the 6DOFs values. Furthermore, for a better understanding of the design results, the values associated with the use of 6DOFs analysis are again proposed between brackets.
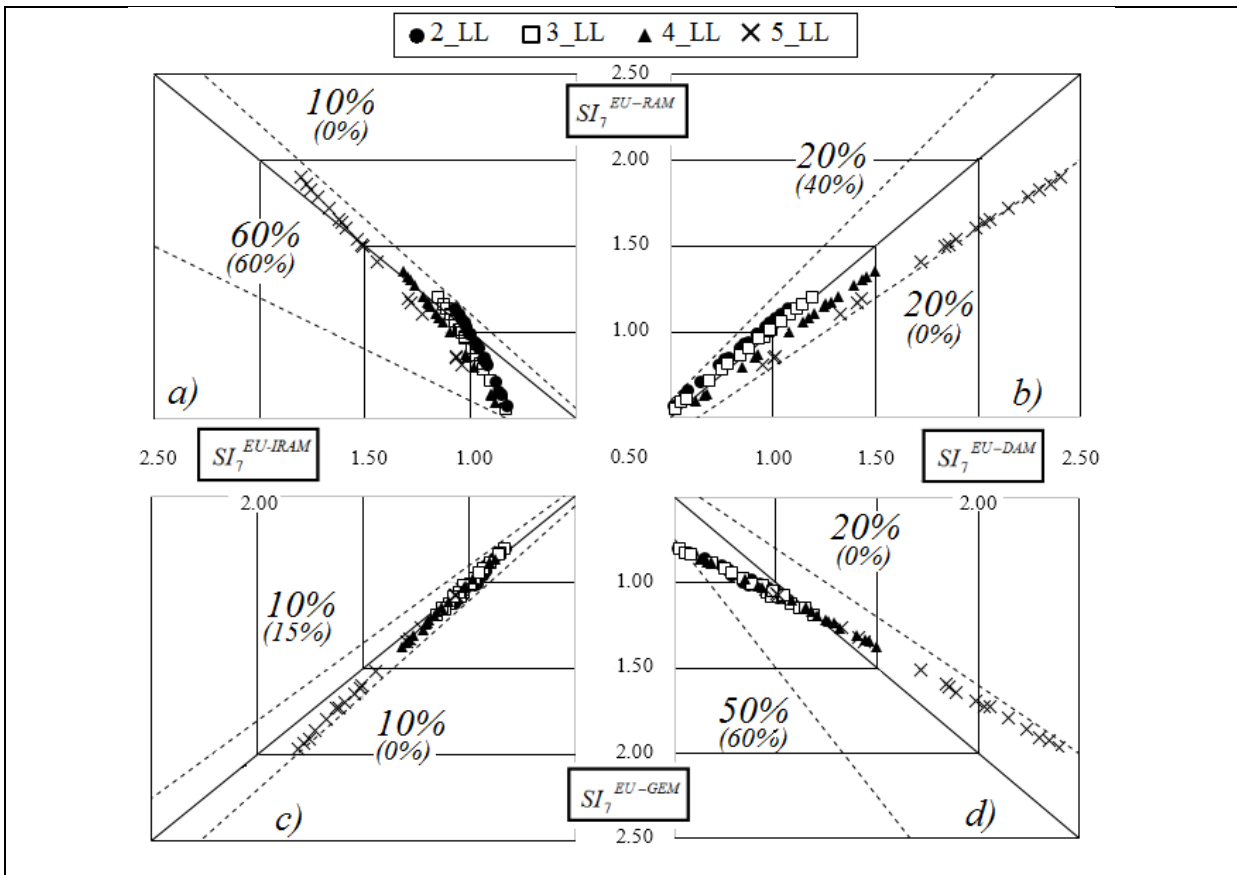
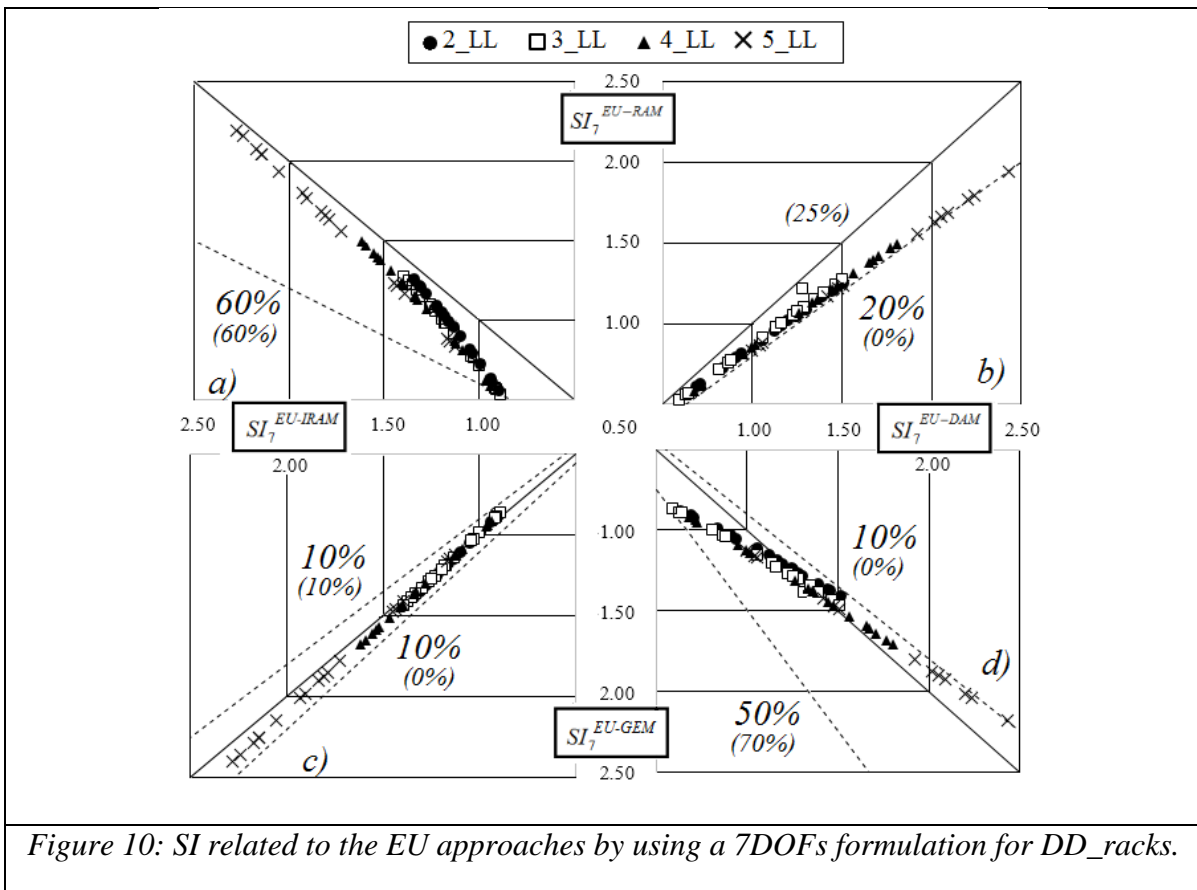*Figure 9: SI related to the EU approaches by using a 7DOFs formulation for MM_racks.*

1

Figure 10: SI related to the EU approaches by using a 7DOFs formulation for DD_racks.
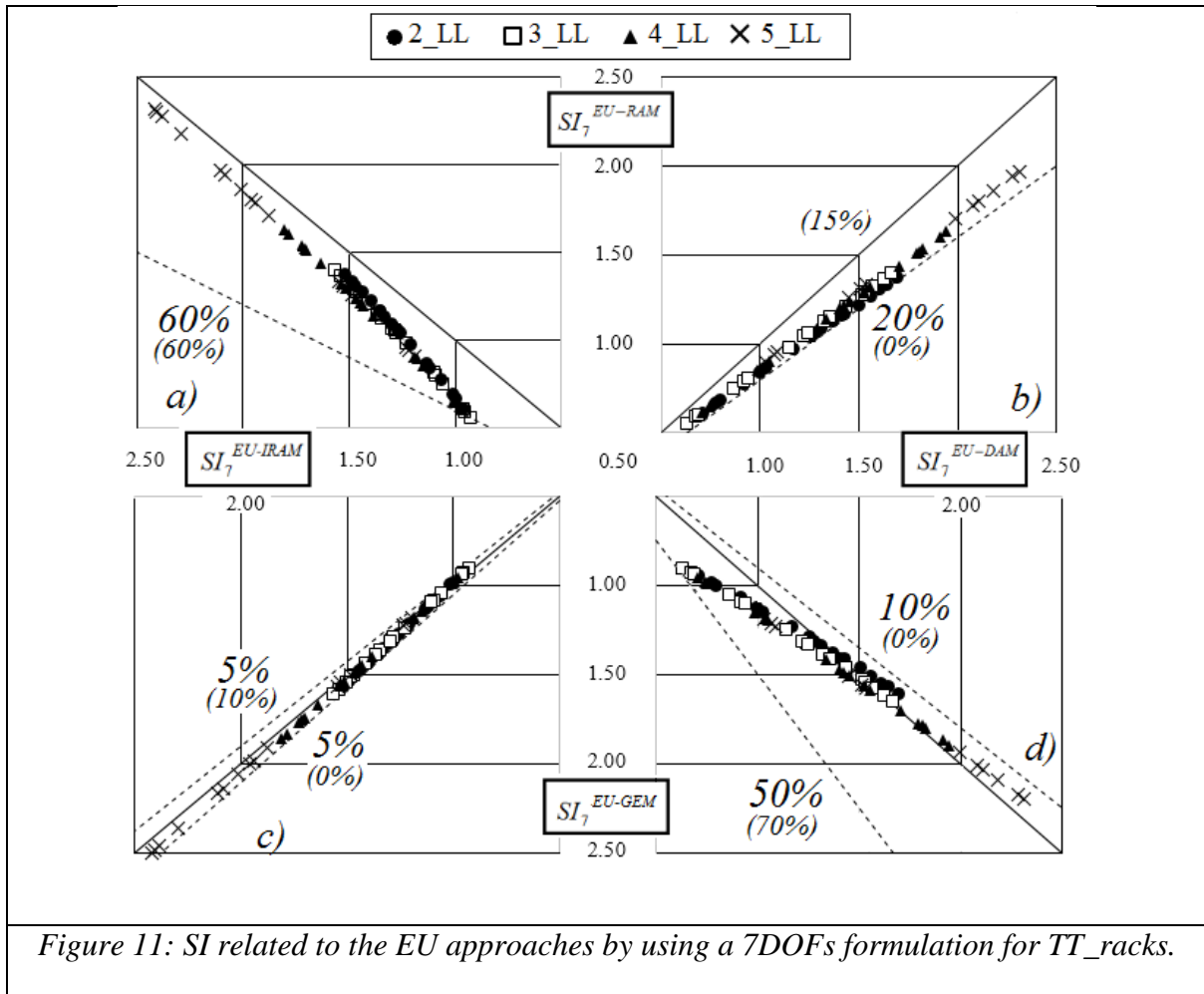
1

Figure 11: SI related to the EU approaches by using a 7DOFs formulation for TT_racks.

2 In general, it can be stated that the differences between the considered methods observed with

3 reference to the 6DOFs results are confirmed and hence the choice of the effective length plays a non-

4 negligible role in determining of the rack performance. Furthermore, owing to the introduction of the

5 7th DOF, in several cases the *SI* values vary remarkably from the 6DOFs value, depending on the

6 assumptions on which the method is based. In particular, it can be noted that:

7 - the EU-DAM approach is most influenced by the *beam* formulation, owing to the presence of

8   the non-negligible warping contribution;

9 - the EU-GEM and EU-IRAM approaches yield to similar results; however these differ to the

10   results obtained using EU-RAM and EU-DAM approaches that are similar to one another.

11   Both the sets of methods are generally characterized by differences of SI values in the range

12   of ±15% with a very limited number of exceptions having difference up to 20%;

13 - the minimum value of the safety index is always associated with the EU-RAM approach, with

14   the exception of MM_racks with 2 and 3 load levels, where EU-DAM gives the most

15   advantageous performance;

1　• SI values associated with the EU design methods become closer with increasing lateral
2　stiffness of the frames, especially for TT_racks. For the highest degree of stiffness of the
3　beam-to-column joints the maximum discrepancy between the methods is almost 25%;

4　• compared with the 6DOFs analysis results, differences between EU-GEM and EU-DAM
5　approaches decrease significantly in these 7DOFs cases, also for the highest values of $\rho_{j,btc}$;

6　• the EU-IRAM and EU-RAM approaches shows relevant differences, especially when the
7　more flexible racks are considered (in the case of the MM_rack having $\rho_{j,btc}=1.0$ and
8　$\rho_{j,base}=0.15$ the difference is up to 60%).

9　As to the US design approaches, reference can be made to figure 12, which is equivalent to the figure
10　7, where the SI values the according to both the US alternatives can be directly compared. Also in
11　these cases, the values between the brackets are related to the 6DOFs limit lines. It can be noted that,
12　also for design according to US code, the differences in the 7DOFs SI decreases, especially for the
13　higher values of beam-to-column stiffness. Furthermore, it is confirmed that for the more flexible
14　racks, the SI associated with the US-ELM approach is slightly greater than the US-NOLM one and
15　differences are never greater than 40%. However, when the beam-to-column joint stiffness increases,
16　the use of the more refined FE formulation gives, in the US-NOLM approach, values up to 15% more
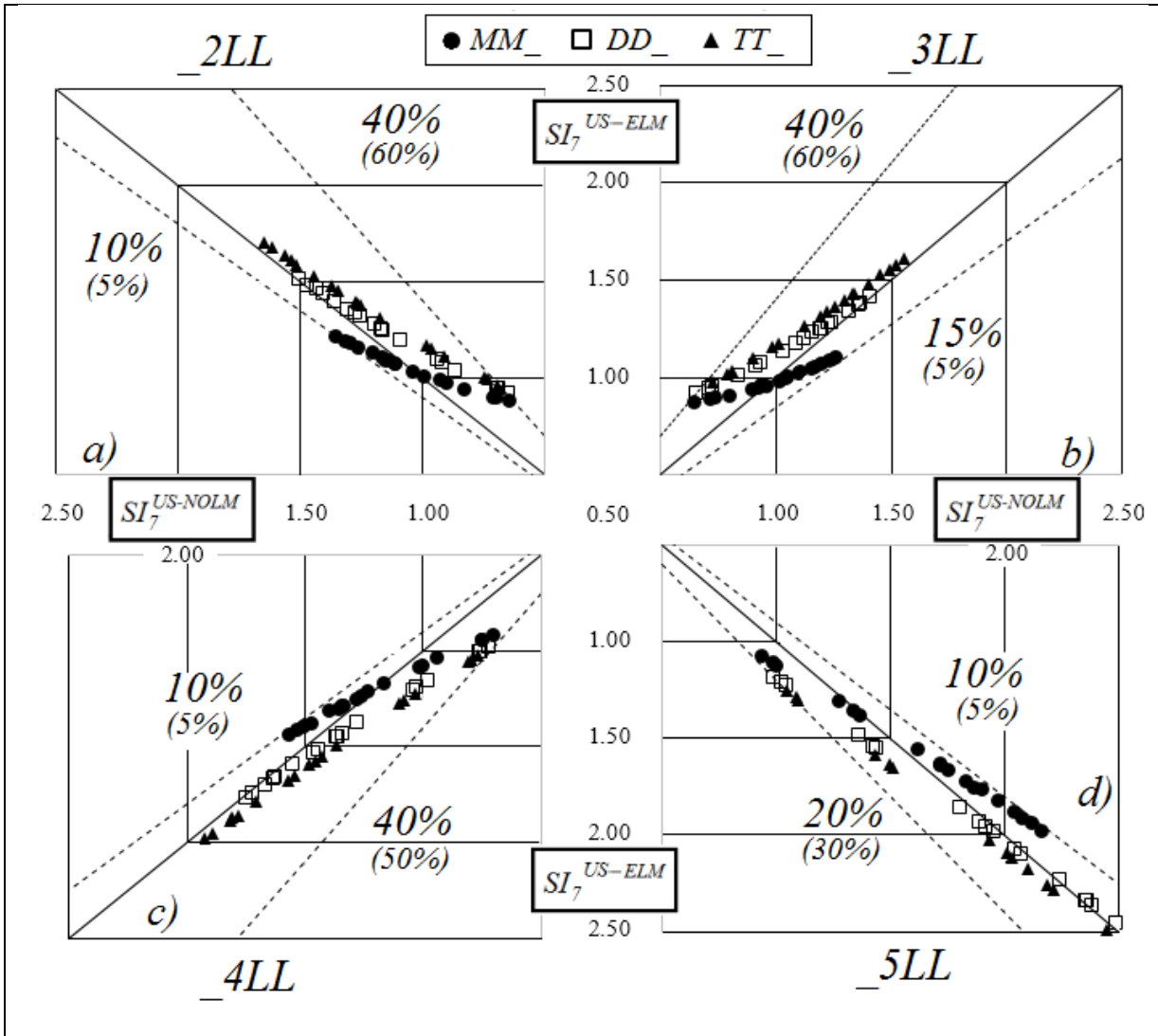17　conservative.

Figure 12: SI related to the US approaches, by using a 7DOFs formulation.

1   Also in the case of 7DOFs design, for appreciating the differences in terms of rack performance

2   between the EU and US approaches, table 6 can be considered, where all data have been grouped and

3   summarized on the basis of the upright cross-section. Also when warping is accounted for it seems

4   reasonable to associate the effective load carrying capacity of the rack to the mean value of the safety

5   index SI_7^{VM} obtained as the mean value of the EU-IRAM, EU.GEM and US-ELM SI values. It can

6   be noted that, as an effect of the bimoment presence, the ratios now become closer to unity; focussing

7   on the EU-IRAM, EU-GEM and US-ELM approaches, the maximum differences are associated with

8   the EU-GEM approach but are limited to 11%.

9   *Table 6: comparison between the EU and US approaches for a 7DOFs based design.*

		$\dfrac{SI_7^{US-ELM}}{SI_7^{EU-IRAM}}$	$\dfrac{SI_7^{US-ELM}}{SI_7^{EU-GEM}}$	$\dfrac{SI_7^{VM}}{SI_7^{EU-IRAM}}$	$\dfrac{SI_7^{VM}}{SI_7^{EU-GEM}}$	$\dfrac{SI_7^{VM}}{SI_7^{US-ELM}}$	$\dfrac{SI_7^{VM}}{SI_7^{EU-DAM}}$	$\dfrac{SI_7^{VM}}{SI_7^{EU-RAM}}$	$\dfrac{SI_7^{VM}}{SI_7^{US-NOLM}}$
MM_	mean	1.06	1.04	1.02	1.01	0.97	1.09	1.13	0.98
	st. dev.	0.0210	0.0282	0.0166	0.0194	0.0107	0.2047	0.1395	0.1144
	Max	1.09	1.11	1.06	1.05	1.00	1.64	1.53	1.31

	min	1.02	0.99	1.00	0.97	0.95	0.80	0.97	0.87
DD_	mean	1.06	1.04	1.03	1.00	0.97	1.06	1.25	1.06
	st.	*0.0216*	*0.0166*	*0.0160*	*0.0136*	*0.0075*	*0.1587*	*0.1529*	*0.1151*
	Max	1.09	1.07	1.06	1.03	0.98	1.51	1.69	1.38
	min	1.02	1.00	1.00	0.98	0.96	0.86	1.09	0.95
TT_	mean	1.07	1.04	1.03	1.01	0.97	1.08	1.26	1.07
	st. dev.	*0.0207*	*0.0158*	*0.0142*	*0.0118*	*0.0081*	*0.1401*	*0.1465*	*0.1011*
	Max	1.10	1.07	1.05	1.03	0.98	1.48	1.68	1.35
	min	1.02	1.01	0.99	0.99	0.96	0.91	1.08	0.96
All	mean	1.06	1.04	1.03	1.01	0.97	1.08	1.21	1.04
	st. dev.	*0.0211*	*0.0202*	*0.0156*	*0.0150*	*0.0087*	*0.1678*	*0.1463*	*0.1102*
	Max	1.10	1.11	1.06	1.05	1.00	1.64	1.69	1.38
	min	1.02	0.99	1.00	0.97	0.95	0.80	0.97	0.88

As to the EU-RAM method, the associated ratios are always greater than those associated with the three equivalent approaches, despite being generally lower than the values corresponding to the EU-DAM approach. Furthermore, it is worth mentioning that the presence of one axis of symmetry imposes consideration of flexural-torsional buckling modes, hence increasing the differences between the EU-DAM and EU-RAM approaches. As in the case of 6DOFs analysis, the degree of reliability of the racks designed according to EU-RAM approach, identified with the $\frac{SI_7^{VM}}{SI_7^{EU-RAM}}$ ratio, could be significantly overestimated, leading to unsafe racks entering the market (mean errors up to 21%, with a maximum overestimation of the safety index up to 69%).

The addition of the bimoment contribution in the equation governing the EU-DAM method significantly reduces the differences the EU-IRAM, EU-GEM and US-ELM 7DOFs approaches; it is confirmed by the mean value of $\frac{SI_7^{VM}}{SI_7^{EU-DAM}}$ ratio, always less than 9% but however very high values can be again observed, up to 64% from the unsafe side, in case of racks with more flexible joints. The US-NOLM method follows the same trend of the EU-DAM approach, in fact if the $\frac{SI_7^{VM}}{SI_7^{US-NOLM}}$ ratio is considered, it can be noted that mean values are always less than 8% but maximum values are too large, overestimating rack performance up to 43%.

6. CONCLUDING REMARKS

A research project has been developed to compare the design approaches admitted for steel racks according to both EU and US steel design provisions. Initially, attention has been focussed on bi-symmetrical cross-section members and the associated research outcomes already discussed by Bernuzzi et al. [6, 7] showed the limits in term of reliability of two European (EU-DAM and EU-RAM) approaches permitted for both the more traditional steel framed system and medium-rise pallet racks comprised of bi-symmetric cross-section members.
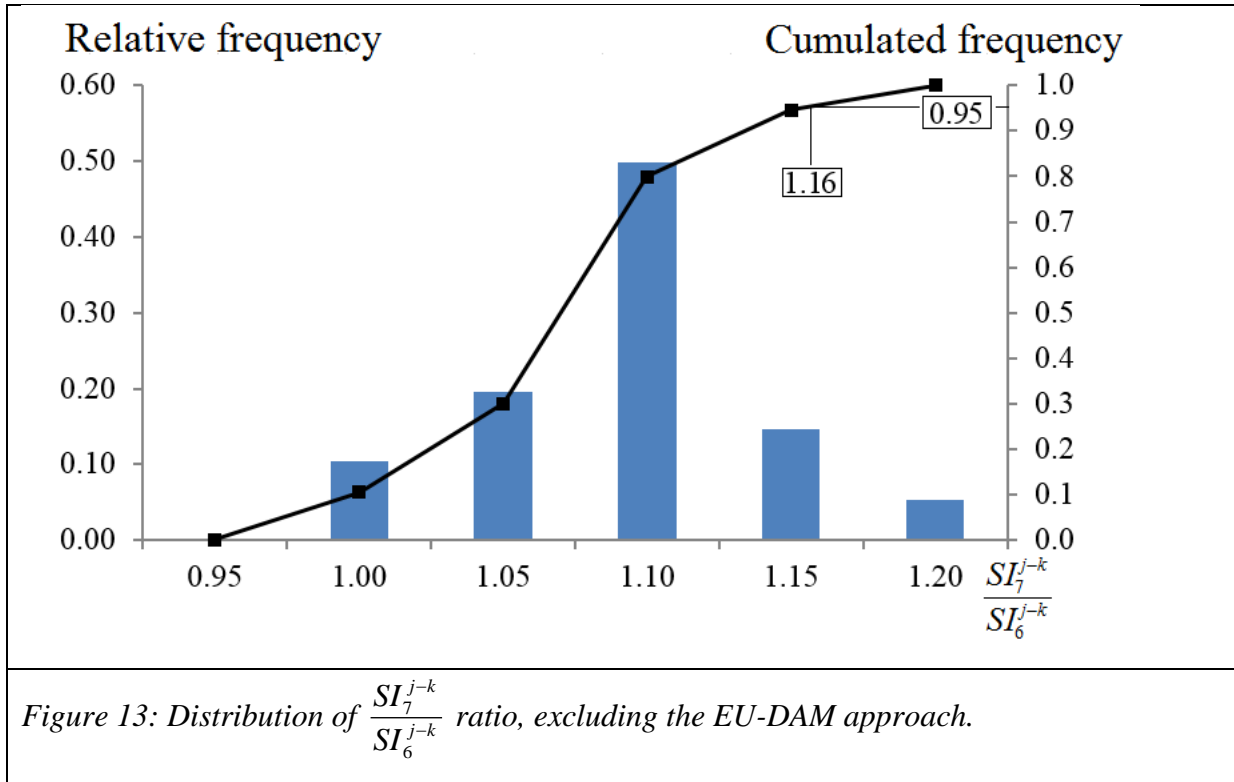
1   The core of the last phase of the research, which has been discussed and summarized in the present

2   paper, is represented by pallet racks built up with mono-symmetric cross-section members. It is

3   confirmed that a role of paramount importance, as shown also in cases of frames with bi-symmetric

4   cross-section members, is played by the choice of the system length, when stability checks are

5   required. The use of the system length recommended by the EU-RAM approach could lead to an

6   overestimation of the rack performance if not balanced by the definition of suitable local

7   imperfections and/or reduction of the member stiffness (as in the US-NOLM approach).

8   Furthermore, another aspect of relevance is related to the capabilities of FE analysis software. The

9   use of two different FE analysis packages shows that FE *beam* formulation neglecting warping effects

10   leads to an estimation of set of displacements, internal forces and moments not correct because of the

11   relevant warping effects are ignored. As shown in table 7, which presents the ratio of $\dfrac{SI_7^{j-k}}{SI_6^{j-k}}$ for all

12   considered j^{th} and k^{th} code approaches, the influence of warping torsion cannot be neglected,

13   especially in the methods based on the resistance checks (totally in the EU-DAM and partially in the

14   EU-GEM). As already mentioned in the paper, maximum differences are related to the EU-DAM

15   approach, despite the fact that in the other cases the use of traditional design in general overestimates

16   the rack performance. Excluding this approach, the differences between the SI associated with the

17   other alternatives always remains greater than unity, but it is however limited, as shown in figure 13

18   in which the ratio $\dfrac{SI_7^{j-k}}{SI_6^{j-k}}$ is presented with a large number of occurrences is in the range $1.05 - 1.15$.

19

20   *Table 7: Influence of the warping (7th DOFs) on the values of the safety index.*

		$\dfrac{SI_7^{EU-DAM}}{SI_6^{EU-DAM}}$	$\dfrac{SI_7^{EU-RAM}}{SI_6^{EU-RAM}}$	$\dfrac{SI_7^{EU-IRAM}}{SI_6^{EU-IRAM}}$	$\dfrac{SI_7^{EU-GEM}}{SI_6^{EU-GEM}}$	$\dfrac{SI_7^{US-NOLM}}{SI_6^{US-NOLM}}$	$\dfrac{SI_7^{US-ELM}}{SI_6^{US-ELM}}$
MM_	mean	1.43	1.06	0.98	1.12	1.07	0.98
	st. dev	*0.0095*	*0.0057*	*0.0052*	*0.0310*	*0.0082*	*0.0134*
	Max	1.50	1.09	1.00	1.18	1.10	1.01
	min	1.29	1.01	0.95	1.03	1.04	0.95
DD_	mean	1.40	1.07	1.05	1.13	1.07	1.05
	st. dev	*0.0181*	*0.0063*	*0.0162*	*0.0362*	*0.0072*	*0.0198*
	Max	1.46	1.10	1.09	1.19	1.11	1.09
	min	1.33	1.02	1.01	1.04	1.04	1.00
TT_	mean	1.36	1.08	1.06	1.12	1.08	1.07
	st. dev	*0.0170*	*0.0067*	*0.0174*	*0.0345*	*0.0083*	*0.0196*
	Max	1.42	1.11	1.11	1.18	1.11	1.10
	min	1.28	1.02	1.01	1.03	1.05	1.02
All	mean	1.39	1.07	1.03	1.12	1.07	1.03
	st. dev	*0.0149*	*0.0062*	*0.0129*	*0.0339*	*0.0079*	*0.0176*

	Max	1.50	1.11	1.11	1.19	1.11	1.10
	min	1.28	1.01	0.95	1.03	1.04	0.95

Figure 13: Distribution of $\dfrac{SI_7^{j-k}}{SI_6^{j-k}}$ ratio, excluding the EU-DAM approach.

As a general conclusion, it can be stated that:

- rack design codes could mandatorily require the use of finite element analysis packages offering suitable FE 7DOFs *beam* formulations;

- with reference to the European Provisions, it is expected that the EU-RAM approach will be eliminate as soon as possible. Furthermore, a better definition of the base assumptions of the EU-DAM alternative is expected. The EU-GEM and the EU-IRAM seem to be very promising approaches that lead to a safe design of the racks, independent of the type of cross-section uprights used;

- with reference to the US alternatives, the US-ELM gives more or less the same results as the EU-IRAM and EU-GEM approaches and seems to be a good design method. Despite the stiffness reduction, the US-NOLM approach seems to be an efficient method only when racks joints are not very flexible, otherwise this approach leads to non-negligible differences with respect to the other approaches.

Finally, it should be noted that all the proposed data, presented in term of SI and/or ratios between them, allow for a detailed appraisal of the safety and of the competitiveness of the racks on the market. As an example, a difference of the safety index of 50% corresponds to a 20% of difference in the

weight of the rack, which corresponds to a non-negligible reduction in cost and hence special attention is required to define design rules that are safe and lead to an optimal use of components.

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1   **APPENDIX A. Design example**

2   The scope of the present Appendix is to reproduce the main computations associated with the design

3   approaches discussed and applied in the paper, which should be useful as a benchmark for the design

4   and validation of the FE packages for racks with mono-symmetric cross-section uprights.

5   The considered case has been taken randomly from the numerical analyses described in section 3 and

6   refers to the more stressed internal upright of the DD_5LL rack (fig. 2) with $\rho_{j,btc} = 1.0$ and

7    $\rho_{j,base} = 0.3$. As it also appears from figure 2, the system length of the upright is 1330 mm (L_y) in the

8   longitudinal (down-aisle) direction and reference is made to Z-panels in the transverse (cross-aisle)

9   direction having height of 1200mm (L_z) forming the upright frames. Blocked warping (WP case) has

10   been assumed at base plate connections. The main cross-section data of the DD uprights are reported

11   in table 2. The material is S355 steel grade [27] with a yielding strength (f_y) equal to 355 MPa.

12   As to the key design values for the component checks, reference has to be made to table A1, where

13   the axial force, N_{Ed}, the bending moments, $M_{y,Ed}$, $M_{z,Ed}$ and the bimoment B_{Ed}, at the bottom

14   (BOT) and the top (TOP) of the more highly stressed upright are reported. Values are obtained from

15   second-order analyses according to the discussed design approaches. Both the set of data associated

16   with 6DOFs and 7DOFs FE *beam* formulations are reported, confirming, as mentioned in the paper,

17   that differences in terms of set of internal forces and bending moments are non-negligible from the

18   design point of view.

19   *Table A1 – Summary of the key results of the second-order FE analysis.*

		DOFs	N_{Ed} [kN]	$M_{y,Ed}^{BOT}$ [kNm]	$M_{y,Ed}^{TOP}$ [kNm]	$M_{z,Ed}^{BOT}$ [kNm]	$M_{z,Ed}^{TOP}$ [kNm]	B_{Ed}^{BOT} [kNm^2]	B_{Ed}^{TOP} [kNm^2]
EU	DAM	6	129.19	1.201	0.226	-0.095	-0.009	-	-
		7	131.68	1.330	0.340	-0.260	-0.009	-0.053	-0.025
	RAM IRAM GEM	6	129.06	0.942	0.123	-0.036	-0.006	-	-
		7	131.68	0.980	0.240	-0.260	-0.009	-0.008	-0.005
US	NOLM	6	122.07	1.685	1.038	-0.119	-0.004	-	-
		7	126.28	1.715	0.450	-0.260	-0.009	-0.020	0.020
	ELM	6	122.09	1.001	0.826	-0.116	-0.004	-	-
		7	126.24	1.210	0.290	-0.260	-0.009	-0.010	0.010

20   The reference uniform load is $q_s = 9.28$ *N/mm* and the elastic buckling load multipliers are

21   $\alpha_{cr}^6 = 1.87$ and $\alpha_{cr}^7 = 1.79$, which have been evaluated by means of the 6DOFs and 7DOFs FE *beam*

22   formulations, respectively.

23

24   **A1. The European approaches.**

In accordance with the requirements of EC3 [1], Young's modulus (E) has been assumed equal to 210000 MPa. Furthermore, as to the stability checks, reference is made to an imperfection factor $\alpha = 0.34$ and material safety factors, γ_{Mj}, have been assumed equal to unity.

EU–DAM: only resistance checks are required by the EU-DAM approach. In figure A1 an example of the distribution of bimoment and 6DOFs and 7DOFs bending moment in the down-aisle direction is presented, with reference to the more stressed external and internal uprights. Design checks are referred to the more stressed element that is an internal upright at the base of the cross-section.

With reference to the two considered FE *beam* formulations, the values of the safety index SI^{EU-DAM} are:

$$SI_6^{EU-DAM} = \left|\frac{129.19\cdot10^3}{600.95\cdot355}\right| + \left|\frac{1.20\cdot10^6}{22350\cdot355}\right| +$$
$$+ \left|\frac{-0.1\cdot10^6}{16115\cdot355}\right| = 0.61 + 0.15 + 0.01 = 0.77$$

$$SI_7^{EU-DAM} = \left(\left|\frac{131.68\cdot10^3}{600.95\cdot355}\right| + \left|\frac{1.33\cdot10^6}{22350\cdot355}\right| +\right.$$
$$\left.+ \left|\frac{-0.3\cdot10^6}{16115\cdot355}\right|\right) + \left|\frac{-0.053\cdot10^9}{717785.8\cdot355}\right| =$$
$$(0.62 + 0.17 + 0.05) + 0.21 = 0.83 + 0.21 = 1.04$$
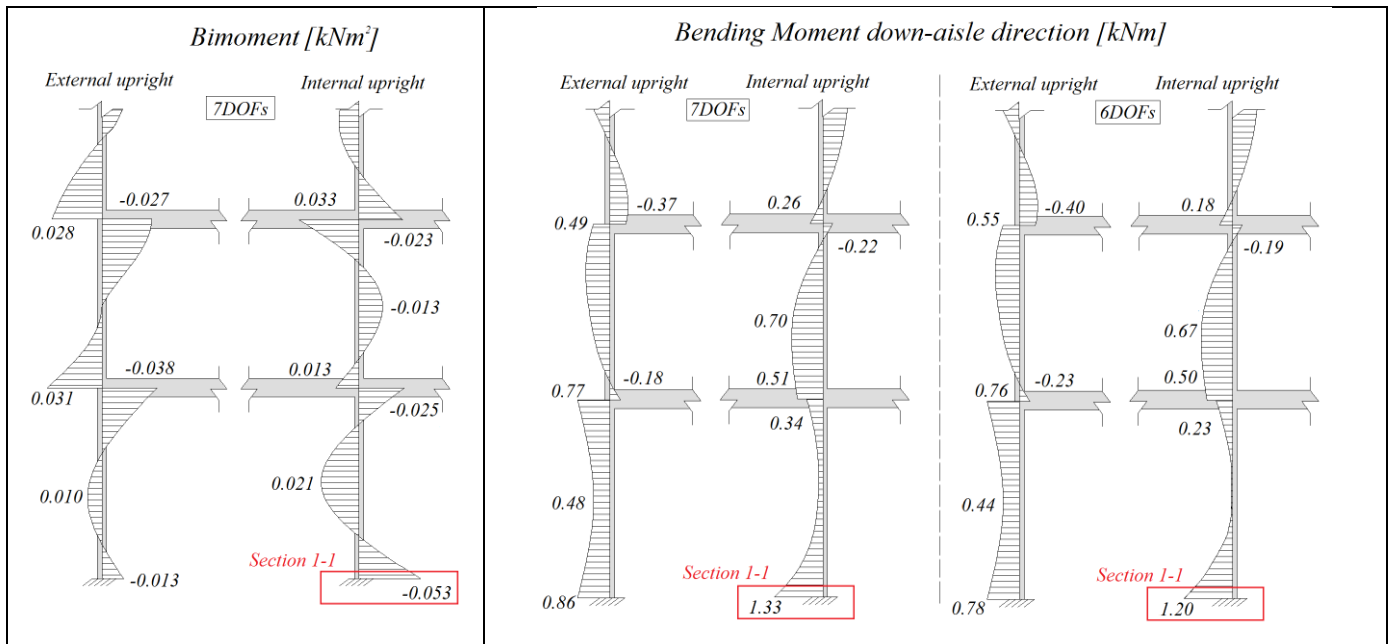
Figure A1: Distributions of the bimoment (a) and bending moment (b) in the down-aisle direction associated with the EU-DAM approach.

Figure A2 presents, for the cross-section in figure A1, the normal stresses distribution in the gross cross-section. In particular, for each critical point of the cross-section (corner or edge point) the values of the normal stress associated with 6DOFs (σ_{tot}^6) and 7DOFs (σ_{tot}^7) FE formulation are presented. In addition, the value of the constant stress due to the axial load (σ_N^7), practically independent of the

1 FE *beam* formulation, is reported in the figure together with the stress associated with the bimoment

2 (σ_B^7). It can be noted that significant differences can be observed in the values of the total normal

3 stresses, depending not only on the presence/absence of the bimomnet but also by the different values
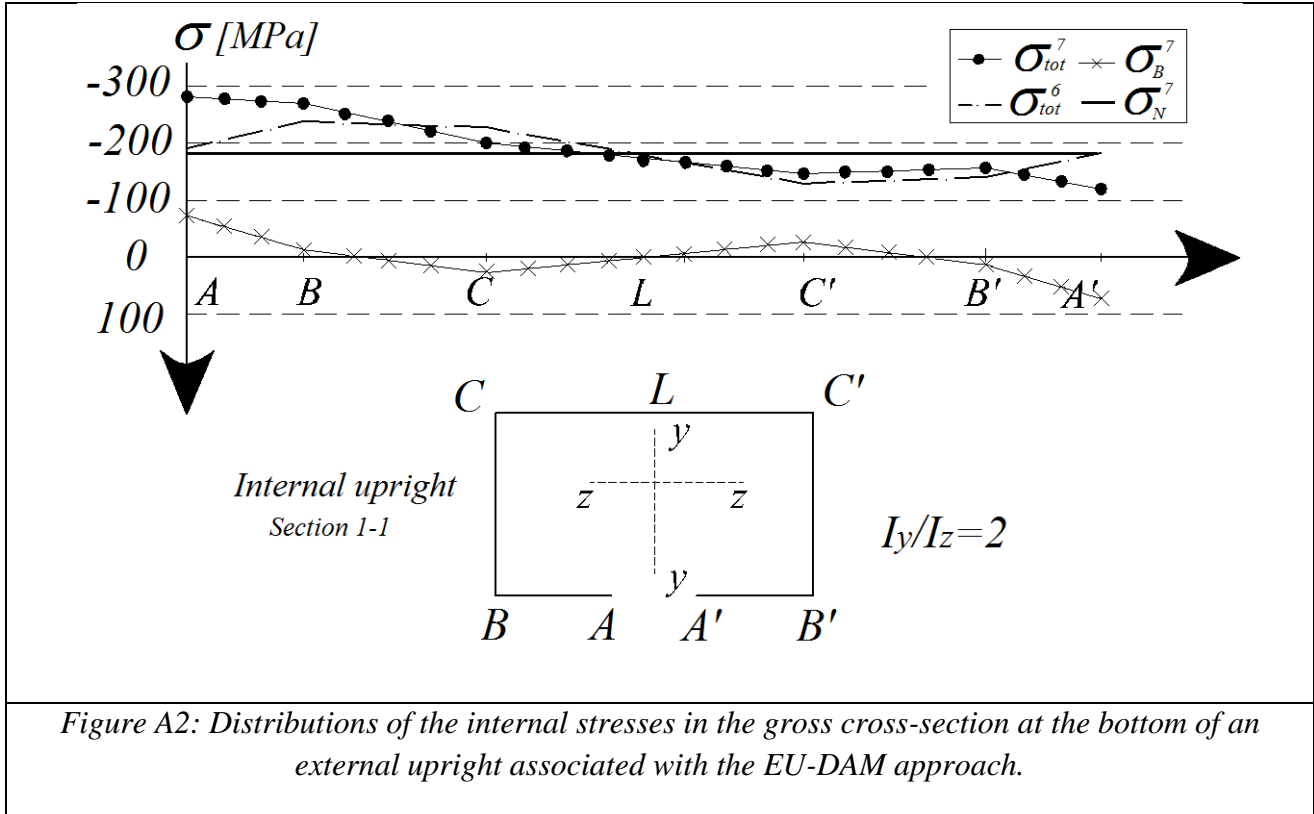
4 of the bending moments.

Figure A2: Distributions of the internal stresses in the gross cross-section at the bottom of an external upright associated with the EU-DAM approach.

EU–RAM: main contributions of the safety index, not depending on FE formulation, need to be preliminarily evaluated.

$$\text{Flexural buckling load } N_{cr,k} = \frac{\pi^2 E \cdot I_k}{L_{0,k}^2}$$

Down - aisle direction | Cross - aisle direction

$$N_{cr,y} = \frac{\pi^2 \cdot 210000 \cdot 1368668}{1330^2} = 1603.66 \cdot 10^3 \, N \quad \Big| \quad N_{cr,z} = \frac{\pi^2 \cdot 210000 \cdot 666368}{1200^2} = 959.11 \cdot 10^3 \, N$$

$$\text{Torsional buckling load: } N_{cr,T} = \frac{1}{i_0^2} \cdot \left[GI_t + \left(\frac{\pi^2 EI_w}{L_{0,T}^2} \right) \right]$$

$$N_{cr,T} = \frac{1}{98.16^2} \cdot \left[80769 \cdot 1061 + \left(\frac{\pi^2 210000 \cdot 5.17 \cdot 10^9}{(0.7 \cdot 1200)^2} \right) \right] = 1586.16 \cdot 10^3 \, N$$

with $i_0 = \sqrt{i_y^2 + i_z^2 + y_c^2} = \sqrt{44^2 + 30.7^2 + 82.2^2} = 98.16 \, mm$

Flexural-torsional buckling load: $N_{cr,FT} = \dfrac{N_{cr,y}}{2\beta} \cdot \left[1 + \dfrac{N_{cr,T}}{N_{cr,y}} - \sqrt{\left(1 - \dfrac{N_{cr,T}}{N_{cr,y}}\right)^2 + 4\dfrac{N_{cr,T}}{N_{cr,y}}\left(\dfrac{y_c}{i_0}\right)^2} \right]$

$N_{cr,FT} = \dfrac{1603.66 \cdot 10^3}{0.598} \cdot \left[1 + 0.99 - \sqrt{(1-0.99)^2 + 4 \cdot 0.99 \left(\dfrac{82.20}{98.16}\right)^2} \right] = 867.99 \cdot 10^3 \, N$

with $\beta = 1 - \left(\dfrac{y_c}{i_0}\right)^2 = 1 - \left(\dfrac{82.20}{98.16}\right)^2 = 0.298$

$$\overline{\lambda}_k = \sqrt{\dfrac{A_{eff} \cdot f_y}{N_{cr,k}}}$$

$\overline{\lambda}_y = \sqrt{\dfrac{600.95 \cdot 355}{1603.66 \cdot 10^3}} = 0.365$

$\overline{\lambda}_z = \sqrt{\dfrac{600.95 \cdot 355}{959.11 \cdot 10^3}} = 0.472$

$\overline{\lambda}_{FT} = \sqrt{\dfrac{600.95 \cdot 355}{867.99 \cdot 10^3}} = 0.495$

$$\varphi = 0,5 \cdot [1 + \alpha(\overline{\lambda} - 0,2) + \overline{\lambda}^2]$$

$\varphi_y = 0.5 \cdot [1 + 0.34 \cdot (0.365 - 0.2) + 0.365^2] = 0.595$

$\varphi_z = 0.5 \cdot [1 + 0.34 \cdot (0.472 - 0.2) + 0.472^2] = 0.657$

$\varphi_{FT} = 0.5 \cdot [1 + 0.34 \cdot (0.495 - 0.2) + 0.495^2] = 0.673$

$$\chi_k = \dfrac{1}{\varphi_k + \sqrt{\varphi_k^2 - \overline{\lambda}_y^2}}$$

$\chi_y = \dfrac{1}{0.595 + \sqrt{0.595^2 - 0.365^2}} = 0.940$

$\chi_z = \dfrac{1}{0.657 + \sqrt{0.657^2 - 0.472^2}} = 0.897$

$\chi_{FT} = \dfrac{1}{0.673 + \sqrt{0.673^2 - 0.495^2}} = 0.886$

1 $\qquad \chi_{min} = \min\{\chi_y; \chi_z; \chi_{FT}\} = 0.886$

2 Evaluation of the critical moment M_{cr} associated with the lateral buckling of the column is required.

3

$M_{cr} = C_1 \dfrac{\pi^2 EI_z}{(k_z L_{0,z})^2} \cdot \left\{ \sqrt{\left(\dfrac{k_z}{k_w}\right)^2 \dfrac{I_w}{I_z} + \dfrac{(k_z L_{0,z})^2 GI_t}{\pi^2 EI_z} + (C_2 z_g - C_3 z_j)^2} - (C_2 z_g - C_3 z_j) \right\}$

$M_{cr} = 2.49 \dfrac{\pi^2 210000 \cdot 666368}{(1 \cdot 1200)^2} \cdot \left\{ \sqrt{\left(\dfrac{1}{0.7}\right)^2 \dfrac{5.17 \cdot 10^9}{6.67 \cdot 10^5} + \dfrac{(1 \cdot 1200)^2 80796 \cdot 1061}{\pi^2 210000 \cdot 666368}} \right\} = 2.45 \cdot 10^8 \, Nmm$

4 $\quad \overline{\lambda}_{LT} = \sqrt{\dfrac{W_{eff,y} \cdot f_y}{M_{cr}}} = \sqrt{\dfrac{22350.7 \cdot 355}{2.45 \cdot 10^8}} = 0.18 < 0.20 \quad \rightarrow \quad \chi_{LT} = 1.0$

As to the bending moment k_{LT} and k_z coefficients, it is necessary to assess the equivalent uniform moment factors, $\beta_{M,y}$ and $\beta_{M,z}$ with reference to the effective moment distribution along the system length, about the y- and z-axis, respectively.

EU-RAM$_6$:

$$\beta_{M,y} = 1.8 - 0.7\psi = 1.8 - 0.7\frac{222623}{940602} = 1.63$$

$$\mu_{LT} = 0.15 \cdot \bar{\lambda}_z \cdot \beta_{M,y} - 0.15 = -0.03 < 0.9$$

$$k_{LT} = 1 - \frac{\mu_{LT} N_{Ed}}{\chi_z A_{eff} \cdot f_y} = 1 - \frac{-0.03 \cdot 129065}{0.897 \cdot 600.95 \cdot 355} = 1.0$$

$$\beta_{M,z} = 1.8 - 0.7\psi = 1.8 - 0.7\frac{-6464}{-35746} = 1.67$$

$$\mu_z = \bar{\lambda}_z(2\beta_{M,z} - 4) = 0.472 \cdot (2 \cdot 1.67 - 4) = -0.31 < 0.9$$

$$k_z = 1 - \frac{\mu_z N_{Ed}}{\chi_z A_{eff} \cdot f_y} = 1 - \frac{-0.31 \cdot 129065}{0.897 \cdot 600.95 \cdot 355} = 1.21 > 1.0$$

$$\rightarrow k_z = 1$$

$$SI_6^{EU-RAM} = \left|\frac{129.06 \cdot 10^3}{0.886 \cdot (600.95 \cdot 355)}\right| +$$

$$+ 1.0\left|\frac{0.94 \cdot 10^6}{1.0 \cdot (22350.7 \cdot 355)}\right| + 1.0\left|\frac{-0.04 \cdot 10^6}{16115.04 \cdot 355}\right| =$$

$$= 0.68 + 0.12 + 0.01 = 0.81$$

EU-RAM$_7$:

$$\beta_{M,y} = 1.8 - 0.7\psi = 1.8 - 0.7\frac{240000}{980000} = 1.63$$

$$\mu_{LT} = 0.15 \cdot \bar{\lambda}_z \cdot \beta_{M,y} - 0.15 = 0.04 < 0.9$$

$$k_{LT} = 1 - \frac{\mu_{LT} N_{Ed}}{\chi_z A_{eff} \cdot f_y} = 1 - \frac{0.04 \cdot 131676}{0.897 \cdot 600.95 \cdot 355} = 0.97 < 1$$

$$\beta_{M,z} = 1.8 - 0.7\psi = 1.8 - 0.7\frac{-9000}{-260000} = 1.56$$

$$\mu_z = \bar{\lambda}_z(2\beta_{M,z} - 4) = 0.472 \cdot (2 \cdot 1.56 - 4) = -0.415 < 0.9$$

$$k_z = 1 - \frac{\mu_{LT} N_{Ed}}{\chi_z A_{eff} \cdot f_y} = 1 - \frac{-0.415 \cdot 131676}{0.897 \cdot 600.95 \cdot 355} = 1.28 > 1$$

$$\rightarrow k_z = 1$$

$$SI_7^{EU-RAM} = \left|\frac{131.68 \cdot 10^3}{0.886 \cdot (600.95 \cdot 355)}\right| +$$

$$+ 0.97\left|\frac{0.98 \cdot 10^6}{1.0 \cdot (22350.7 \cdot 355)}\right| + 1.0\left|\frac{-0.26 \cdot 10^6}{16115.04 \cdot 355}\right| =$$

$$= 0.70 + 0.12 + 0.04 = 0.86$$

EU–IRAM: stability check is based on the effective length here evaluated by means of the use of the critical load multiplier α_{cr}, obtained from a FE buckling analysis. In case of 6DOFs analysis, the critical load results associated with flexural mode $N_{cr,y} = N_{cr,z} = \alpha_{cr}^6 \cdot N_{Ed} = 241.35 \cdot 10^3 \, N$. Based on this value it is possible to obtain the flexural-torsional critical buckling load as:

$$N_{cr,FT} = \frac{N_{cr,y}}{2\beta} \cdot \left[1 + \frac{N_{cr,T}}{N_{cr,y}} - \sqrt{\left(1 - \frac{N_{cr,T}}{N_{cr,y}}\right)^2 + 4\frac{N_{cr,T}}{N_{cr,y}}\left(\frac{y_c}{i_0}\right)^2}\right] =$$

$$= \frac{1603.66 \cdot 10^3}{0.598} \cdot \left[1 + 6.57 - \sqrt{(1 - 6.57)^2 + 4 \cdot 6.57\left(\frac{82.20}{98.16}\right)^2}\right] = 217.19 \cdot 10^3 \, N$$

1 It is worth noticing that the critical load based on the buckling length is approximately 23% of the

2 one based on the system length assumed in the EU-RAM approach and results slightly greater

3 (approximately 8%) than the one associated with the 6DOFs approach.

4 For what concerns the 7DOFs analysis, it is found that $N_{cr} = \alpha_{cr}^{7} N_{Ed} = 234.38 \cdot 10^{3} \ N$ that directly

5 accounts for the flexural-torsional behavior.

6 EU-IRAM$_6$ EU-IRAM$_7$

$$\bar{\lambda}_{min} = \sqrt{\frac{600.95 \cdot 355}{217.19 \cdot 10^{3}}} = 0.991 \qquad\qquad \bar{\lambda}_{min} = \sqrt{\frac{600.95 \cdot 355}{234.38 \cdot 10^{3}}} = 0.954$$

$$\chi_{min} = 0.603 \qquad\qquad\qquad\qquad\qquad \chi_{min} = 0.626$$

$$\bar{\lambda}_{z} = \sqrt{\frac{600.95 \cdot 355}{241.35 \cdot 10^{3}}} = 0.940 \qquad\qquad \chi_{z} = 0.897$$

$$\chi_{z} = 0.635 \qquad\qquad\qquad\qquad\qquad k_{LT} = 1.0$$

$$k_{LT} = 1 - \frac{\mu_{LT} N_{Ed}}{\chi_{z} A_{eff} \cdot f_{y}} = 1 - \frac{0.08 \cdot 129065}{0.603 \cdot 600.95 \cdot 355} = 0.92 < 1 \qquad k_{z} = 1.0$$

$$k_{z} = 1 - \frac{\mu_{z} N_{Ed}}{\chi_{z} A_{eff} \cdot f_{y}} = 1 - \frac{-0.61 \cdot 129065}{0.635 \cdot 600.95 \cdot 355} = 1.55 > 1.0$$

$$\rightarrow k_{z} = 1$$

$$SI_{6}^{EU-IRAM} = \left| \frac{129.06 \cdot 10^{3}}{0.603 \cdot (600.95 \cdot 355)} \right| + \qquad\qquad SI_{7}^{EU-IRAM} = \left| \frac{131.68 \cdot 10^{3}}{0.626 \cdot (600.95 \cdot 355)} \right| +$$

$$+ 0.92 \left| \frac{0.94 \cdot 10^{6}}{1.0 \cdot (22350 \cdot 355)} \right| + 1.0 \left| \frac{-0.04 \cdot 10^{6}}{16115 \cdot 355} \right| = \qquad + 1.0 \left| \frac{0.98 \cdot 10^{6}}{1.0 \cdot (22350 \cdot 355)} \right| + 1.0 \left| \frac{-0.26 \cdot 10^{6}}{16115 \cdot 355} \right| =$$

$$= 1.00 + 0.10 + 0.01 = 1.11 \qquad\qquad\qquad = 0.99 + 0.12 + 0.04 = 1.15$$

7 It is worth noting that $\chi_{LT} = 1.0$ has been assumed, as previously calculated

8

9 **EU–GEM:** only the effects of global (sway) imperfection are accounted for also in this case:

EU-GEM$_6$ EU-GEM$_7$

$$\alpha_{ult,k,6} = \frac{1}{\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}}} = \qquad\qquad \alpha_{ult,k,7} = \frac{1}{\left(\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} \right) + \frac{B_{Ed}}{B_{Rd}}} =$$

$$= \frac{1}{0.728} = 1.371 \qquad\qquad\qquad\qquad = \frac{1}{0.787 + 0.031} = \frac{1}{0.818} = 1.222$$

$$\bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ultm,k}}{\alpha_{cr}}} = \sqrt{\frac{1.371}{1.87}} = 0.856 \qquad\qquad \bar{\lambda}_{op} = \sqrt{\frac{\alpha_{ultm,k}}{\alpha_{cr}}} = \sqrt{\frac{1.222}{1.79}} = 0.826$$

$$\chi_{op} = 0.689 \qquad\qquad\qquad\qquad\qquad \chi_{op} = 0.708$$

$$SI_6^{EU-GEM} = \frac{1}{\chi_{op} \cdot \alpha_{ult,k}} = 1.07 \qquad\qquad SI_7^{EU-GEM} = \frac{1}{\chi_{op} \cdot \alpha_{ult,k}} = 1.15$$

1 **A2. The United States approaches.**

2 It has been decided to develop US computation according to the international system units, and as a

3 consequence, Young's modulus E= 199950 MPa has been assumed.

4 **US–NOLM:** check has to be done assuming an effective length factor with $K = 1$.

5 Reduction of the yield strength for compression:

6 critical flexural stress:

$$F_{ex} = \frac{\pi^2 E}{\left(KL/r_x\right)^2} = \frac{1973920.88}{\left(1 \cdot 1330/44\right)^2} = 2160\,\text{MPa} \qquad\qquad F_{ey} = \frac{\pi^2 E}{\left(KL/r_y\right)^2} = \frac{1973920.88}{\left(1 \cdot 1200/30.7\right)^2} = 1292\,\text{MPa}$$

7 critical torsional stress:

8 $$F_{ez} = \left(\frac{\pi^2 E C_w}{\left(KL\right)^2} + GJ\right)\frac{1}{A_g \cdot r_0^2} = \left(\frac{1973920.88 \cdot 5.17 \cdot 10^9}{\left(KL\right)^2} + 80769 \cdot 1061\right)\frac{1}{707 \cdot 9635.25} = 2137\,\text{MPa}$$

9 critical felxural-torsional stress:

10 $$F_e = \left(\frac{F_{ey} + F_{ez}}{2H} + GJ\right)\left(1 - \sqrt{1 - \frac{4 F_{ey} F_{ez} H}{\left(F_{ey} + F_{ez}\right)^2}}\right) = 1169\,\text{MPa}$$

11 with:

12 $$\bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g} = 82.2^2 + \frac{1368668 + 666368}{707} = 9635.25$$

13 $$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} = 1 - \frac{82.2^2}{9635.25} = 0.299$$

14 $$F_{el} = \min\{F_{ex}; F_{ey}; F_{ez}; F_e\} = 1169 MPa$$

15 $$\lambda_c = \sqrt{\frac{F_y}{F_e}} = \sqrt{\frac{355}{1169}} = 0.551 < 1.5$$

16 $$F_n = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{355}{1169}}\right] 355 = 312.63\,\text{MPa}$$

17 $$A_e = \left(1 - (1 - Q_{US}^N)\left(\frac{F_n}{F_y}\right)^{Q_{US}^N}\right) A_{net} = \left(1 - (1 - 0.85)\left(\frac{313}{355}\right)^{0.85}\right) 707 = 612\,mm^2$$

1. $P_n = A_e \cdot F_n = 312.63 \cdot 612 = 191.33\,\text{kN}$

2. For the bending resistance it is necessary to check if the member should be subjected to lateral-

3. torsional instability, according to the AISI [28] formula:

4. $F_{e,m} = \dfrac{C_b \cdot r_0 A_g}{S_f}\sqrt{F_{ey} \cdot F_{ez}} = \dfrac{1.23 \cdot 98.15 \cdot 707}{24033}\sqrt{1292 \cdot 2137} = 7587\,MPa$

5. $F_{e,m} = 7587\,MPa > 987\,MPa = 2.78 \cdot F_y$

6. with:

7. $C_b = \dfrac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C} = 1.23$

8. with the relationship being fulfilled, the members are not subject to lateral torsional buckling, as

9. already observed for the EU procedures in which $\chi_{LT} = 1.0$ was computed. The upright resistance is

10. given by:

11. $M_{nx} = S_{e,x} \cdot F_y = 22350.7 \cdot 355 = 7.93\,kNm$

12. Safety index $SI^{US-NOLM}$ is:

$$SI_6^{US-NOLM} = \frac{P_r}{\phi_c P_n} + \frac{M_x}{\phi_b M_{nx}} + \frac{M_y}{\phi_b M_{ny}} =$$
$$= \left|\frac{122.07 \cdot 10^3}{0.9 \cdot 191327}\right| + \left|\frac{1.69}{0.9 \cdot 7.93}\right| + \left|\frac{-0.12}{0.9 \cdot 5.72}\right| =$$
$$= 0.71 + 0.24 + 0.02 = 0.97$$

$$SI_7^{US-NOLM} = \frac{P_r}{\phi_c P_n} + \frac{M_x}{\phi_b M_{nx}} + \frac{M_y}{\phi_b M_{ny}} =$$
$$= \left|\frac{126.28 \cdot 10^3}{0.9 \cdot 191327}\right| + \left|\frac{1.71}{0.9 \cdot 7.93}\right| + \left|\frac{-0.26}{0.9 \cdot 5.72}\right| =$$
$$= 0.73 + 0.24 + 0.05 = 1.02$$

13.

14. **US–ELM:** upright check has to be performed with the value of effective length derived from finite

15. element buckling analysis.

US-ELM$_6$

$$F_{ey} = \frac{\alpha_{cr}^6 N_{Ed}}{A_g} = \frac{218554}{707} = 309\,\text{MPa}$$

the flexural-torsional one is

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H} + GJ\right)\left(1 - \sqrt{1 - \frac{4 F_{ey} F_{ez} H}{(F_{ey} + F_{ez})^2}}\right) = 280\,\text{MPa}$$

$$\lambda_c = \sqrt{\frac{F_y}{F_e}} = \sqrt{\frac{355}{280}} = 1.126 < 1.5$$

$$F_n = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{355}{280}}\right] 355 = 208.8\,\text{MPa}$$

US-ELM$_7$

$$L_{eff} = \sqrt{\frac{\pi^2 EI}{\alpha_{cr}^7 N_{Ed}}} = 3457.7\,mm$$

$$K = \frac{L_{eff}}{L} = \frac{3458}{1330} = 2.6$$

$$F_e = \frac{\alpha_{cr}^7 N_{Ed}}{A_g} = \frac{225970}{707} = 319.6\,\text{MPa}$$

$$\lambda_c = \sqrt{\frac{F_y}{F_e}} = \sqrt{\frac{355}{319.6}} = 1.11 < 1.5$$

$$F_n = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.658^{\frac{355}{319.6}}\right] 355 = 223\,\text{MPa}$$

$$A_e = \left(1 - (1 - Q_{US}^N)\left(\frac{F_n}{F_y}\right)^{Q_{US}^N}\right)A_{net} =$$

$$= \left(1 - (1 - 0.85)\left(\frac{209}{355}\right)^{0.85}\right)707 = 639\ mm^2$$

$$P_n = A_e \cdot F_n = 208.8 \cdot 639 = 133.44\ \text{kN}$$

$$SI_6^{US-ELM} = \frac{P_r}{\phi_c P_n} + \frac{M_x}{\phi_b M_{nx}} + \frac{M_y}{\phi_b M_{ny}} =$$

$$= \left|\frac{122.07 \cdot 10^3}{0.9 \cdot 133445}\right| + \left|\frac{1.00}{0.9 \cdot 7.93}\right| + \left|\frac{-0.12}{0.9 \cdot 5.72}\right| =$$

$$= 1.02 + 0.14 + 0.02 = 1.18$$

$$A_e = \left(1 - (1 - Q_{US}^N)\left(\frac{F_n}{F_y}\right)^{Q_{US}^N}\right)A_{net} =$$

$$= \left(1 - (1 - 0.85)\left(\frac{223}{355}\right)^{0.85}\right)707 = 636\ mm^2$$

$$P_n = A_e \cdot F_n = 223 \cdot 636 = 141.83\ \text{kN}$$

$$SI_7^{US-ELM} = \frac{P_r}{\phi_c P_n} + \frac{M_x}{\phi_b M_{nx}} + \frac{M_y}{\phi_b M_{ny}} =$$

$$= \left|\frac{126.25 \cdot 10^3}{0.9 \cdot 141835}\right| + \left|\frac{1.21}{0.9 \cdot 7.93}\right| + \left|\frac{-0.26}{0.9 \cdot 5.72}\right| =$$

$$= 0.99 + 0.17 + 0.02 = 1.21$$

A3. Resistance check

As already discussed in the paper, in the case of mono-symmetric cross-sections, there is the important contribution due to the bimoment and it is essential to also check the resistance of the more stressed cross-section, which should govern design, especially in cases of partially loaded racks. As an example, reference is made to the EU-RAM, EU-GEM and EU-IRAM approaches, which have the same values of internal forces and moments. The resistance checks have to be referred to the more stressed upright cross-section, like in EU-DAM, but due to the presence of the sole overall (sway) imperfection, different distributions of the moments are expected, as it appears from the comparison between figures A1 and A3. Furthermore, figure A4 is related to the normal stress distribution on the more stressed gross cross-section.

$$SI_6^{\text{Re}x} = \left|\frac{129.06 \cdot 10^3}{600.95 \cdot 355}\right| + \left|\frac{0.941 \cdot 10^6}{22350 \cdot 355}\right| + \left|\frac{-0.04 \cdot 10^6}{16115 \cdot 355}\right| = 0.60 + 0.12 + 0.01 = 0.73$$

$$SI_7^{\text{Re}x} = \left(\left|\frac{131.68 \cdot 10^3}{600.95 \cdot 355}\right| + \left|\frac{0.98 \cdot 10^6}{22350 \cdot 355}\right| + \left|\frac{-0.24 \cdot 10^6}{16115 \cdot 355}\right|\right) + \left|\frac{-0.008 \cdot 10^9}{717785 \cdot 355}\right| =$$

$$= (0.61 + 0.13 + 0.04) + 0.03 = 0.78 + 0.03 = 0.81$$

It can be observed that, as in the cases considered in the numerical study, the resistance checks are less severe than the stability ones and it is a general remark associated with the case of totally loaded racks.
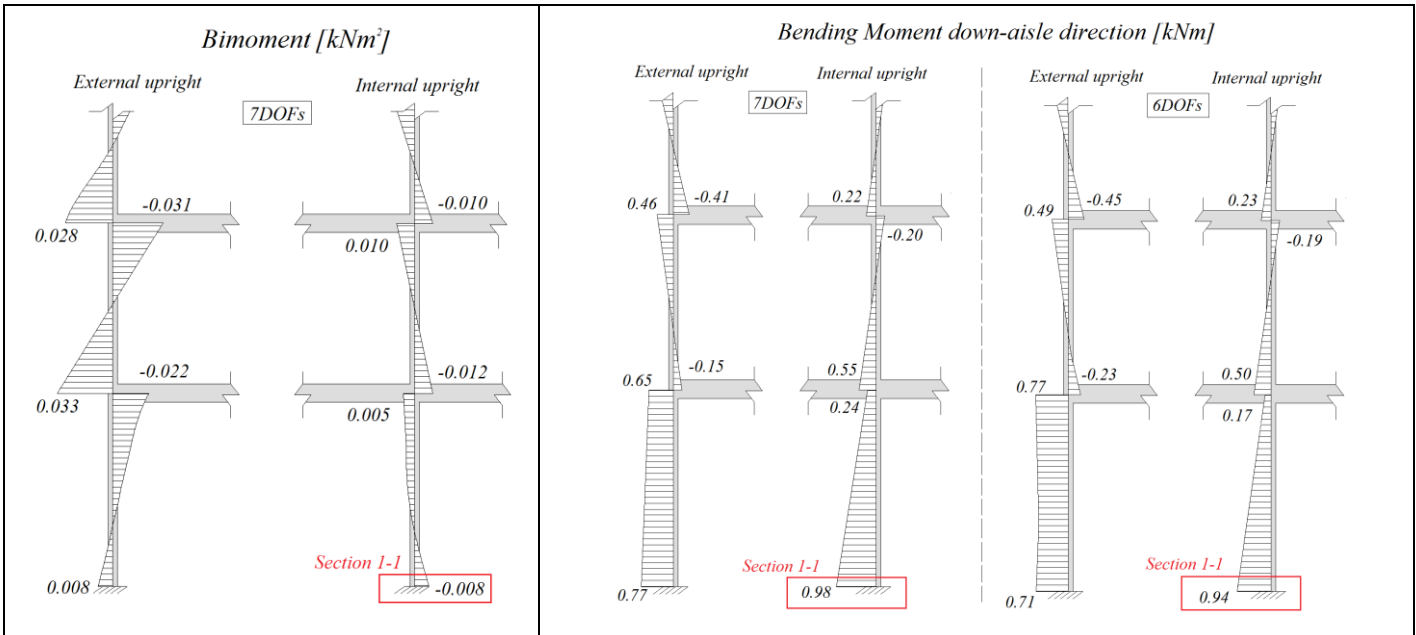
Figure A3: Distributions of the bimoment (a) and bending moment (b) in the down-aisle direction associated with the EU-RAM, EU-GEM and EU-IRAM approaches.

2   **A4. Result comparison**
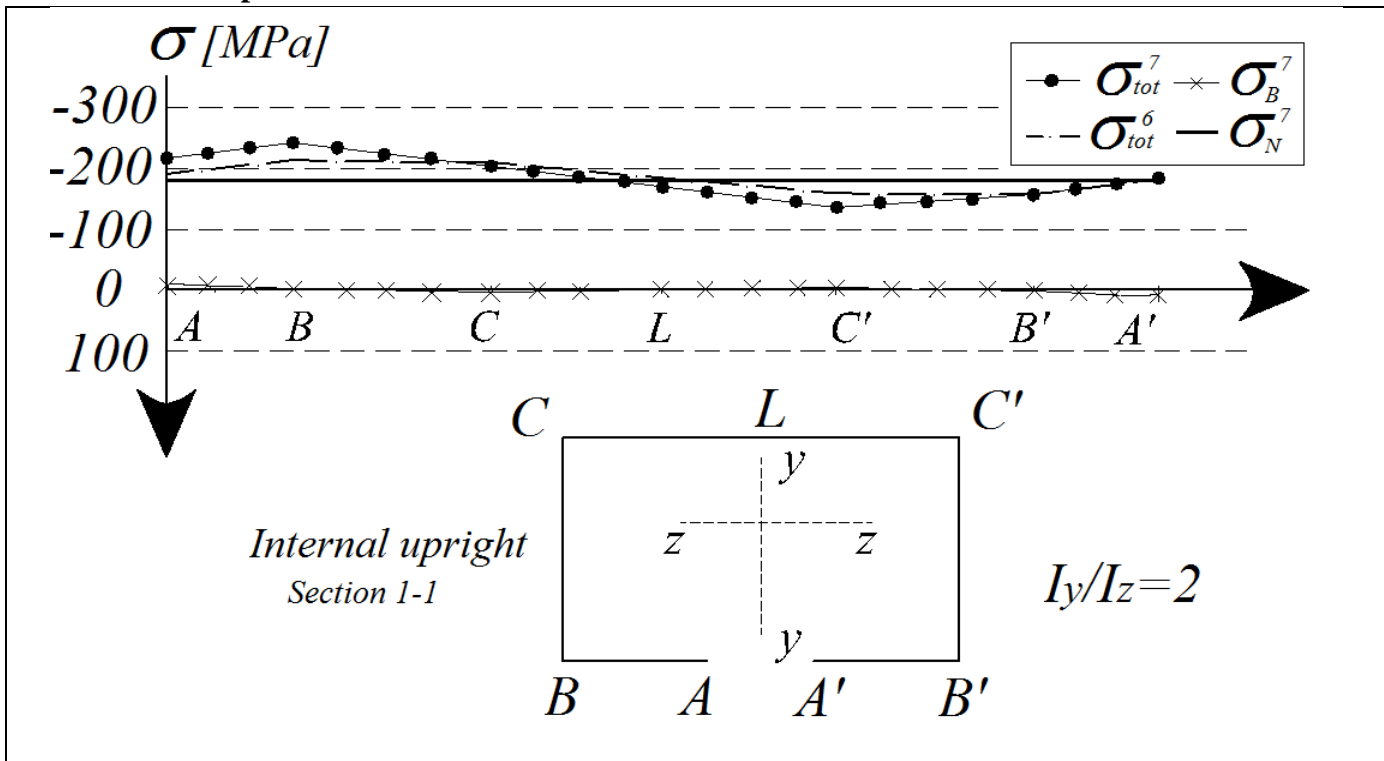
Figure A4: Distributions of the internal stresses in the gross cross-section at the bottom of an external upright associated with the EU-RAM, EU-GEM and EU-IRAM approaches.

3

4   Main steps related to the calculations associated with the six considered design alternatives have been

5   presented in the Appendix and for a general comparison, reference can be made to the safety index

6   values: in some cases, SI values are greater than unity but the scope of the present Appendix is to

1 propose a comparison independently from the acceptability or not from a designer's point of view of

2 the verification checks.

3 Table A2 summarizes key final results of the design procedures in terms of safety index (SI), reporting

4 also the terms related to the axial load (SI_N), bending moments along the y- and z-axis (SI_{My} and SI_{Mz},

5 respectively) and bimoment (SI_B). In general, it can be noted that the contribution due to bending

6 moments is more limited with respect to the one associated with the axial load, especially for bending

7 moments (M_z) along the cross-aisle direction. Furthermore, as expected, the influence of the

8 bimoment is non negligible on methods based on the resistance checks, like the EU-DAM and the

9 EU-GEM and of the same order of magnitude of the bending contribution.

10

11

12

13

14

15 *Table A2 – Summary of the key verification results*

Method			SI_N	SI_{My}	SI_{Mz}	SI_B	SI	SI_7/SI_6	SI^{max}/SI_i
EU	DAM	6	0.606	0.151	0.017	-	0.77	1.351	*1.571*
		7	0.617	0.168	0.045	0.210	1.04		*1.163*
	RAM	6	0.683	0.119	0.006	-	0.81	1.062	*1.494*
		7	0.697	0.124	0.044	-	0.86		*1.407*
	IRAM	6	1.003	0.109	0.006	-	1.11	1.037	*1.109*
		7	0.985	0.122	0.044	-	1.15		*1.051*
	GEM	6			-		1.07	1.075	*1.131*
		7					1.15		*1.052*
US	NOLM	6	0.714	0.236	0.020	-	0.97	1.052	*1.247*
		7	0.734	0.237	0.050	-	1.02		*1.186*
	ELM	6	1.017	0.140	0.023	-	1.18	1.025	*1.025*
		7	0.990	0.169	0.050	-	1.21		*1.000*

16

17 A quite wide dispersion of the results can be noted with reference to the EU approaches: in the case

18 of 6DOFs formulation, the maximum difference of 38% is reduced to 19% when warping effects are

19 considered. As to the US approaches, the safety index associated with the US-NOLM method is very

20 close to the SI associated with EU-IRAM and EU-GEM ones, as already remarked in the paper, and

21 the dispersion of the data is more limited: the ratio between the US-ELM and US-NOLM ranges

22 between 1.18 and 1.21. Furthermore, the US-ELM approach provides a more conservative evaluation

23 of the member performance, with the safety index being equal to 1.21 on the safe side.

24 **APPENDIX B: List of symbols**

1   <u>Latin upper case letters</u>

2   A = gross cross-section area.

3   A_{eff} = effective cross-section area.

4   AISC = American Institute of Steel Construction.

5   ANSI = American National Standards Institute.

6   B = bimoment.

7   B_{Rd} = bimoment resistence.

8   DAM = Direct Analysis Method.

9   E = Modulus of elasticity of steel.

10   E_d = design value.

11   EC3 = EN 1993-1-1 Eurocode 3 "Design of Steel Structures".

12   ELM = Effective Length Method.

13   RMI = Rack Manufacturers Institute.

14   EU = Europe, European.

15   F_n = critical stress.

16   F_{el} = elastic buckling stress.

17   F_y = US yielding strength.

18   G = shear modulus.

19   GEM= GEneral Method.

20   LL = load levels.

21   L = member length.

22   L_{eff} = effective buckling length.

23   L_u = member length for lateral buckling instability.

24   I_t = Saint-Venant torsion constant.

25   I_w = warping constant.

26   I_y, I_z = second moment of area.

27   K = effective length factor.

28   IRAM = improved rigorous analysis method.

29   M_{cr} = critical bending moment.

30        $M_{Ed}, M_{y,Ed}, M_{z,Ed}$ = design bending moment.

31        $M_n, M_{y,n}, M_{z,n}$ = nominal bending resistance.

32   M_{Rk} = characteristic bending resistance.

33   N_{cr} = critical load for the i-member.

34   N, N_{Ed} = member axial load.

35   N_{Rk} = characteristic axial resistance.

36        $N_{b,Rd}$ = axial stability resistance.

37   P_c = design axial strength.

38   P_n = nominal resistance strength for compression.

39        $Q, Q^N, Q_{EU}^N, Q_{US}^N$ = reduction factor for axial load.

40   Q_{EU}^{Mz}, Q_{EU}^{My} = reduction factor bending moment.

41   RAM= Rigorous Analysis Method.

42   R_d = resistance.

43        $S_{j,btc}, S_{j,base}$ = stiffness of connection.

44        $SI_{j,btc}^{EC3-LB}$ = lower bound of EC3.

1	$SI_{j,btc}^{EC3-UB}$ = upper bound of EC3.
2	$SI_{j,base}^{EC3-UB}$ = upper bound of EC3 for base-plate connections.
3	SI^{j-k} = safety index associated with the *j*-code and the *k*- design approach.
4	SI , SI^{EU} , SI^{US} = design safety index.
5	US = United State of America.
6	W_{eff} , $W_{eff,y}$ $W_{eff,z}$ = effective cross-section modulus.
7	WP = prevented warping at base plate connection.
8	WF = free warping at base plate connection.

9 <u>Latin lower case letters</u>

10	e=eccentricity.
11	h = interstorey height.
12	k_j, k_z, k_y = bending interaction factor.
13	*Max* = maximum value.
14	*min* = minimum value.
15	f_y = specified minimum yield stress strength.

16 <u>Greek case letters</u>

17	α = imperfection coefficient associated with the relevant buckling curve.
18	α_{cr} = buckling overall frame multiplier obtained via a finite element buckling analysis.
19	$\alpha_{ult,k}$ = minimum load multiplier evaluated with reference to the cross-section resistance.
20	β_{Mj} = bending moment distribution coefficient.
21	ψ= gradient moment coefficient.
22	$\bar{\lambda}_{op}$ = relative slenderness of the whole structure.
23	$\bar{\lambda}_C$ = slenderness factor.
24	μ_j = non-dimensional term for beam-column verification check.
25	$\rho_{j,btc}$ = parameter to define the elastic rotational stiffness of beam-to-column joints.
26	$\rho_{j,base}$ = parameter to define the elastic rotational stiffness of base-plate joints.
27	χ = reduction factor for the relative buckling curve.
28	χ_{LT} = reduction factor due to lateral buckling.
29	σ = normal stress.
30	χ_{op} = buckling reduction factor referred to the overall structural system.
31	$\gamma_M = \gamma_{M1}$ =material safety factor.

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Table B1 – Comparison between EU and US codes terminology.

EU	Term	US
N_{Ed}	axial force demand	P_r
$N_{b,Rd}$	design axial strength	P_n
$M_{y,Ed}$, $M_{z,Ed}$	required flexural strength about centroidal axes.	M_{rx}, M_{ry}
$M_{y,Rk}$, $M_{z,Rk}$	design flexural strength about centroidal axes.	M_{cx}, M_{cy}
N_{cr}	elastic critical buckling load	P_e

W_{eff}	elastic section modulus of effective cross-section	S_e
I_y , I_z	second moment of Area about centroidal axes	I_x , I_y
I_t	Saint-Venant torsion constant	J
I_w	torsional warping constant of cross-section	C_w
i_y , i_z	radius of gyration about symmetry centroidal axes.	r_x , r_y
f_y	specified minimum yield stress strength	F_y
y-y	cross-section symmetry axis	x-x
z-z	cross-section non symmetry axis	y-y

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i_y , i_z	radius of gyration about symmetry centroidal axes.	r_x , r_y
f_y	specified minimum yield stress strength	F_y